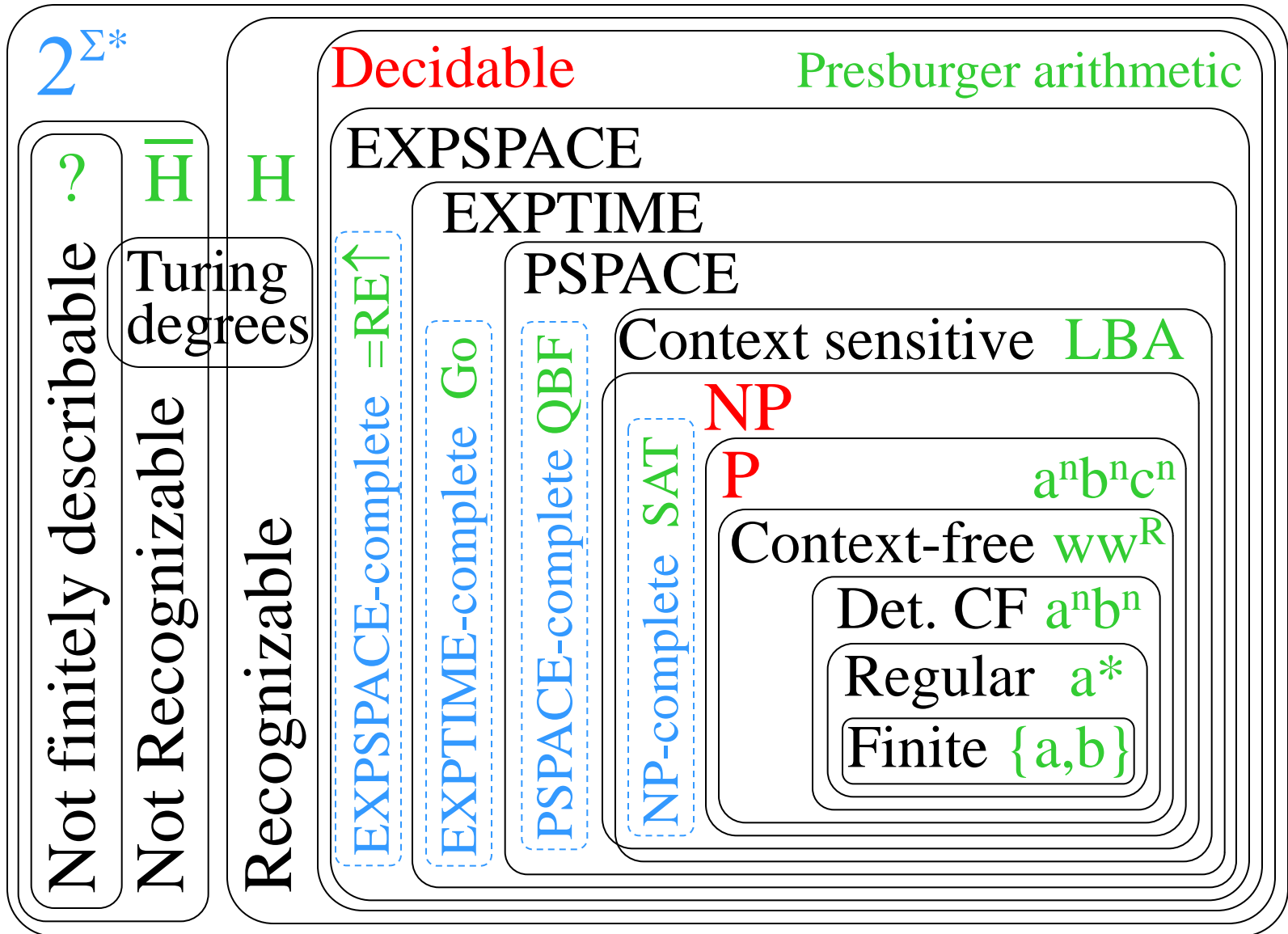


The Extended Chomsky Hierarchy



Computational Universality

Theorem: Many other systems are equivalent to Turing machines.



• Grammars $cS \rightarrow aNbc \mid S$

• λ -calculus $(\lambda X. X + 1)$

• Post tag systems $A \rightarrow bc$

• μ -recursive functions $\mu(f)(x,y) = z$

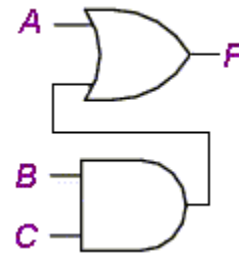
• Cellular automata

• Boolean circuits

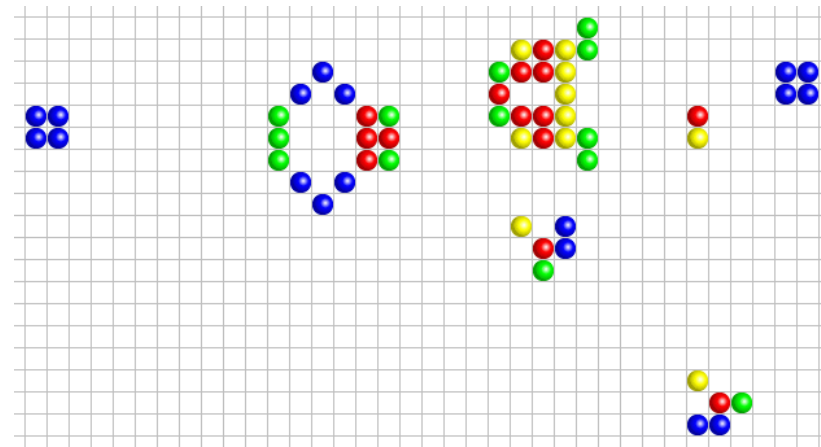
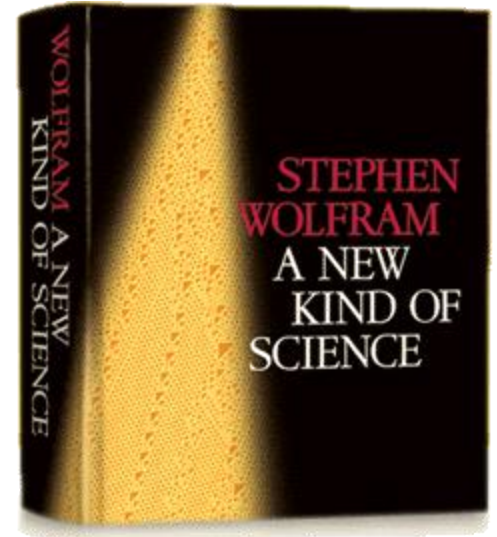
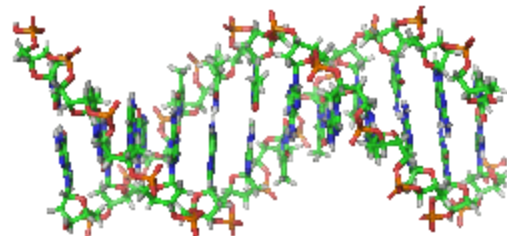
• Diophantine equations

• DNA

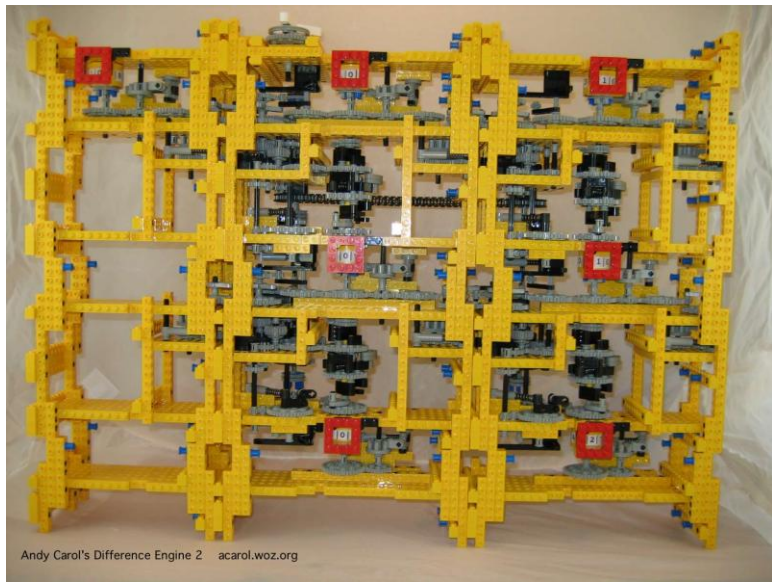
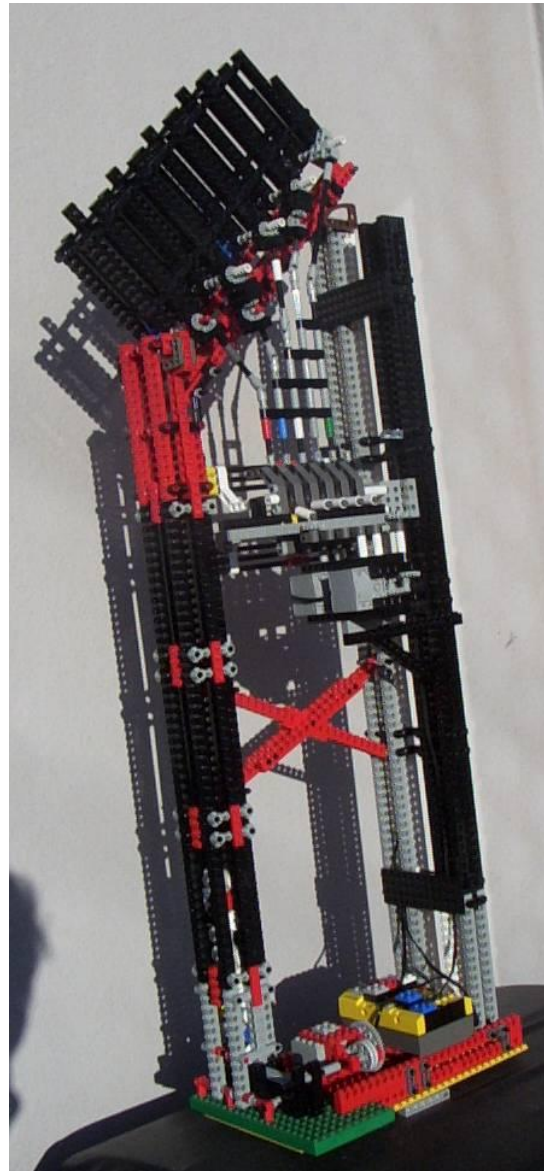
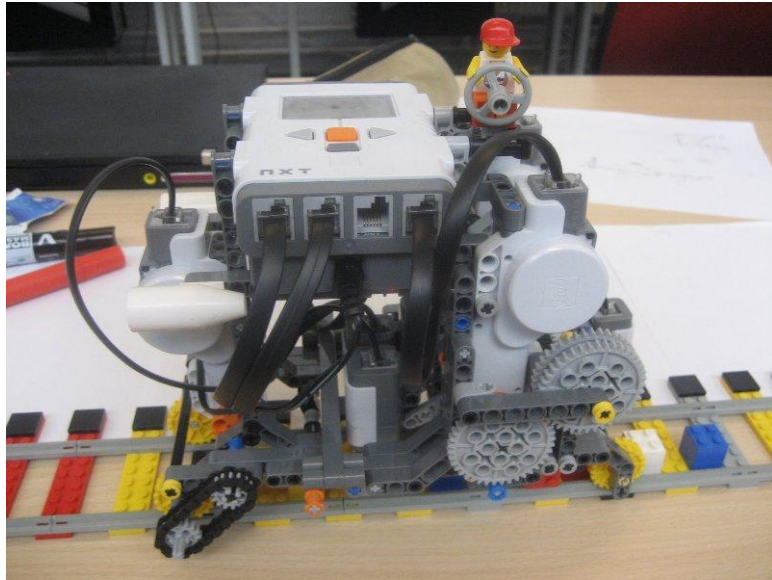
• Billiards!



$$x^3 + y^3 + z^3 = 33$$



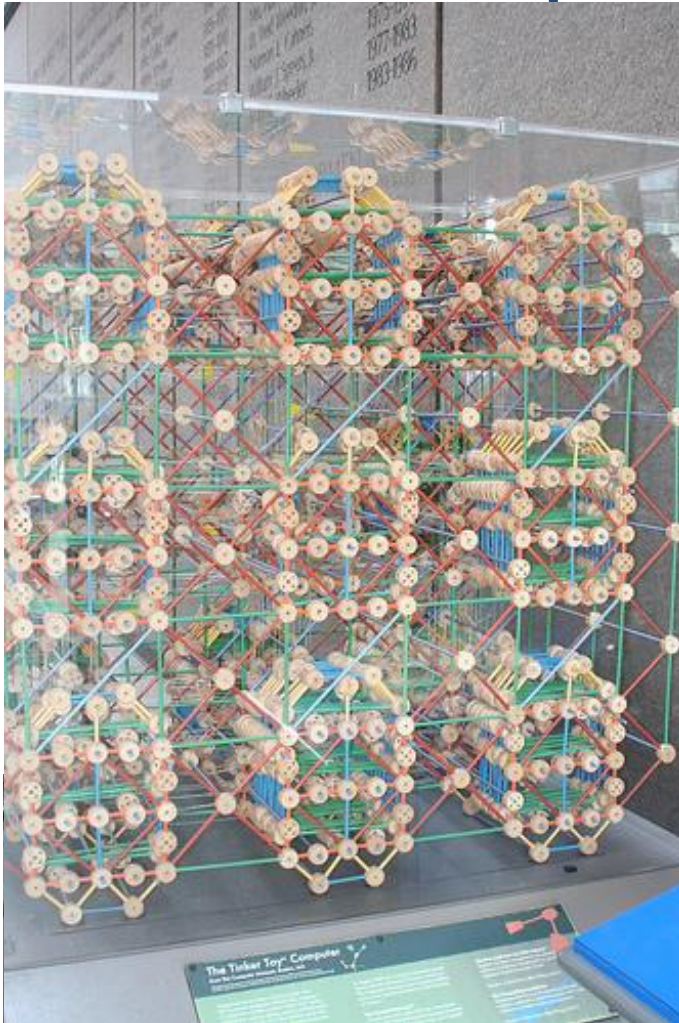
Computational Universality



LEGO Turing machines

Mechano computers

Computational Universality



12 THE PATTERN ON THE STONE

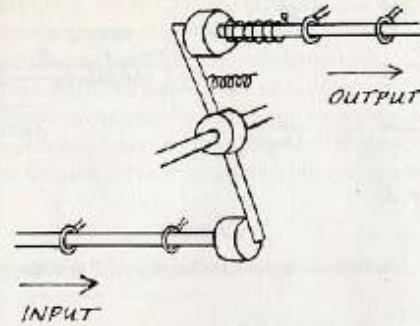
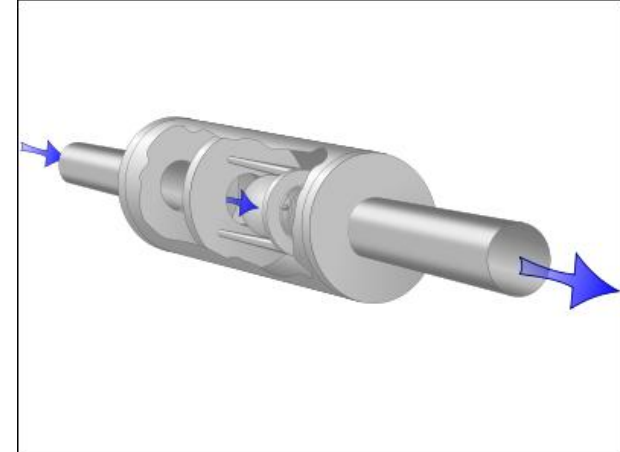
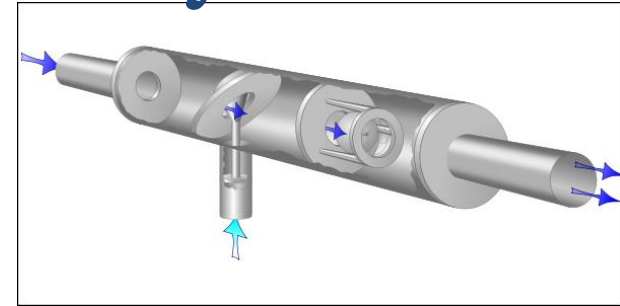


FIGURE 5
Mechanical inverter



NUTS AND BOLTS 11

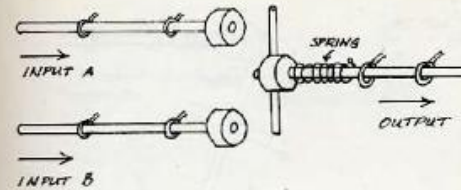
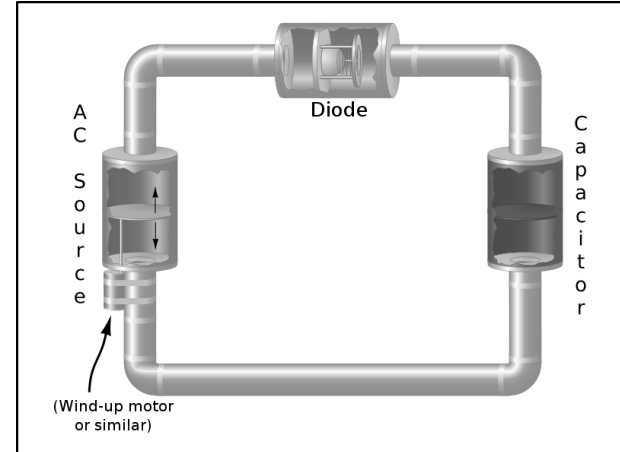
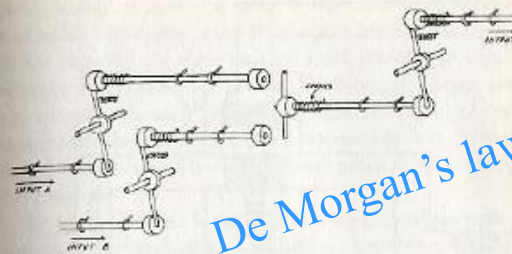


FIGURE 4
Mechanical implementation of the OR function



NUTS AND BOLTS 13



De Morgan's law!

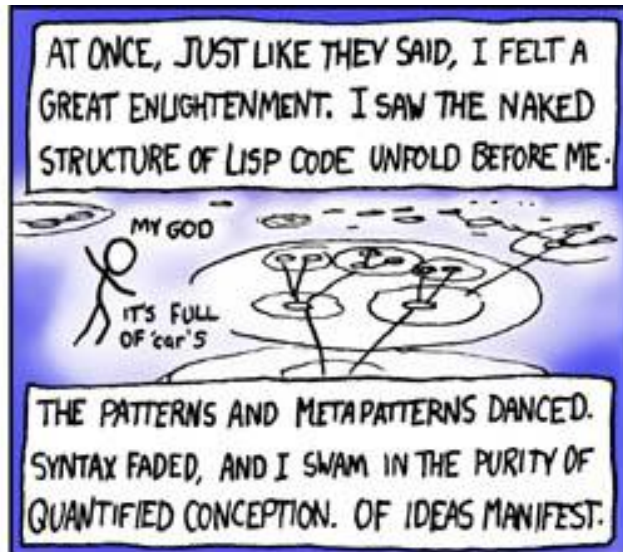
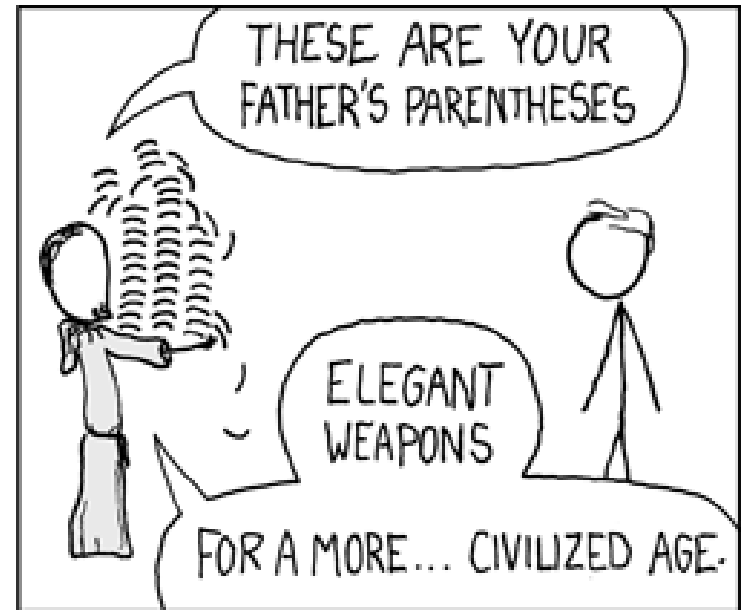
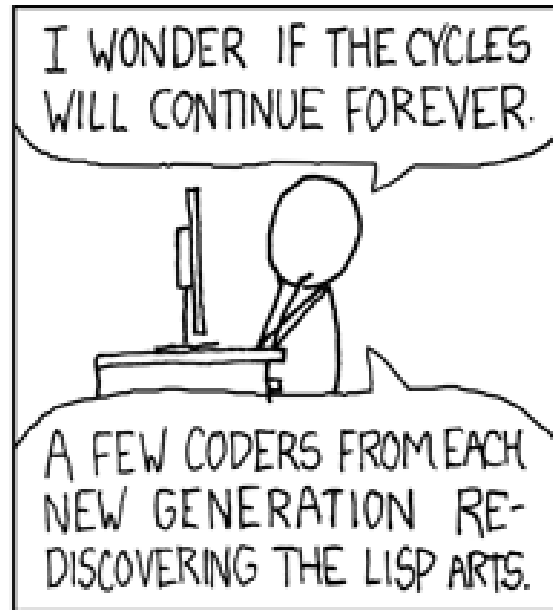
FIGURE 6
An And block constructed by connecting an Or block to inverters

Tinker toy computers

Nuts-and-bolts computers

Hydraulic computers

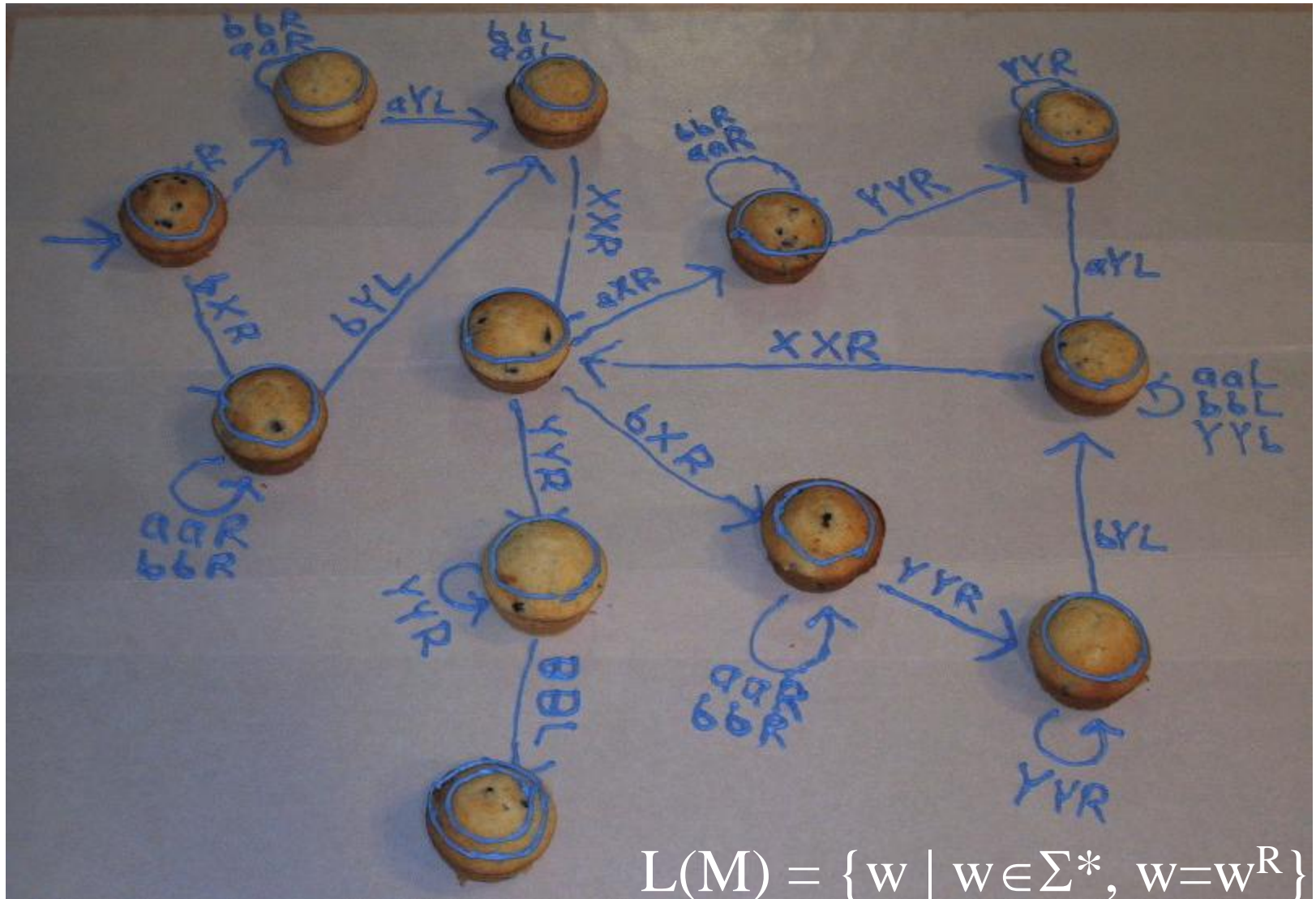
λ -Calculus and LISP



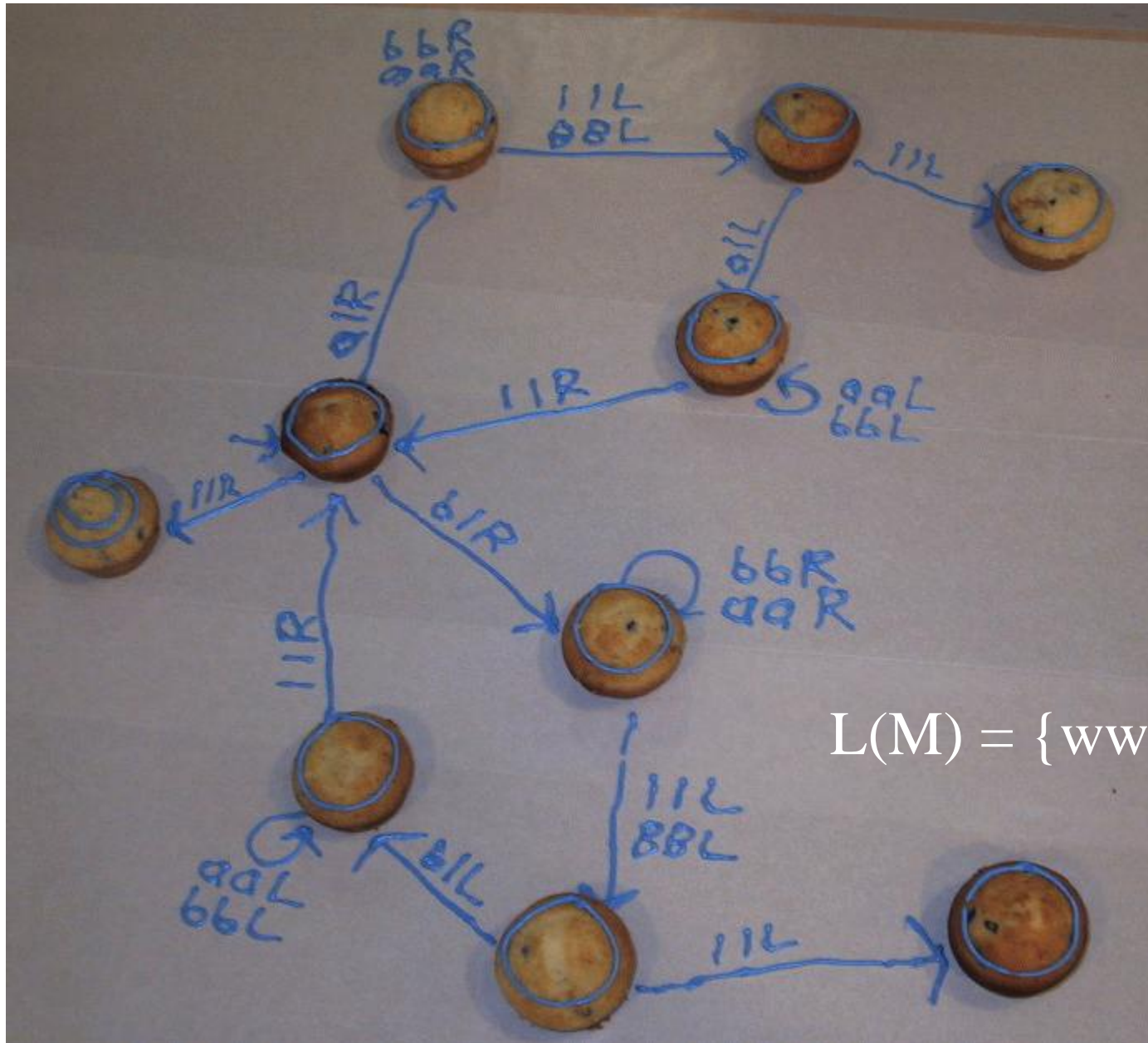
TRULY, THIS WAS THE LANGUAGE FROM WHICH THE GODS WROUGHT THE UNIVERSE.



Blueberry Muffin Turing Machines



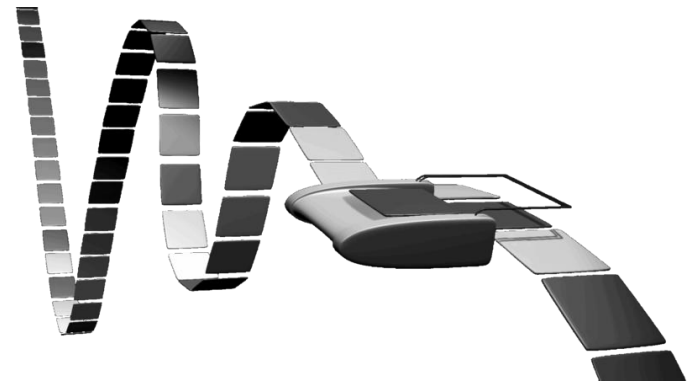
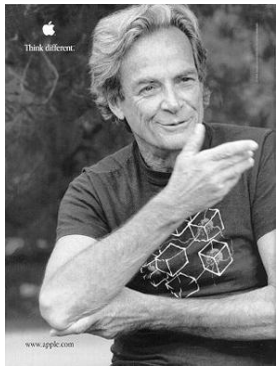
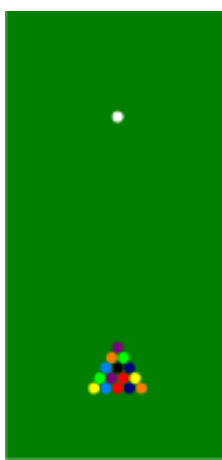
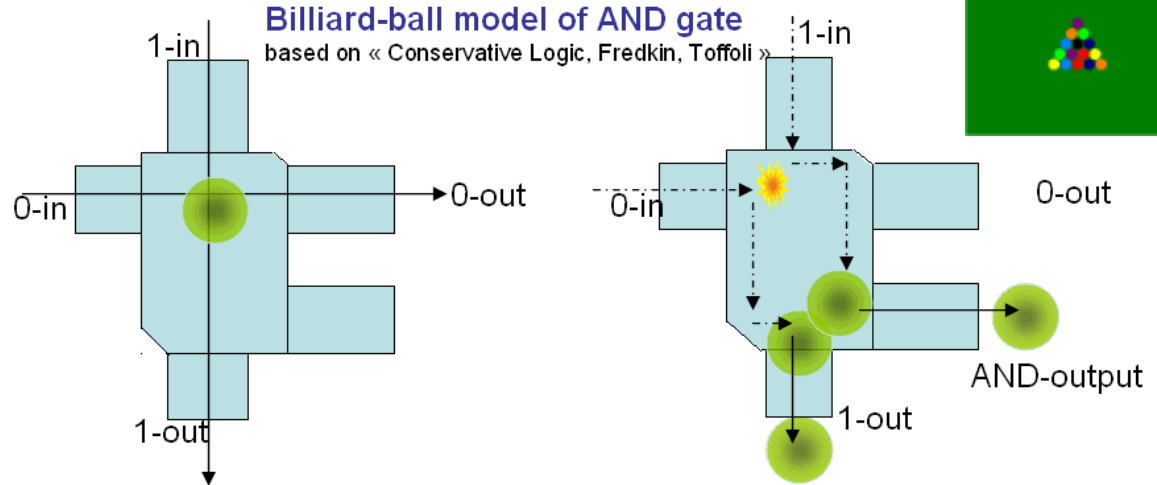
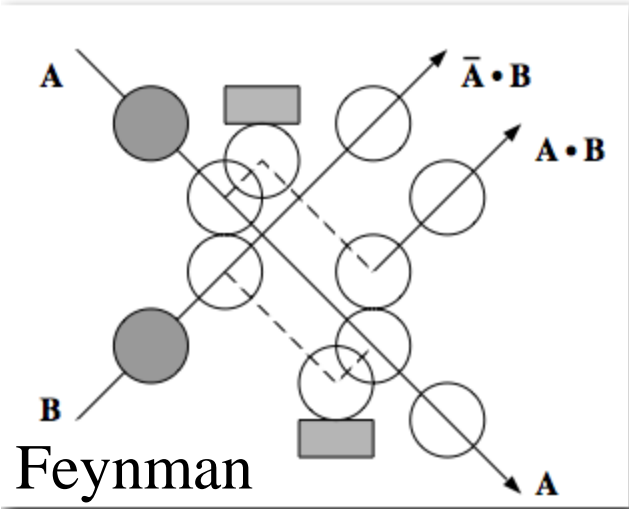
Blueberry Muffin Turing Machines



$$L(M) = \{ww \mid w \in \Sigma^*\}$$

Universality of Billiards

Theorem: Billiards is computationally universal!

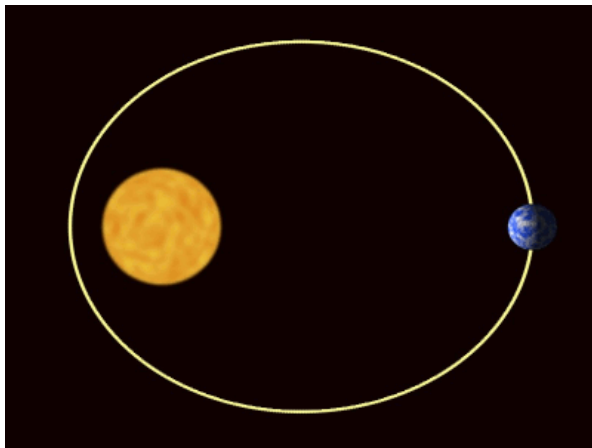


Corollary: Pool is “undecidable”!

Corollary: Newtonian mechanics is universal!

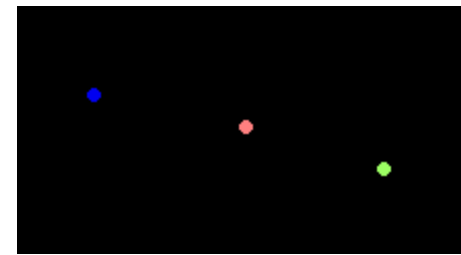
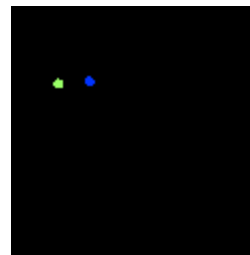
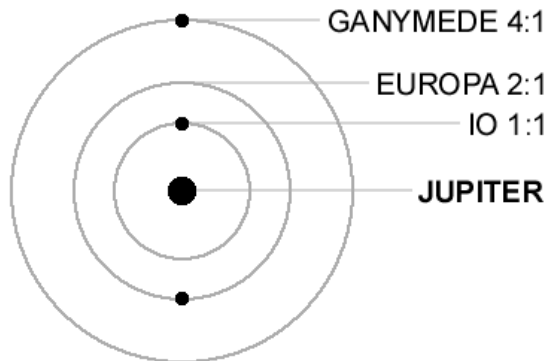
Orbital Mechanics

New solutions to gravitational N-body problems:



Lunar Transit of Earth
NASA's EPOXI Spacecraft

Range to Earth = 31 million miles
Infrared-Green-Blue Color Composite



Observation: Planetary systems are like “3D billiards”.

Theorem: Gravitational systems are chaotic & undecidable!

JOHANNES KEPLER'S UPHILL BATTLE

...SO, YOU SEE, THE ORBIT OF A PLANET IS ELLIPTICAL.

WHAT'S AN ORBIT?

WHAT'S A PLANET?

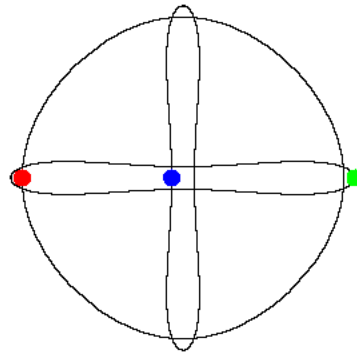
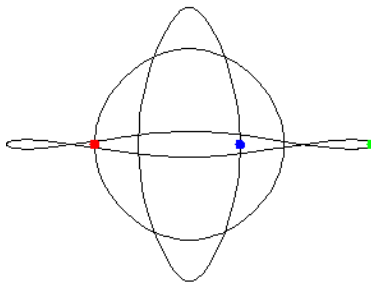
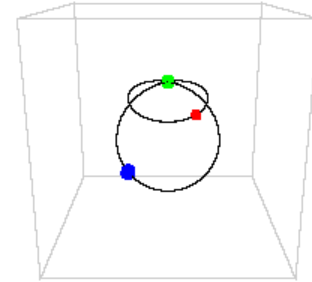
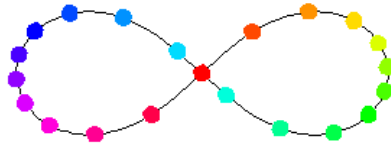
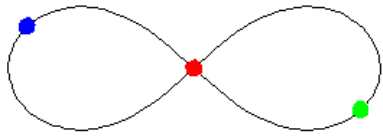
WHAT'S 'ELLIPTICAL'?



J. HARRIS

Orbital Mechanics

New solutions to the gravitational N-body problem:



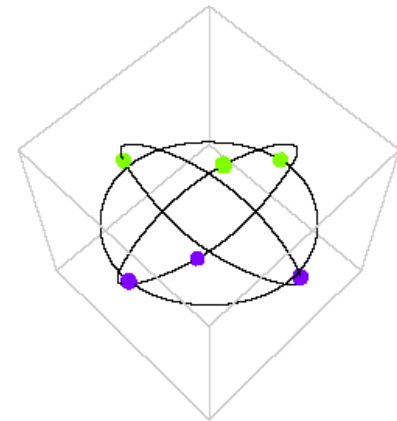
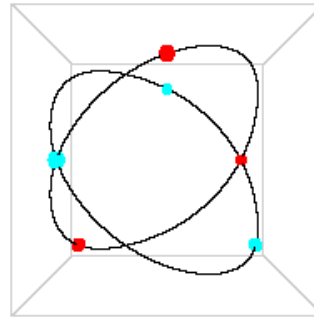
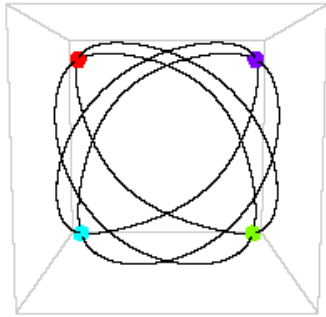
Theorem: These orbits are stable!

Chris Moore: <http://www.santafe.edu/~moore/gallery.html>



Orbital Mechanics

New solutions to the gravitational N-body problem:



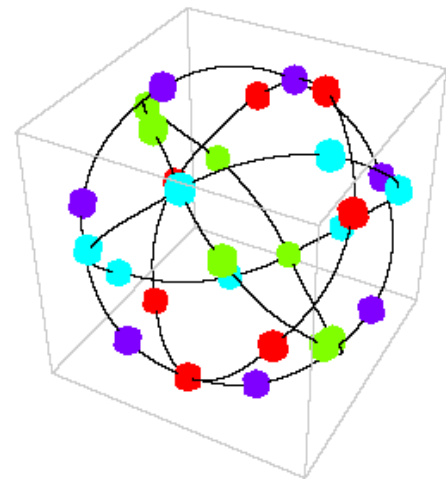
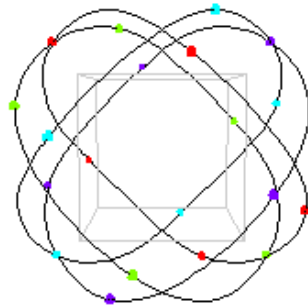
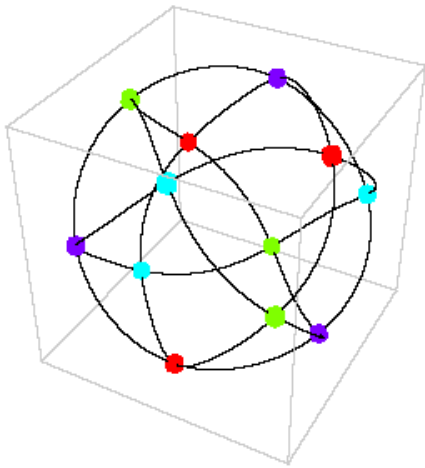
Theorem: These orbits are stable!

Chris Moore: <http://www.santafe.edu/~moore/gallery.html>



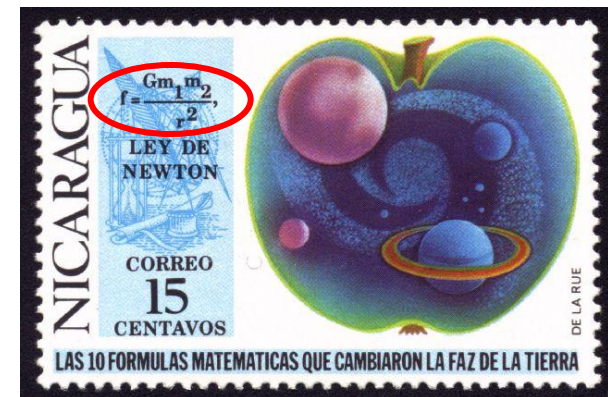
Orbital Mechanics

New solutions to the gravitational N-body problem:



Theorem: These orbits are stable!

Chris Moore: <http://www.santafe.edu/~moore/gallery.html>



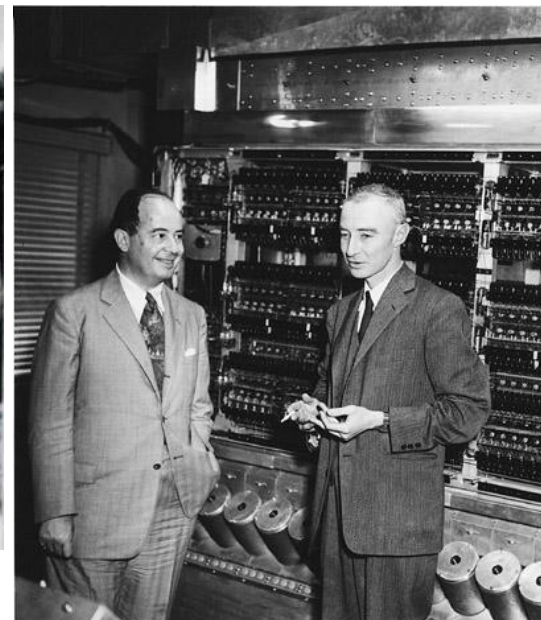
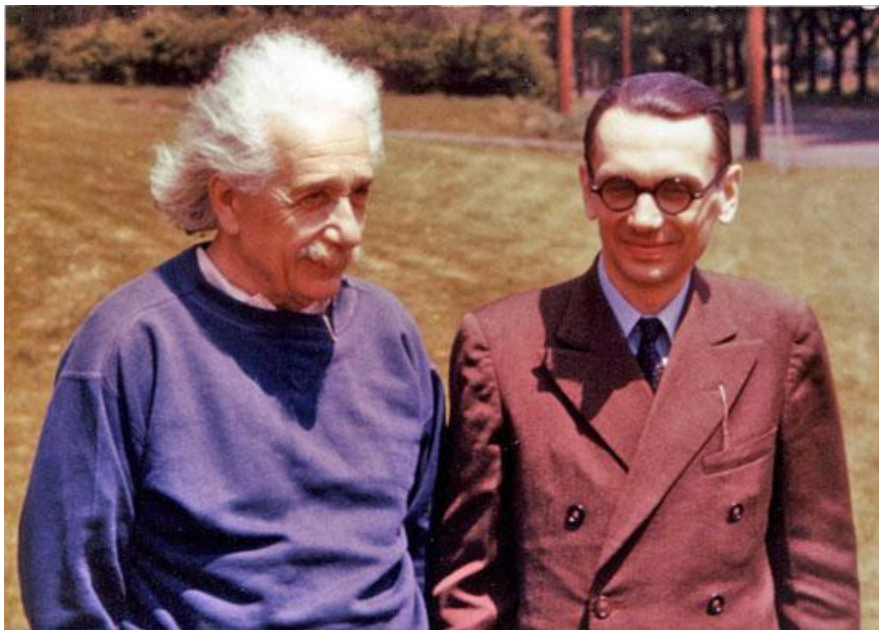
HIGH-GRAVITY BASEBALL



The Church-Turing Thesis



Princeton / Los Alamos / ENIAC

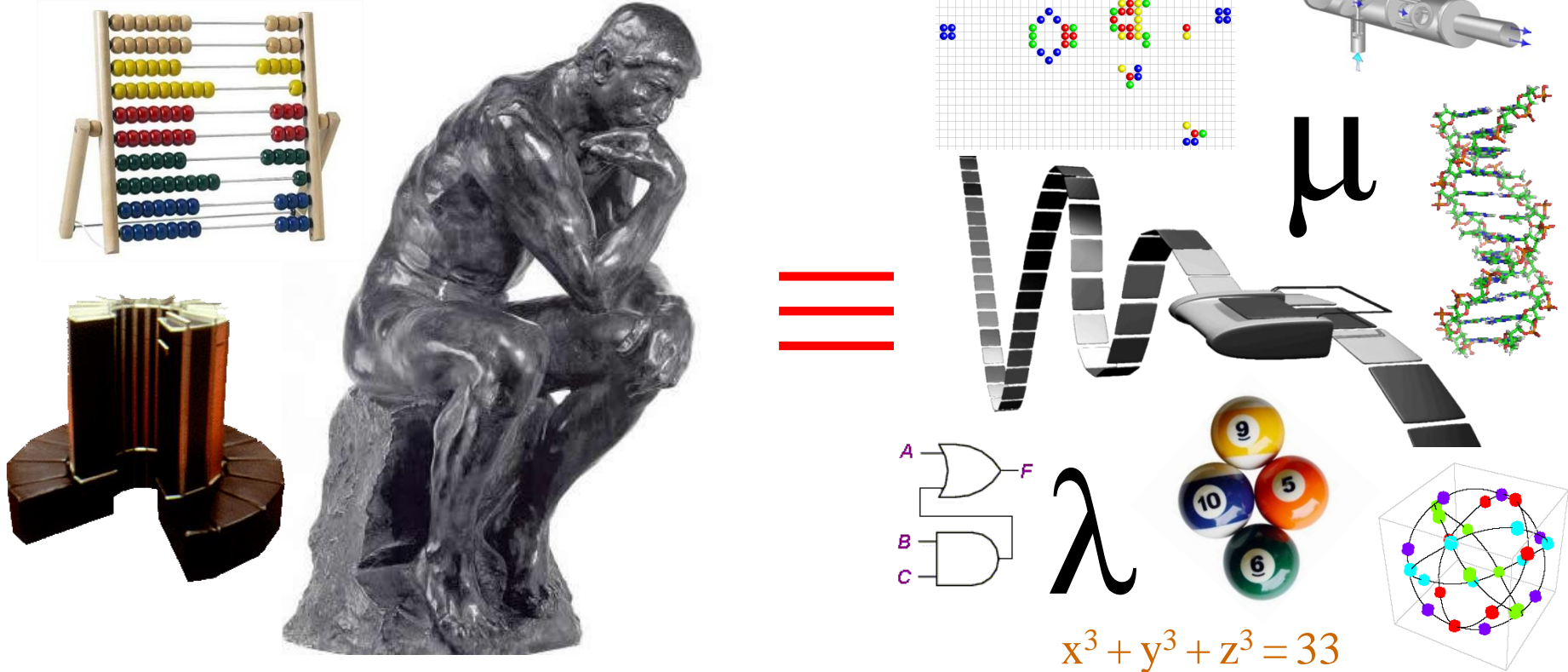


Church - Turing - Gödel - Einstein - von Neumann - Ulam - Oppenheimer - Feynman



The Church-Turing Thesis

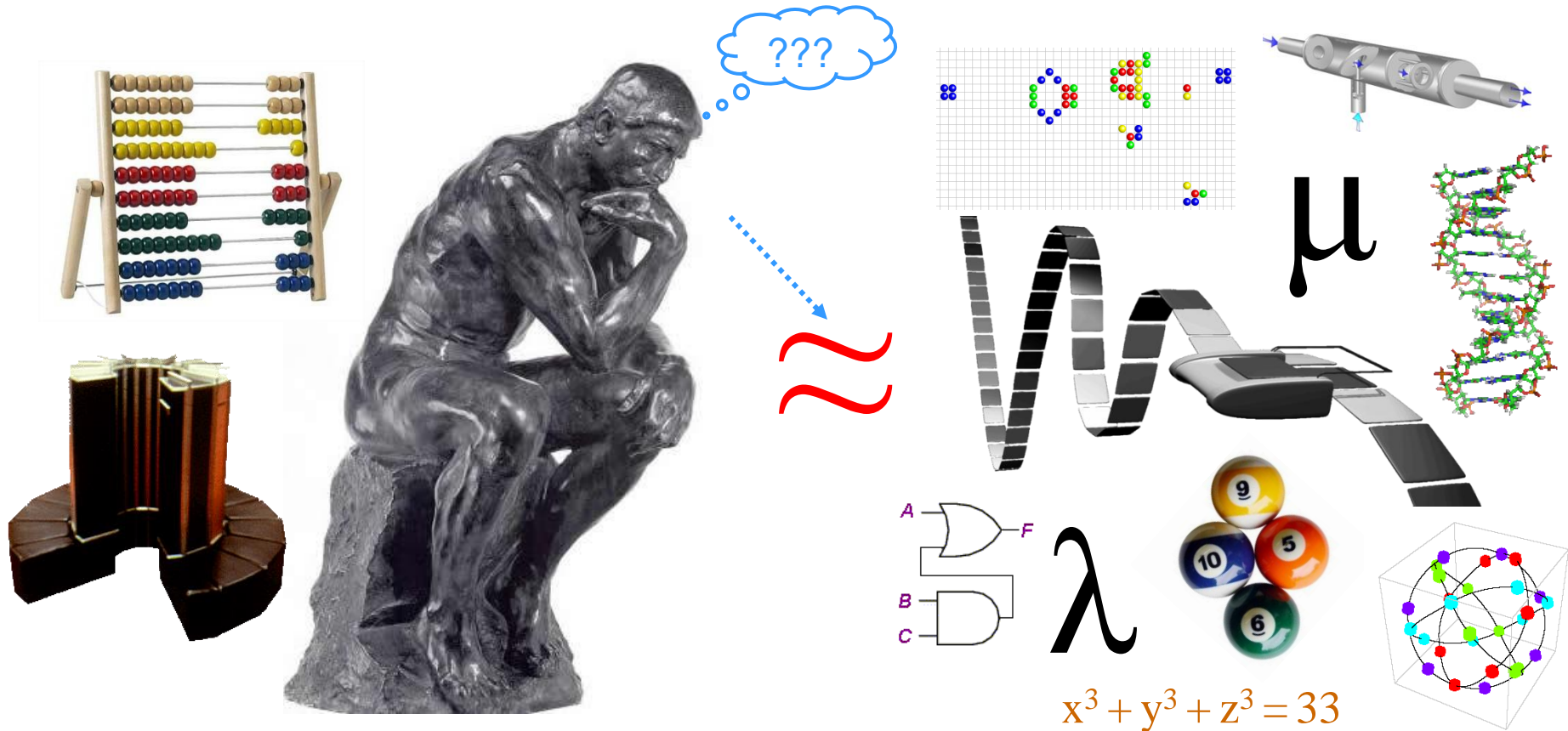
Q: What does it mean “to be computable”?



The Church-Turing Thesis: Anything that is “intuitively computable” is also Turing-machine computable.

The Church-Turing Thesis

Q: Why “thesis” and not “theorem”?



Undefined / informal tasks: produce (or even identify) good music, art, poetry, humor, aesthetics, justice, truth, etc.

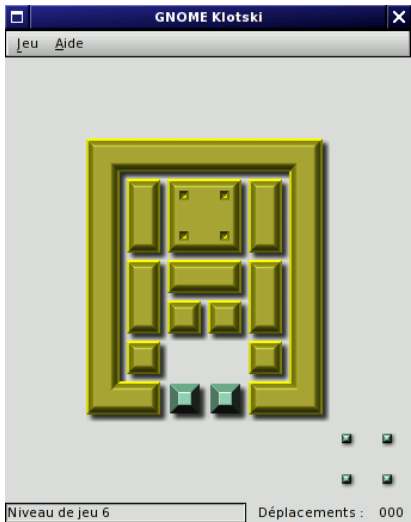
Things We Can Compute

Puzzles:



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

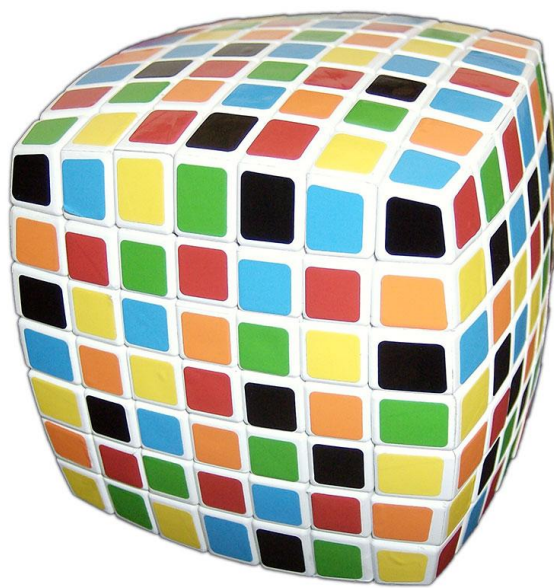
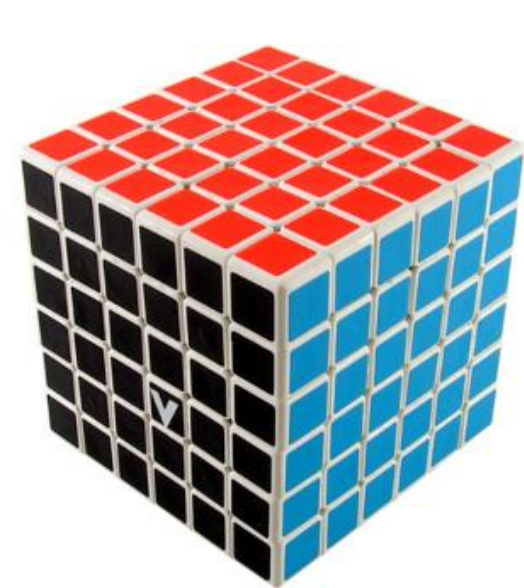
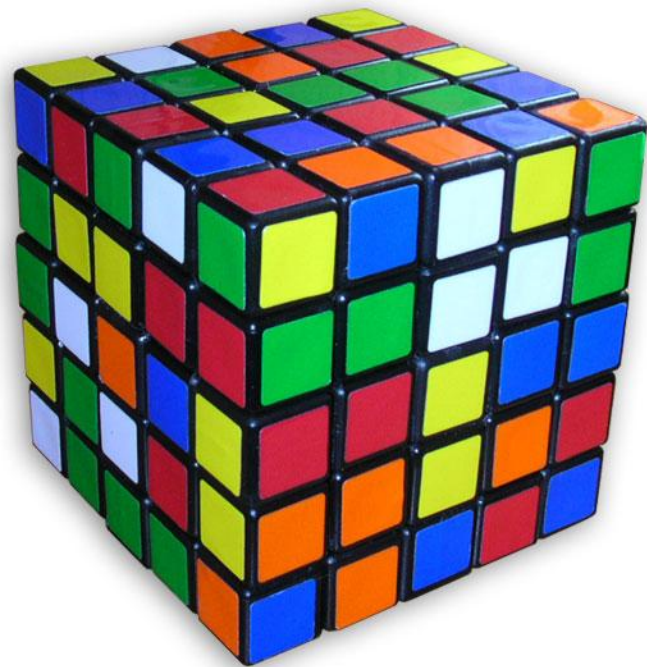
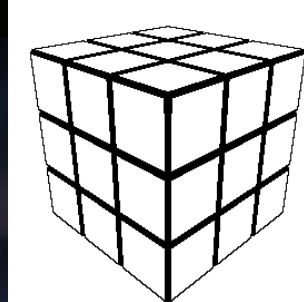
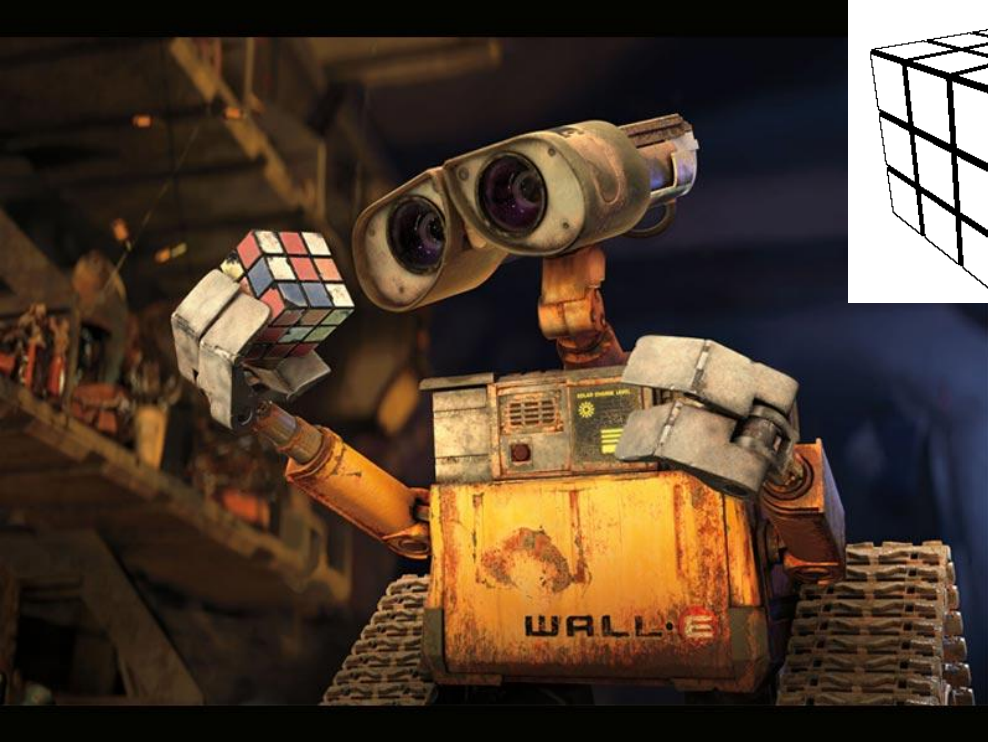
A collection of crossword puzzle grids. The top grid is empty. Below it is a small square grid. To the right is a large grid with numbers indicating starting points for words: 1 1, 1 1 3 3, 3 2 1 3 2 3 5 1 2 3 2 2, 1 2 2 1 1 2 1 6 9 3 2 1 3 2 1 2 2, 2 2 3 3 1 1 2 3 4 1 2 2 7 2 4 2 1 2 1 1.



5	3			7				
6			1	9	5			
	9	8				5	6	
8				6			3	
4		6	8		3	7	1	
7		3		2			6	
	6					2	8	
	8		4	1	9		5	
				8			7	9

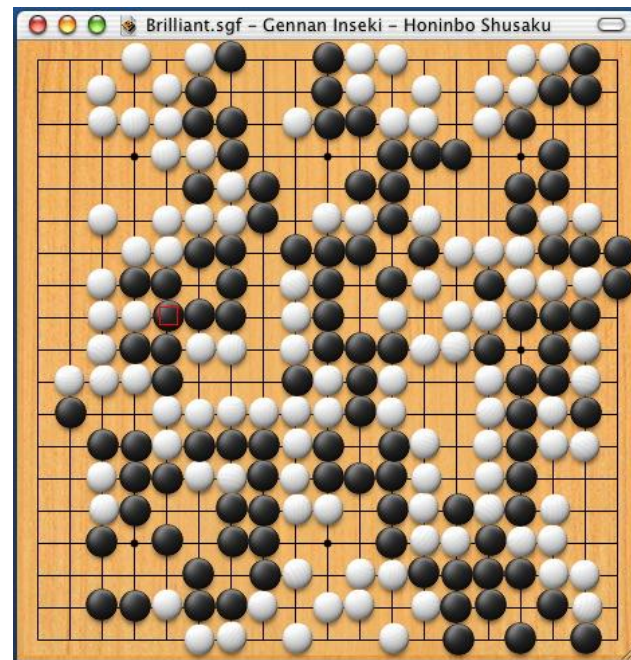
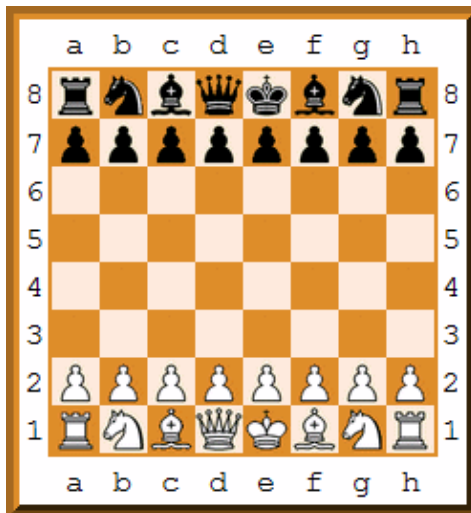
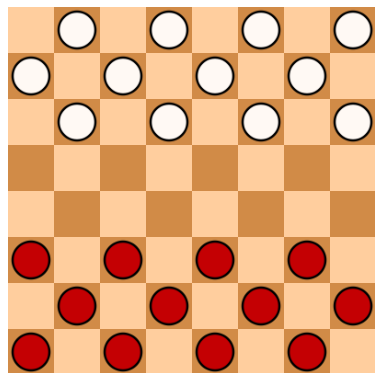
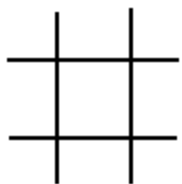


Well-defined (albeit large) discrete solution spaces



Things We Can Compute

Games:

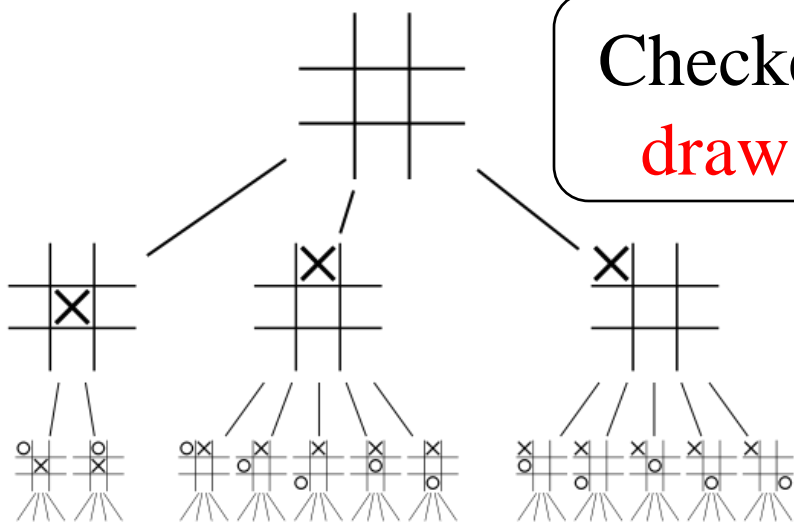


Tree size:
765

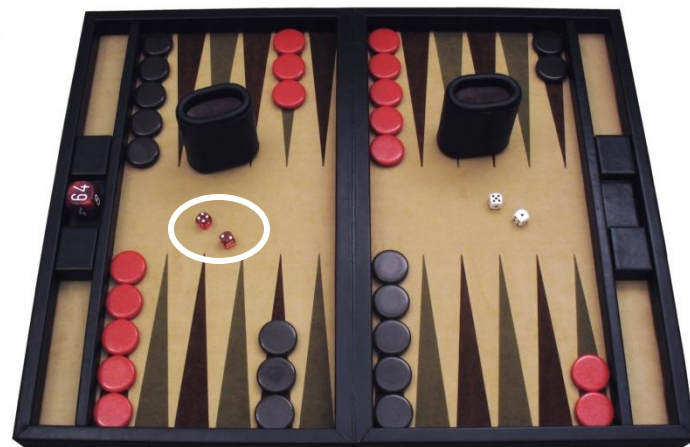
10^{20}

10^{50}

10^{171}

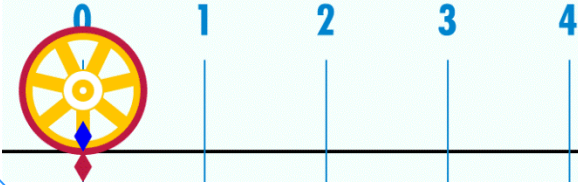


Checkers proven
draw in 2007!



Things We Can Compute

Digits of Pi: $\sum_{n=0}^{\infty} \frac{4(-1)^n}{2n+1} = \pi$



$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} \dots$$

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \dots$$

$$\pi = \sqrt{12} \left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots \right)$$

$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239}$$

$$\pi = 3 + \frac{1^2}{6 + \frac{3^2}{6 + \frac{5^2}{6 + \frac{7^2}{\dots}}}}$$

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \dots$$

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

$$\pi = \sum_{k=0}^{\infty} \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right),$$

$$\frac{426880\sqrt{10005}}{\pi} = \sum_{k=0}^{\infty} \frac{(6k)!(13591409+545140134k)}{(3k)!(k!)^3(-640320)^{3k}}$$

$$\pi = \frac{1}{2^6} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{10n}} \left(-\frac{2^5}{4n+1} - \frac{1}{4n+3} + \frac{2^8}{10n+1} - \frac{2^6}{10n+3} - \frac{2^2}{10n+5} - \frac{2^2}{10n+7} + \frac{1}{10n+9} \right)$$

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230
 664709384460955058223172535940812848111745028410270193852110555964462294895493038196442881097566593344612847564823378
 678316527120190914564856692346034861045432664821339360726024914127372458700660631558817488152092096282925409171536436
 789259036001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274956
 735188575272489122793818301194912983367336244065664308602139494639522473719070217986094370277053921717629317675238467
 481846766940513200056812714526356082778577134275778960917363717872146844090122495343014654958537105079227968925892354
 201995611212902196086403441815981362977477130996051870721134999999837297804995105973173281609631859502445945534690830
 264252230825334468503526193118817101000313783875288658753320838142061717766914730359825349042875546873115956286388235
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 530185296899577362259941389124972177528347913151557485724245415069595082953311686172785588907509838175463746493931925
 506040092770167113900984882401285836160356370766010471018194295559619894676783744944825537977472684710404753464620804
 66842590694912 ...

Things We Can Compute

Prime numbers:

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Prime numbers

Theorems:

\exists an infinity of primes

\exists # primes $\leq n \rightarrow n / \log_e n$

\exists arbitrarily large prime gaps

Open problems:

\exists an infinity of prime pairs? (i.e., p & $p+2$) ?

Goldbach's conjecture (verified for all $n < 10^{18}$):

every even integer > 2 is the sum of two primes ?

Largest known prime: $2^{43,112,609} - 1$ (12,978,189 digits)



HOLD ON THERE, MR. WEBSTER. 1677 ISN'T PRIME - IT'S DIVISIBLE BY 43.

53

223

547

1033

1677

AA

1677

1677

1677

is

Things We Can Compute

More prime numbers theorems:

No polynomial yields **only** primes.

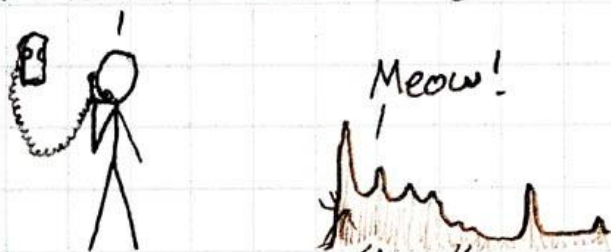
N^2+n+41 yields **40 consecutive primes** for $0 \leq n \leq 39$.

The set of **primes coincides exactly** with the positive values of the following 26-variable polynomial:

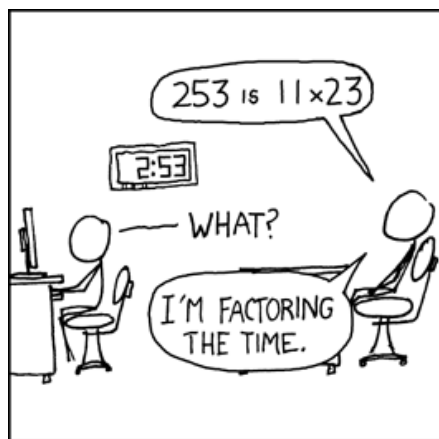
$$(k+2)(1 - [wz + h + j - q]^2 - [(gk + 2g + k + 1)(h + j) + h - z]^2 - [16(k+1)^3(k+2)(n+1)^2 + 1 - f^2]^2 - [2n + p + q + z - e]^2 - [e^3(e+2)(a+1)^2 + 1 - o^2]^2 - [(a^2 - 1)y^2 + 1 - x^2]^2 - [16r^2y^4(a^2 - 1) + 1 - u^2]^2 - [n + l + v - y]^2 - [(a^2 - 1)l^2 + 1 - m^2]^2 - [ai + k + 1 - l - i]^2 - [((a + u^2(u^2 - a))^2 - 1)(n + 4dy)^2 + 1 - (x + cu)^2]^2 - [p + l(a - n - 1) + b(2an + 2a - n^2 - 2n - 2) - m]^2 - [q + y(a - p - 1) + s(2ap + 2a - p^2 - 2p - 2) - x]^2 - [z + pl(a - p) + t(2ap - p^2 - 1) - pm]^2)$$

as a, b, c, \dots, z range over the nonnegative integers!

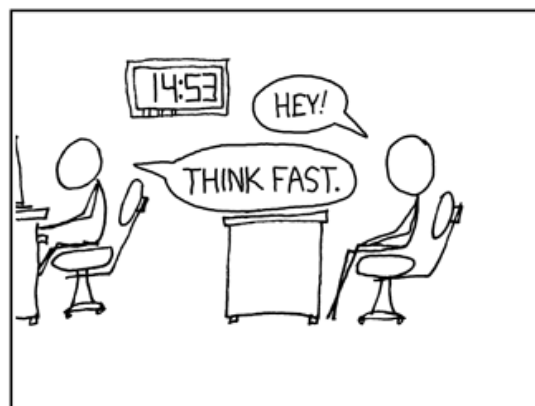
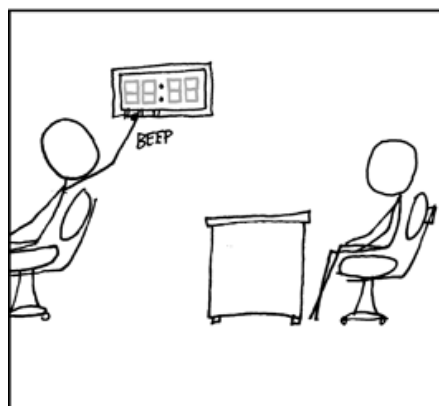
Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took
the Fourier transform of my cat...



$\pi = 3.14159265$
3589793help
imtrappedin
a universe fac
tory7108914...



I HAVE NOTHING TO DO, SO I'M TRYING
TO CALCULATE THE PRIME FACTORS OF THE
TIME EACH MINUTE BEFORE IT CHANGES.
IT WAS EASY WHEN I
STARTED AT 1:00, BUT
WITH EACH HOUR THE
NUMBER GETS BIGGER.
I WONDER HOW
LONG I CAN KEEP UP.



HEY, CHECK IT OUT: $e^\pi - \pi$ IS
19.999099979. THAT'S WEIRD.

YEAH. THAT'S HOW I
GOT KICKED OUT OF
THE ACM IN COLLEGE.

... WHAT?



DURING A COMPETITION, I
TOLD THE PROGRAMMERS ON
OUR TEAM THAT $e^\pi - \pi$
WAS A STANDARD TEST OF FLOATING-
POINT HANDLERS -- IT WOULD
COME OUT TO 20 UNLESS
THEY HAD ROUNDING ERRORS.



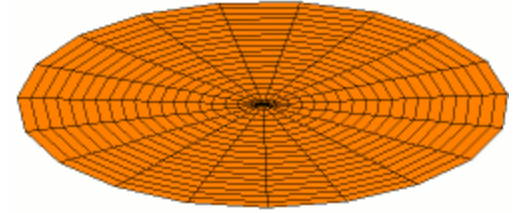
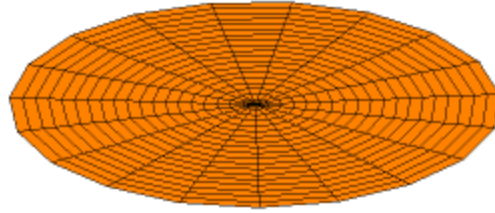
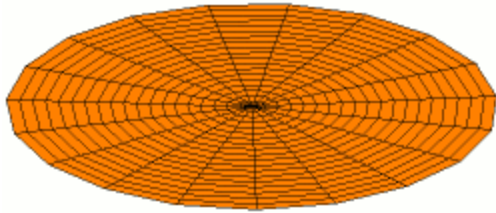
THAT'S
AWFUL.

YEAH, THEY DUG THROUGH
HALF THEIR ALGORITHMS
LOOKING FOR THE BUG
BEFORE THEY FIGURED
IT OUT.

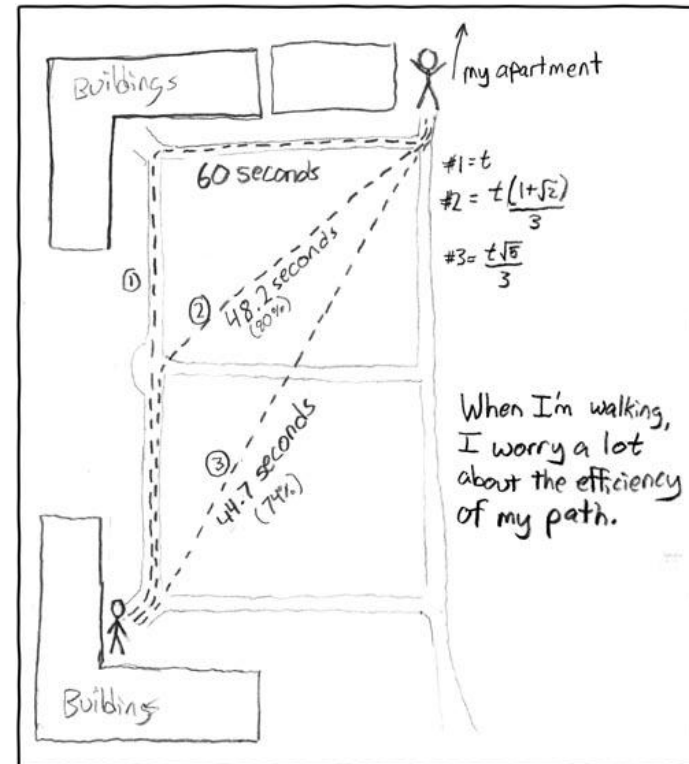
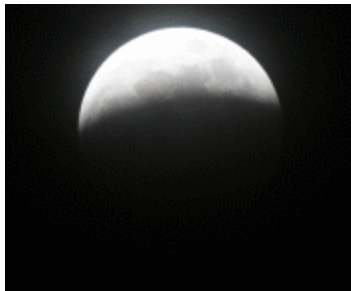


Things We Can Compute

Harmonics:

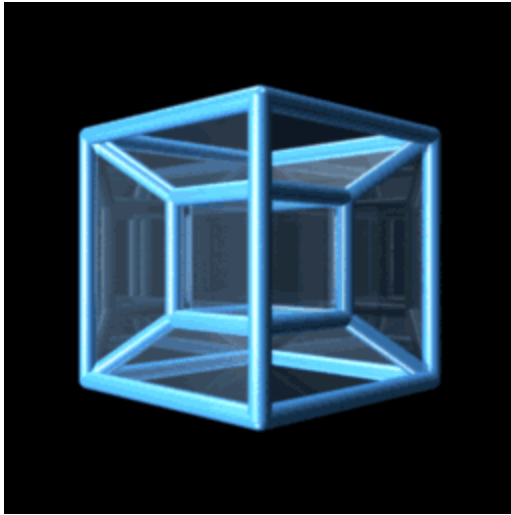


Eclipses:

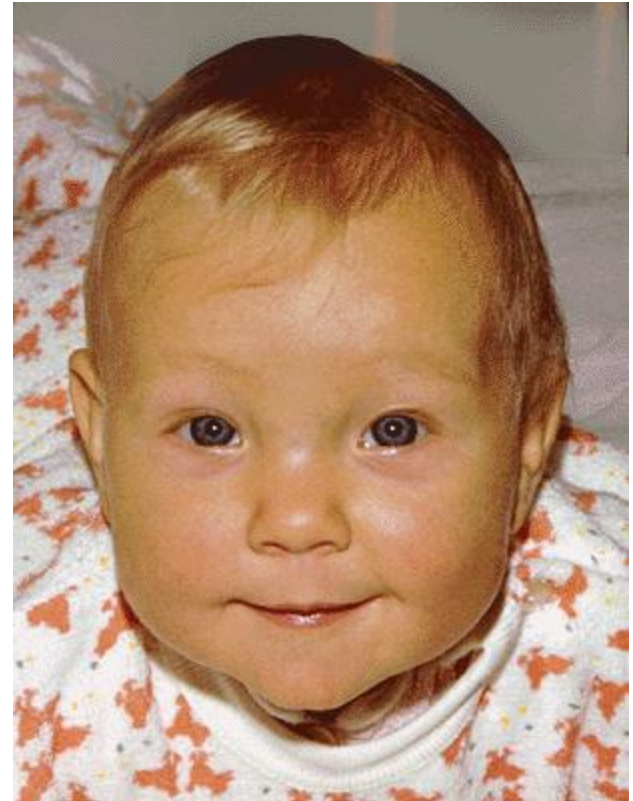


Things We Can Compute

Visualization:

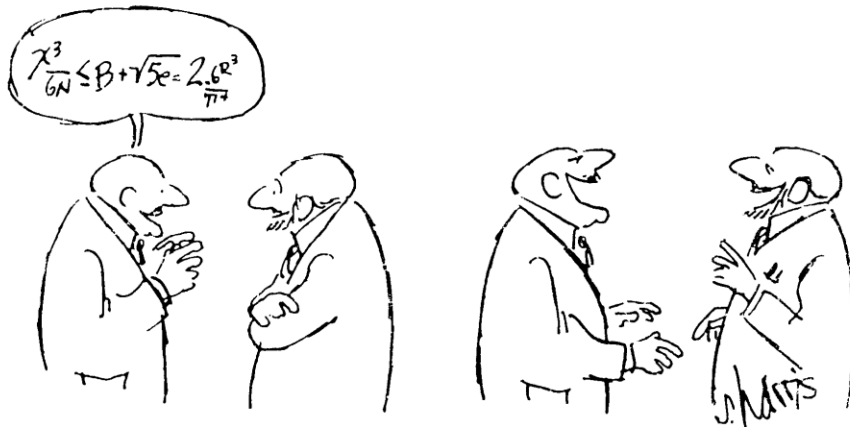
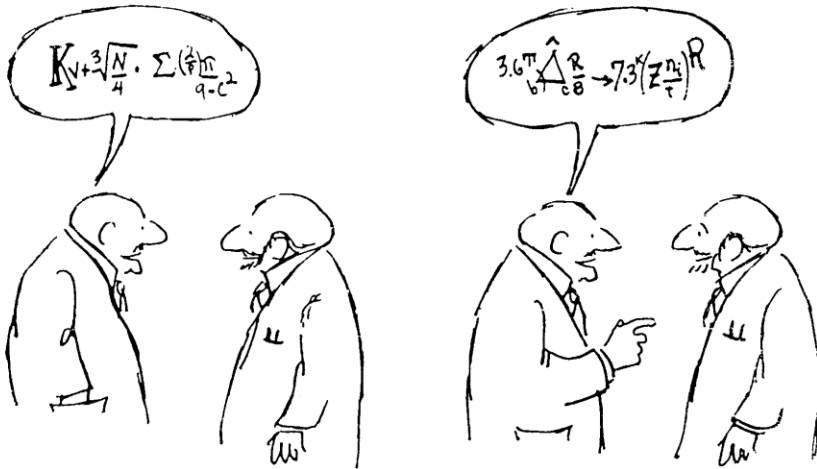


Morphing:



What Can't We Compute and Why?

Humor:



© 1973 The New Yorker Magazine, Inc.

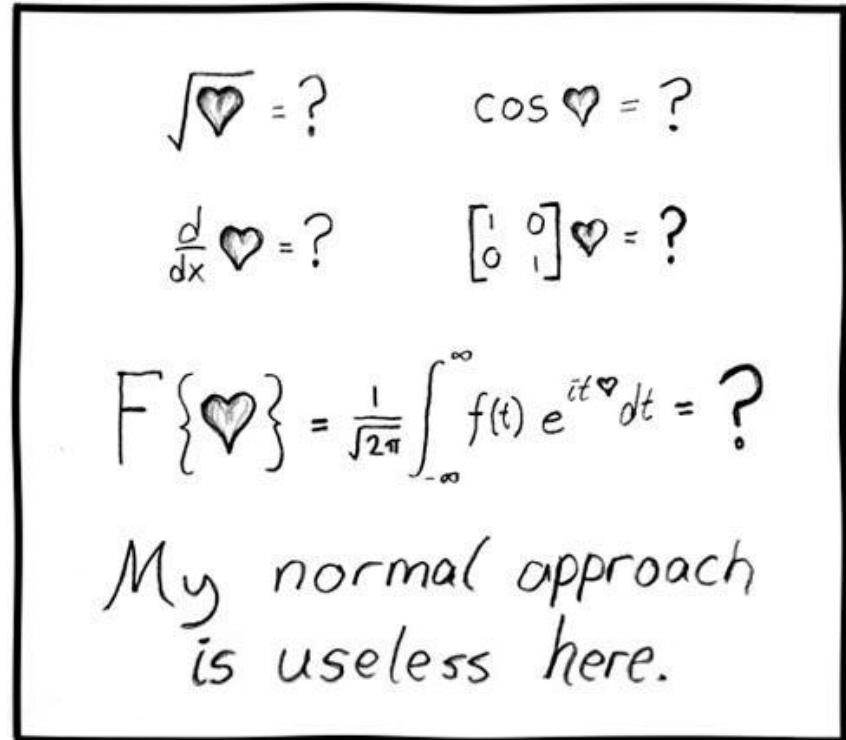


"THERE ARE ESSENTIALLY FOUR BASIC FORMS FOR A JOKE — THE CONCEALING OF KNOWLEDGE LATER REVEALED, THE SUBSTITUTION OF ONE CONCEPT FOR ANOTHER, AN UNEXPECTED CONCLUSION TO A LOGICAL PROGRESSION AND SLIPPING ON A BANANA PEEL."

Issues: not well-defined, subjective, ambiguous

What Can't We Compute and Why?

Emotions:

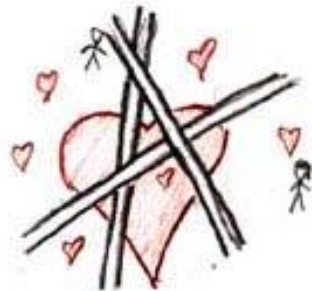


Issues: not well-defined, subjective, ambiguous



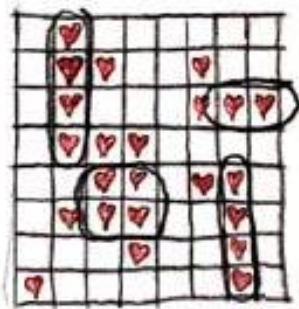
You make me feel so much
it all runs together
I wish I could tell you

So few words
for so many feelings
Crisscrossing my heart



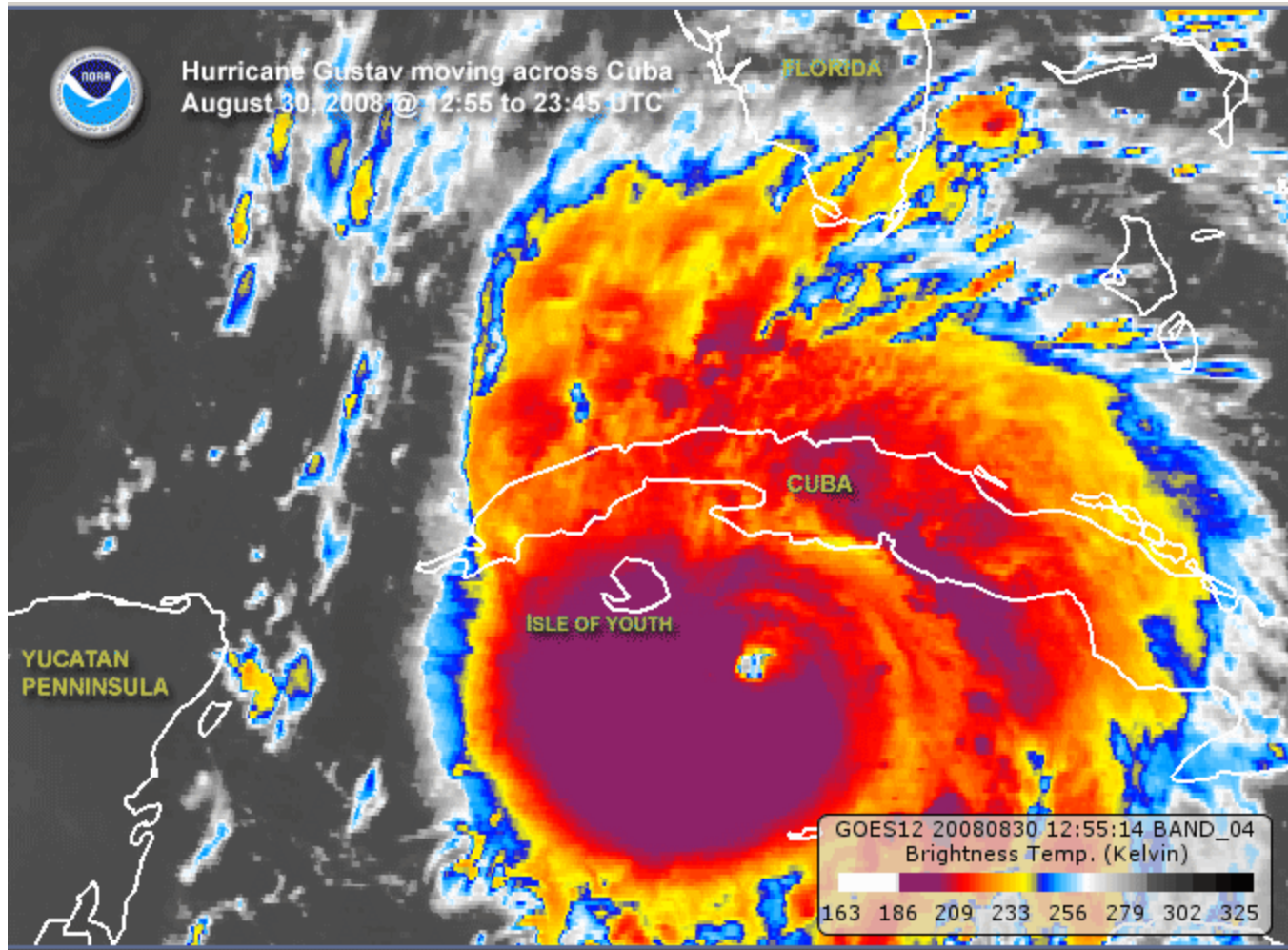
A matrix of desire
Tangled relations
I can't simplify

I wish I could find
the Karnaugh map
for love.



What Can't We Compute and Why?

Weather:



Issues: chaos, insufficient data, undecidability

What Can't We Compute and Why?

Tsunamis:

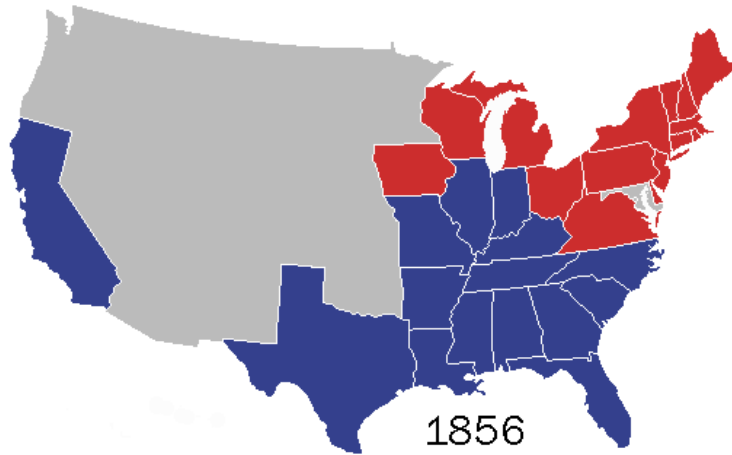


Dec 2004 trunami, 225,000 dead
Energy: 9.5 teratons, 100-ft waves

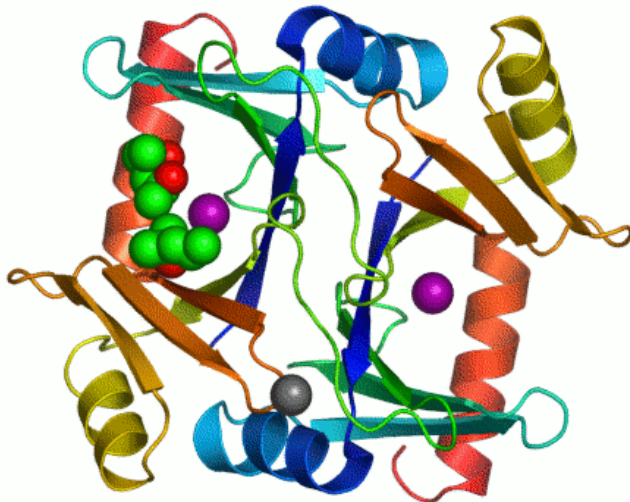
Issues: chaos, insufficient data, undecidability

What Can't We Compute and Why?

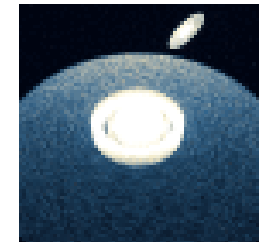
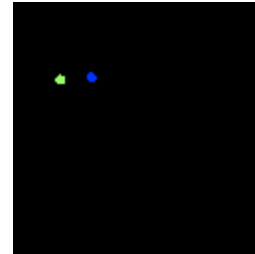
Elections:



Protein folding:

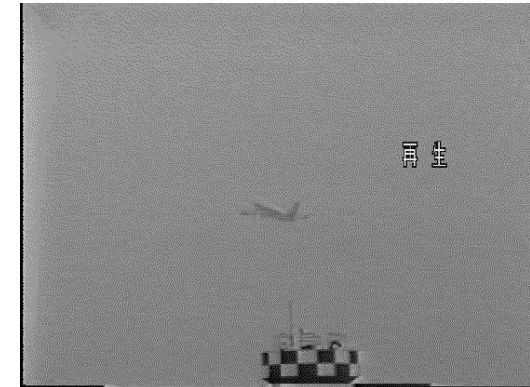


N-Body systems:



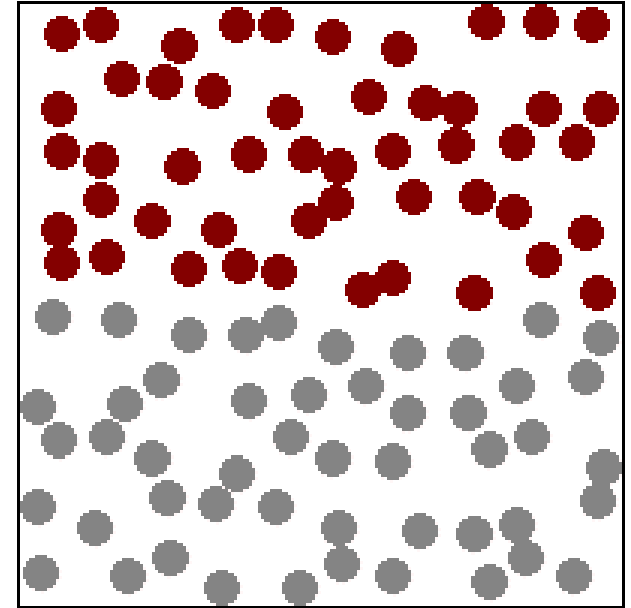
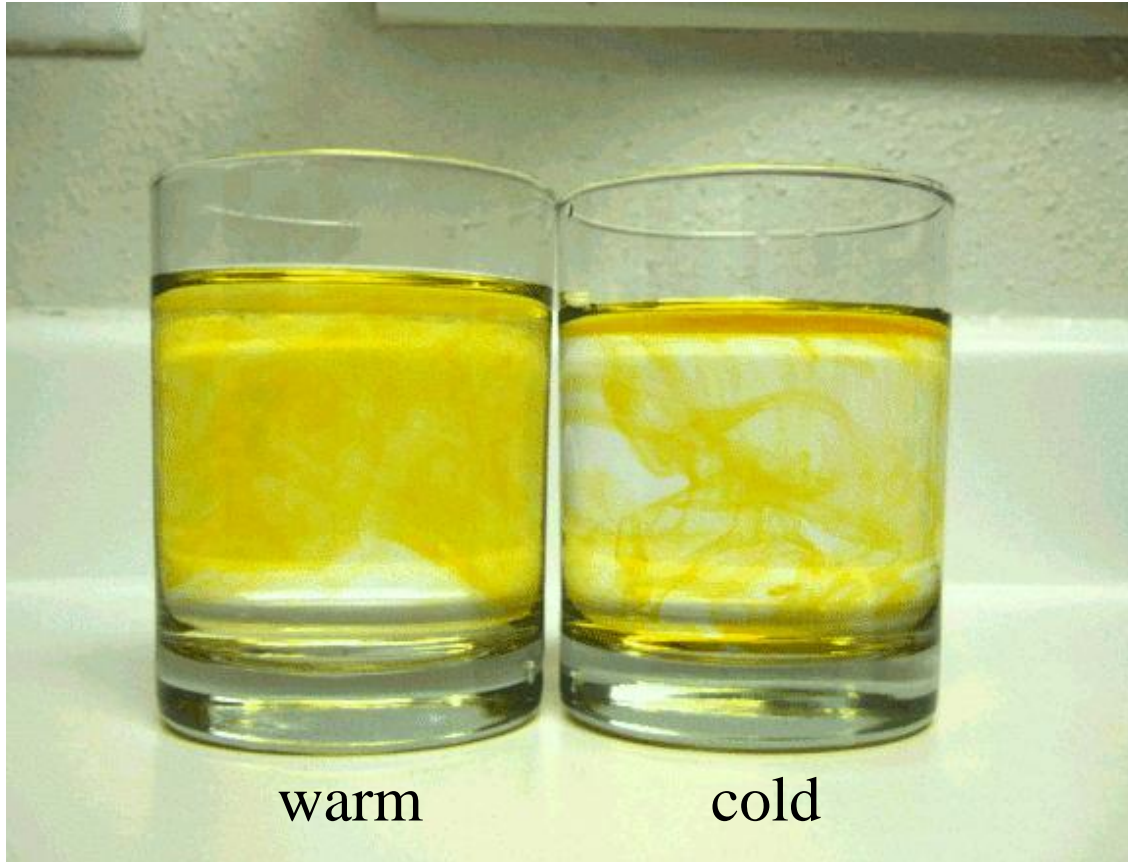
Galactic collisions

Lightning:



What Can't We Compute and Why?

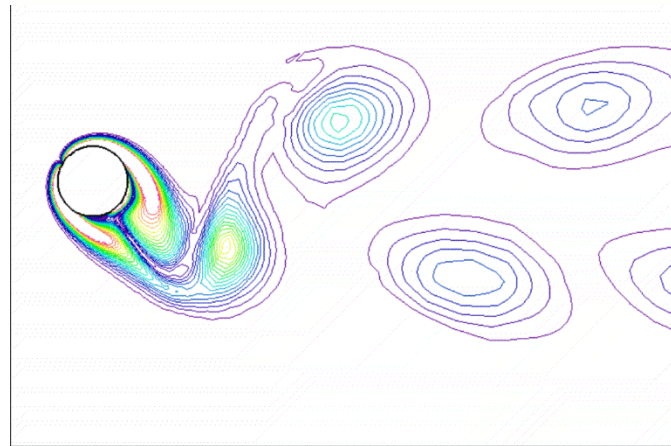
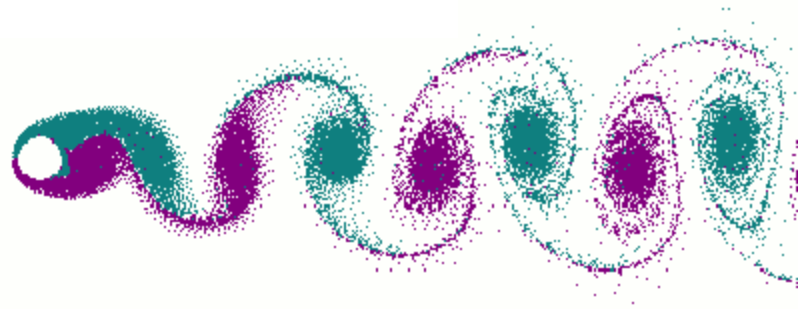
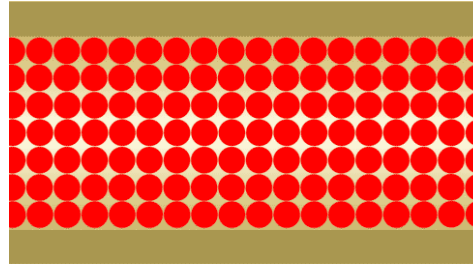
Diffusion:



Issues: chaos, insufficient data, undecidability

What Can't We Compute and Why?

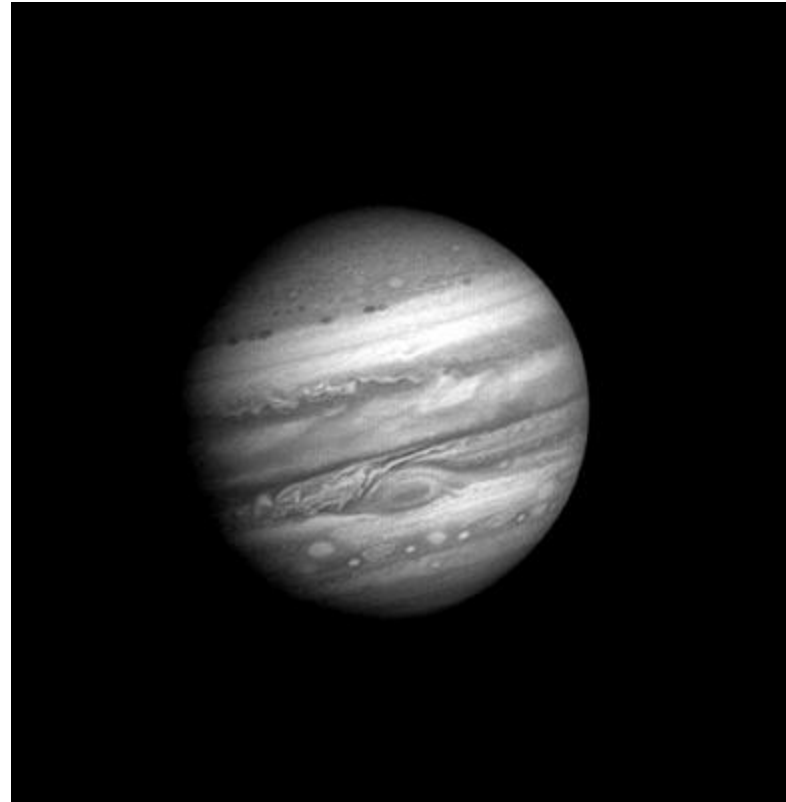
Turbulence:



Issues: chaos, insufficient data, undecidability

What Can't We Compute and Why?

Turbulence:



Issues: chaos, insufficient data, undecidability

The Euler and Navier–Stokes equations describe the motion of a fluid in \mathbb{R}^n ($n = 2$ or 3). These equations are to be solved for an unknown velocity vector $u(x, t) = (u_i(x, t))_{1 \leq i \leq n} \in \mathbb{R}^n$ and pressure $p(x, t) \in \mathbb{R}$, defined for position $x \in \mathbb{R}^n$ and time $t \geq 0$. We restrict attention here to incompressible fluids filling all of \mathbb{R}^n . The *Navier–Stokes* equations are then given by

$$(1) \quad \frac{\partial}{\partial t} u_i + \sum_{j=1}^n u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x, t) \quad (x \in \mathbb{R}^n, t \geq 0),$$

$$(2) \quad \operatorname{div} u = \sum_{i=1}^n \frac{\partial u_i}{\partial x_i} = 0 \quad (x \in \mathbb{R}^n, t \geq 0)$$

with initial conditions

$$(3) \quad u(x, 0) = u^\circ(x) \quad (x \in \mathbb{R}^n).$$

Here, $u^\circ(x)$ is a given, C^∞ divergence-free vector field on \mathbb{R}^n , $f_i(x, t)$ are the components of a given, externally applied force (e.g. gravity), ν is a positive coefficient (the viscosity), and $\Delta = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$ is the Laplacian in the space variables. The *Euler equations* are equations (1), (2), (3) with ν set equal to zero.

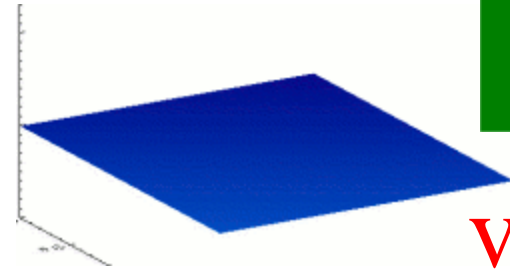
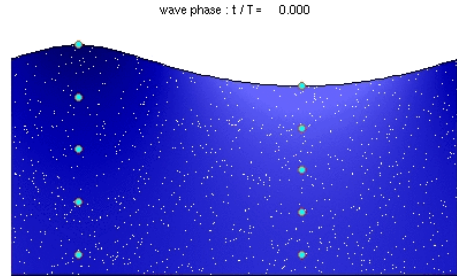
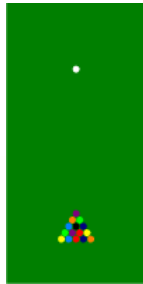
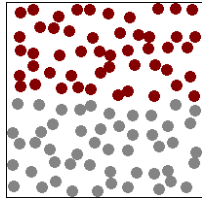
Equation (1) is just Newton's law $f = ma$ for a fluid element subject to the external force $f = (f_i(x, t))_{1 \leq i \leq n}$ and to the forces arising from pressure and friction. Equation (2) just says that the fluid is incompressible. For physically reasonable

Theory vs. Reality Chasms

Navier–Stokes equations:
$$\frac{\partial}{\partial t} u_i + \sum_{j=1}^n u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x, t)$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \nabla \cdot \mathbb{T} + \mathbf{f}$$

$$\operatorname{div} u = \sum_{i=1}^n \frac{\partial u_i}{\partial x_i} = 0$$



VS.





Clay Mathematics Institute

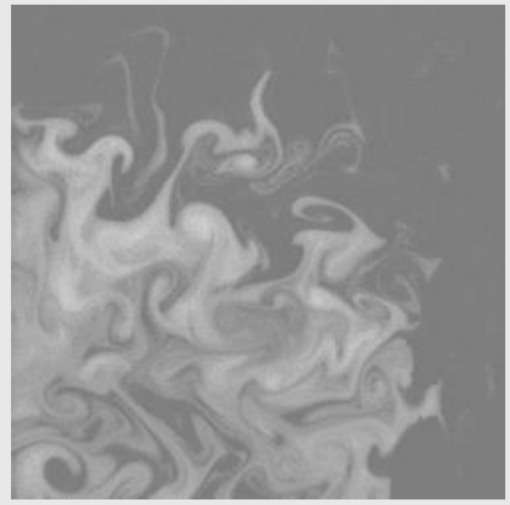
Dedicated to increasing and disseminating mathematical knowledge

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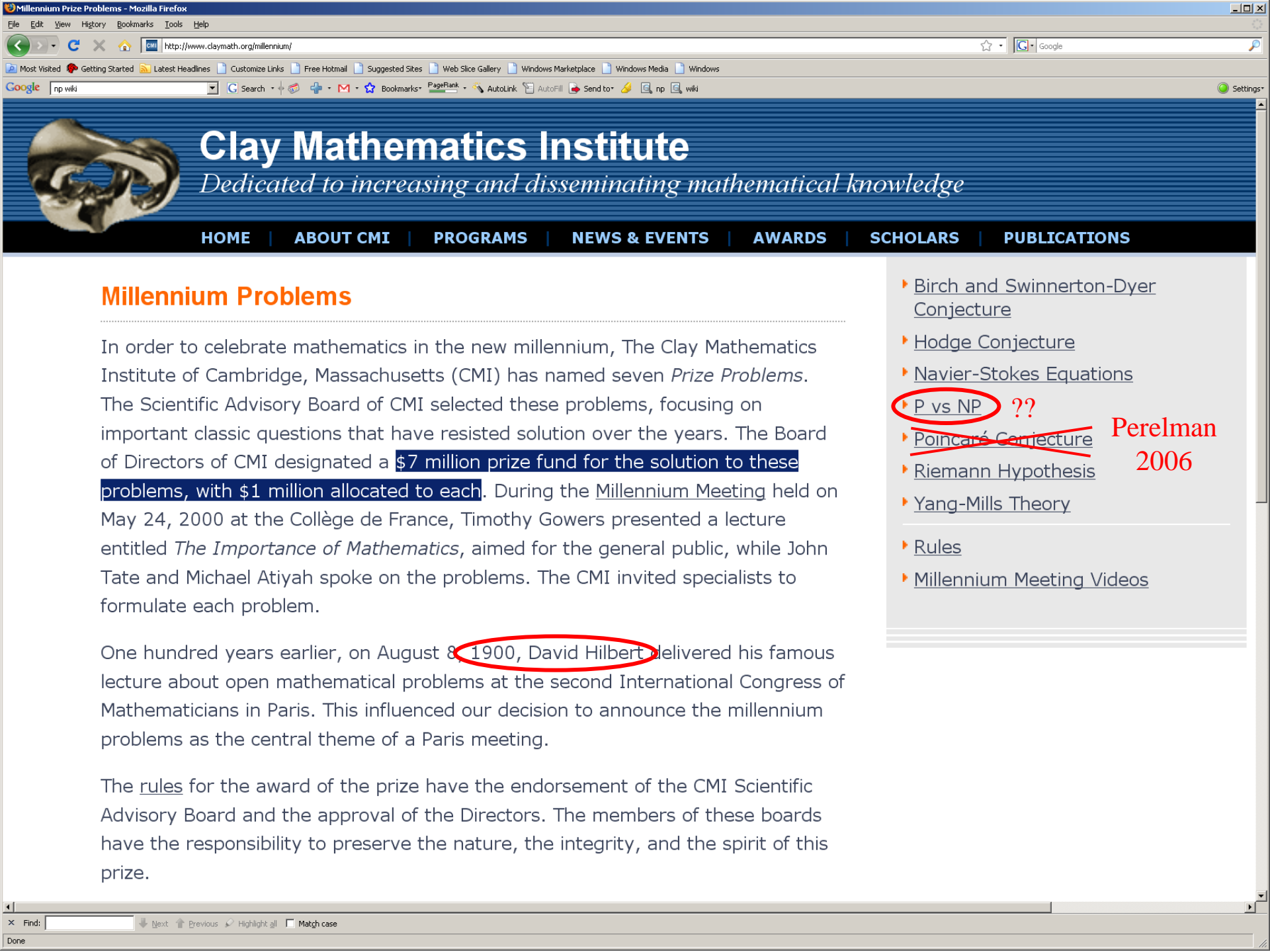
Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, **our understanding of them remains minimal**. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

- ▶ [The Millennium Problems](#)
- ▶ [Official Problem Description — Charles Fefferman](#)
- ▶ [Lecture by Luis Caffarelli \(video\)](#)



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Millennium Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven *Prize Problems*. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a **\$7 million prize fund for the solution to these problems, with \$1 million allocated to each.** During the Millennium Meeting held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled *The Importance of Mathematics*, aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

One hundred years earlier, on August 8, **1900, David Hilbert** delivered his famous lecture about open mathematical problems at the second International Congress of Mathematicians in Paris. This influenced our decision to announce the millennium problems as the central theme of a Paris meeting.

The rules for the award of the prize have the endorsement of the CMI Scientific Advisory Board and the approval of the Directors. The members of these boards have the responsibility to preserve the nature, the integrity, and the spirit of this prize.

- ▶ [Birch and Swinnerton-Dyer Conjecture](#)
- ▶ [Hodge Conjecture](#)
- ▶ [Navier-Stokes Equations](#)
- ▶ **P vs NP ??**
- ▶ ~~Poincaré Conjecture~~ **Perelman 2006**
- ▶ [Riemann Hypothesis](#)
- ▶ [Yang-Mills Theory](#)
- ▶ [Rules](#)
- ▶ [Millennium Meeting Videos](#)

nano? REAL PROGRAMMERS USE emacs



HEY. REAL PROGRAMMERS USE vim.



WELL, REAL PROGRAMMERS USE ed.



NO, REAL PROGRAMMERS USE cat.



REAL PROGRAMMERS USE A MAGNETIZED NEEDLE AND A STEADY HAND.



EXCUSE ME, BUT REAL PROGRAMMERS USE BUTTERFLIES.



THEY OPEN THEIR HANDS AND LET THE DELICATE WINGS FLAP ONCE.

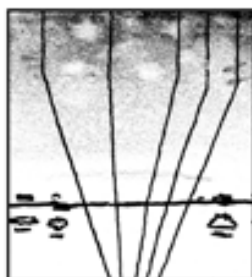


THE DISTURBANCE RIPPLES OUTWARD, CHANGING THE FLOW OF THE EDDY CURRENTS IN THE UPPER ATMOSPHERE.



THESE CAUSE MOMENTARY POCKETS OF HIGHER-PRESSURE AIR TO FORM,

WHICH ACT AS LENSES THAT DEFLECT INCOMING COSMIC RAYS, FOCUSING THEM TO STRIKE THE DRIVE PLATTER AND FLIP THE DESIRED BIT.



NICE. 'COURSE, THERE'S AN EMACS COMMAND TO DO THAT.

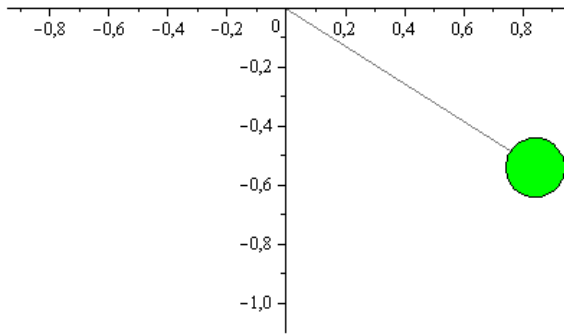
OH YEAH! GOOD OL' C-x M-c M-butterfly...



DAMMIT, EMACS.

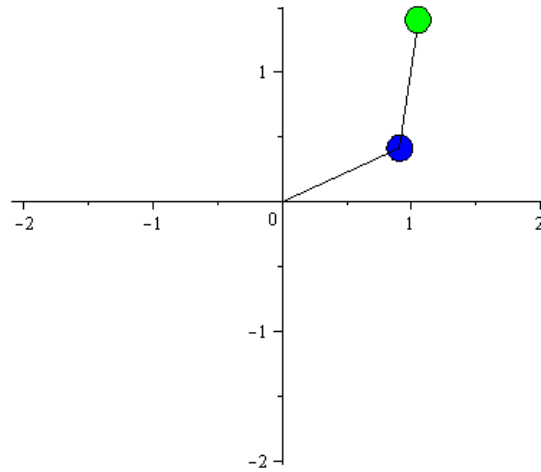
Simple Chaotic Systems

Compound pendulums:



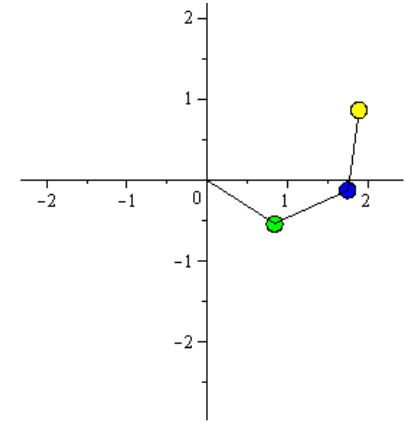
$$\frac{d^2}{dt^2} \phi(t) + \frac{g}{l} \phi(t) = 0 \Rightarrow \phi(t) = \phi(0) \cos\left(\sqrt{\frac{g}{l}} t\right)$$

$l = 1 \text{ m}; \phi(0) = 1 \text{ rad}; \frac{d}{dt}\phi(0) = 0; T = 2\pi \sqrt{\frac{1 \text{ m}}{g}} \sim 2 \text{ s}$



$$l_1 = l_2 = 1 \text{ m}; m_1 = m_2 = 1 \text{ kg};$$

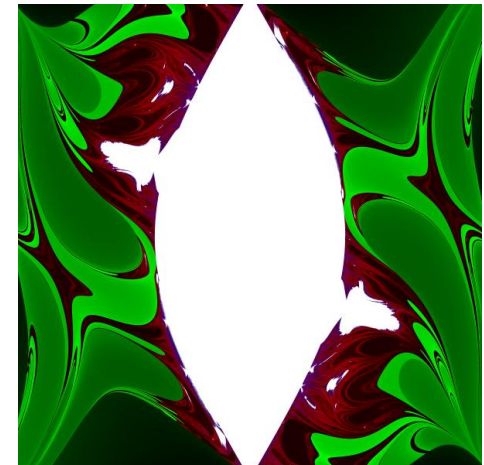
$$\phi_1(0) = 2 \text{ rad}; \phi_2(0) = 3 \text{ rad}; \frac{d}{dt}\phi_1(0) = \frac{d}{dt}\phi_2(0) = 0$$



$$l_1 = l_2 = l_3 = 1 \text{ m}; m_1 = m_2 = m_3 = 1 \text{ kg};$$

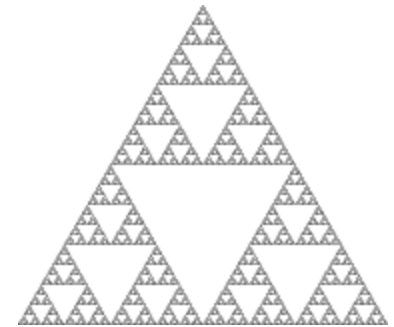
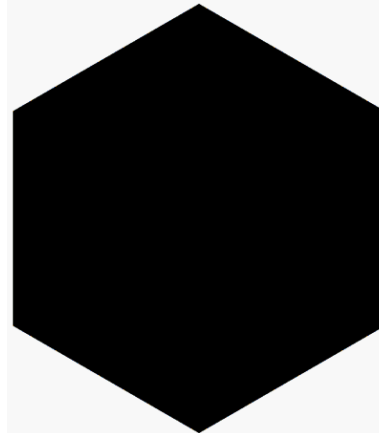
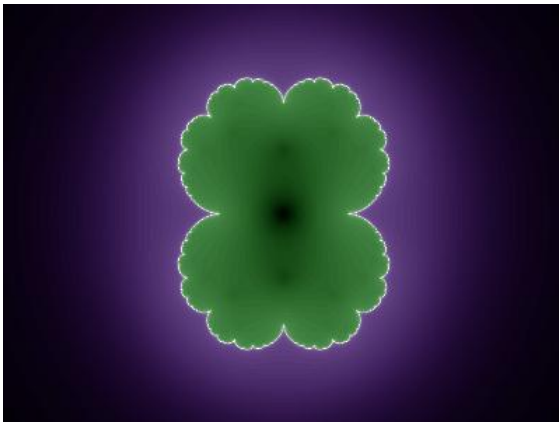
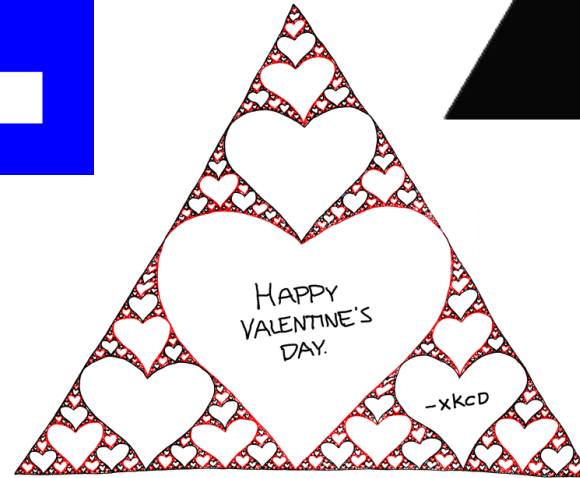
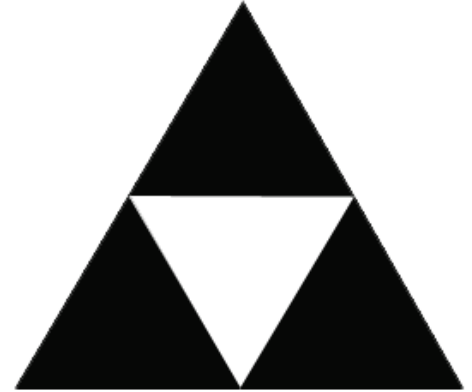
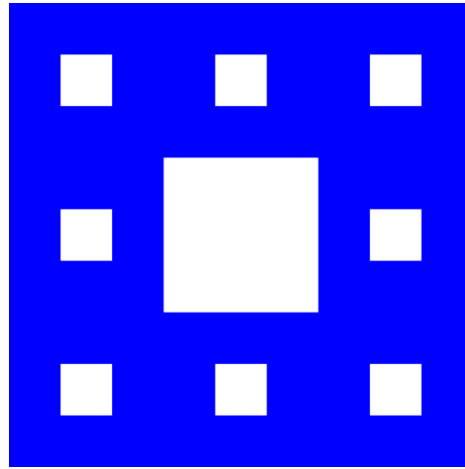
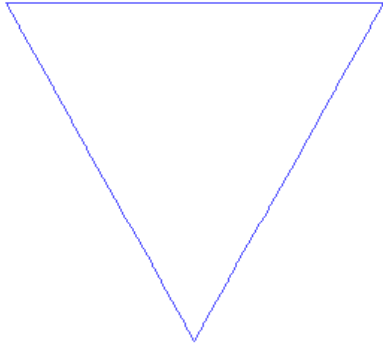
$$\phi_1(0) = 1 \text{ rad}; \phi_2(0) = 2 \text{ rad}; \phi_3(0) = 3 \text{ rad};$$

$$\frac{d}{dt}\phi_1(0) = \frac{d}{dt}\phi_2(0) = \frac{d}{dt}\phi_3(0) = 0$$

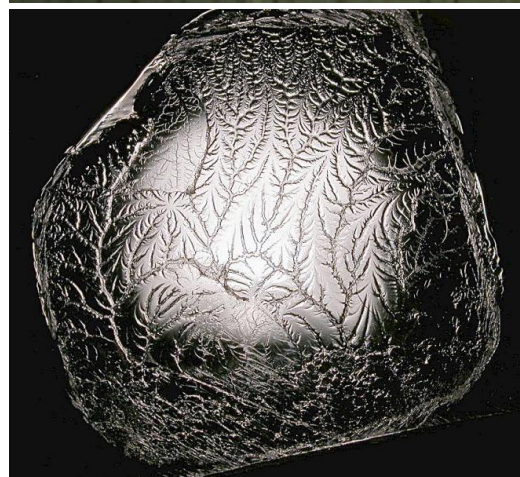
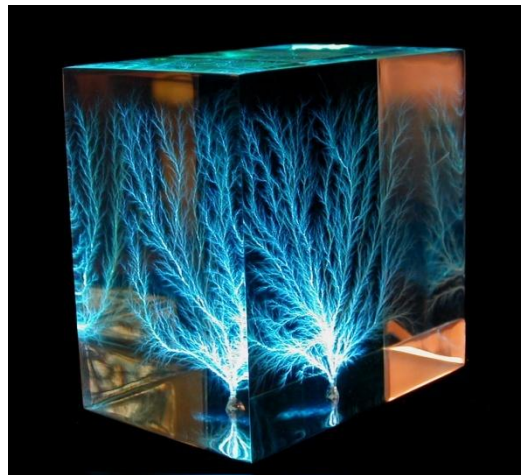
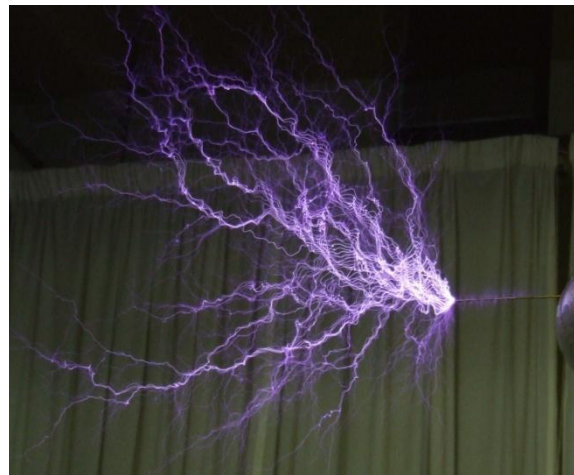


Issues: chaos, undecidability

Simple Chaotic Systems: Fractals

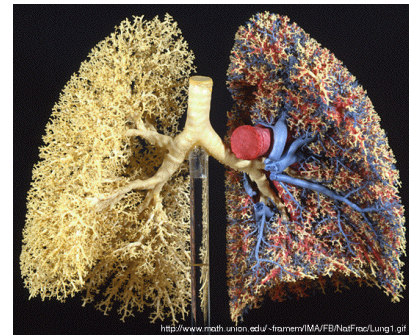
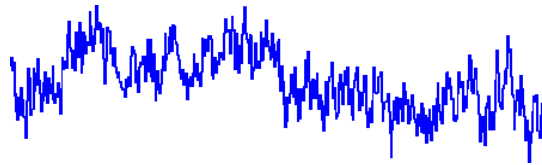
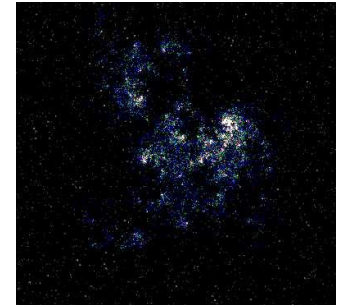
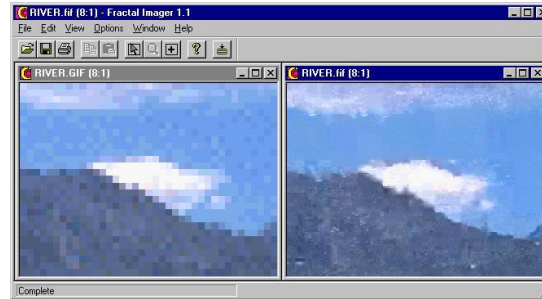




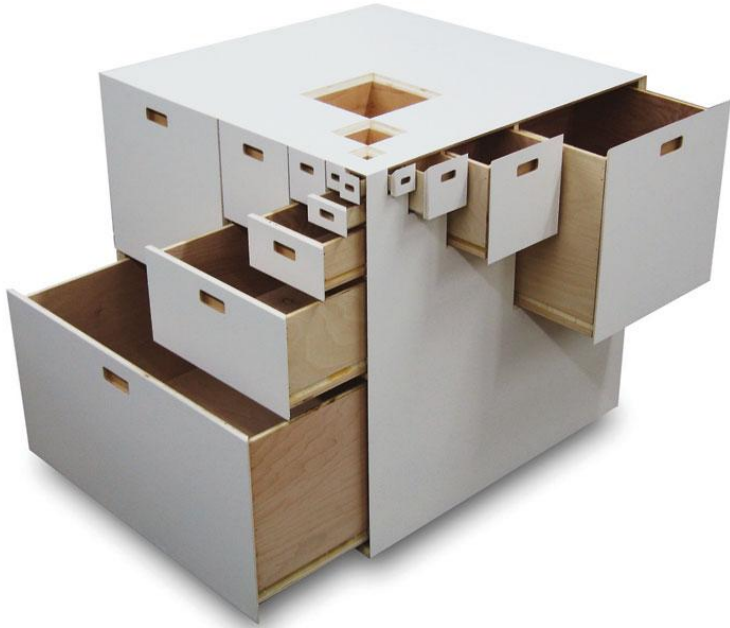


Applications of Fractals

- Compressing images
- Simulating galaxies
- Analyzing markets
- Generating music
- Modeling weather
- Movie special effects
- Designing video games
- Describing crystal growth
- Understanding anatomy
- Explaining plant forms
- Tracking populations
- Fashion design



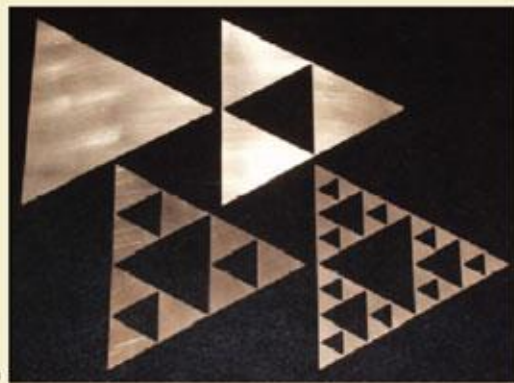
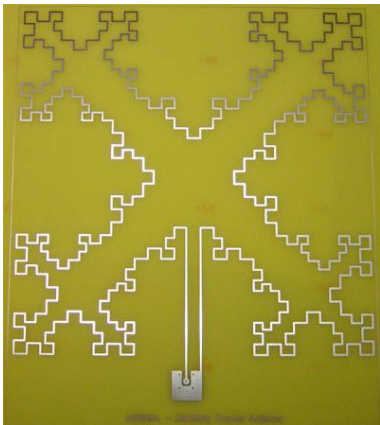
Applications of Fractals





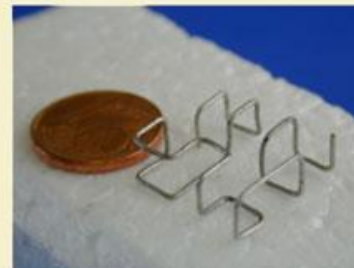
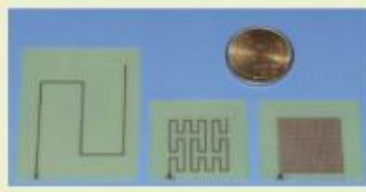
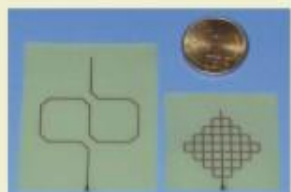
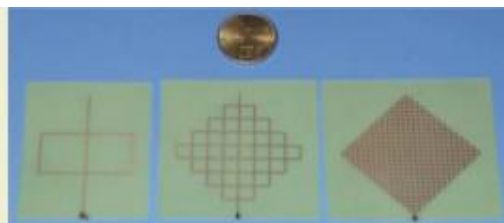
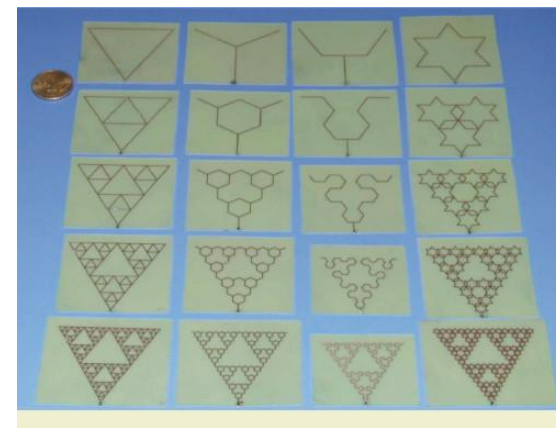
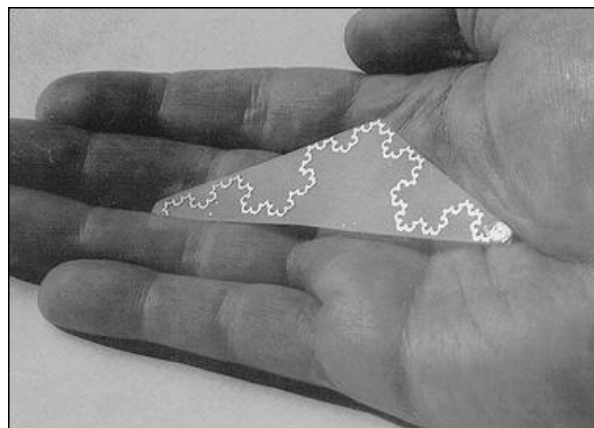
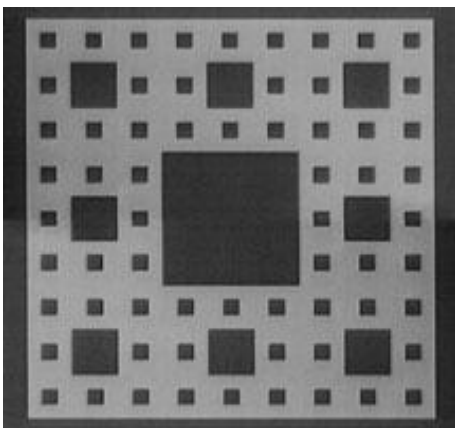
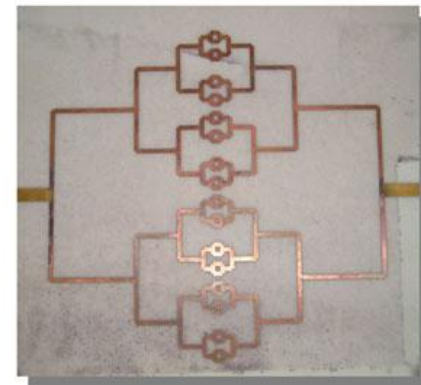
"WE DID THE WHOLE ROOM OVER
IN FRACTALS."

Fractal Antennas



a)

b)

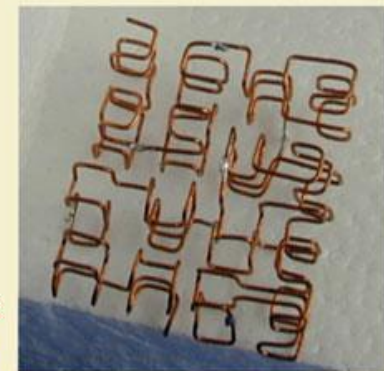


2nd iteration

$h=5$ mm
 $s=17$ mm

3rd iteration

$h=10$ mm
 $s=23$ mm



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Fractal Antenna's proven products and embedded components power some of today's most innovative applications—from RFID, to telematics, to electronic warfare and signal intelligence. The company's systematic approach to product development ensures delivery of the optimal and practical antenna solution for every application.

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Metacloak: Fractal Antenna Systems, "See-thru" Wideband Invisibility Cloak - Mozilla Firefox

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METACLOAK™

On March 27, 2009 history was made. Fractal Antenna, the world's pioneering innovator of antennas and fractal electronics, did what others said couldn't be done: demonstrate a working, "see-thru" wideband invisibility cloak.

Now while the cloak works only at microwaves, and a well-known "see-thru" microwave cloak was demonstrated several years ago at a very narrow band, it is Fractal Antenna's wideband cloak that opens the opportunity for the science of "metamaterials" to become practical. Simply put by analogy, it is the difference between a telescope that only sees purple compared to one that sees all of nature's colors—and a bunch that go beyond the range of our vision.

As we are scientists and engineers working in a small company setting, we are doing our best to meet the desire for information against the ongoing needs of our valued customers. This means that we will be expanding this page in the coming weeks, and invite you to drop on by periodically to see what new info we have up. Also, we welcome queries on the cloak, but can't answer them directly. We will be glad to present your questions and comments in the FAQ section if they apply to a wide audience (fractalcloak@aol.com). Kindly keep business queries through the Fractal Antenna Systems website. We welcome you to explore this exciting breakthrough and the opportunities that arise from it. Enjoy our Metacloak™ page!

Object (cloaked region)

FRACTAL's Wideband Invisibility Cloak FLASHLIGHT ANALOGY

Direct
Obstacle ('Object')
Object & Cloak

Object

FRACTAL's Wideband Invisibility Cloak

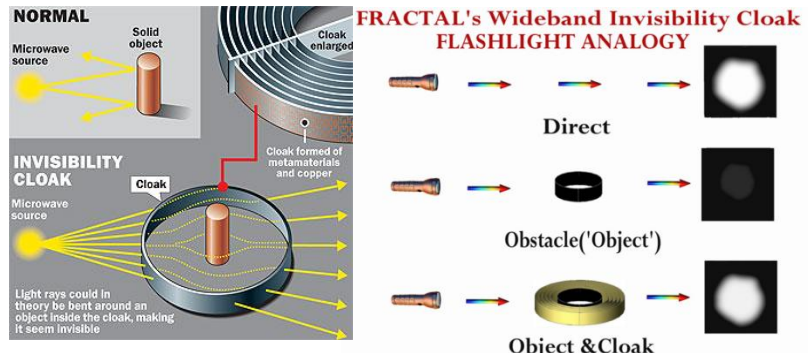
Wideband invisibility Cloak (Object in Black)

- Press Kit
- The Cloak and the Data Summary
- Cloaking Myths and Realities
- Metamaterials and Metafractal™ Technology Available July 4
- What Does It Mean? Available July 4
- Metacloak FAQs Email your questions to fractalcloak@aol.com.
- Harry Potter Realized: Next Steps? Available July 31
- Metacloak: The Video Available July 4
- Fractal Antenna's web site

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Find: Next Previous Highlight all Match case

Done



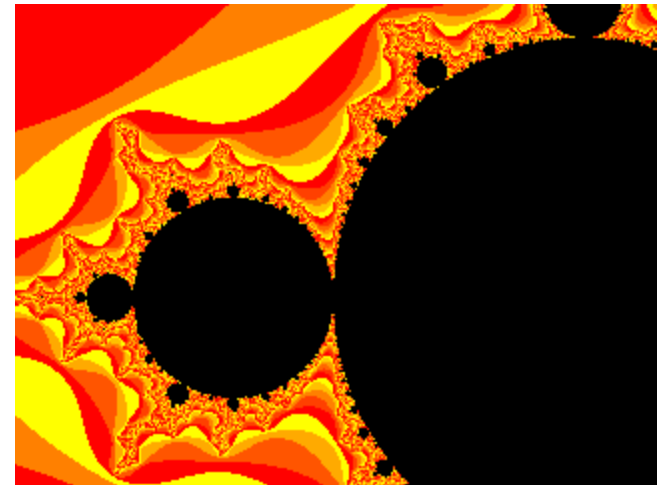
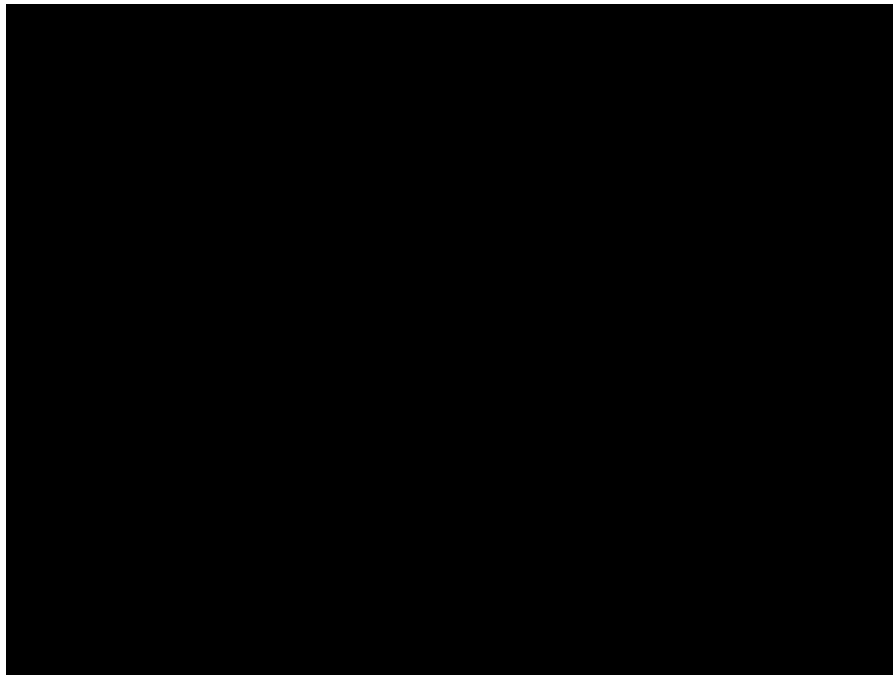
Simple Chaotic Systems: Fractals

$$P_c(z) = z^2 + c$$

$$P_c: \mathbb{C} \rightarrow \mathbb{C}$$

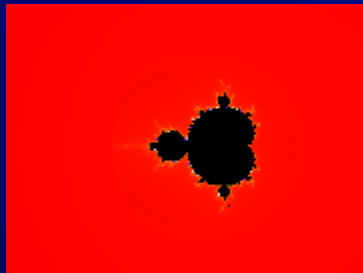
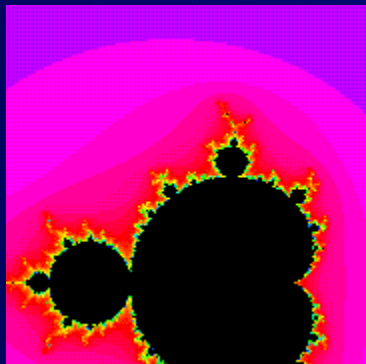
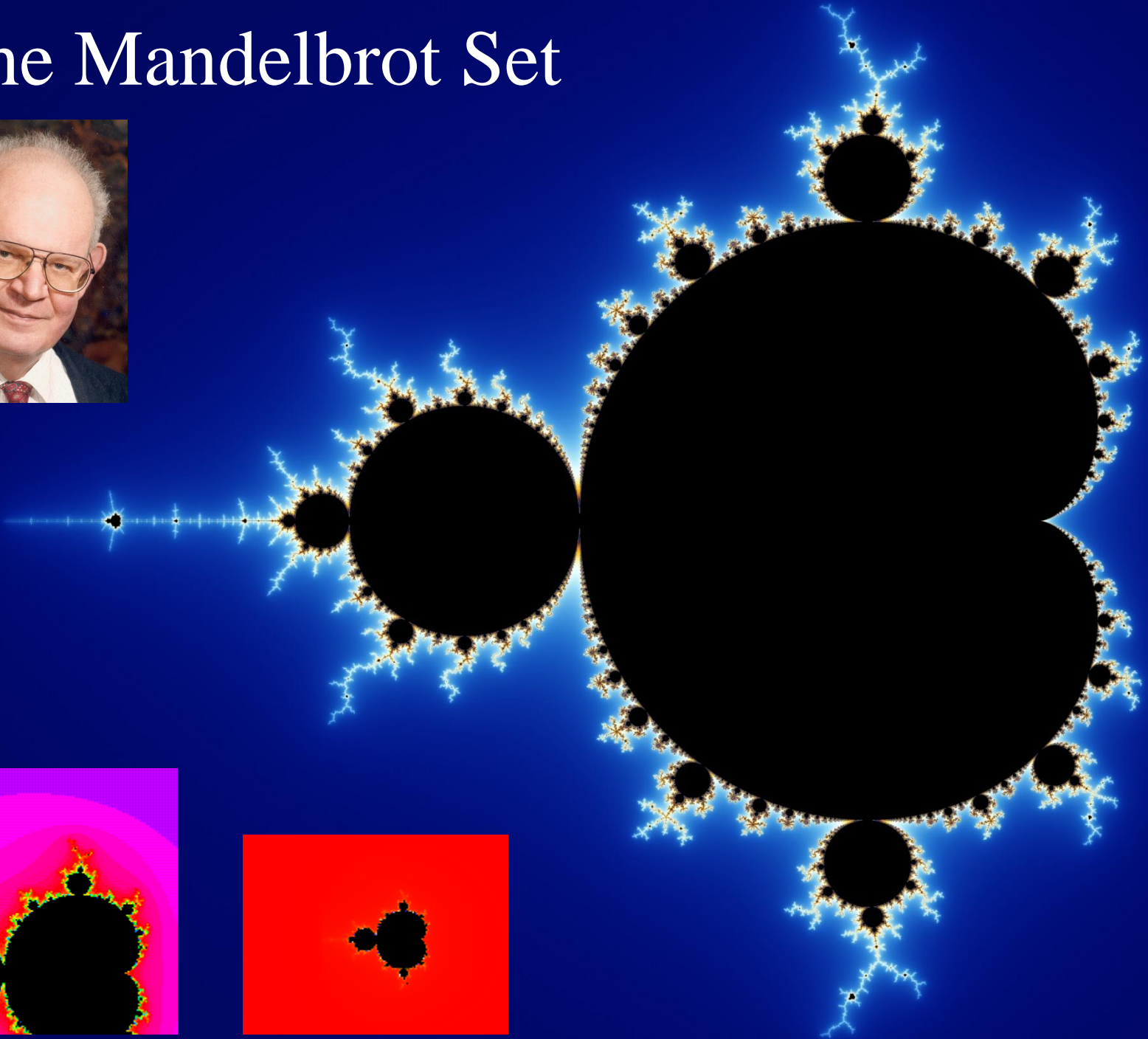
$$Q(c, n) = P_c(Q(c, n-1)) \quad Q(c, 0) = 0$$

$$\text{Mandelbrot set} = \{c \in \mathbb{C} \mid Q(c, n) < 2 \quad \forall n\}$$

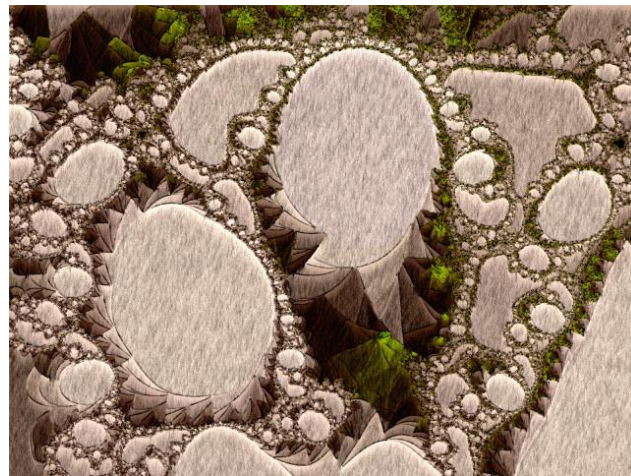
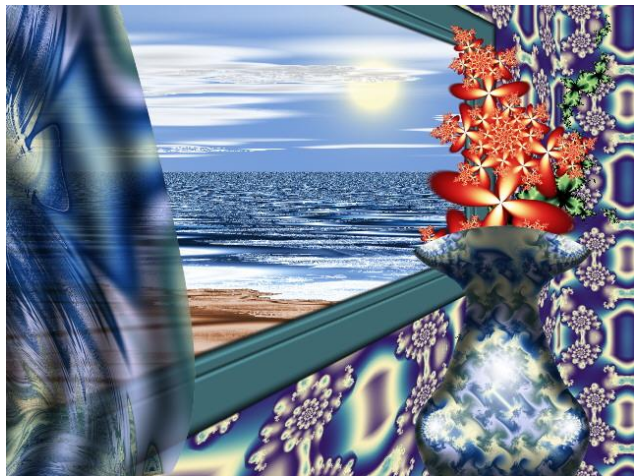
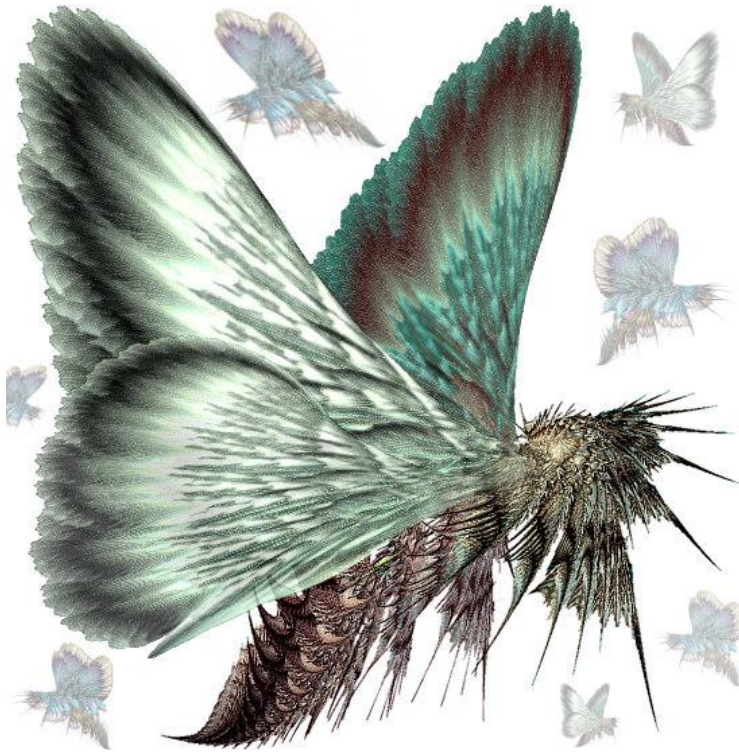


Issues: chaos, undecidability, incompleteness

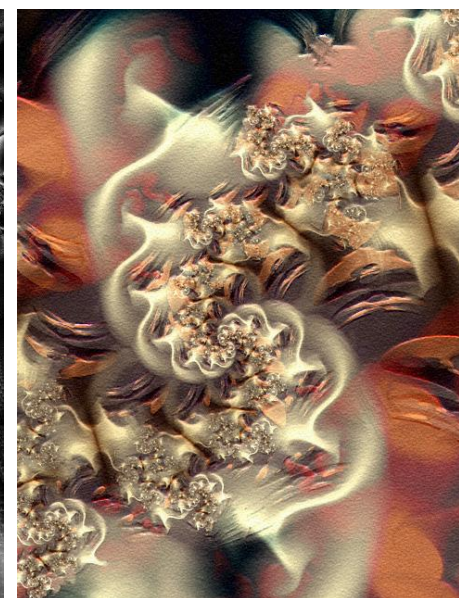
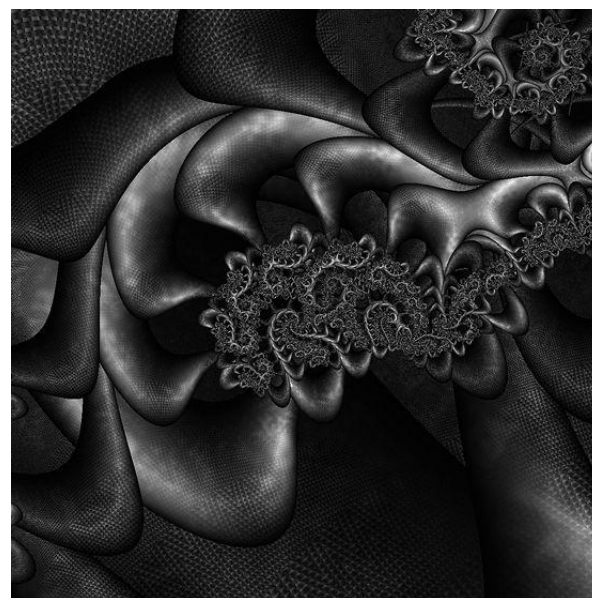
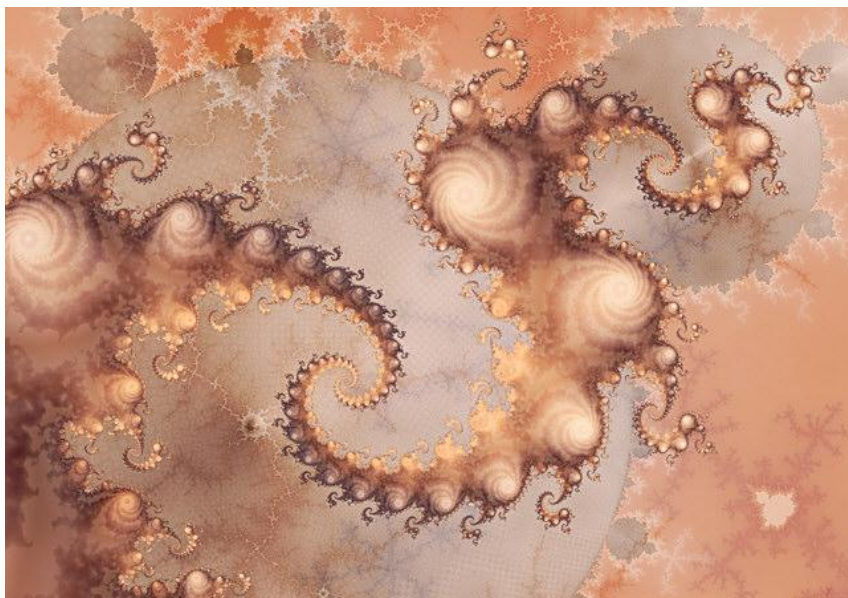
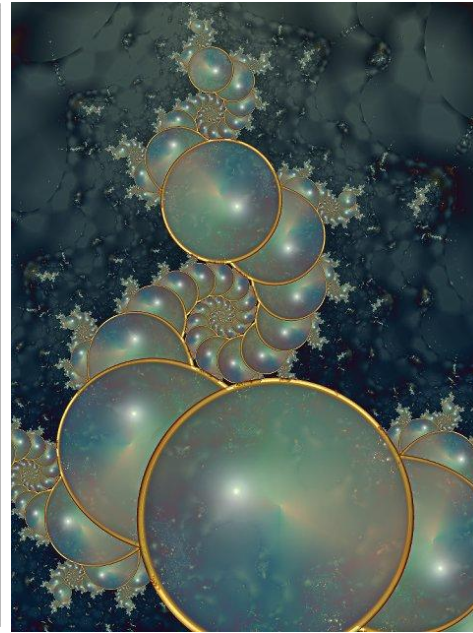
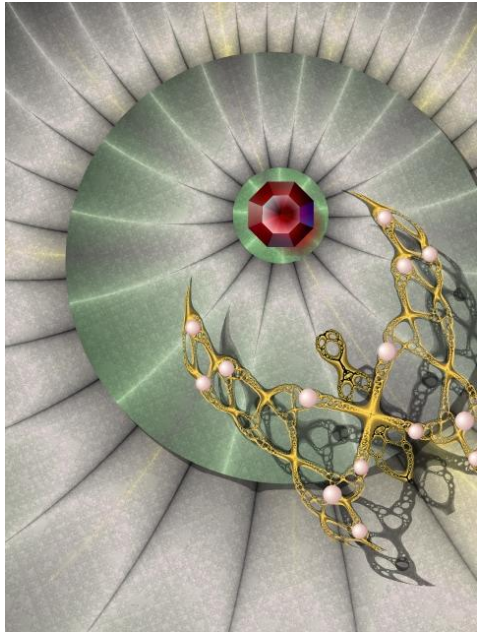
The Mandelbrot Set



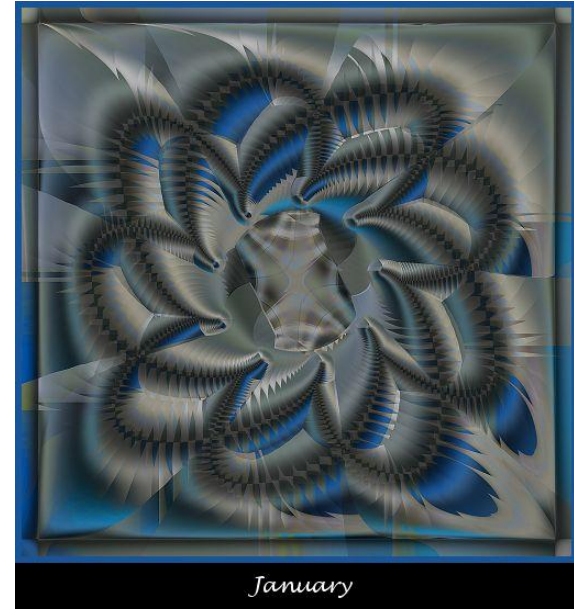
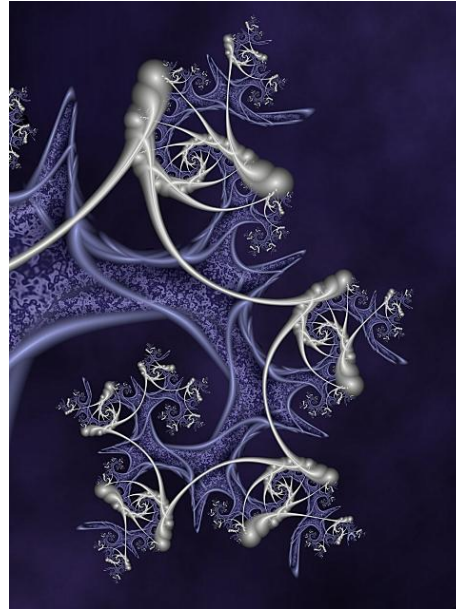
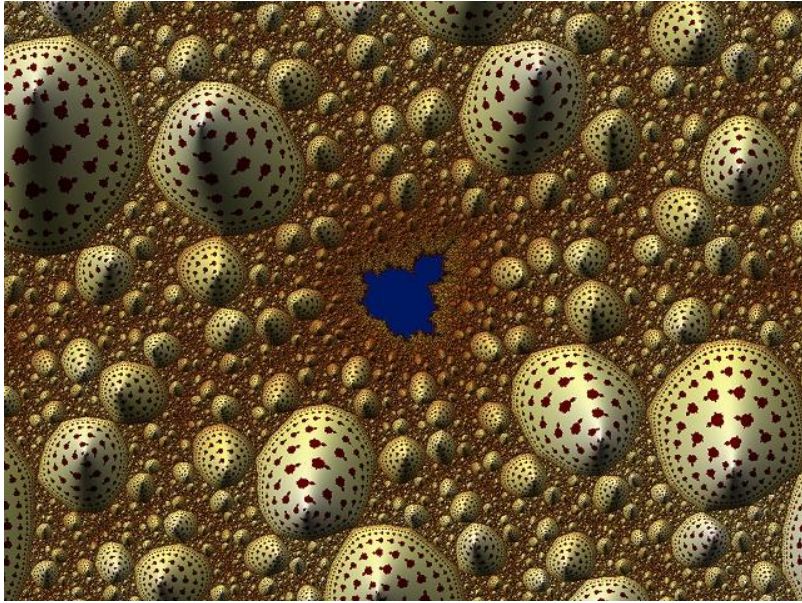
Fractal Art



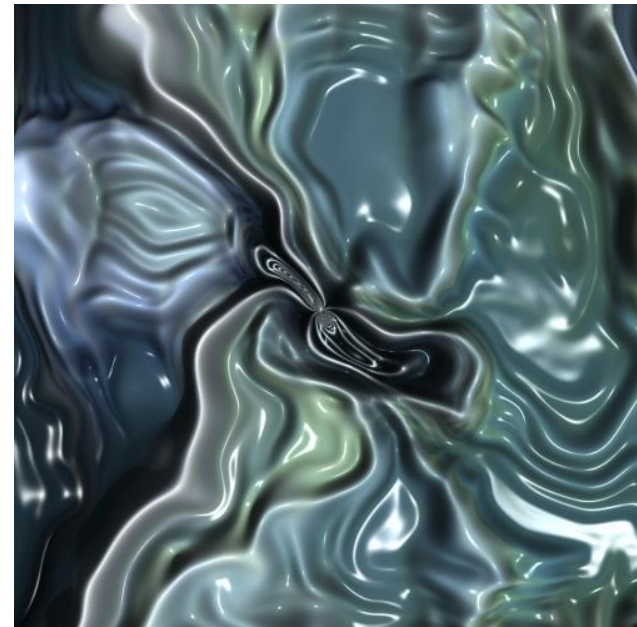
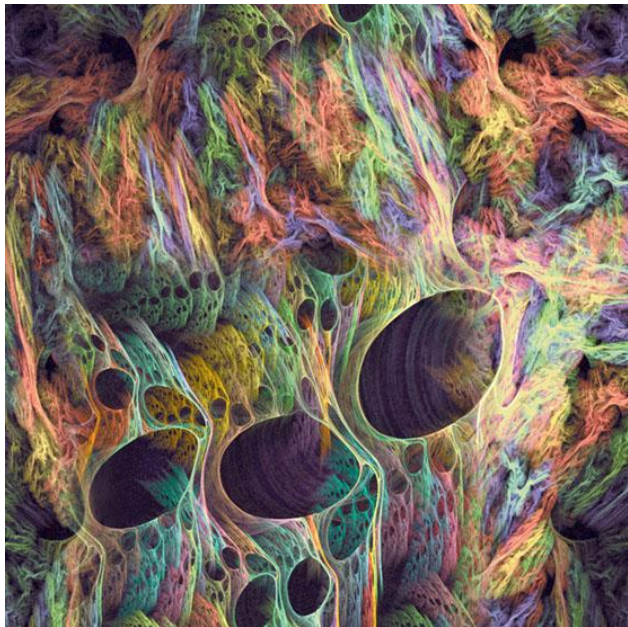
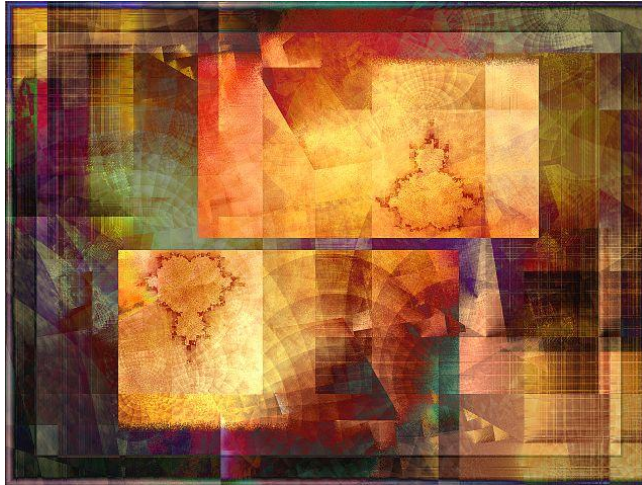
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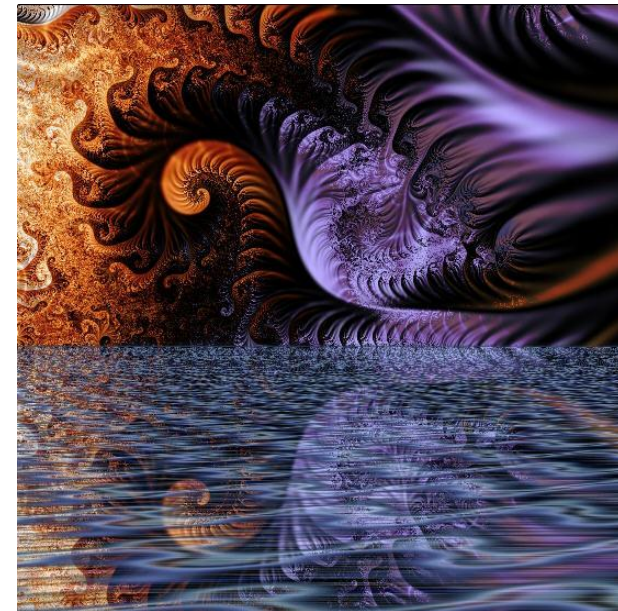
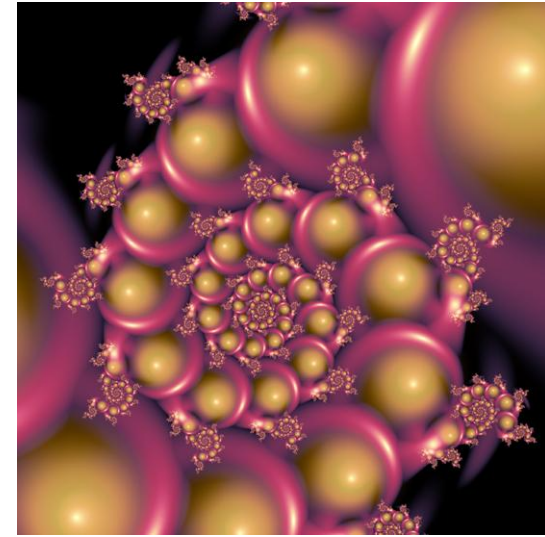
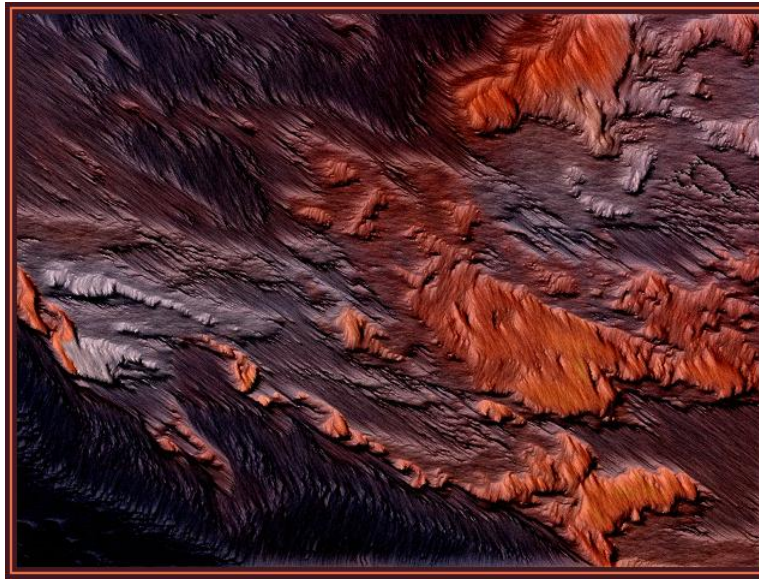
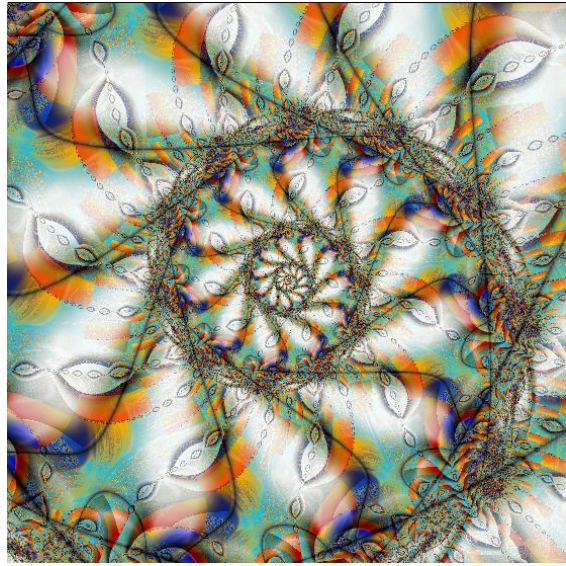
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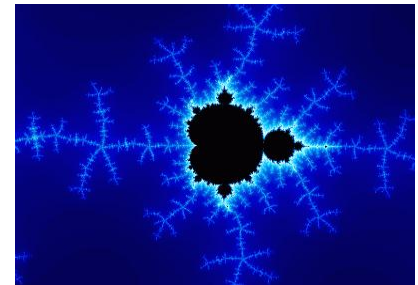
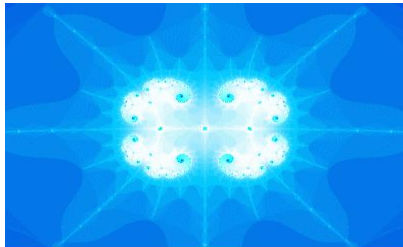
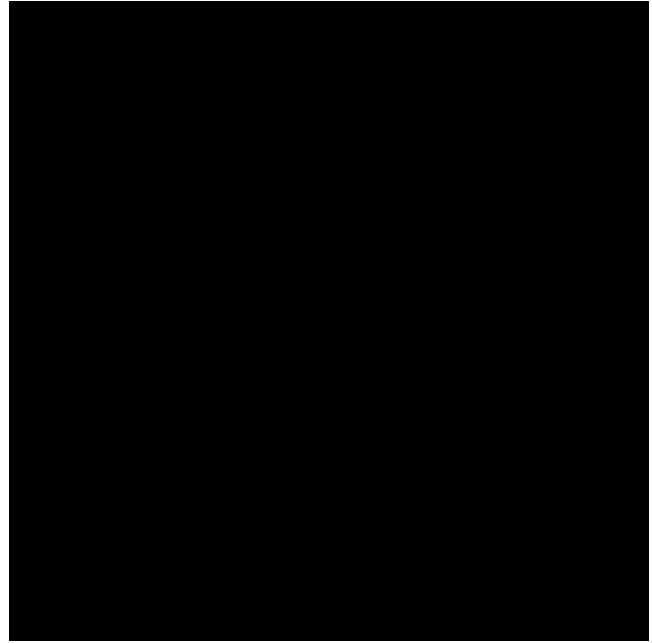
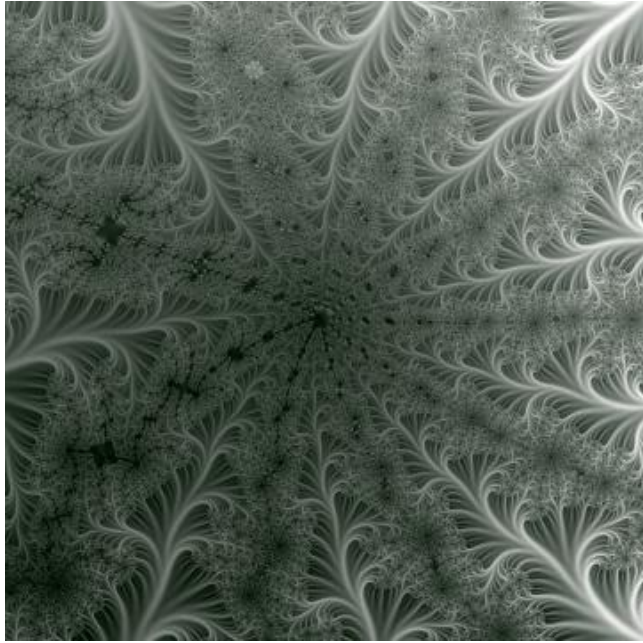
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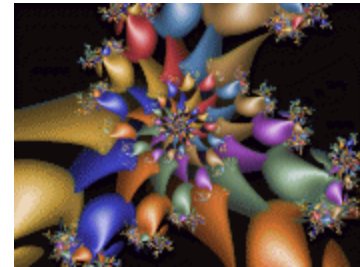
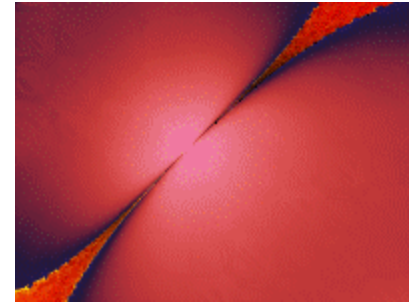
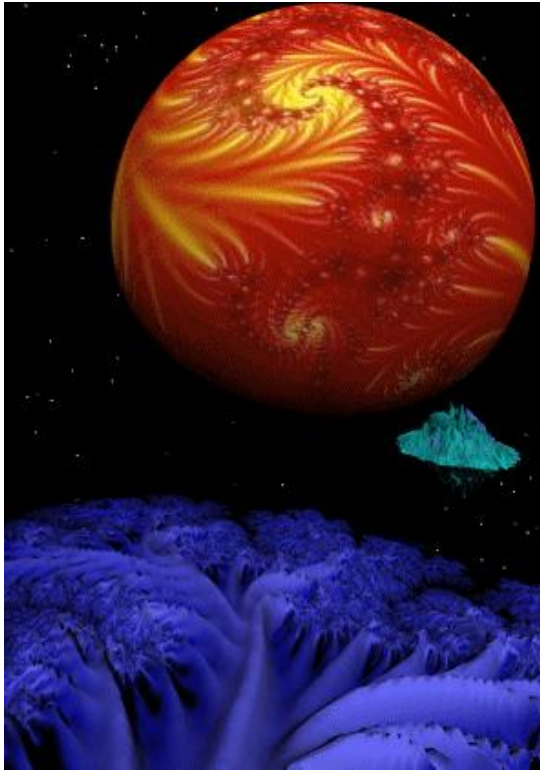
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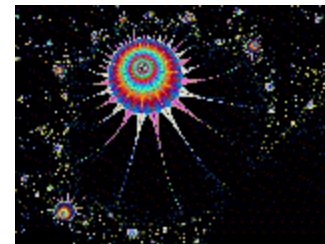
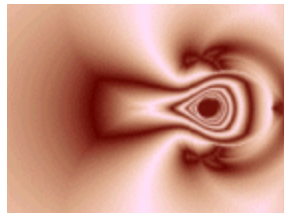
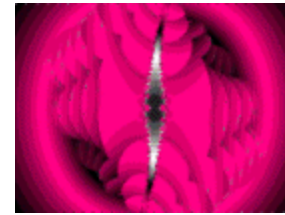
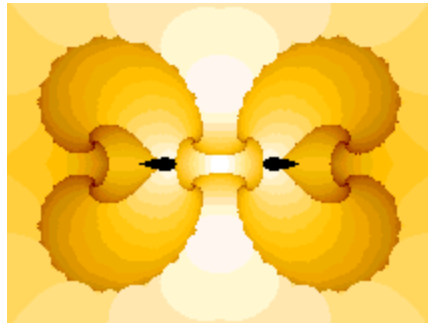


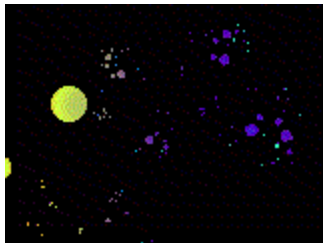
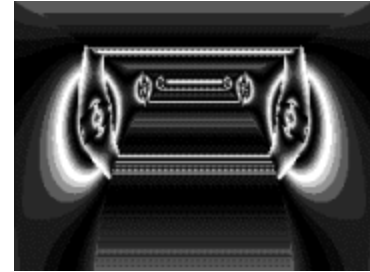
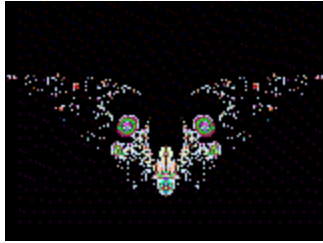
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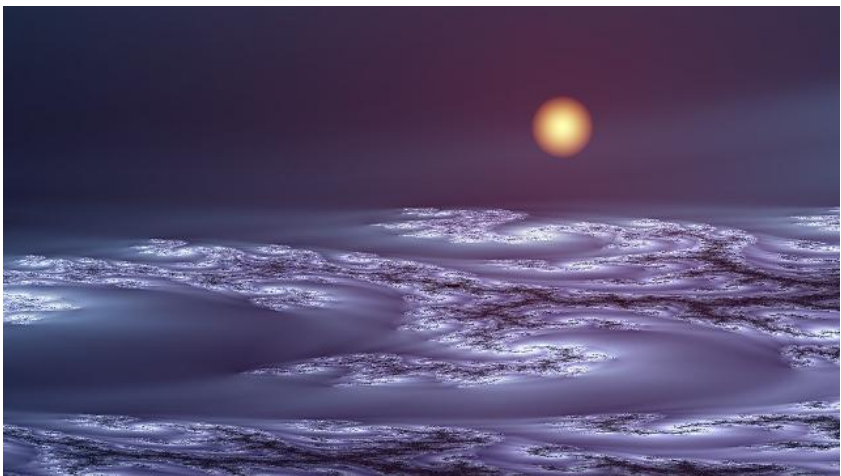
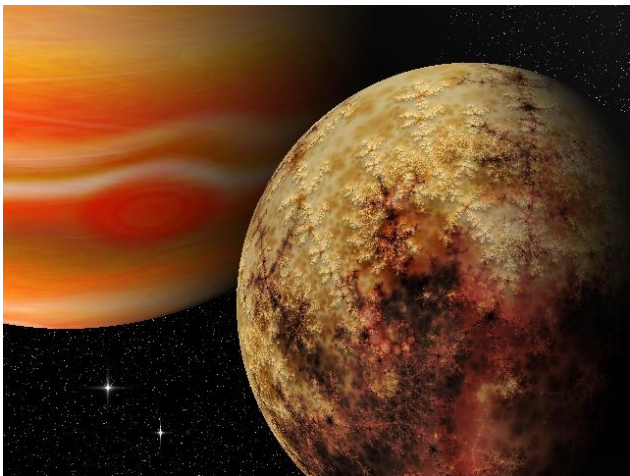


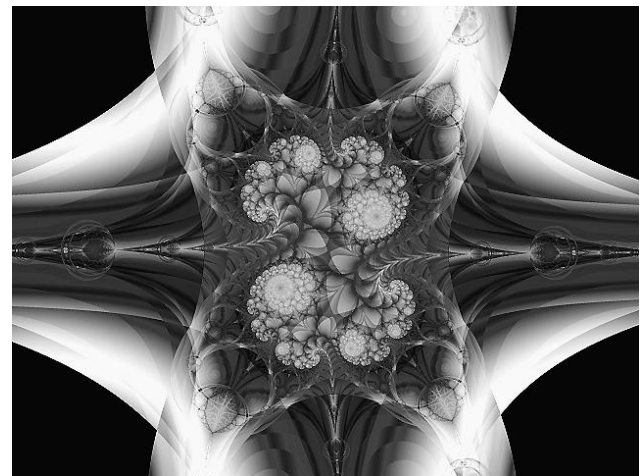
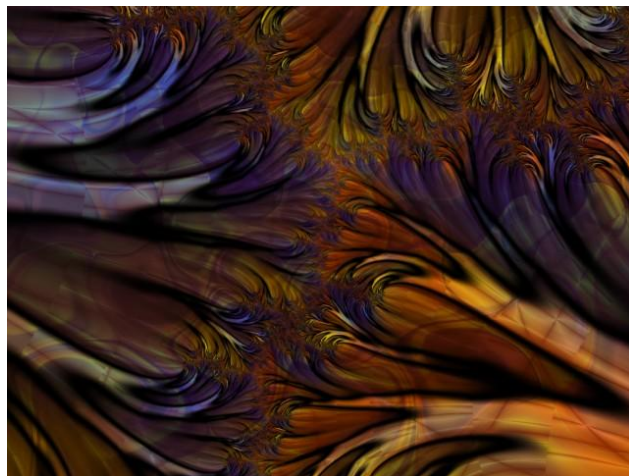
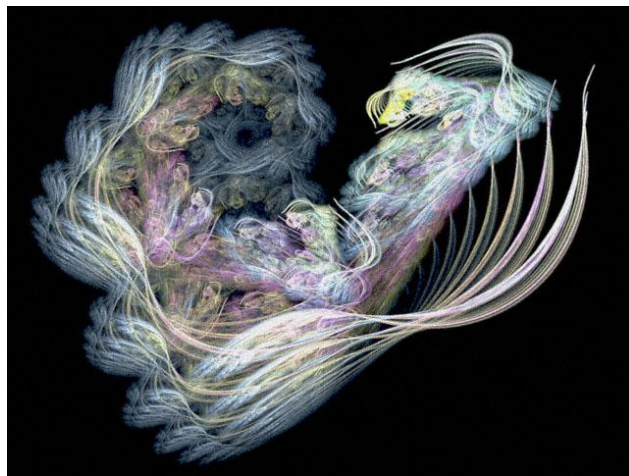
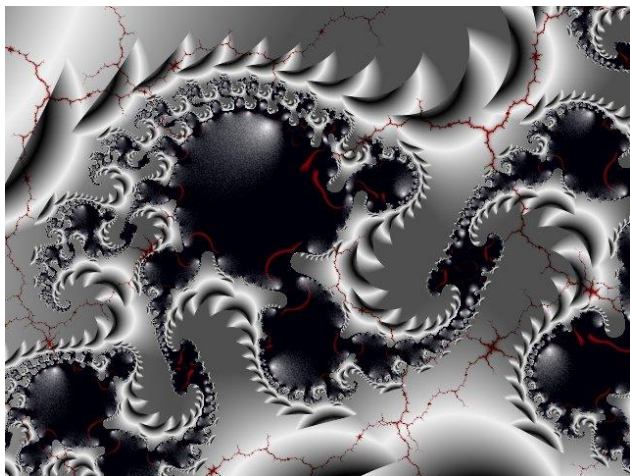
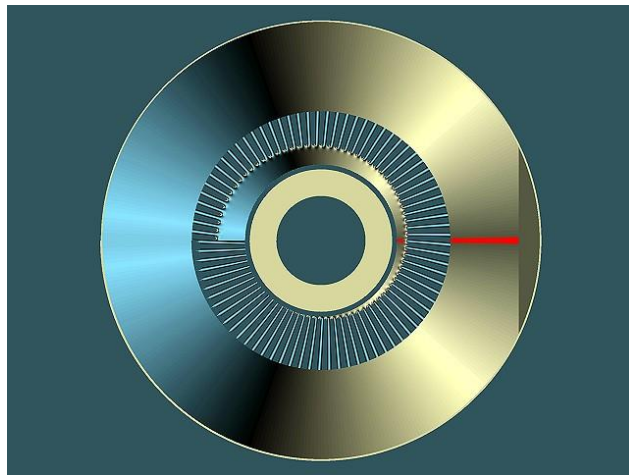
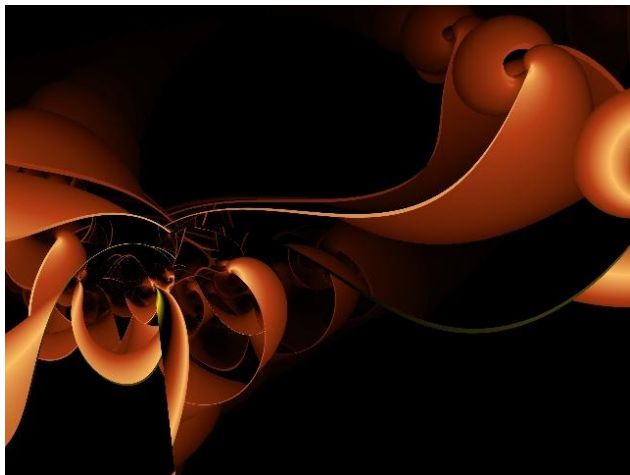
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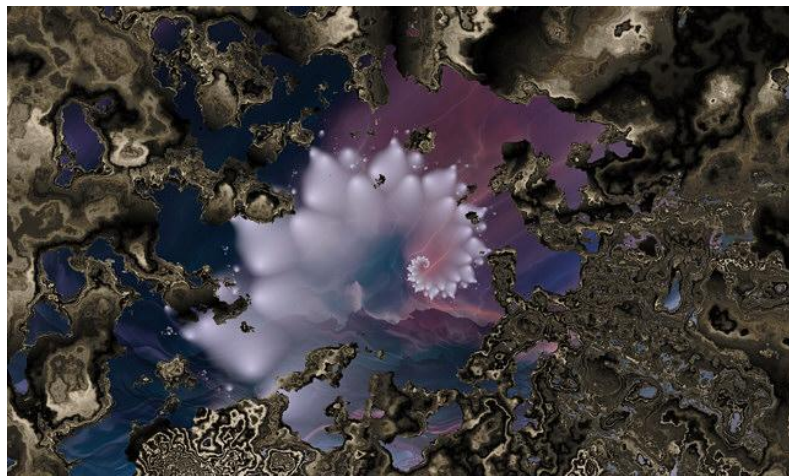
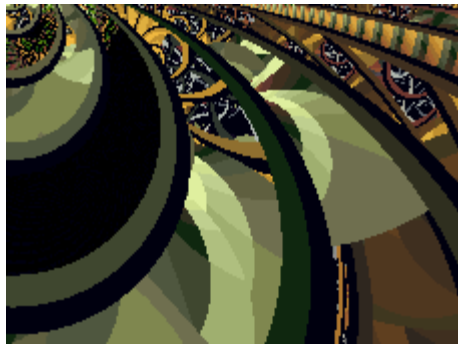
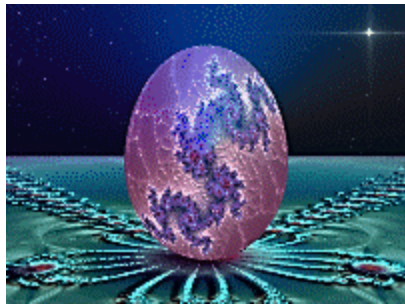
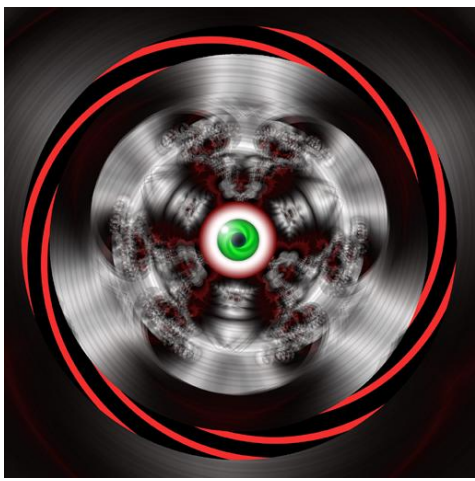
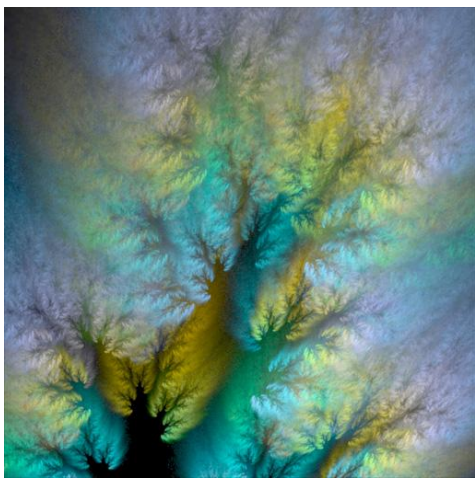
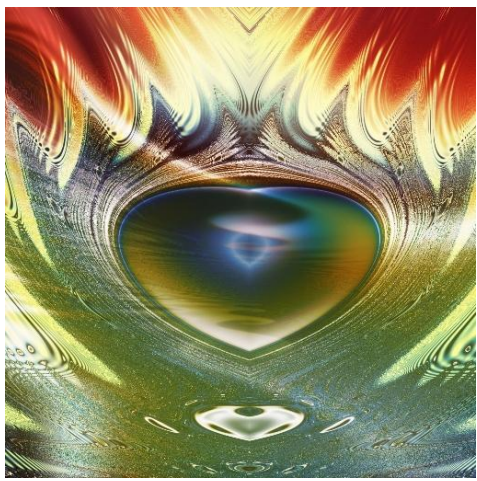
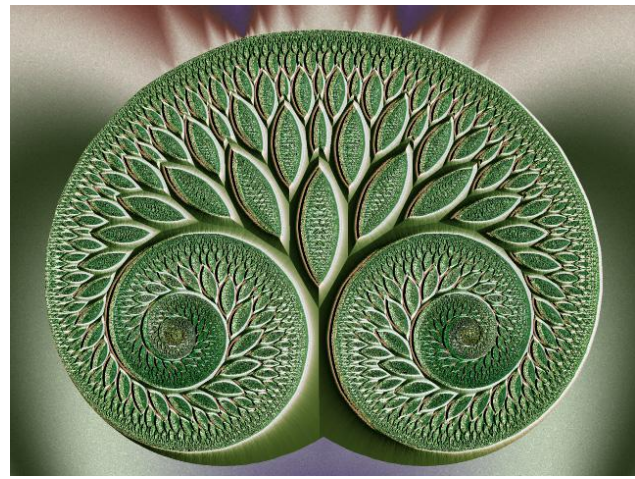
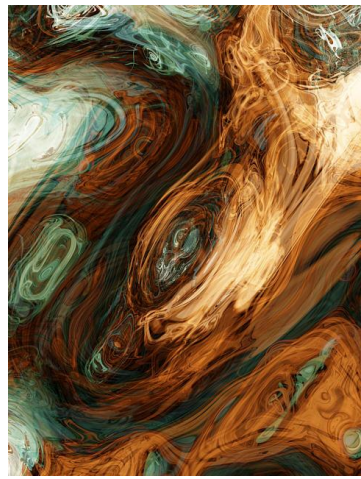


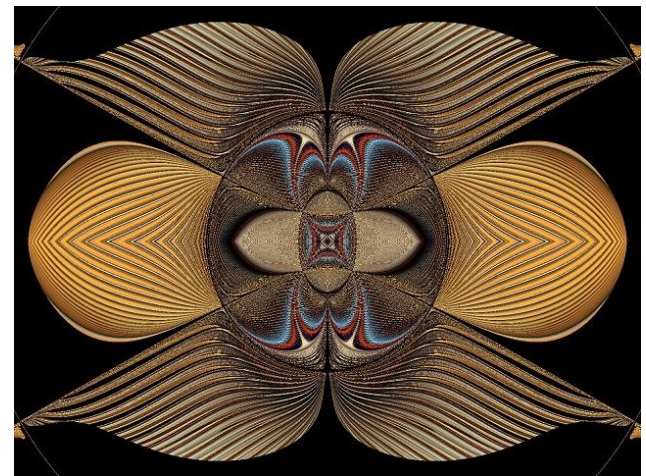
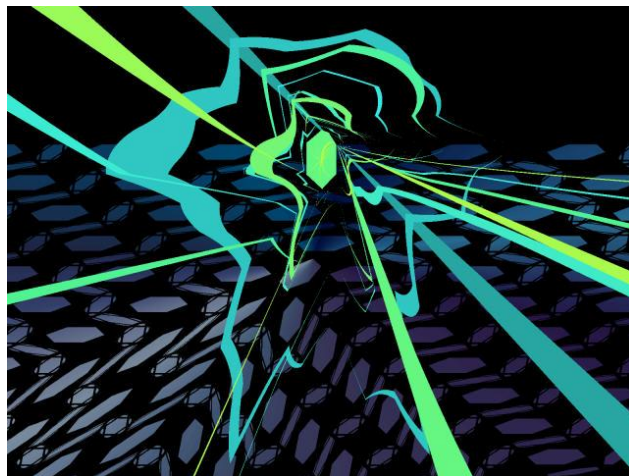
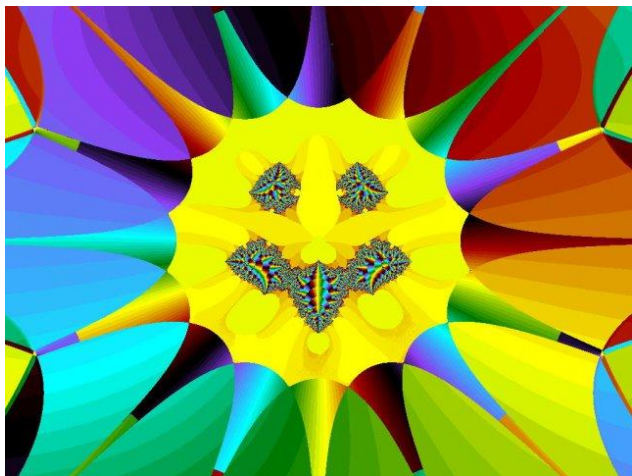
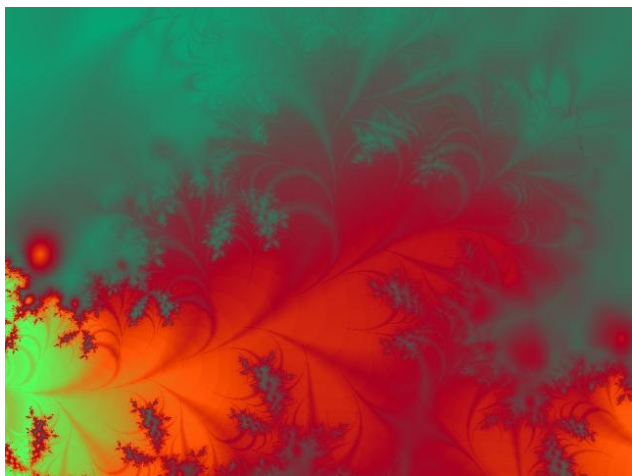
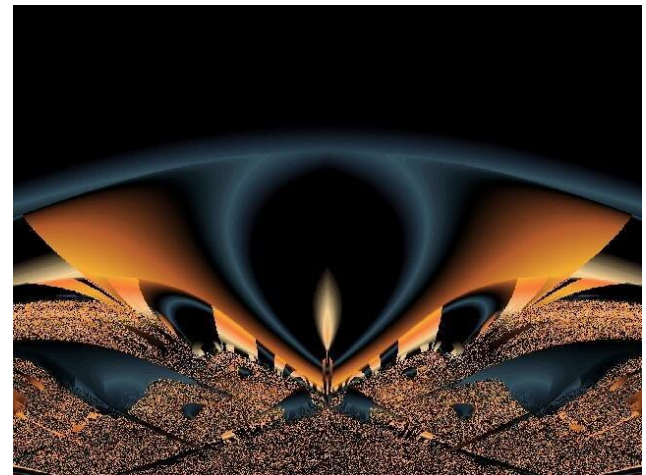
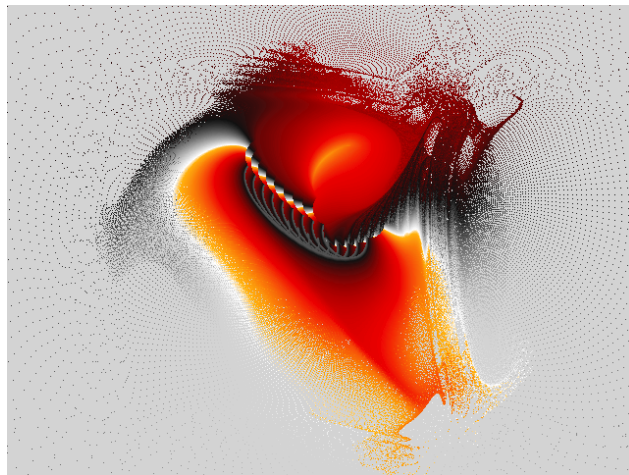
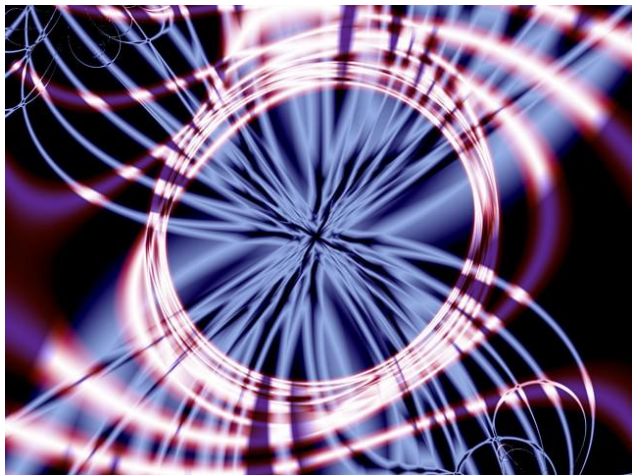


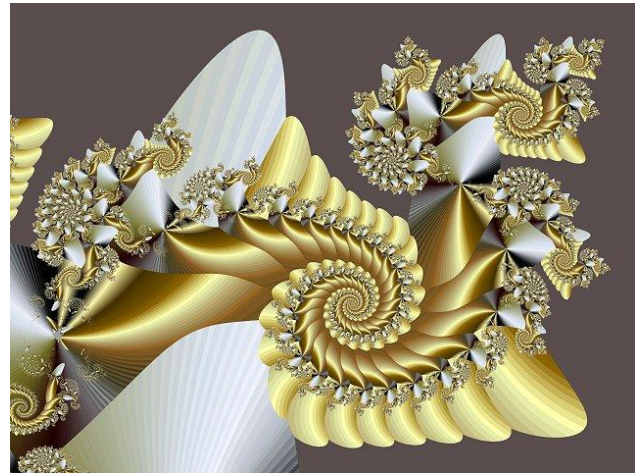
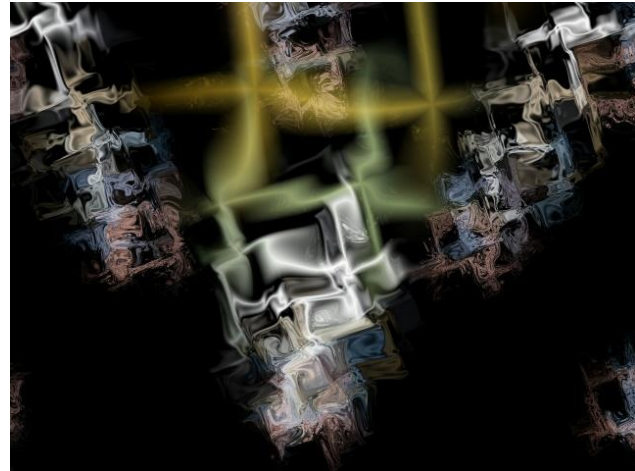
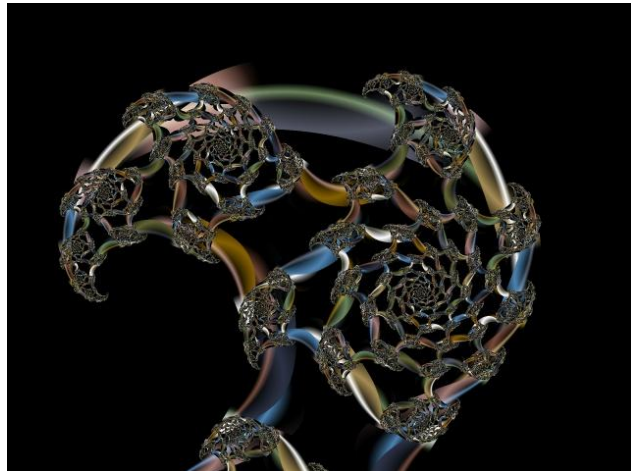
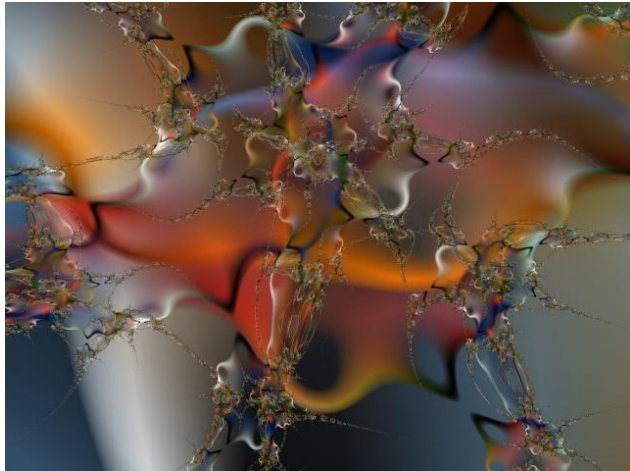
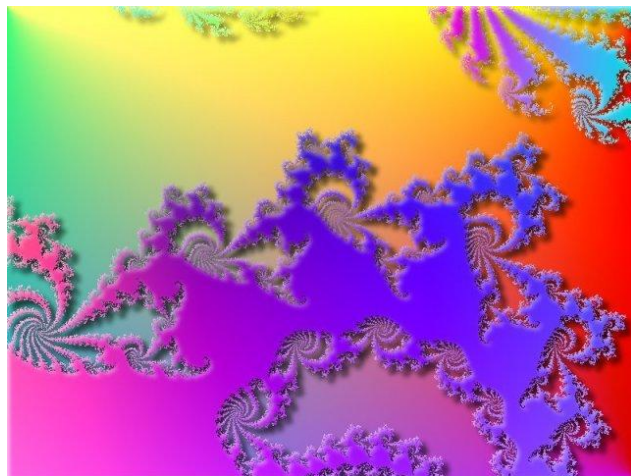
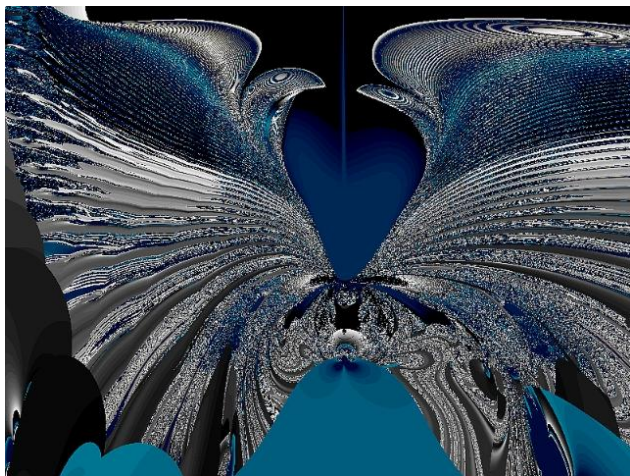


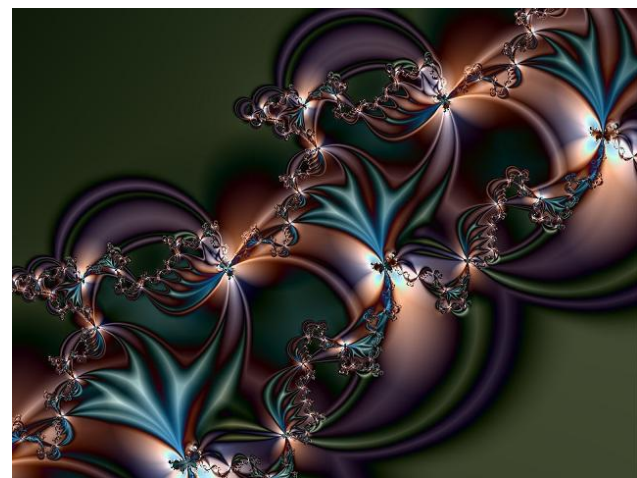
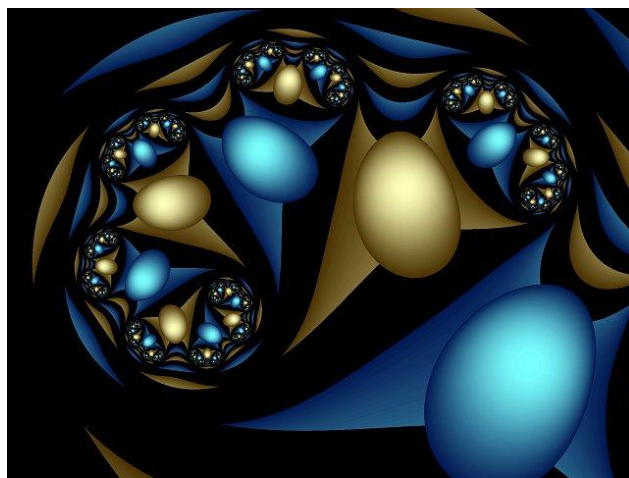
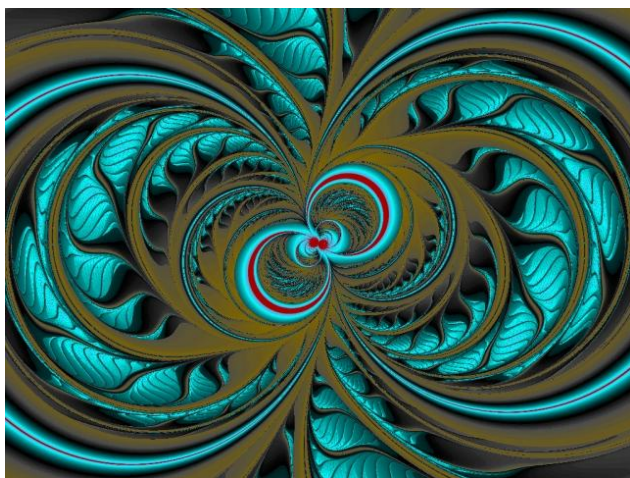
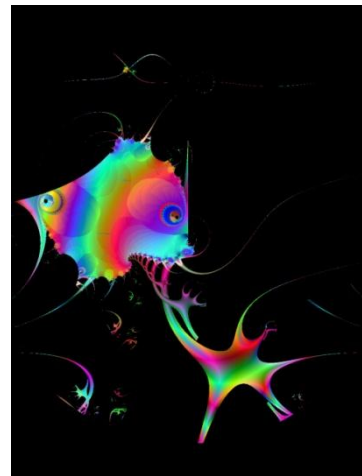
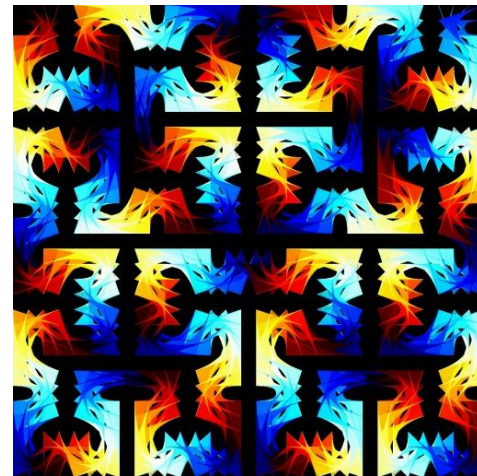
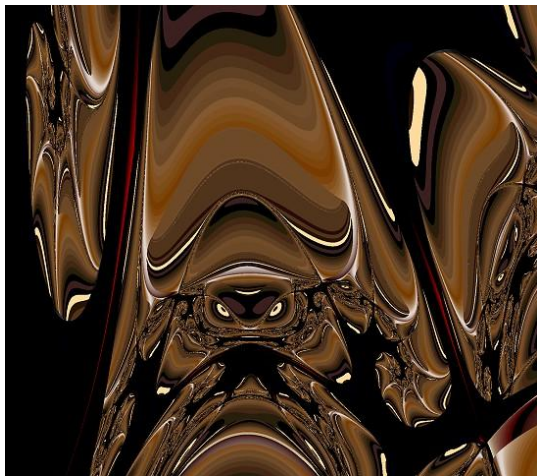
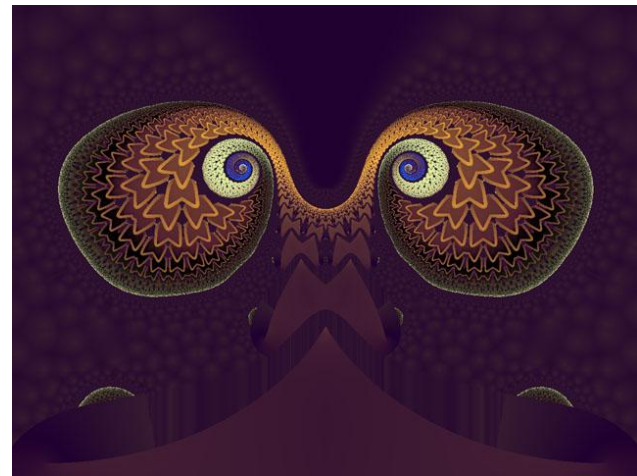
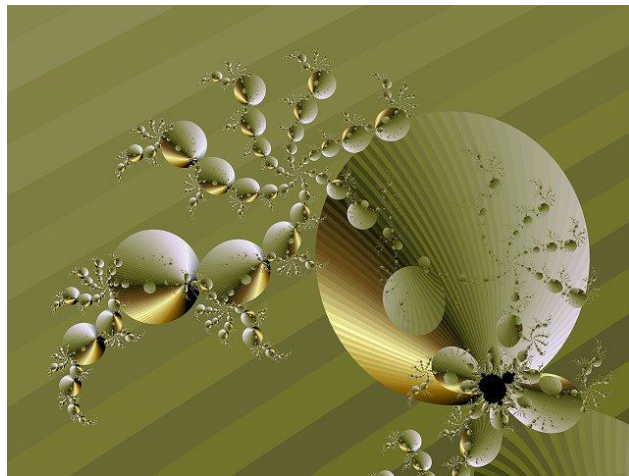
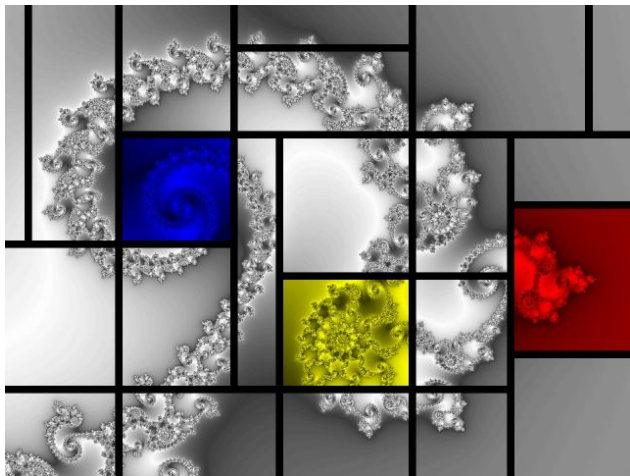


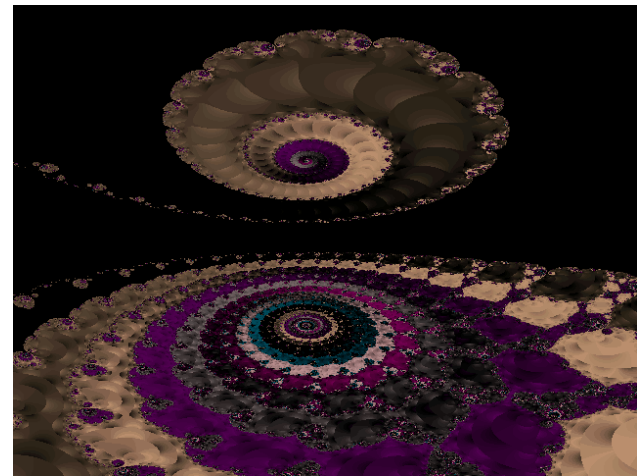
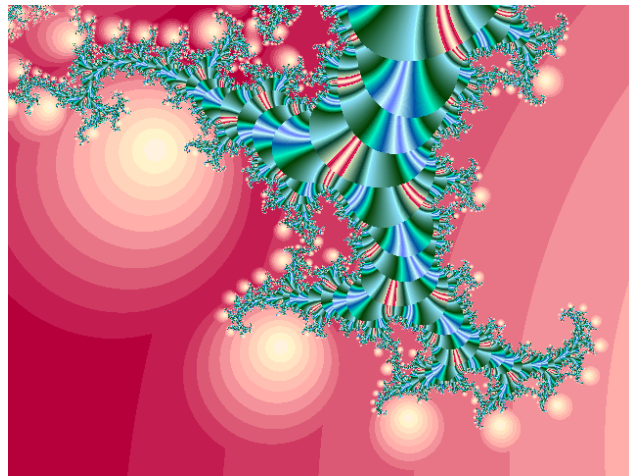
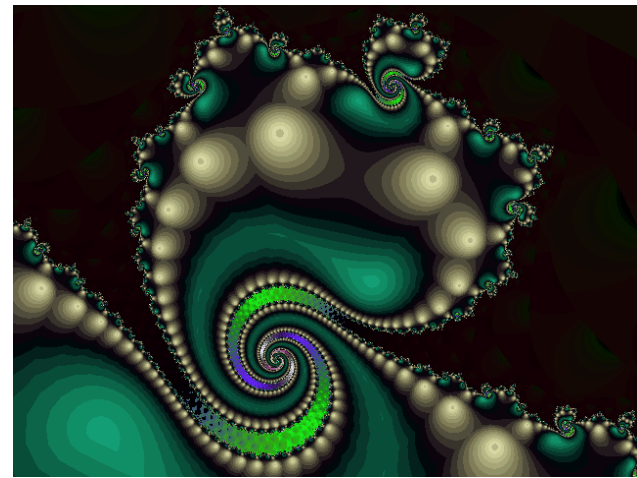
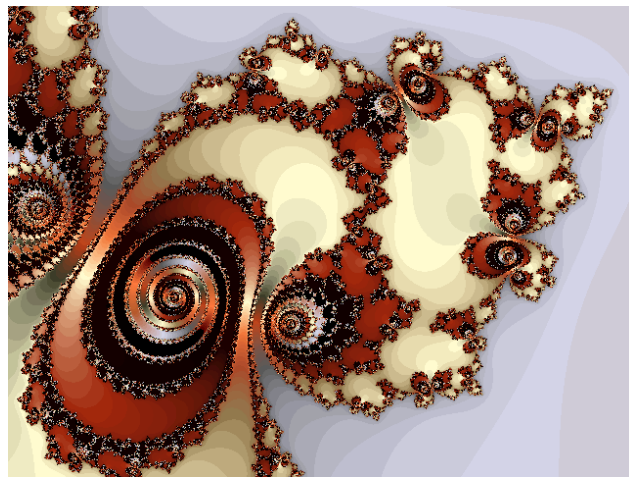
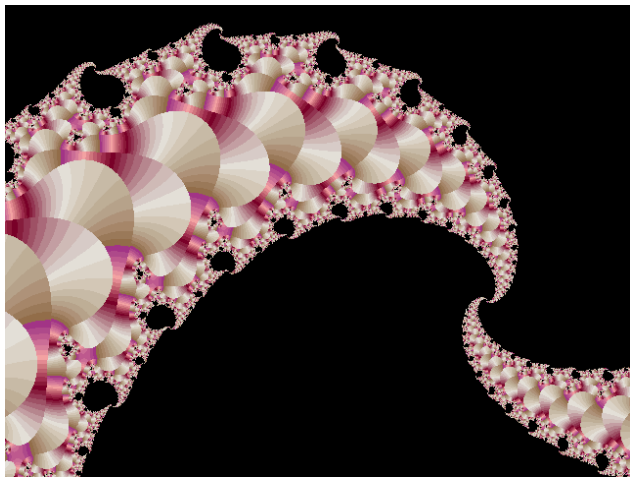
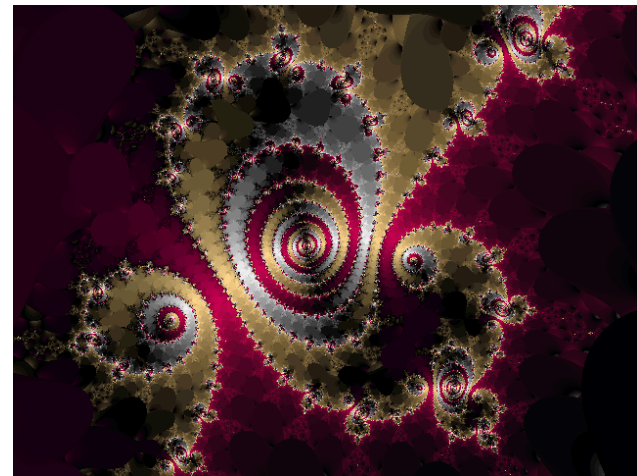
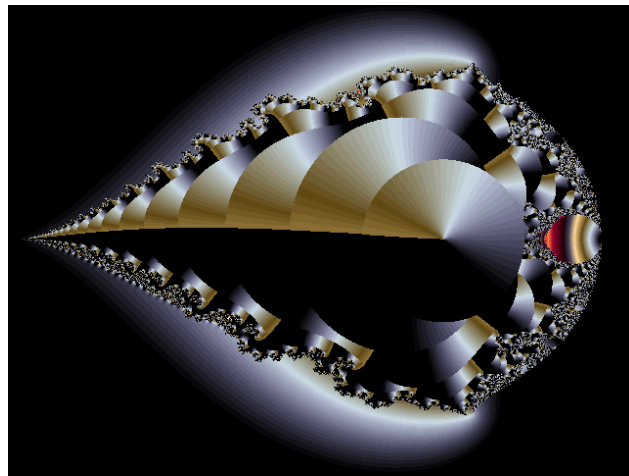
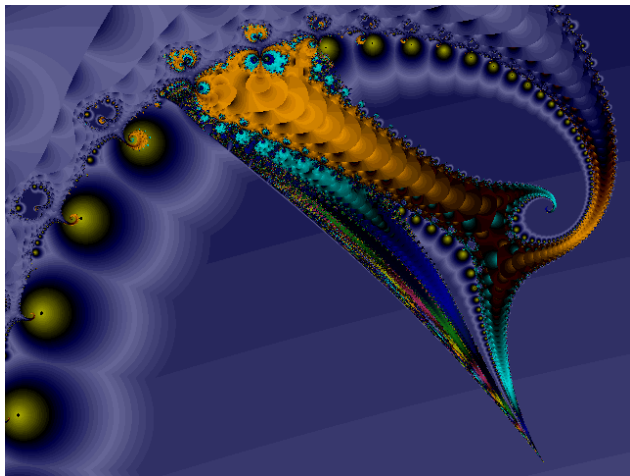


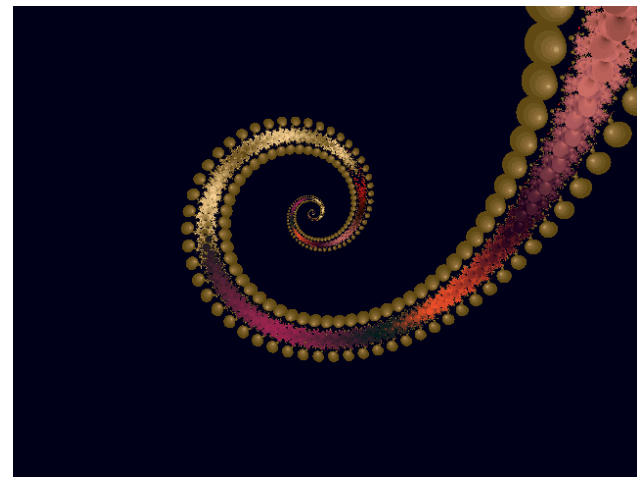
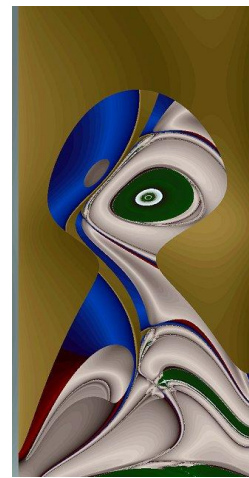
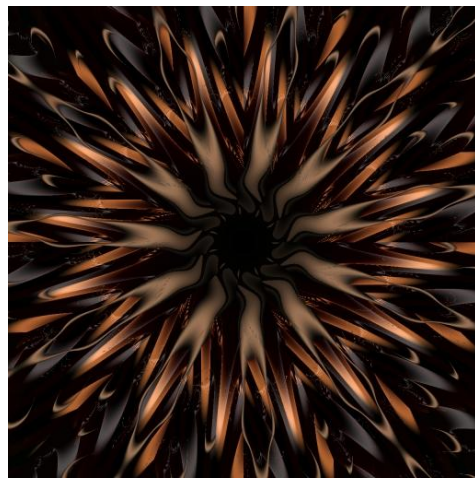
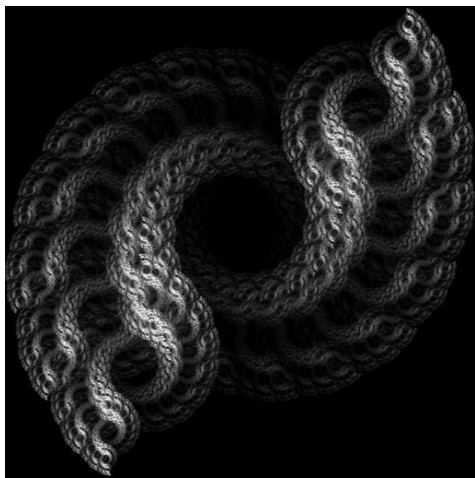
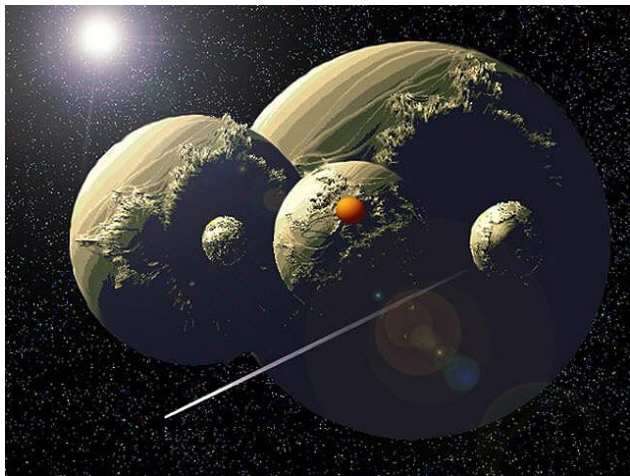








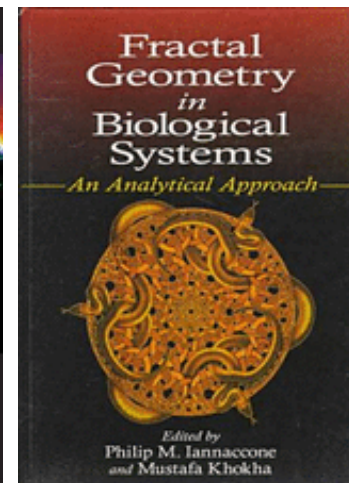
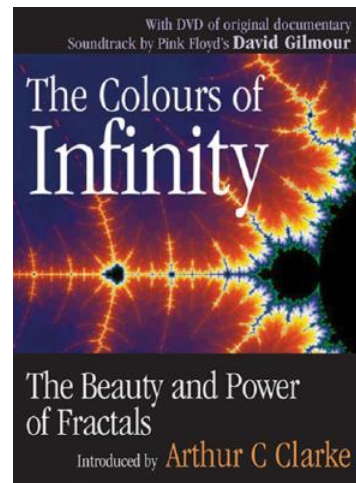
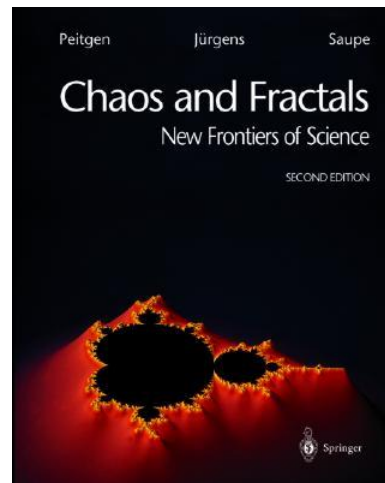
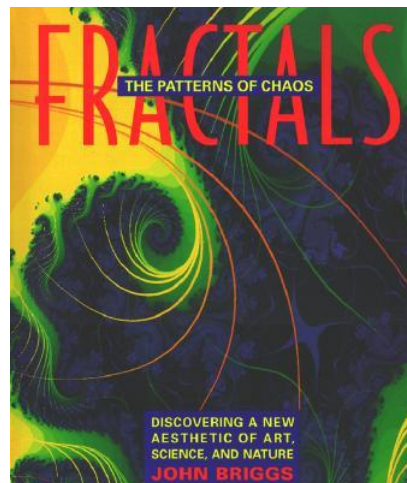
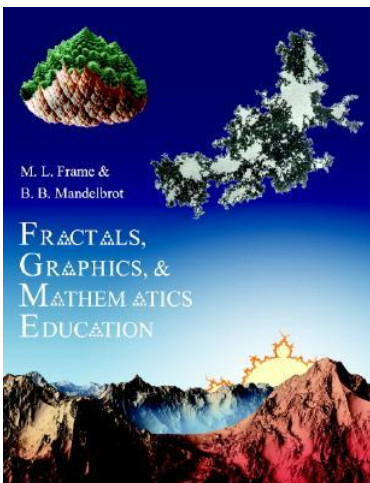
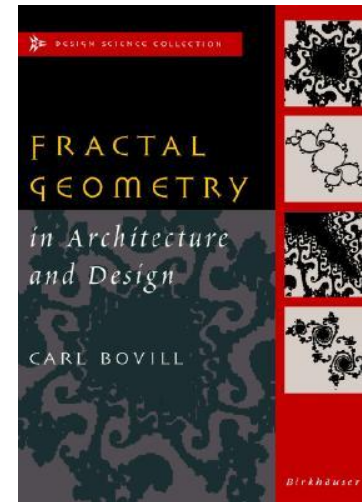
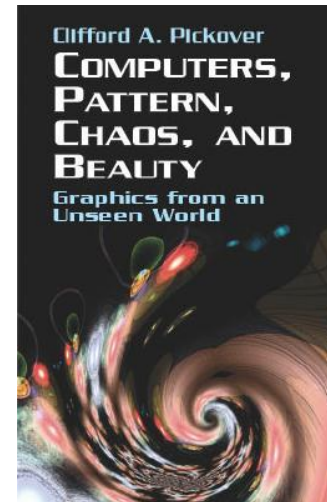
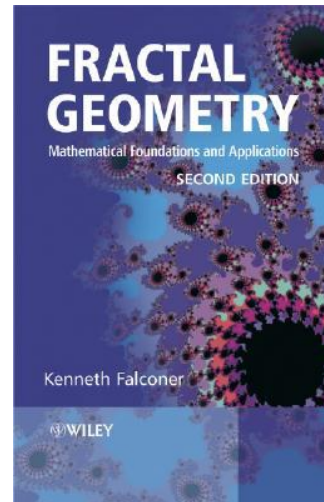
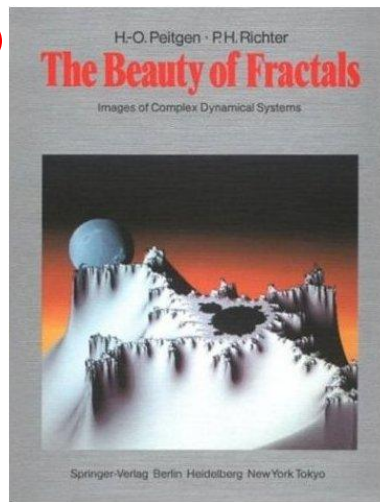
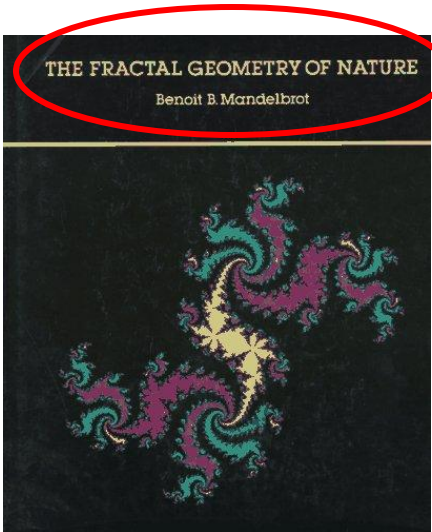




More on Fractals

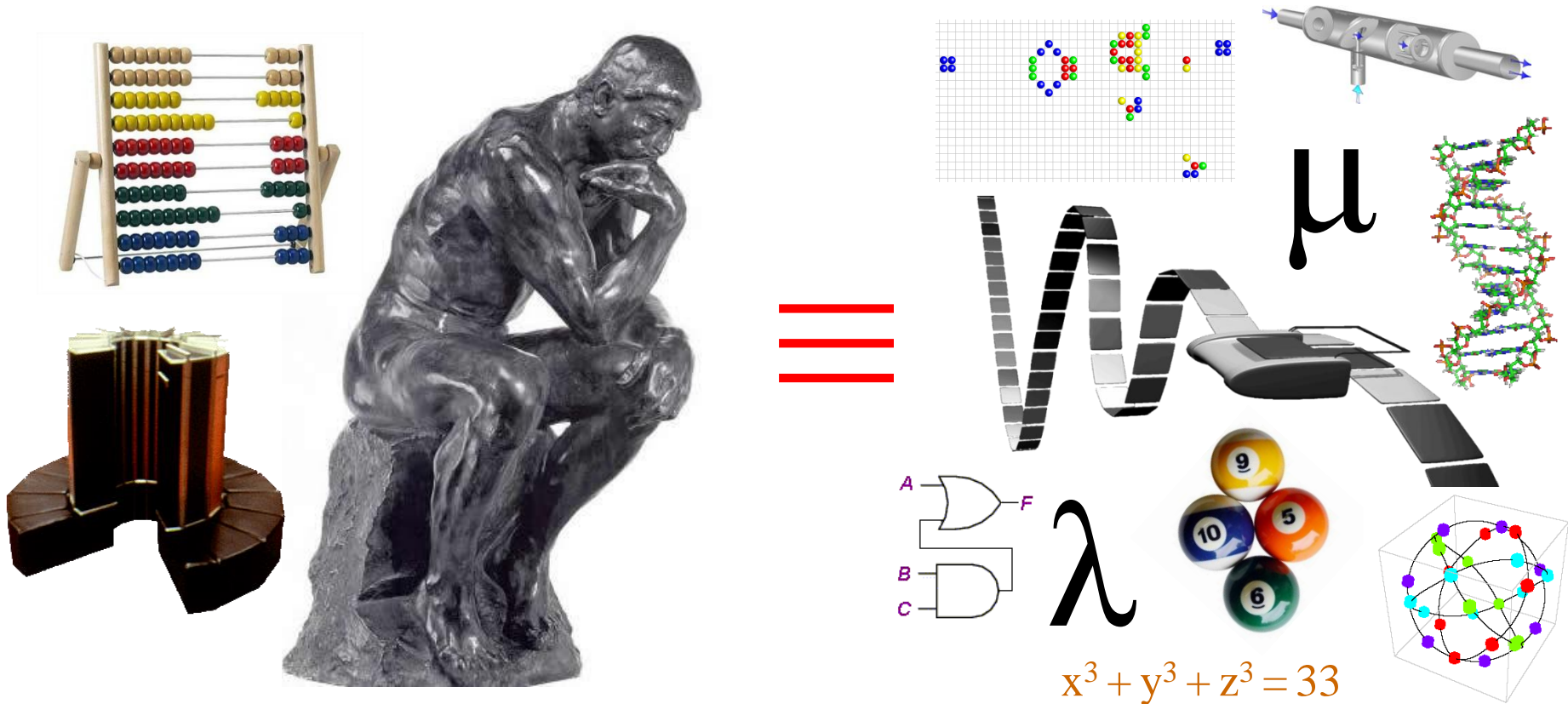
Fractal Art Contests: www.fractalartcontests.com

www.wikipedia.org/wiki/Mandelbrot_set



The Church-Turing Thesis

Q: What does it mean “to be computable”?



The Church-Turing Thesis: Anything that is “intuitively computable” is also Turing-machine computable.

“Le Penseur”
Auguste Rodin, 1902
Musée Rodin, Paris

