







Vectors
matrices
derivatives
integrals

Vector $\begin{cases} \text{multidimensional numbers} \\ \text{direction + length} \end{cases}$

$$(x, y) + (a, b) = (x+a, y+b)$$

$$(3, 1, 3.75, 12, -2, 0) + \text{element-wise}$$

$$[3 \quad 1 \quad 3.75 \quad 12 \quad -2 \quad 0]$$

$$\begin{bmatrix} 3 \\ 1 \\ 3.75 \\ 12 \\ -2 \\ 0 \end{bmatrix} \begin{matrix} x \\ y \\ z \\ w \\ r \\ g \\ b \\ s \\ t \\ \vdots \end{matrix}$$

Scalar = number

$$(x, y, z) * S = (x \cdot S, y \cdot S, z \cdot S)$$

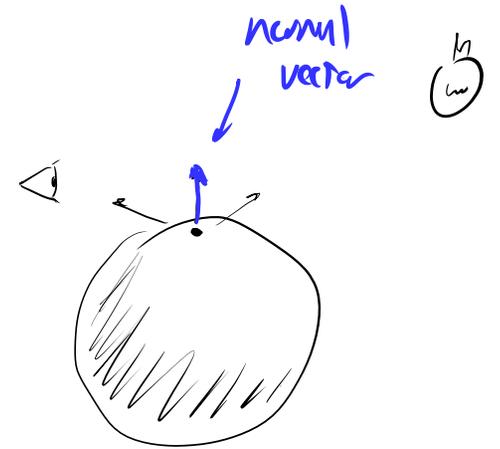
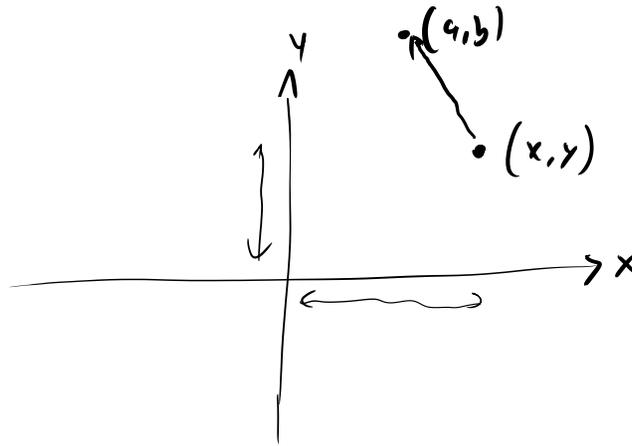
Vector

• convention; math

• points

Vector • Separations $P_2 - P_1$

• directions



length of vector $(3, 4)$

2 (or dimension)

$(3, 4, 5, 6, 0, -2)$

$$5 \sqrt{3^2 + 4^2}$$

ell-2 norm

L_2

$$\sqrt{3^2 + 4^2 + 5^2 + 6^2 + 0^2 + (-2)^2}$$

normalize

$$\frac{\vec{d}}{\|\vec{d}\|}$$

$$\vec{d} * \frac{1}{\|\vec{d}\|}$$

Unit vector

$$\| (3, 4) \|_2 = 5$$

Normal \triangleq perpendicular to a surface

$$\vec{x} \quad \vec{x} \quad \times \quad x$$

[]

Trig

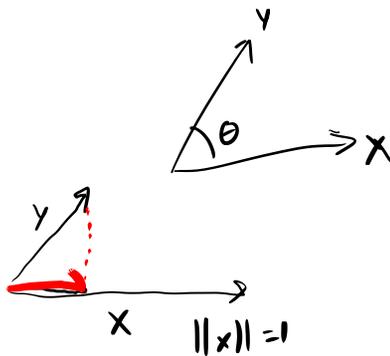
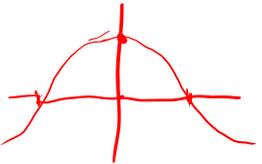
$$\|\vec{x}\| = \sqrt{\vec{x} \cdot \vec{x}}$$

dot
(inner)

$$(x, y, z, w) \cdot (a, b, c, d) =$$

$$x \cdot a + y \cdot b + z \cdot c + w \cdot d$$

$$\vec{x} \cdot \vec{y} = \|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos(\theta)$$



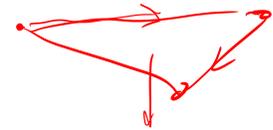
$(\vec{x} \cdot \vec{y}) \vec{x}$ = projection of \vec{y} onto \vec{x}
(assume \vec{x} is unit vector)

cross 3D only
(2D fake version)

$$(x, y, z) \times (a, b, c) =$$

$$(yb - cz, za - cx, xb - ay)$$

$\vec{a} \times \vec{b}$ perp to both \vec{a} & \vec{b}



$$\|\vec{x} \times \vec{y}\| = \|\vec{x}\| \cdot \|\vec{y}\| \cdot \sin(\theta)$$

Matrix

$$\begin{bmatrix} x+4y+2z \\ -y+3z \\ y \end{bmatrix} = \begin{bmatrix} 1 & 4 & 2 \\ 0 & -1 & 3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

identity matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 21 & 38 & 45 \\ 17 & 19 & 21 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 2 \\ 0 & -1 & 3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

1. linear system of eq

$$= A\vec{x} = \vec{b}$$

2. SVD $\begin{matrix} \text{rotates} \\ \downarrow \end{matrix}$ $\begin{matrix} \text{Scale} \\ \downarrow \end{matrix}$ $\begin{matrix} \text{rotates} \\ \downarrow \end{matrix}$

$$A = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

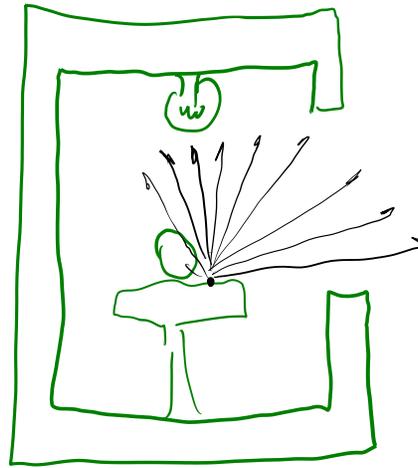
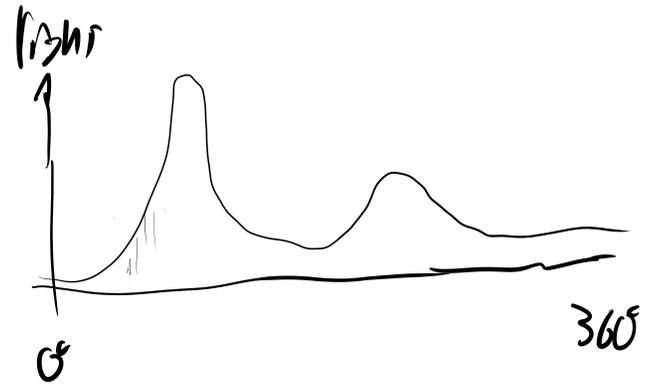
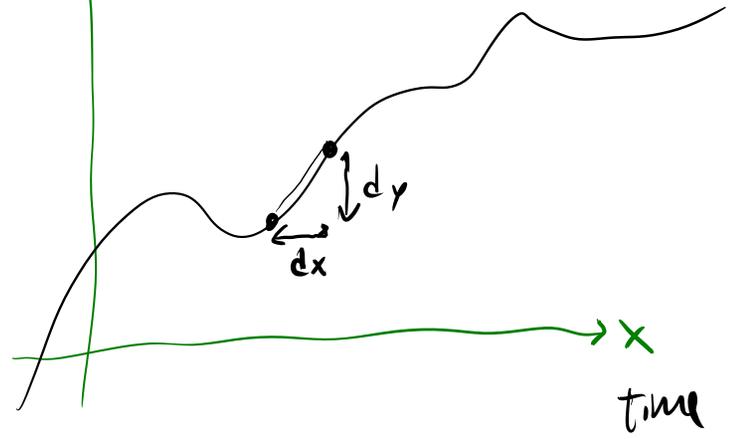
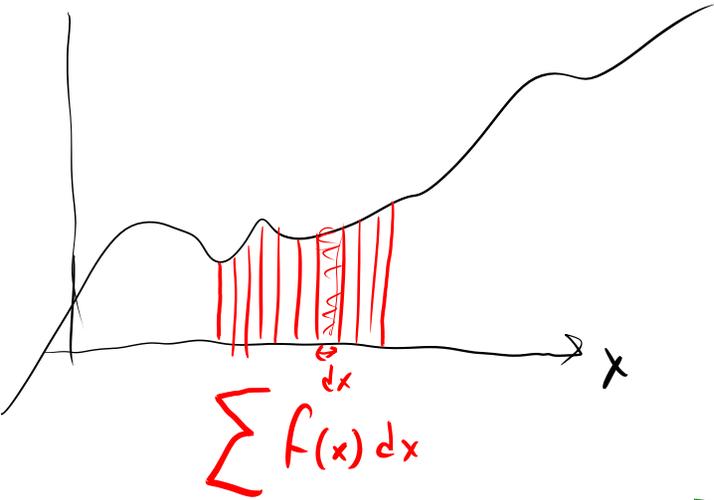
derivative
differentiation

Posit
 $y = f(x)$
 $\frac{dy}{dx} \rightarrow \text{speed}$
as $dx \rightarrow 0$

limit

integral
integration

$y = f(x)$





$$\iint F_{\text{inc}}(\theta, \phi) d\theta d\phi$$

$$\sum \iiint F(x, y, z) dx dy dz$$

$$P(t) = \int_{t=0}^t \text{motion } dt$$

$$\int \text{accel } dt dt$$

