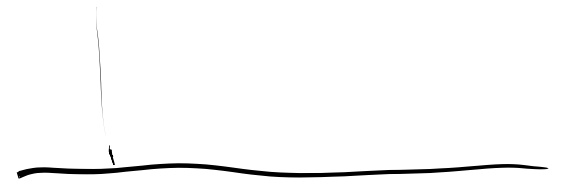


α

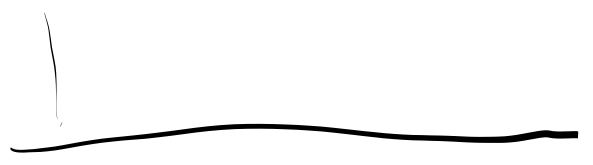
opacity

50%



25%

100%



50%

- $\frac{1}{4}$

- $\frac{3}{8}$

- $\frac{5}{8}$

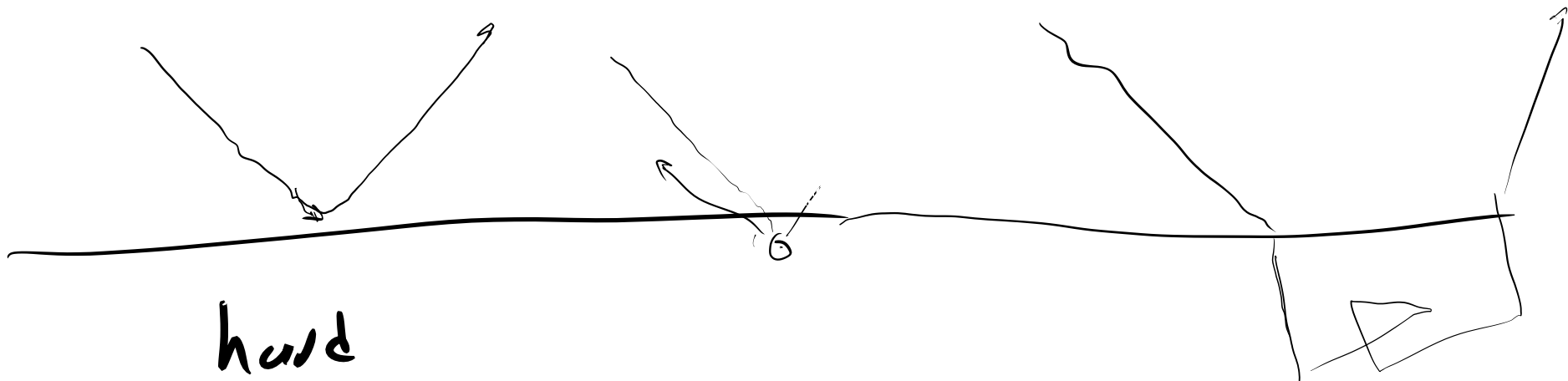
= 62.5%

$(.5)(.25) + 1(.5)(.75)$

$.625$

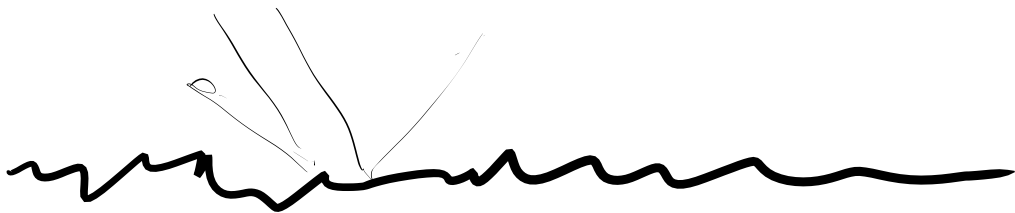
$\frac{3}{8}$

Glare



hard

smooth



Vector - list of numbers

$$\vec{x} = (1, 3, 0, -2.1)$$

$$\vec{y} = (3, 3, 1, 1)$$

$$\vec{x} + \vec{y} = (4, 6, 1, -1.1)$$

$$\vec{x} - \vec{y} = (-2, 0, -1, -3.1)$$

$$2\vec{x} = (2, 6, 0, -4.2)$$

$$\vec{x} \cdot \vec{y} = 3 + 9 + 0 + -2.1 = 9.9$$

dimension: # of values
length / magnitude: $\sqrt{\vec{x} \cdot \vec{x}}$
 $\|\vec{x}\|$

$$\vec{x} = (x_1, x_2, x_3)$$

Orthogonal (perpendicular) $x \cdot y = 0$

dot inner

$$3D: \vec{x} \times \vec{y} = (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1)$$

$$2D: (x_1, x_2) \cdot (-x_2, x_1) = 0$$

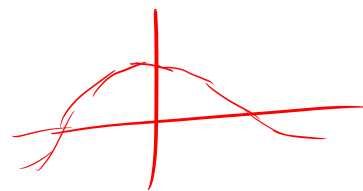
$$\vec{x} \times \vec{y} = x_1 y_2 - x_2 y_1$$

cross product

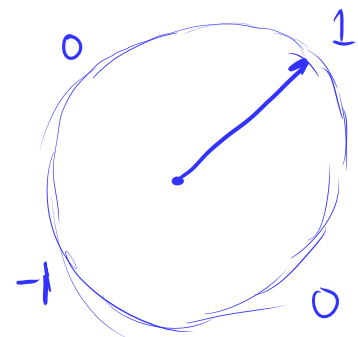
$$(\vec{x} \times \vec{y}) \cdot \vec{x} = 0$$

$$(\vec{x} \times \vec{y}) \cdot \vec{y} = 0$$

$$\vec{x} \cdot \vec{y} = \|\vec{x}\| \|\vec{y}\| \cos(\theta)$$



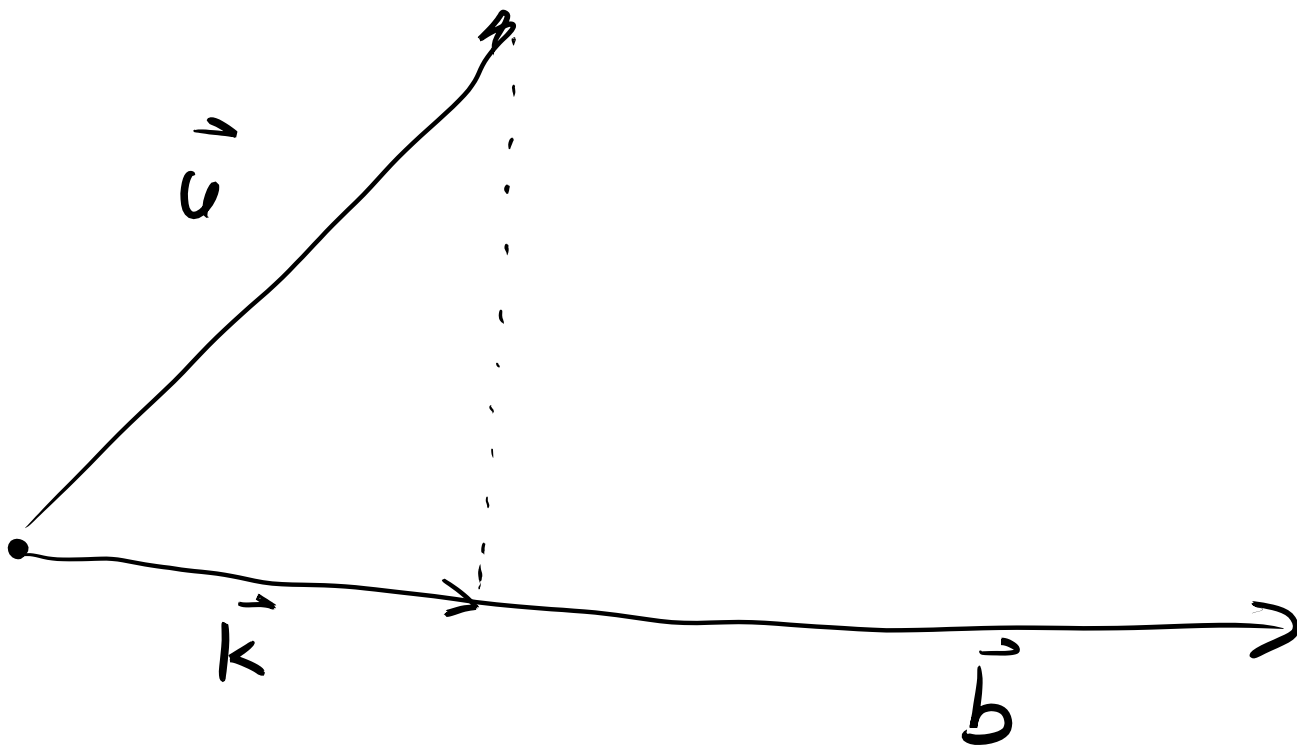
$$\|\vec{x} \times \vec{y}\| = \|\vec{x}\| \|\vec{y}\| \sin(\theta)$$



Normalize \vec{x} by $\frac{\vec{x}}{\|\vec{x}\|}$

$\|\vec{x}\| = 1$, unit vector

Normal vector perpendicular to surface



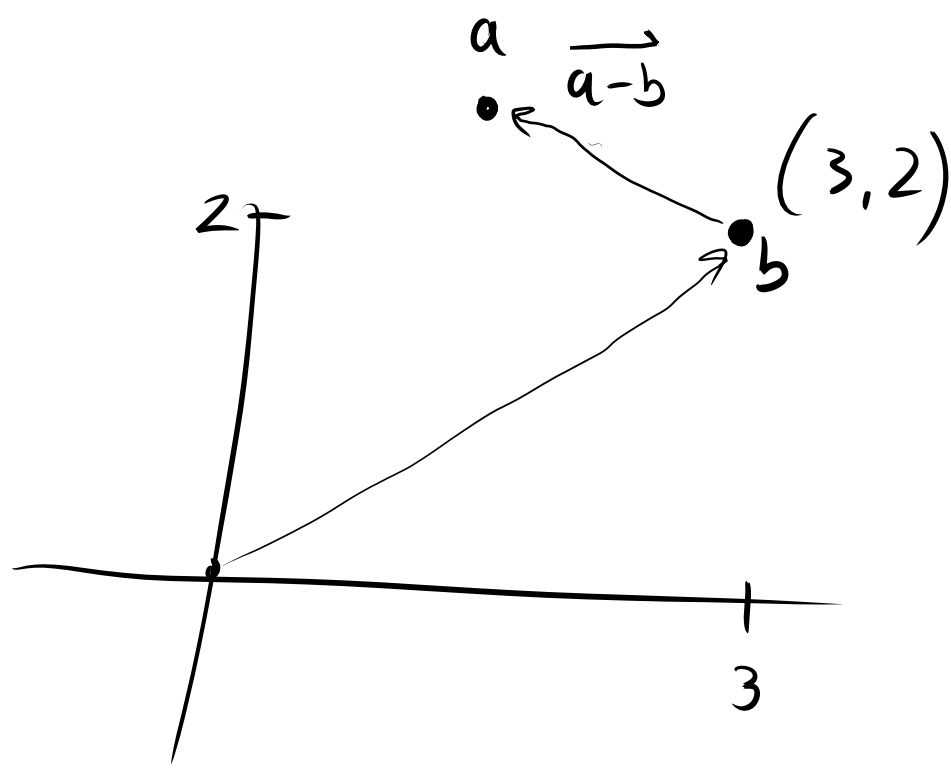
$$\vec{k} = (\vec{a} \cdot \vec{b}) \vec{b}$$

if

$$\|\vec{b}\| = 1$$

Vector

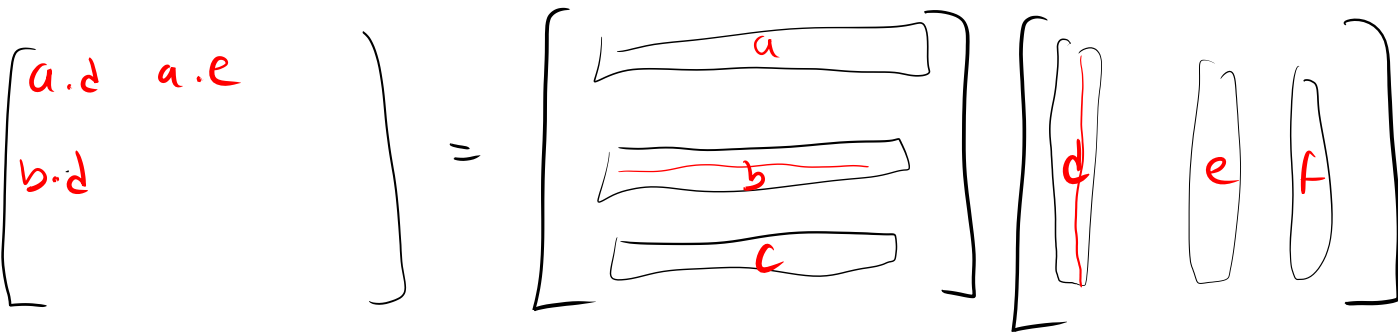
- main lib of #s
↳ point
- offset
- homogenous



Matrix grid of numbers

$$A \vec{x}$$
$$\vec{x}^T A$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & -4 & 0 \\ 3 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & 0 & 3 \\ 1 & -4 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$



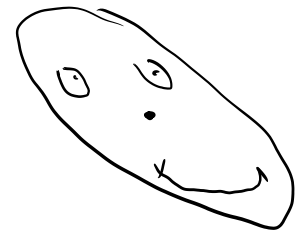
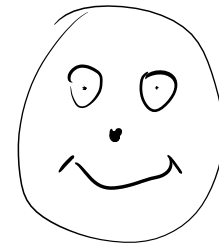
Singular Value Decomposition

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} & & \\ & M_{3 \times 3} & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$U \cdot U^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

orthogonal diagonal orthogonal

$$A = U S V$$



$$\begin{bmatrix} 3x \\ 2y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$3x - 2y + z = 0$$

$$x + 3z = -2$$

$$y = 1$$

$$\begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\boxed{A \vec{x} = \vec{b}}$$

$$A^{-1}A = AA^{-1} = I$$

$$\vec{x} = A^{-1} \vec{b}$$

Calculus

derivative

$f(x)$

$$\frac{df}{dx} = \frac{\delta f}{\delta x} = f' = \dot{f} = f_x = \text{rate of change}$$

$$x(t) = \text{position}$$

$$\dot{x}(t) = \text{velocity}$$

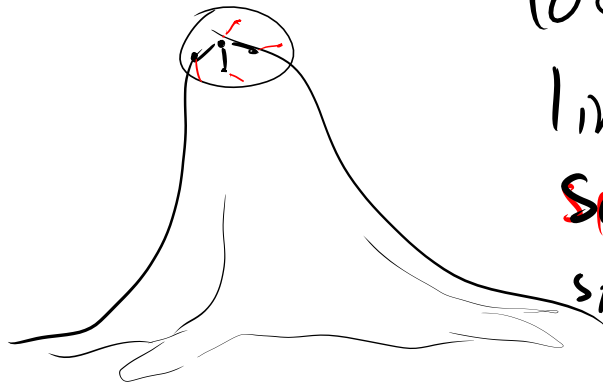
$$\ddot{x}(t) = \text{acceleration}$$

∫ Integral = Area under curve



Diff EQ

$$\ddot{x}(t) = f(x(t))$$



localize

linearize at t

~~Solve~~ $A\vec{x} = \vec{b}$

stop time

$$A\vec{x} = \vec{b}$$

↑
sparse

Symmetric

positive-definite

Conjugate gradient