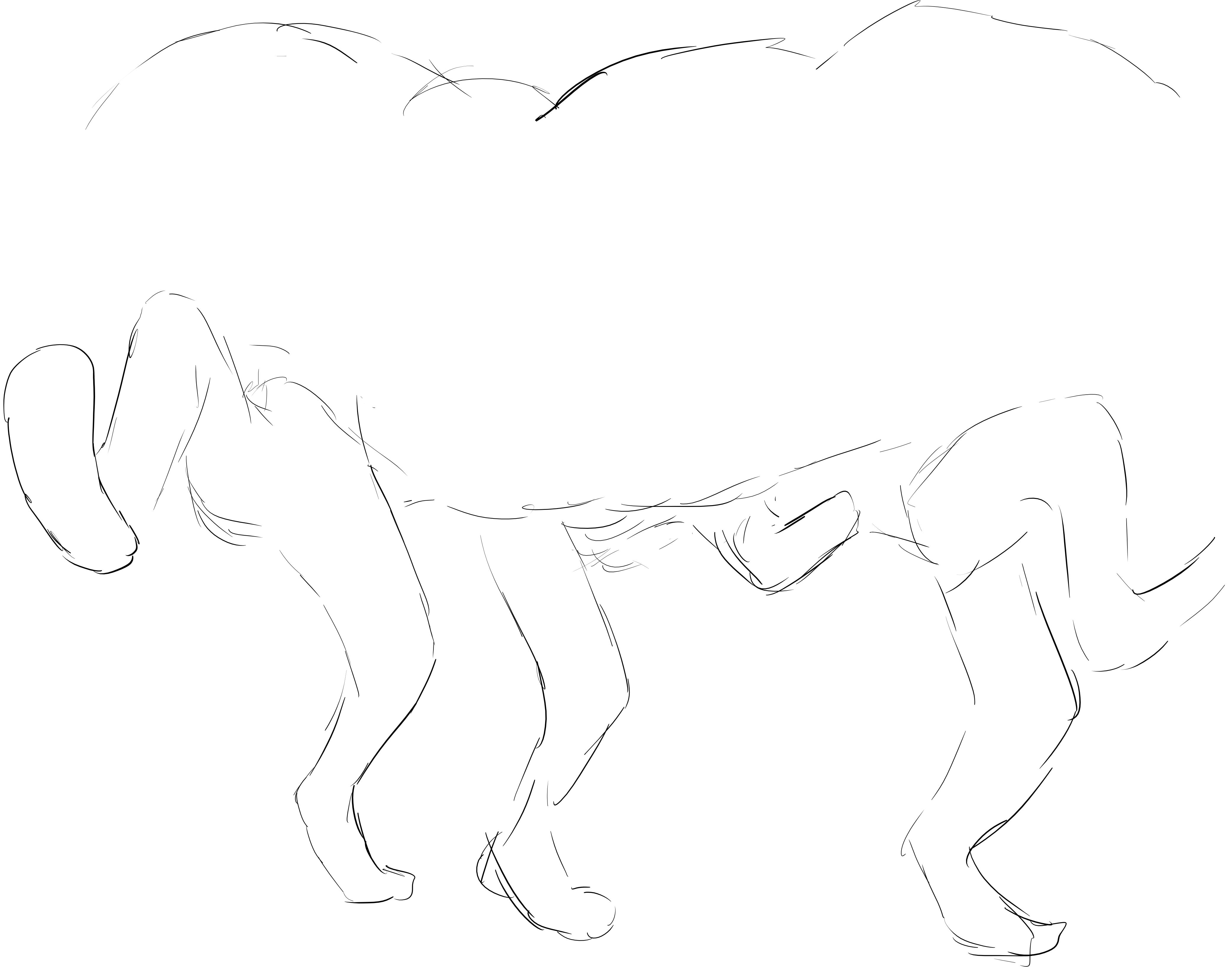

[redacted]

[redacted]!

$$\binom{127}{3}$$

$\log_{base>1}(x)$ monotonic

$\boxed{x = \frac{\log_2(a\cdot b)}{\log_2(a)}} \times \log_2(a)$

$\log_2(a\cdot b) = \log_2(a^x)$

$>$

$\log_{0.5}(a)$

$-\log_2(a) < -\log_2(b)$

$\log_{2^{-1}}(a)$

$\log_2(a) > \log_2(b)$

[redacted] a^x

$\left(\frac{1}{x}\right)$

$\frac{1}{-1}\log_2(a)$

$\log_{a^b}(c) = \boxed{\frac{1}{b}}\log_a(c)$

$-\log_2(a)$

$f(a,b) = \frac{1}{\frac{\log_2(a\cdot b)}{\log_2(a)}}$

$\leftarrow$ $\log_{a\cdot b}(c)$

Thm: $\log_2(3) \notin \mathbb{Z}$

Proof

Contr:

$$2^{\log_2(3)} = 2^{x}$$

$$3 = 2^{x}$$

FToA

$$\log_2(3) \notin \mathbb{Q}$$

$$\log_2(3) = \frac{x}{y} \quad \neq 0$$

$$y\log_2(3) = x$$

$$\log_2(3^y) = x$$

$$3^y = 2^x$$

$$\cancel{x \neq 0} \vee \boxed{y \neq 0}$$

FToA

Thm $\forall x \in \mathbb{Z}. \left[\underbrace{\left(\log_3(x) \in \mathbb{Q} \right)} \to \left(\exists n \in \mathbb{Z} . \ x = 3^n \right) \right]$

$$\log_3(x) = \frac{a}{b} \qquad\qquad b \log_3(x) = a$$

$$\log_3(x^b) = a$$

$$x^b = 3^a$$

$$(x)^b = \left(3^{a/b}\right)^b$$

$$x^b = 3^a$$

FToA

$$3^{k/b} \in \mathbb{Z} \to k/b \in \mathbb{Z}$$

$$x^b = 3^k$$

$$x = 3^{\boxed{k/b}}$$

$\forall a,b \in \mathbb{Z} \, . \quad 3^{\frac{a}{b}} \in \mathbb{Z} \rightarrow \frac{a}{b} \in \mathbb{Z}$

Cont assum $\quad 3^{a/b} \in \mathbb{Z} \wedge \frac{a}{b} \notin \mathbb{Z}$

$3^{a/b} = x \qquad \boxed{a \neq kb}$

$3^a = x^b$

$\log_3 3^a = \log_3 x^b \qquad \log_3 x \notin \mathbb{Z}$

$\boxed{a = b \log_3 x}$

$\forall . x \in \mathbb{Z} \left[\left(\log_3(x) \in \mathbb{Q} \rightarrow \exists n \in \mathbb{Z} . x = 3^n \right) \right]$

Cont Assm $\left(\log_3(x) \in \mathbb{Q}\right) \wedge \left(\nexists n \in \mathbb{Z} .. x = 3^n\right)$

Cons arb x (univ. inst)

$\log_3(x) = \frac{a}{b}$ $a, b \in \mathbb{Z}$

$x^b = 3^a$

FToA

x's Racr all 3

$\exists n \in \mathbb{Z} .. 3^n = x$

Cont

⋮

br x arb, $\forall x \in \mathbb{Z}$ ...

$\perp$

$\forall x \in \mathbb{Z}^+ . \log_2 x < x$

$a \leq b < c$

$a < c$

induction

Base case $x = 1$ $\log_2(1) = 0$ $x = 1$

$\log_2(k+1) \leq \log_2(2k) < k+1$

$<$

$0 < 1$ ✓

inductive. assume $(\log_2(k) < (k)$, $k \geq 1$

$k+1 \leq 2k$

goal $\log_2(k+1) < (k+1)$

$1 + \log_2 k < 1 + k$

$2k \geq k+1$

$\log_2$ monotonic ... $\log_2(x) > \log_2(y)$

$\equiv$

$x > y$

$\log_2 2 + \log_2 k < 1 + k$

$\log_2(2k) < 1 + k$

$\log_2(2k) \geq \log_2(k+1)$

$\log_2(k+1) \leq \log_2(2k)$

Transitivity of $<$

$\log_2(k+1) < (k+1)$