CS3102 Theory of Computation
Homework 1
Department of Computer Science, University of Virginia

Gabriel Robins

Please solve the problems below and prove all your answers. Informal arguments are acceptable, but please make them precise / detailed / convincing enough so that they can be easily made rigorous. To review notation and definitions, please read the "Basic Concepts" summary posted on the class Web site (at http://www.cs.virginia.edu/~robins/cs3102/basics.pdf), and also read the first two chapters from the Sipser textbook. Please turn in your solutions in **hardcopy** (not by Email) at the beginning of class (2:00pm) on Thursday Sept 28, 2017. (Please staple together all your solution sheets.)

1.   Solve problem 0.5 on page 26 of [Sipser, Second Edition].

2.   Solve problem 0.12 on page 27 of [Sipser, Second Edition].

3.   True or false: a countable union of countable sets is countable.

4.   True or false: if T is countable, then the set $\{S \mid S \subseteq T, S \text{ finite}\}$ is also countable.

5.   What is the cardinality of each of the following sets ?
        (i.e., finite, countably infinite, or uncountably infinite)
        a.   The set of all possible Java programs
        b.   The set of all finite strings over the alphabet $\{0,1,2\}$
        c. $\{\emptyset, \mathbf{N}, \mathbf{Q}, \mathbf{R}\}$
        d. $\mathbf{R} - \mathbf{Q}$

6.   Prove without using induction that $n^4 - 4n^2$ is divisible by 3 for all $n \geq 0$.

7.   How many distinct boolean functions on N variables are there?
        In other words, what is the cardinality of $|\{f \mid f : \{0,1\}^N \rightarrow \{0,1\}\}|$?

8.   Prove or disprove: every regular language is countable.

9.   Prove or disprove: the set of all regular languages is countable.

10.   Solve problems 1.6(b), 1.6(h), 1.6(i) on page 84 of [Sipser, Second Edition].
        Use JFLAP (http://www.jflap.org/) to implement and test each of these DFAs, and include in your answers screen shots which show what each of these DFAs looks like inside JFLAP.

11.   Solve problem 1.17 on page 86 of [Sipser, Second Edition].
        Use JFLAP (http://www.jflap.org/) to implement and test the DFA and NFA of both parts of this question, and include in your answers JFLAP screen shots which show both of these automata.