

Skills:

- ① Design an adder.

Outline

- ① Discuss the adder
- ② Review number representation.
- ③ Masks

Add two binary
 Constrain we only want to use logic operations

Table the position of the binary numbers

Input 1	x_n	x_{n-1}	x_2	x_1	x_0	Input 2	y_n	y_{n-1}	y_2	y_1	y_0
	0	...	0	0	0		0	0	0	0	0
	output										
	z_n	...	z_2	z_1	z_0						
	0		0	0	0						

Let's consider the case of add two binary number that are 1 bit long

$x_0 + y_0$

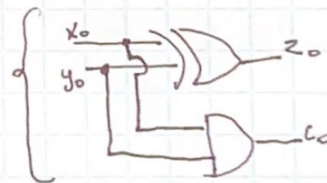
Let write out the truth table for this addition

x_0	y_0	z_0	c_0
0	0	0	-
0	1	1	-
1	0	1	-
1	1	0	1

Special case because the result is 10 we call this extra one the carry bit

Question can we design a circuit that outputs z_0

$x_0 \oplus y_0 = z_0$
 ↑
 XOR



But what about the carry bit
 $x_0 \wedge y_0 = c_0$

could you design

* Gate reminder

What about binary number that are longer than 2 bits

Consider case with two binary number

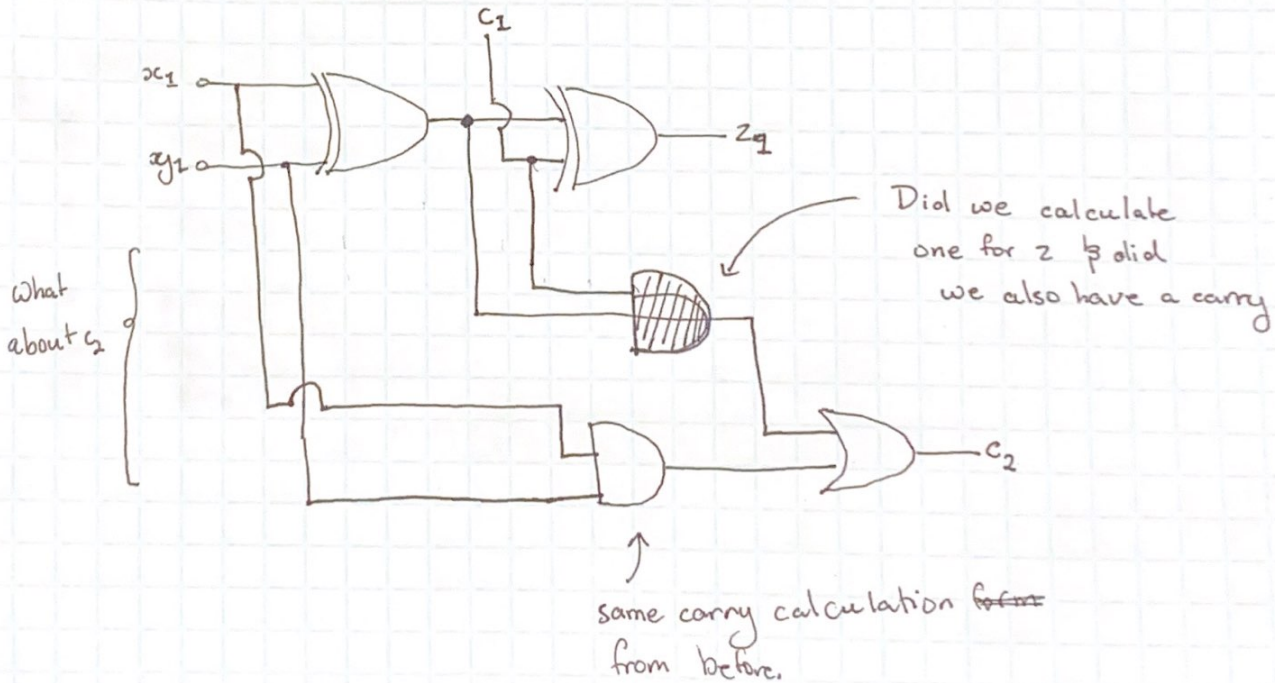
$$\begin{array}{r}
 \text{carry (out)} \\
 \text{c}_2 \quad \left. \begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right\} \begin{array}{r} 1 \\ 1 \\ \hline 11 \\ + 11 \\ \hline 110 \end{array} \\
 \text{c}_1
 \end{array}$$

Example 2.1

Let's write out the truth table.

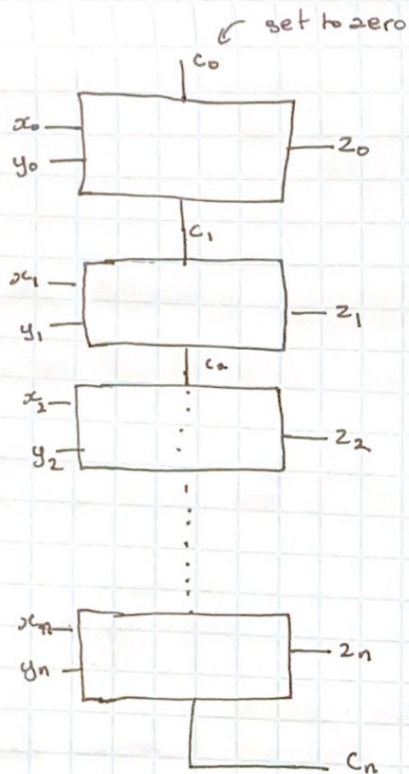
c ₁	x ₁	y ₁	z ₁	c ₂
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

← The case we looked from example 2.1



Then how do we scale to number of bits

Page 3



Commonly referred to as a ripple carry adder.

Now we finally have our adding box



* review of ~~the~~ bitwise operators, $\frac{1}{2}$ Masking

* Maybe sum bit puzzles.

Operators

2

& Bitwise and

| Bitwise or

^ Bitwise xor.

\gg Bitwise to the right no sign extension

\ll Bitwise shift to the left no sign extension.

Masking

001110000 construct a sequence of one $\&$ zero that you can use to extract or flip information / binary values

Consider the following examples.

$x = 110111 \leftarrow$ Set the forth bit of this number to one

$x \mid 001000$
 \leftarrow we could ~~use~~ ^{or} with the following mask,

$$\begin{array}{r} 110111 \\ \text{or } 001000 \\ \hline 111111 \end{array}$$

\leftarrow we've set the forth bit to one.

How could we construct the mask

~~001000~~

\downarrow

~~001000~~ $\rightarrow 1 \ll 4$

$x \mid = 1 \ll 4 \leftarrow$ Set the forth bit of x to one

$x = x \mid 1 \ll 4$

Clear (we normal use ands)

$x = 1111$

~~1111~~ clear the first, third and forth bits

~~we~~ we could do that by anding with 0010

$$\begin{array}{r} 1111 \\ \wedge 0010 \\ \hline 0010 \end{array}$$

$x \& 1 = 1 \ll 1$

Clear (and)

$x = 1111$ clear just the second bit

we could and with 1101

$$\begin{array}{r}
 \cancel{0010} \\
 1101 \\
 \wedge 1111 \\
 \hline
 1101
 \end{array}$$

← just clears the second bit

How do we create the mask 1101

$y = 0001$

$y \ll 1 = 0010 \leftarrow y \ll 1$

$\sim y = 1101 \leftarrow \neg y$ (not of y) write $\sim y$ in c.

Use or to set
Use and to clear

But how would you flip.

1101 → 1111
1111 → 1101 } Think about this for a second.

mask = 0010
 $x = 1101$

$$\begin{array}{r}
 x \wedge \text{mask} = 1101 \\
 \oplus 0010 \\
 \hline
 1111 \\
 \uparrow \text{flipped}
 \end{array}$$

~~mask = 1101~~
 mask = 0010
 $x = 1101$

$$\begin{array}{r}
 x \wedge \text{mask} = 0010 \\
 \oplus 1111 \\
 \hline
 1101 \\
 \uparrow \text{flipped}
 \end{array}$$

Parity bit

even odd.

01101

0

1

~~even parity number~~

parity $\left\{ \begin{array}{l} 0 \text{ if even number of } 1 \\ 1 \text{ if odd number of } 1 \end{array} \right.$

parity = 0

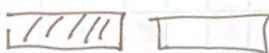
repeat 32

parity $\wedge = (x \gg 1)$
 $x \ll 1$


0
1

Puzzle.

x y

x' = 

$\gg \gg$

y' = 

$\ll \ll$

or together.

⊕

$* 0 = 1$
 $\wedge 0 = 0$

S 0000 0000

S 1111 0000

S 1111

↑

two complement
flip +1
add.