Confronting Science's Logical Limits

The mathematical models now used in many scientific fields may be fundamentally unable to answer certain questions about the real world. Yet there may be ways around these problems

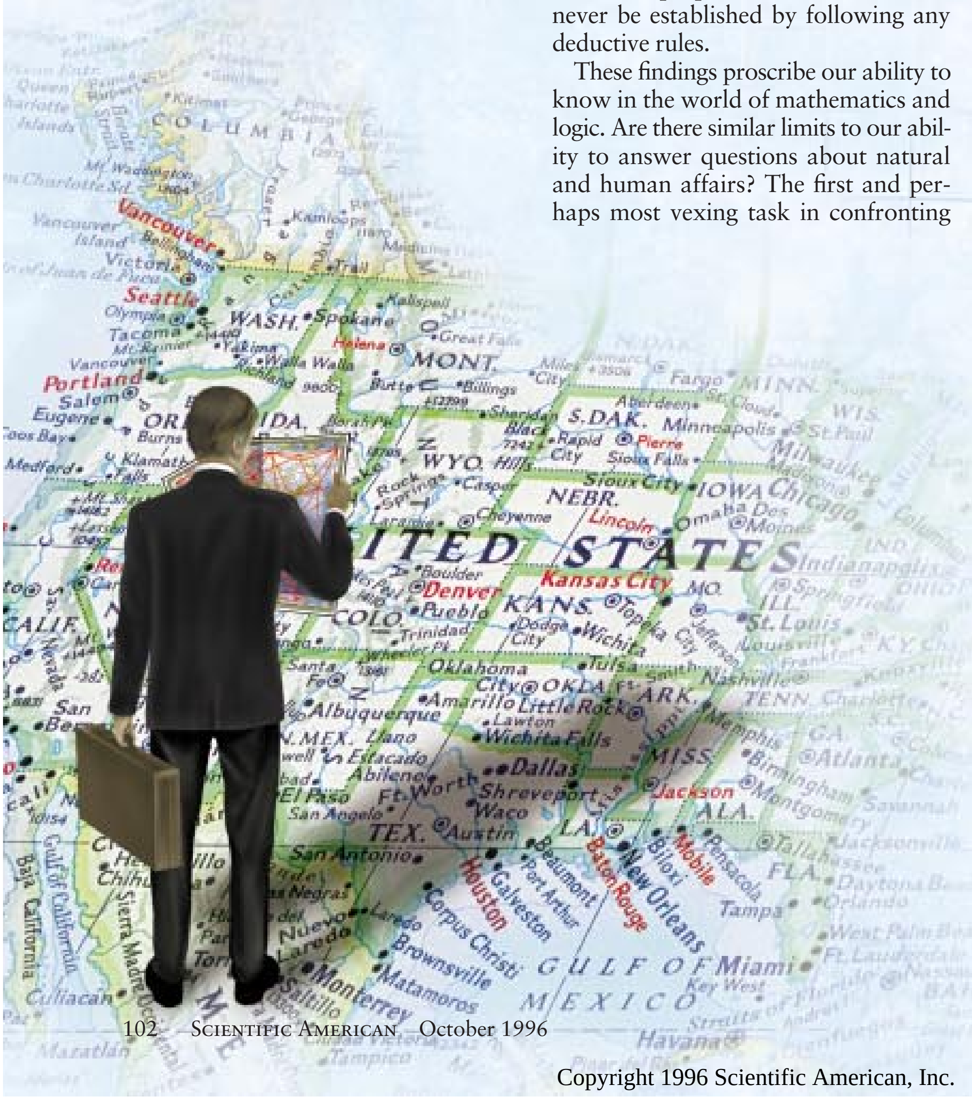
by John L. Casti

To anyone infected with the idea that the human mind is unlimited in its capacity to answer questions, a tour of 20th-century mathematics must be rather disturbing. In 1931 Kurt Gödel set forth his incompleteness theorem, which established that no system of deductive inference can answer all questions about numbers. A few years later Alan M. Turing proved an equivalent assertion about computer programs, which states that there is no systematic way to determine whether a given program will ever halt when processing a set of data. More recently, Gregory J. Chaitin of IBM has found arithmetic propositions whose truth can never be established by following any deductive rules.

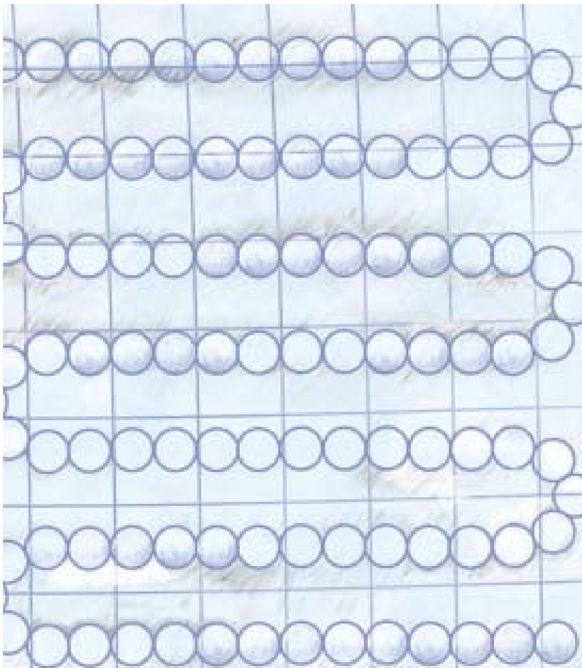
These findings proscribe our ability to know in the world of mathematics and logic. Are there similar limits to our ability to answer questions about natural and human affairs? The first and perhaps most vexing task in confronting this issue is to settle what we mean by "scientific knowledge." To cut through this philosophical Gordian knot, let me adopt the perhaps moderately controversial position that a scientific way of answering a question takes the form of a set of rules, or program. We simply feed the question into the rules as input, turn the crank of logical deduction and wait for the answer to appear.

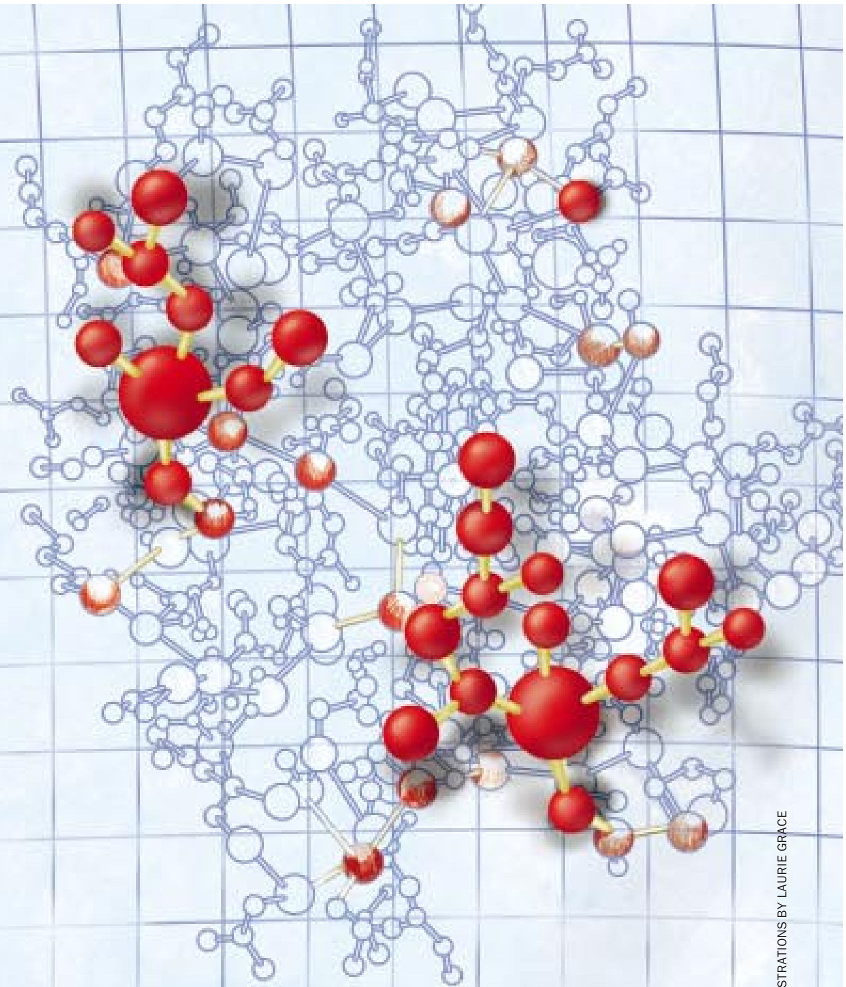
Thinking of scientific knowledge as being generated by what amounts to a computer program raises the issue of computational intractability. The difficulty of solving the celebrated traveling-salesman problem, which involves finding the shortest route connecting a large number of cities, is widely believed to increase exponentially as the number of destinations rises. For example, pinpointing the best itinerary for a salesman visiting 100 cities would require examining $100 \times 99 \times 98 \times 97 \times \ldots \times 1$ possibilities—a task that would take even the fastest computer billions of years to complete.

But such a computation is possible—at least in principle. Our focus is on questions for which there exists no program at all that can produce an answer. What would be needed for the world of physical phenomena to display the kind of logical unanswerability seen in mathematics? I contend that nature would have to be either inconsistent or incomplete, in the following senses. Consistency means that there are no true para-



TRAVELING SALESMAN would need the world's fastest computer running for billions of years to calculate the shortest route between 100 destinations. Scientists are now seeking ways to make such daunting problems more tractable.





ILLUSTRATIONS BY LAURIE GRACE

PROTEIN-FOLDING PROBLEM considers how a string of amino acids (*left*) folds up almost instantaneously into an extraordinarily complex, three-dimensional protein (*right*). Biologists are now trying to unravel the biochemical "rules" that proteins follow in accomplishing this feat.

doxes in nature. In general, when we encounter what appears to be such a paradox—such as jets of gas that seemed to be ejected from quasars at faster than light speeds—subsequent investigation has provided a resolution. (The "superluminal" jets turned out to be an optical illusion stemming from relativistic effects.)

Completeness of nature implies that a physical state cannot arise for no reason whatsoever; in short, there is a cause for every effect. Some analysts might object that quantum theory contradicts the claim that nature is consistent and complete. Actually, the equation governing the wave function of a quantum phenomenon provides a causal explanation for every observation (completeness) and is well defined at each instant in time (consistency). The notorious "paradoxes" of quantum mechanics arise because we insist on thinking of the quantum object as a classical one.

A Triad of Riddles

It is my belief that nature is both consistent and complete. On the other hand, science's dependence on mathematics and deduction hampers our ability to answer certain questions about the natural world. To bring this issue into sharper focus, let us look at three well-known problems from the areas of physics, biology and economics.

• Stability of the solar system. The most famous question of classical mechanics is the *N*-body problem. Broadly speaking, this problem looks at the behavior of a number, *N*, of point-size masses moving in accordance with Newton's law of gravitational attraction. One version of the problem addresses whether two or more of these bodies will collide or whether one will acquire an arbitrarily high velocity in a finite time. In his 1988 doctoral dissertation, Zhihong (Jeff) Xia of Northwestern University showed how a single body moving back and forth between two binary systems (for a total of five masses) could approach an arbitrarily high velocity and be expelled from the system. This result, which was based on a special geometric configuration of the bodies, says nothing about the specific case of our solar system. But it does suggest that perhaps the solar system might not be stable. More important, the finding offers new tools with which to investigate the matter.

• Protein folding. The proteins making up every living organism are all formed as sequences of a large number of amino acids, strung out like beads on a necklace. Once the beads are put in the right sequence, the protein folds up rapidly into a highly specific three-dimensional structure that determines its function in the organism. It has been estimated that a supercomputer applying plausible rules for protein folding would need 10^{127} years to find the final folded form for even a very short sequence consisting of just 100 amino acids. In fact, in 1993 Aviezri S. Fraenkel of the University of Pennsylvania showed that the mathematical formulation of the protein-folding problem is computationally "hard" in the same way that the traveling-salesman problem is hard. How does nature do it?

• Market efficiency. One of the pillars on which the classical academic theory of finance rests is the idea that financial markets are "efficient." That is, the market immediately processes all information affecting the price of a stock or commodity and incorporates it into the current price of the security. Consequently, prices should move in an unpredictable, essentially random fashion, discounting the effect of inflation. This, in turn, means that trading schemes based on any publicly available information, such as price histories, should be useless; there can be no scheme that performs better than the market as a whole over a significant interval. But actual markets do not seem to pay much attention to academic theory. The finance literature is filled with such market "anomalies" as the low price–earnings ratio effect, which states that the stocks of firms whose prices are low relative to their earnings consistently outperform the market overall.

The Unreality of Mathematics

Our examination of the three questions posed above has yielded what appear to be three answers: the solar system may not be stable, protein folding is computationally hard, and financial markets are probably not completely efficient. But what each of these putative "answers" has in common is that it involves a mathematical *representation* of the real-world question, not the question itself. For instance, Xia's solution of the *N*-body problem does not explain how real planetary bodies move in accordance with real-world gravitational forces. Similarly, Fraenkel's conclusion that protein folding is computationally hard fails to address the issue of how real proteins manage to do their job in seconds rather than eons. And, of course, canny Wall Street operators have thumbed their noses at the efficient-market hypothesis for decades. So to draw any conclusions about the inability of science to deal with these questions, we must either justify the mathematical model as a faithful representation of the physical situation or abandon the mathematics altogether. We consider both possibilities in what follows.

What these examples show is that if we want to look for scientifically unanswerable questions in the real world, we must carefully distinguish between the world of natural and human phenomena and mathematical and computational models of those worlds. The objects of the real world consist of directly observable quantities, such as time and position, or quantities, such as energy, that are derived from them. Thus, we consider parameters such as the measured position of planets or the actual observed configuration of a protein. Such observables generally constitute a discrete set of measurements taking their values in some finite set of numbers. Moreover, such measurements are generally not exact.

In the world of mathematics, on the other hand, we have symbolic representations of such real-world observables, where the symbols are often assumed to belong to a continuum in both space and time. The mathematical symbols representing attributes such as position and speed usually have numerical values that are integers, real numbers or complex numbers, all systems containing an infinite number of elements. In mathematics the concept of choice for characterizing uncertainty is randomness.

Finally, there is the world of computation, which occupies the curious position of having one foot in the real world of physical devices and one foot in the world of abstract mathematical objects. If we think of computation as the execution of a set of rules, or algorithm, the process is a purely mathematical one belonging to the world of symbolic objects. But if we regard a computation as the process of turning switches on or off in the memory of an actual computing machine, then it is a process firmly rooted in the world of physical observables.

One way to demonstrate whether a given question is logically impossible to answer by scientific means is to restrict all discussion and arguments solely to the world of natural phenomena. If we follow this path, we are forbidden to translate a question such as "Is the solar system stable?" into a mathematical statement and thereby to generate an answer with the logical proof mechanism of mathematics. We then face the problem of finding a substitute in the physical world for the concept of mathematical proof.

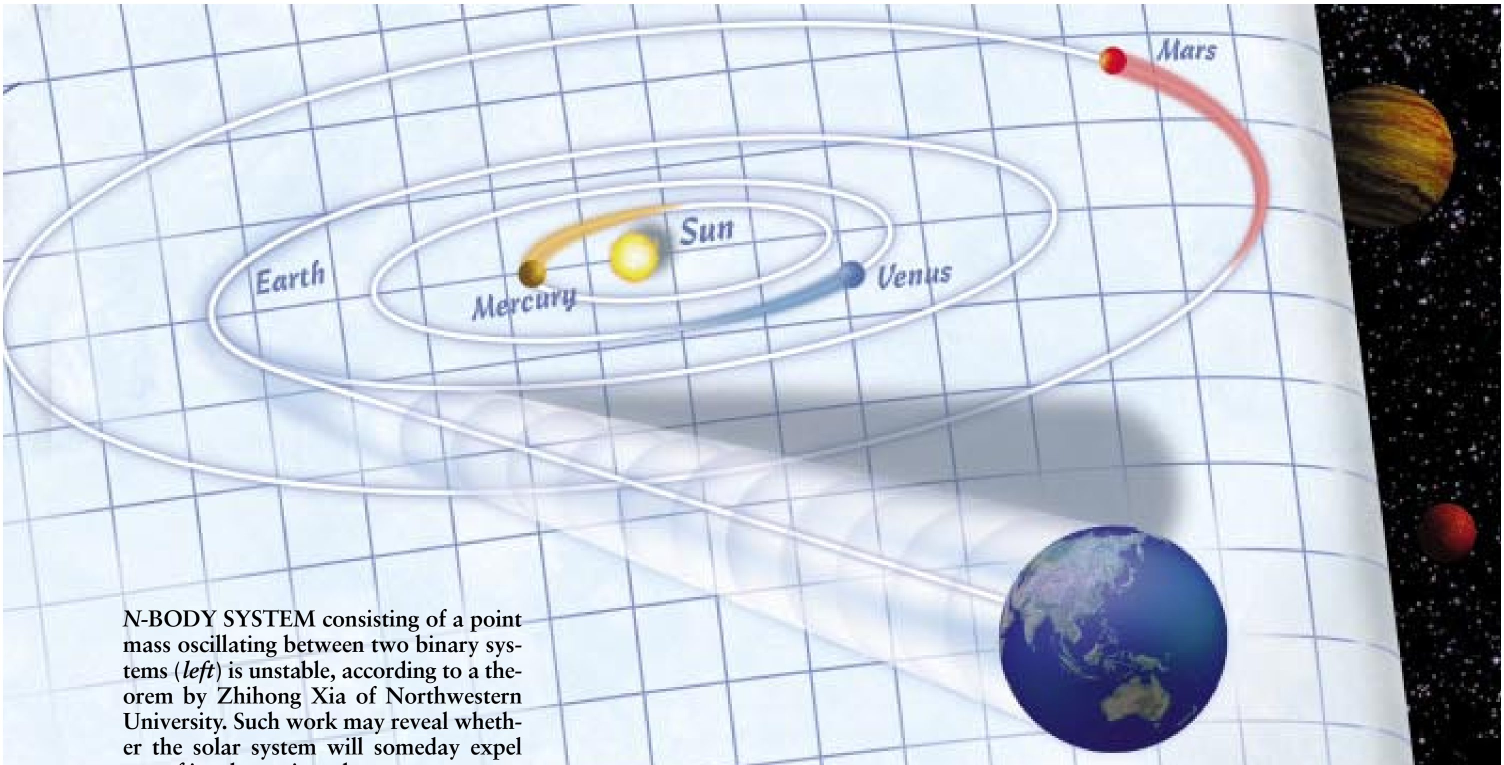
A good candidate is the notion of causality. A question can be considered scientifically answerable, in principle, if it is possible to produce a chain of causal arguments whose final link is the answer to the question. A causal argument need not be expressed in mathematical terms. For example, the standard deductive argument "All men are mortal; Socrates is a man; therefore, Socrates is mortal" is a causal chain. There is no mathematics

involved, just plain English. On the other hand, constructing a convincing causal argument without recourse to mathematics may be a daunting task. In the case of the stability of the solar system, for example, one must find compelling nonmathematical definitions of the planets and gravity.

Given these difficulties, it seems wise to consider approaches that mix the worlds of nature and mathematics. If we want to invoke the proof machinery of mathematics to settle a particular real-world question, it is first necessary to "encode" the question as a statement in some mathematical formalism, such as a differential equation, a graph or an *N*-person game. We settle the mathematical version of the question using the tools and techniques of this particular corner of the mathematical world, eventually "decoding" the answer (if there is one!) back into real-world terms. One challenge here is establishing that the mathematical version of the problem is a faithful representation of the question as it arises in the real world. How do we know that mathematical models of a natural system and the system itself bear any relation to each other? This is an old philosophical conundrum, entailing the development of a theory of models for its resolution. Moreover, mathematical arguments may be subject to the constraints revealed by Gödel, Turing and Chaitin; we do not know yet whether the real world is similarly constrained.

The Noncomputational Mind

There may be ways to sidestep these issues. The problems identified by Gödel and others apply to number systems with infinite elements, such as the set of all integers. But many real-world



***N*-BODY SYSTEM consisting of a point mass oscillating between two binary systems (*left*) is unstable, according to a theorem by Zhihong Xia of Northwestern University. Such work may reveal whether the solar system will someday expel one of its planets into deep space.**

problems, such as the traveling-salesman problem, involve a finite number of variables, each of which can take only a finite number of possible values.

Similarly, nondeductive modes of reasoning—induction, for instance, in which we jump to a general conclusion on the basis of a finite number of specific observations—can take us beyond the realm of logical undecidability. So if we restrict our mathematical formalisms to systems using finite sets of numbers or nondeductive logic, or both, every mathematical question should be answerable; hence, we can expect the decoded real-world counterpart of such questions to be answerable as well.

Studies of the human mind may reveal other ways to bypass logical limits. Some artificial-intelligence proponents have proposed that our brains are computers, albeit extremely sophisticated ones, that perform calculations in the same logical, step-by-step fashion that conventional computers (and even parallel processors and neural networks) do. But various theorists, notably the mathematical physicist Roger Penrose of the University of Oxford, have argued that human cognitive activity is not based on any known deductive rules and is thus not subject to Gödelian limits.

Recently this viewpoint has been bolstered by studies carried out under the aegis of the Institute for Future Studies in Stockholm by me, the psychologist Margaret A. Boden of the University of Sussex, the mathematician Donald G. Saari of Northwestern University, the economist Åke E. Andersson (the institute's director) and others. Our work strongly suggests that in the arts as well as in the natural sciences and mathematics, the human creative capacity is not subject to the rigid constraints of a computer's calculations. Penrose and other theorists have conjectured that human creativity stems from some still unknown mechanisms or rules, perhaps related to quantum mechanics. By uncovering these mechanisms and incorporating them into the scientific method, scientists may be able to solve some seemingly intractable problems.

Of course, science's ability to plumb nature's secrets is limited by many practical considerations—such as measurement error, length of computation, physical and economic resources, political will and cultural values. But none of these considerations bears on whether there is a logical barrier to our answering a certain question about the natural world. My contention is that there is not. So a tour of 20th-century mathematics need not be so disturbing after all!

The Author

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Further Reading

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