You Don't Know That

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1 A Motivating Example

Take something you are absolutely convinced of. Something you cannot seriously doubt if you tried. Something founded both in logic and in life-long social reinforcement. For example, take

$$2 + 3 = 5$$

We will try to develop some vocabulary to describe this kind of statement in a moment, but for now let us just plow on without that.

What is 2? Does such a thing as 3 exist? Have you ever seen whatever you mean by 5 in real life? In short, what does $2 + 3 = 5$ mean?

Let us go back to the kindergarden example. You have two apples, I give you three more apples; how many apples do you have now? Of course you answer that you have five apples. Nobody would claim that after this little trade you will end up with four or six apples; this is because $2 + 3 = 5$, but $2 + 3 \neq 4$ and $2 + 3 \neq 6$. You know this; I know this. There is no real denying it.

But let me ask another question. You have two apples, I give you three more apples; now how much apple do you have? Suppose the two apples you had totaled twelve ounces of apple, and the three I gave you totaled seventeen ounces of apple. No problem; you can still use your knowledge of + to determine that $12 + 17 = 29$ and assert that you now have twenty-nine ounces of apple.

Now let us suppose that one of the apples I gave you was sweeter and juicer than the other two. Let us suppose that one of the apples you already have has a small bad spot you need to cut away before eating. Also, suppose the apples are old, and since you only planned to eat one a day you will probably have one or two of them spoil before you get to them. However, you have a refrigerator that will keep all the apples fresh, but there's no room in it because someone filled it with a dozen five-gallon sacks of vanilla pudding. You could just take one of the sacks out, but then the owner might want you to replace it if it spoils before use and though you have the money to do that with, you don't know where you could find another five-gallon sack of vanilla pudding. Now, how many usable apples do you have?

2 A Model of "Knowledge"

This section operates under the assumption that you read section 1 and concluded without being able to answer the final question. If this assumption is false, go back to section 1 and add whatever you need to to make the question of uncertain answer in your own mind, and then come back to this section.

Let us now analyze the failure of $2 + 3 = 5$ to apply to my adding three apples to the two you have. I assume you are still completely convinced that $2 + 3$ does indeed equal 5, and that this formula does apply to counting apples; and yet, somehow, it does not *always* apply to counting apples, or perhaps it is not always, in itself, *sufficient* knowledge for the task at hand. So what, then, does knowing $2 + 3 = 5$ mean?

I am going to assert, without proof, that $2 + 3 = 5$ is an instance of a production rule over a formal system*. Not to be confusing, let me define what I mean. A **formal system** is a set of symbols and a set of rules for manipulating those symbols. A **production rule** is part of a formal system that allows us to change one set of symbols into other, equivalent symbols; for example, the addition production rule allows us to change two numbers separated by a plus sign into a single number. An **instance** of a production rule is one particular case where the rule is applied; for example, addition says that $x + y = z$ is true for some, but not all, numbers x, y, and z; one instance where it is true is $2 + 3 = 5$. Knowledge about formal systems are characterized by certainty, universality, and a feeling of rigor; you have probably never heard someone criticize your formal knowledge itself.

Not all of your knowledge is related to a formal system. For example, you know your name, but it is extremely unlikely that you have any formal system where we can define what we mean by "you," "your," or even "name." Likewise, you may know what grade you got on your last test, who the president of the country is, and what sugar tastes like. I categorize this kind of knowledge as information, and treat it differently than formal knowledge.

Before moving further, let us consider what happens when someone asserts something contrary to your knowledge. If someone contradicts your formal knowledge, you will not even begin to believe them. If I state, $2+3 = 4$ you will assume I am either joking, insane, am talking about a formal system with the same symbols but different rules than that you are used to, or I am maliciously trying to confuse you.

If, however, I assert that the person you think is the president is not really the president, a discussion will ensue. There will be evidence and arguments produced by each side; authorities will be appealed to, with reasons that the authorities ought to be believed. The argument will take on the form of establishing common information (e.g., "we both agree that The New York Times says...") and appealing to formal knowledge to support the conclusion of that information (e.g. "humans are afraid of being caught lying. Someone would

*There is a full and rich literature on the theory of formal systems; if you are familiar with it, ignore it for now, as it is not important for this paper.

catch the NYT if it lied about the president. The NYT is run by humans. Thus, the NYT would be afraid to lie about the president"; an appeal to the formal system known as syllogistic logic). If no common information can be established, we call the "knowledge" subjective (for example, the fact that curried okra tastes good* cannot be supported by formal derivation of other info, and so is subjective knowledge) and do not treat it as actually being truth in its own right†.

Thus knowledge can be said to consist of subjective information and well-defined formal knowledge. To discuss what information "means" or to talk "rationally" about what we experience we first translate shared information into our formal system, reason in that system, and then translate our results back into information. This translation process is called **modeling** and is the third part of our model of knowledge, accompanying information and formal knowledge. And it is in the modeling step that our example in section 1 broke down; the formal system (arithmetic) was fine and we had no disagreement about the basic information (which was all hypothetical anyway), but it was not clear how to model the information in terms of our formal system.

3 Failures in Modeling

In this section I will try to convince you that you should not believe anything you know. The reason you should not believe anything you know is because of systemic failures in the modeling process, which are themselves twofold: the philosophic failure and the rigorous failure.

3.1 Philosophic Failure of Modeling

Somehow, in every rational process, we go from experience, or more precisely from memory of experience, to some sort of informal observation, and then on to some formal statements which we can manipulate, after which we reverse the process. This usually goes unnoticed; you are so accustomed to some of the common mappings (counting, for instance) that you probably never thought of it as a modeling process; but it is well worth inspection. If our modeling process is wrong, if it leaves out anything important or introduces anything faulty, then we should have no confidence in our results.

The modeling process we use in ordinary life is based on the idea of "good enough." If knowing the number of apples is enough for our purposes, we count apples. If it isn't, we may weigh them, or chemically test their sugar content, or whatever. The problem here is that "good enough" is itself quite subjective. Even if we were to agree what we mean by "good enough"—say, for example, that we decide we need 20 ounces of edible apple flesh, plus or minus 3 ounces—we cannot measure how close we came to our tolerance without some other

*This *is* a fact, incidentally.

†Which means the real fact is that curried okra tastes good to me, not that it tastes good in and of itself.

formal system to compare our answer too.

Not to be long about this point, we cannot halt this process of needing a formal system to measure our formal system against in any verifiable way. Eventually we have to just say, "this is good enough *for me*" in a subjective abstract sense. All modeling is, by its very nature, rooted in some sort of non-rigourus faith.

We cannot even provide rigorous evidence that our models match reality.

3.2 Rigorous Failure of Modeling

There are some people whose life work consists of coming up with models that seem to match reality as closely as possible. For example, nuclear physics tries to define what the building blocks of reality are, and how they interact in all conceivable cases. Their models are incomplete, but let us assume for a moment that they are both complete and true. In other words, I have a perfectly accurate model from the world into a formal system, and from that formal system back again.

I cannot even begin to use that "perfect" model. It is too messy to think about.

However, remembering our discussion of "good enough," shouldn't we be able to use our "perfect" model to prove that some simpler model is good enough? Weirdly, the answer to this question is usually "no."

What we are really asking is, "are there a sequence of steps I can follow which will take the information I have and produce information that is close to the information I want?" This is the core question of a field known as computational theory, and there are proofs that any formal system either has a lot of questions it cannot answer, including the question of whether the systems itself is sound, or else it contains a contradiction and should not be trusted*. Even if you had a perfect mapping, you could not use it, nor could you create a consistent simpler mapping that you could actually use.

4 Ergo, What?

I have asserted herein that knowledge is made up of

- Formal knowledge, which is relatively undisputed but not directly related to reality.
- Information, which is related to reality but subjective and disputable.

*This proof, known as Gödel's Incompleteness Theorem, as well as associated proofs from computation and complexity theories, are far more elegant than I can express here. Interested readers are referred to Douglas R. Hofstadter's massive work *Gödel, Escher, Bach: An Eternal Golden Braid* or to the smaller and less readable but much more complete *Computability and Complexity Theory* by Homer and Selman.

- Mappings between formality and information, which are themselves subjective and unverifiable and, even if they were actually correct, are far too complex to actually use.

It may, then, look like I am claiming the world is unknowable and advocating some sort of zen-like cessation of any effort to understand anything. Nothing could be further from my heart.

My goal in sharing this has been two-fold. First, I want you to understand that there are, in actual fact, dilemmas in the world; questions to which the answer is not known. Second, I want you to recognize that at some level, all of the things you believe, and I really do mean *all* of them, you chose to believe of your own free will*.

Knowing that "knowledge" is, at the root, a matter of faith and personal choice is, I think, one of the great motivators of life. It removes the victimization from the world and helps to direct ones mind in thoughtful consideration of more than merely how knowledge is applied, but also how it is acquired and why it is believed.

Too often we ignore the mapping. We agree that the formalism should not be disputed, so we assume that any misinformation should be rooted at the source information level. This is sometimes true, but we don't have very good reasons for accepting one mapping between observation and logic over another.

You have two apples. I give you three more apples. How many apples do you have?

Sometimes, the answer isn't five.

*Or, if you have chosen not to believe in free will, by whatever it is that your information→formalism mapping claims is the source of those things that I see as being the purview of free will.