CS3102 Theory of Computation
Homework 3
Department of Computer Science, University of Virginia

Gabriel Robins

- This assignment is due before 11:59pm on Saturday March 10, 2018; late submissions will not be accepted.
- This is an open-book, open notes, pledged homework assignment.
- You may work and brainstorm in groups, but verbatim copying of other people's solutions is disallowed.
- Please carefully read the Cheating Policy on the CS3102 course syllabus.
- Please do not submit answers that you do not fully understand (we reserve the right to ask you to explain any of your answers verbally in person, and we will definitely exercise this option).
- Solve as many of the problems as you can; please explain / prove all answers.
- Shorter explanations / proofs / algorithms are much preferable to longer ones.
- Clearly state the short answer / proof idea first, and then your complete answer / proof.
- Submit only the pages provided (use more sheets only if absolutely necessary).
- Please do not procrastinate / cram, which will not work well for you in this course.
- Please do not put us (and yourself) in an awkward position where you force us to say "we told you so".

Please meet with the TAs often, ask them questions regularly, and attend the weekly problem-solving sessions.

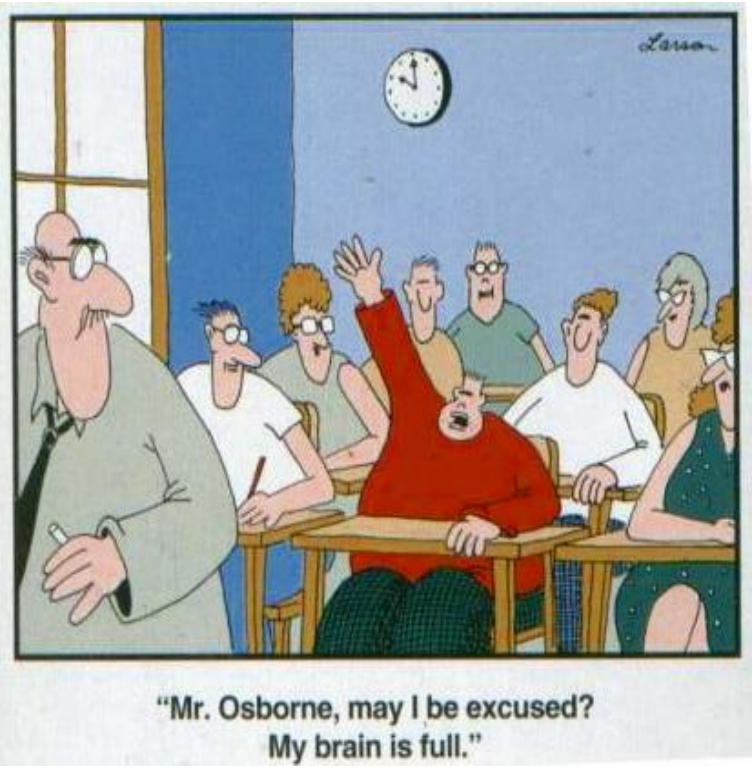
Please submit your solutions through the online Collab system. In the very rare case where that's not possible, then please Email your PDF to the course TAs (and CC Professor Robins).

Name:_____ **UVa Computing ID:**_____

Please pledge and sign here, certifying that you full complied with the Honor Code and the course Cheating Policy summarized on page 3 of the Course Syllabus:_____

_____

_____



Problem 1:	**20**	_____
Problem 2:	**20**	_____
Problem 3:	**20**	_____
Problem 4:	**20**	_____
Problem 5:	**20**	_____
Problem 6:	**20**	_____
Problem 7:	**20**	_____
Problem 8:	**60**	_____
Problem 9:	**50**	_____
Problem 10:	**10**	_____
Total:	**260**	_____

Additional Instructions and Guidelines

Please solve the problems below and prove all your answers. Informal arguments are acceptable, but please make them precise / detailed / convincing enough so that they are rigorous. To review notation and definitions, please read the "Basic Concepts" summary posted on the class Web site and also read Chapter 2 (entitled "Context-Free Languages", pages 99-134) in the [Sipser, Second Edition] textbook.

Please directly edit this homework PDF file, insert your answers there, and submit your completed homework as a PDF attachment on the course Collab page. If you do not have a PDF-file editor, we recommend using for example either the PDF-Xchange Editor or Foxit. Both of these are free, and are powerful enough to complete your homework, including creating diagrams in your answers, if necessary. Many other free PDF editors can work also. If you prefer to edit the MS Word version of this file and then convert it into a PDF document, an MS Word version of this file is available as a link from the class Web site.

Please do not submit answers that you do not fully understand; we reserve the right to ask you to explain any of your answers verbally in person (and we have exercised this option in the past). Please put your name and computing ID on the first page, and sign the pledge that you complied with the UVa Honor Code as well as with the course Cheating Policy summarize on page 3 of the Course Syllabus.

Important: please uniquely name your submitted solutions PDF file using your name and your UVa computing ID, using the file name format: LastName_FirstName_ComputingID_Homework_3.pdf (e.g. "Robins_Gabriel_gr3e_Homework_3.pdf").

If for any reason you cannot find a suitable PDF editor that works for you, or you have trouble editing your homework PDF file, you may edit the Microsoft Word version of the file (available on the class Web site), and then re-generate the PDF with your solutions included in it. Either way, you must use Collab to submit your PDF file (not a hardcopy). In the very rare case that Collab doesn't work for you, please Email your PDF to the course TAs (and CC Professor Robins).

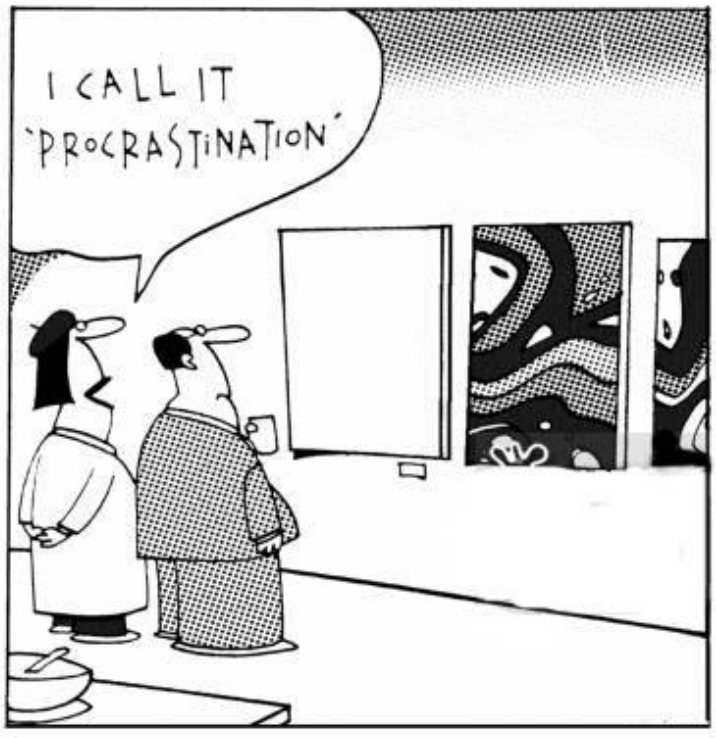
Please turn in your Homework 3 solutions into Collab before 11:59pm on Saturday March 10, 2018. Late submissions will not be accepted (the online Collab system will simply refuse to accept late submissions after the deadline, so you literally will not even be able to turn it in late even if you try). So if you haven't finished the assignment by the deadline, your best strategy is to just turn in the portion that you have finished by that time (rather than be late and not receive any credit for that assignment at all), and then going forward please make sure you are not late in turning in any future assignments.

Please note that if against our advice you plan to push the deadline and try to turn in an assignment a few minutes before the deadline, and something glitches in the system (e.g., network delays or server issues or other logistical problems), that too will be your responsibility since you chose to push the deadline and ignore our instructions to not procrastinate. In such a case you would have missed the opportunity to turn in the current assignment. So your best strategy in general is to turn in an assignment several days ahead of its deadline (these potential glitch-scenarios are already "baked into" the very generous due-date deadlines).

Please do not tell us after the fact that you didn't realize all this, or that you decided to not plan accordingly, because then we will just remind you to re-read these instructions and advice here, and we will also remind you that not getting credit for an assignment is a very small price to pay for such an important life lesson in learning to not procrastinate and taking personal responsibility for your own decisions and actions, and the resulting consequences. This policy is designed to help train people in good planning, avoiding procrastination, resisting the temptation to cheat, and taking personal responsibility for their decisions and actions like the adults that we all are. So whether this is obvious to you or not, these policies are actually designed for your own benefit and will help you become a more effective individual – our gift to you! ☺

Aside from turning in the assigned homeworks, you are expected to also work on the posted problem sets on a daily/weekly basis. Remember that most homework and exam questions in this course will come from the posted problem sets (or will be minor variations thereof). So your best strategy is to solve as many problems as you can during the semester on a daily basis (not only the ones that are assigned on the Homeworks). You should also meet regularly with the course TAs, and attend as many of the problem-solving sessions as possible (hopefully all of them). We estimate that to fully understand and master the material of this course typically requires an average effort of somewhere between at least six and ten hours per week, as well as regular meetings with the TAs and attendance of the weekly problem-solving sessions.

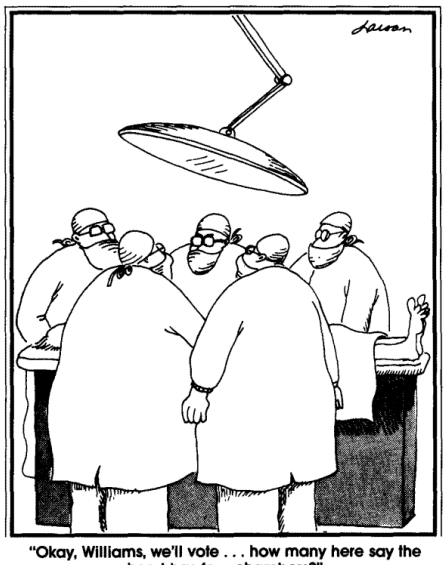
We also observed that historically, people who attend the weekly problem-solving sessions tend to perform more than a full letter grade better in the course, as well as less tempted to cheat, as compared with people who do not attend these weekly meetings. So if you only spend a couple of hours per week on this course, you are already seriously underestimating the amount of effort and practice required to learn this material, and we sincerely ask you to please stay on top of things, not procrastinate, and regularly practice solving lots of problems. Please do not put us (and yourself) in an awkward position where you force us to say "we told you so" (and "we even told you so in writing. Repeatedly.")



1. Solve problem 1.46(a) on page 90 of the [Sipser, Second Edition] textbook.
(This also comes from problem 1 on Problem set 4.)

Proof idea (one short phrase):

Proof:



"Okay, Williams, we'll vote . . . how many here say the heart has four chambers?"

2. Use JFLAP (http://www.jflap.org/) to implement and test the pushdown automaton of problem 1.46(a) on page 90 of the [Sipser, Second Edition] textbook. Describe how your PDA works, and include in your answers screen shots which show visually what your PDA looks like inside JFLAP, as well as examples of your PDAs execution on some input strings.

2a. Draw the PDA of 1.46(a) and explain how it works to recognize its language:

2b. Give JFLAP screenshots of the PDA of 1.46(a) as well as screenshots of some execution examples:

3. Solve problem 1.42 on page 89 of the [Sipser, Second Edition] textbook.
(This also comes from problem 1 on Problem set 4.)

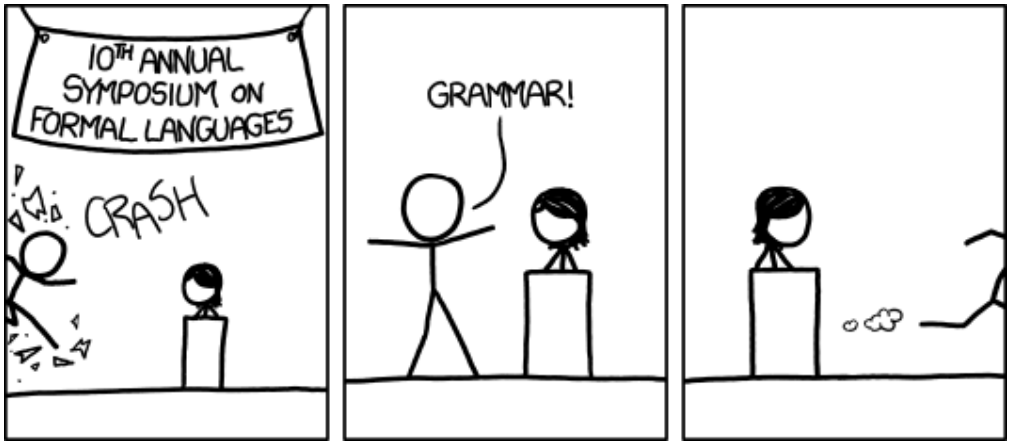
Proof idea (one short phrase):

Proof:

4. Are there two non-finitely-describable languages whose concatenation is regular? (This comes from problem 9 on Problem Set 4.)

Proof idea (one short phrase):

Proof:



5. Let $L = \{0^i 1^j \mid i \neq j\}$. Is L a regular language?
(This is problem 17 from Problem Set 5.)

Short answer (circle one): **yes** **no**

Proof idea (one short phrase):

Proof:

6. Does there exist a context-free grammar for $\{0^i1^j \mid 1 \leq i \leq j \leq 2i\}$?
(This is problem 14 from Problem Set 5.)

Short answer (circle one): **yes** **no**

Proof:

Due to his grammar mistake, Wilbur found a position. It just wasn't the one he wanted.

7. Is the SHUFFLE of two context-free languages necessarily context-free?
(This is problem 19(a) from Problem Set 5.)

"Frankly, I have a lot of trouble relating to all this symbolism."

8. Finish reading Polya's "How to Solve It" book. Describe three general problem-solving techniques discussed in this book, and give detailed example problems from Homeworks 1-3 in this class where each of these three techniques were useful / applicable in solving those problems.

8a. Problem-solving technique 1 from [Polya] and how it works (write short description here):

Examples of usefulness of technique 1 in solving problems from the last three Homeworks:

8b. Problem-solving technique 2 from [Polya] and how it works (write short description here):

Examples of usefulness of technique 2 in solving problems from the last three Homeworks:

8c. Problem-solving technique 3 from [Polya] and how it works (write short description here):

Examples of usefulness of technique 3 in solving problems from the last three Homeworks:

Step One
Understand the Problem

Can you state the problem in your own words?
What are you trying to find or do?
What are the unknowns?
What information do you obtain from the problem?
What information, if any, is missin
or not needed?

Step Two
Make a Plan

Choose the most appropriate strategy (or strategies).
Look for a pattern.
Remember related problems.
Break the problem down into different parts.
Make a table.
Make a diagram.
Write an equation.
Use a guess and check.
Work backward.

George Polya's
Steps to Problem Solving

Step Three
Do the Plan

Implement the plan in Step 2 and perform the necessary math computations.
Check each step of the plan.
Keep an accurate record of your work.
Organize your work into easy to understand visuals.
Double check your math work.

Step Four
Look Back

Check the results in the original problem.
Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable?
Determine whether there is another method of finding the solution.
If possible, determine other related or more general problems for which the techniques will work

9. For each of the following statements, state whether it is always true, never true, or sometimes true and sometimes false.

9a. A superset of a context-free language is not context-free.
(This is comes from problem 3 on Problem Set 5.)

Short answer (circle one): **always true** **sometimes true** **always false**

Proof:

"Make my speech so ambigious it doesn't matter *what* they take out of context!"

9b. A context-free language contains a regular subset.
(This is a minor variation of problem 2 on Problem Set 5.)

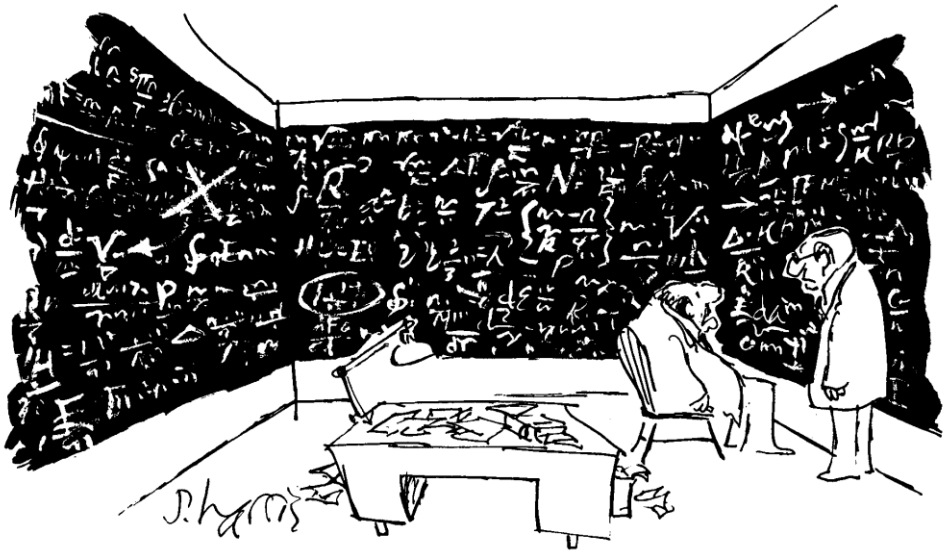
Short answer (circle one): **always true** **sometimes true** **always false**

Proof:

9c. A regular language contains a proper context-free subset.
(This is a minor variation of problem 7 on Problem Set 3.)

Short answer (circle one): **always true** **sometimes true** **always false**

Proof:



"Whatever happened to *elegant* solutions?"

9d. A countably-infinite union of context-free languages is context-free (i.e. does an infinite number of applications of the set union operator to context-free languages preserve context-freeness?) (This comes from problem 4 on Problem Set 5.)

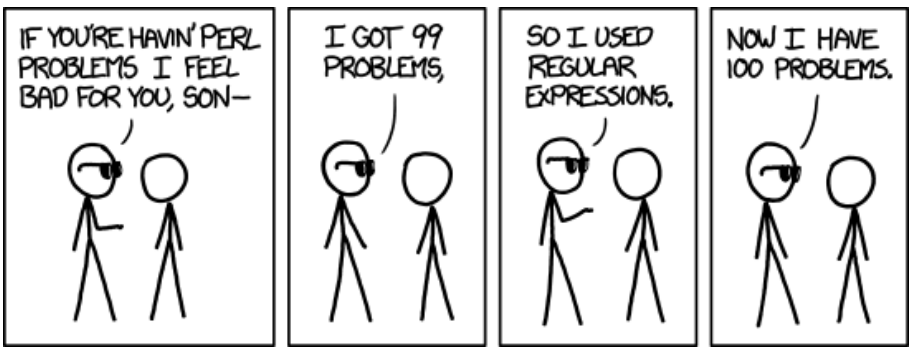
Short answer (circle one): **always true** **sometimes true** **always false**

Proof:

9e. A countably-infinite intersection of context-free languages is regular.
(This is a minor variation of problem 8 on Problem Set 3.)

Short answer (circle one): **always true** **sometimes true** **always false**

Proof:



10. Did you follow all of the instructions on pages 1-3 of this assignment, including the specified file naming convention of your submitted PDF file? (Please note that complying with these instructions is worth 10 points on this assignment. We included this "problem" for credit here since many people didn't read or ignored the instructions on the last homework assignment.)

Short answer (circle one): **yes** **no**