CS3102 – Theory of Computation

Midterm Examination – Spring 2017
University of Virginia

Gabriel Robins

• This is a 6-hour take-home open-book, open notes, pledged exam.
• Note: while for your convenience the “possession time” of this exam is up to 24 hours, the actual “work time” to complete this exam should not exceed 6 contiguous hours.
• No collaborations, no Web searches, nor communications with others are allowed during the exam.
• Do as many of the problems as you can; please explain/prove all answers.
• Shorter algorithms / explanations / proofs are much preferable to longer ones.
• Clearly state the short answer / proof idea first, and then your complete answer / proof.
• Submit only the pages provided (use more sheets only if absolutely necessary).
• Derive answers on scratch paper first, then copy them neatly onto these pages.

During the exam, please feel free to ask clarifying questions using Email; responses will be posted to the class Web page (so please look at the class Web page often during this exam).

When you finish the exam, please slip it under Professor Robins’ office door (406 Rice Hall).

Name:__________________________________________

Problem 1: 20 ________
Problem 2: 20 ________
Problem 3: 20 ________
Problem 4: 20 ________
Problem 5: 20 ________
Problem 6: 20 ________
Problem 7: 30 ________
Total: 150 ________
1) Set cardinalities: Solve problem 2 on problem set 2.

**Short answer (circle one):**

True
False

**Proof:**
2) Regularity: Solve problem 7 on problem set 2.

**Short answer (circle one):**

- **a.** True
- **b.** False

**Proof:**

**Short answer (circle one):**

- **a.** True
- **b.** False

**Proof:**

**Short answer (circle one):**  
Countable  
Not Countable

**Proof:**

**Short answer (circle one):**

- Regular
- Not Regular

**Proof:**

"You want proof? I'll give you proof!"
5) Concatenation: Are there two non-regular languages whose concatenation is regular?

Short answer (circle one): Yes No

Proof:

**Short answer (characterization):**

**Proof:**
Let F denote some finite language, R denote some regular language, C denote some context-free language, and N denote some non-context-free language. For each one of the following statements, prove whether it always holds, sometimes holds, or never holds:

a) RN is regular

Short answer (circle one): always sometimes never

Proof:

b) N – R is regular

Short answer (circle one): always sometimes never

Proof:

c) N \cap F is not regular

Short answer (circle one): always sometimes never

Proof:

"It's too late to correct it," said the Red Queen.
"When you've once said a thing, that fixes it, and you must take the consequences."
d) N – F is regular

Short answer (circle one): always sometimes never

Proof:

e) C* is infinite

Short answer (circle one): always sometimes never

Proof:

f) R is context-free

Short answer (circle one): always sometimes never

Proof:

"Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth."
- Sherlock Holmes (by Sir Arthur Conan Doyle, 1859-1930)