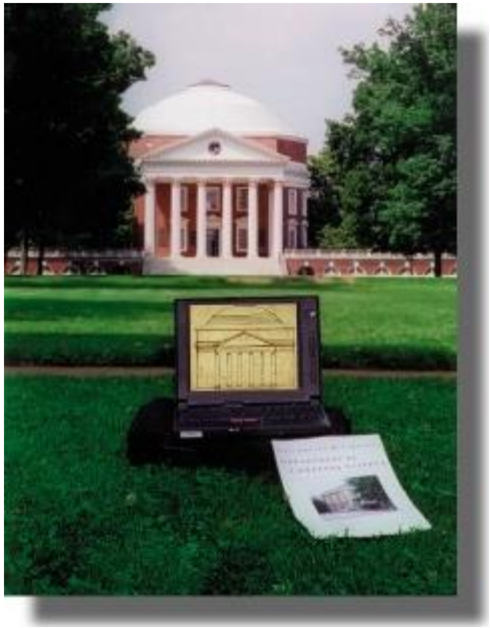


Theory of Computation CS3102

Gabriel Robins

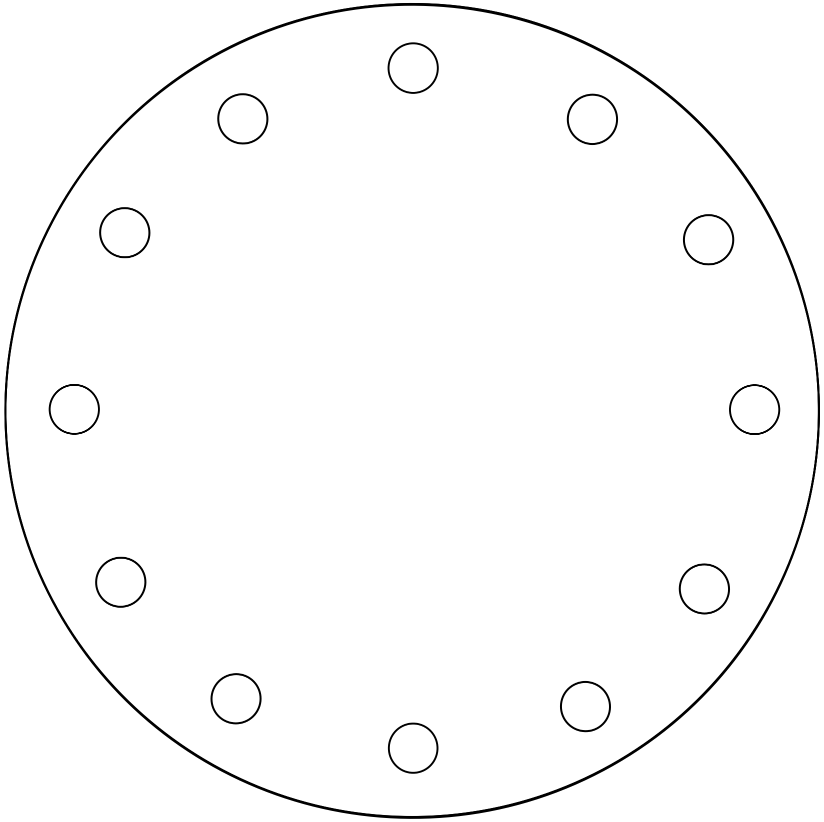
Department of

Computer Science

University of Virginia

www.cs.virginia.edu/robins/theory

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?

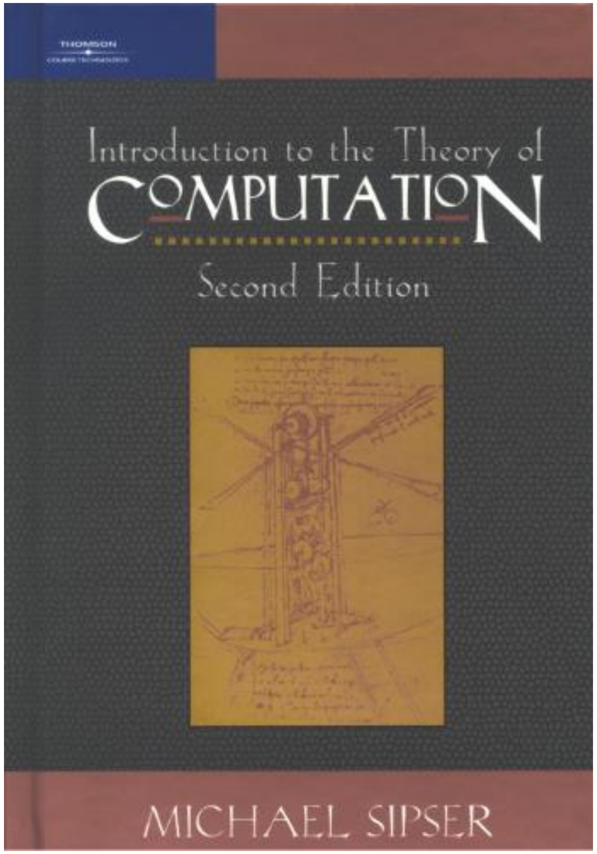


- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Theory of Computation (CS3102) - Textbook

Textbook:

Introduction to the Theory of Computation, by Michael Sipser (MIT), 2nd Edition, 2005



Good Articles / videos:

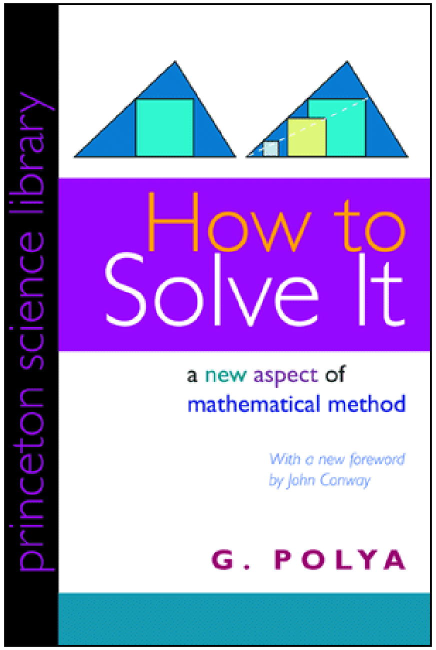
www.cs.virginia.edu/~robins/CS_readings.html

Theory of Computation (CS3102)

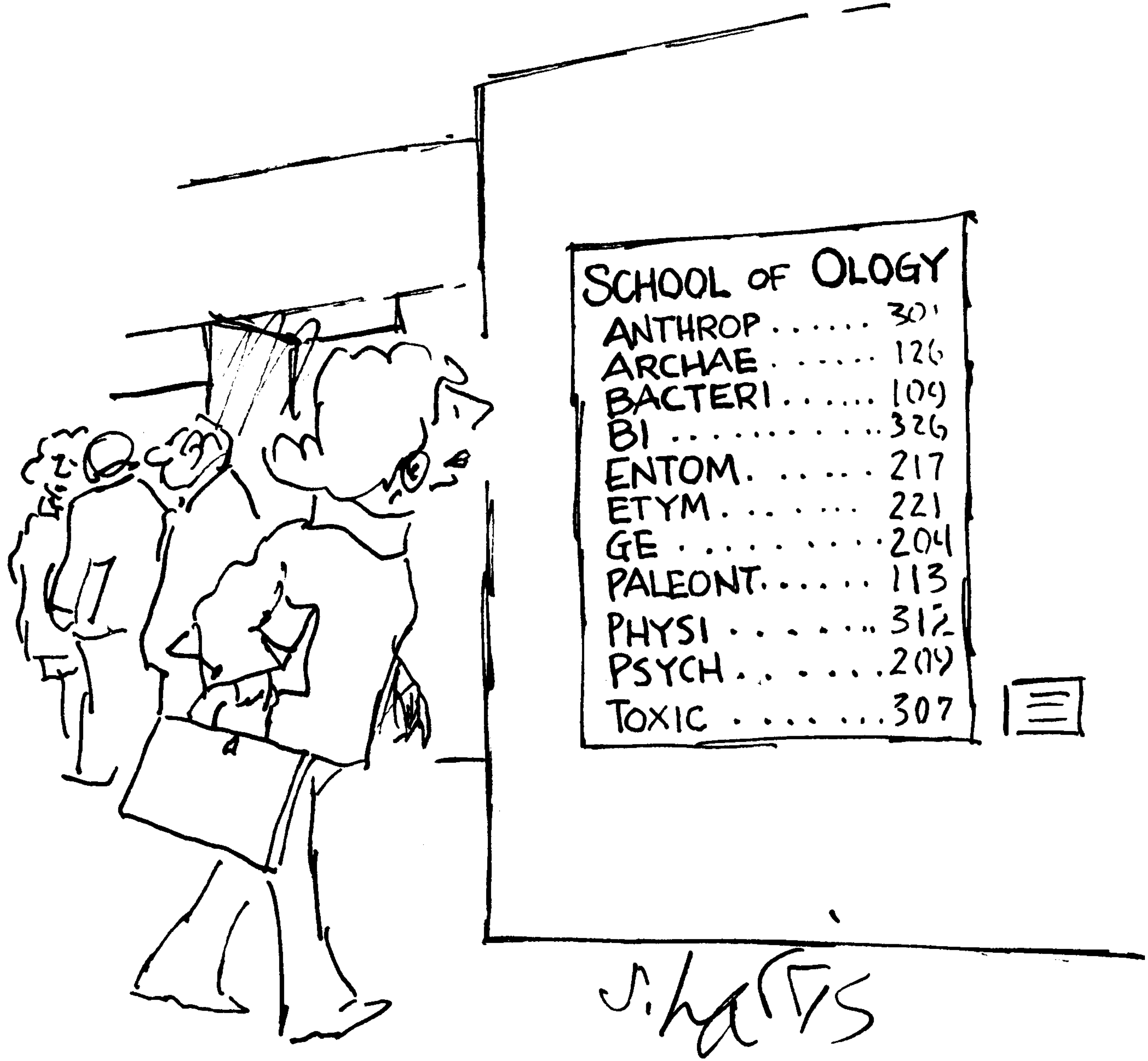
Required reading:

How to Solve It, by George Polya (MIT), Princeton University Press, 1945

- A classic on problem solving



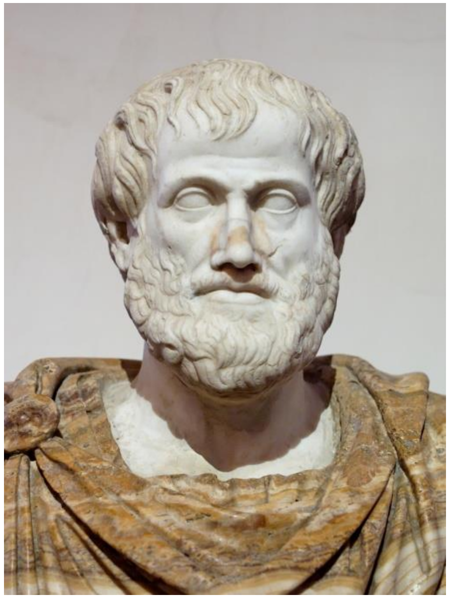
George Polya (1887-1985)

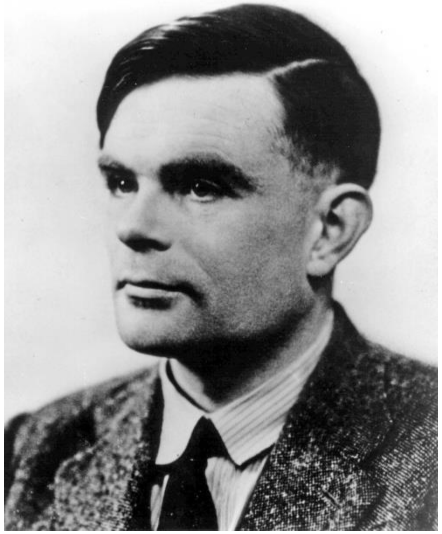

SCHOOL OF OLOGY
ANTHROP 301
ARCHAE 126
BACTERI 109
BI 326
ENTOM 217
ETYM 221
GE 204
PALEONT 113
PHYSI 312
PSYCH 209
TOXIC 307
J. Harris

Theory of Computation (CS3102) - Syllabus

A brief history of computing:

- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan, Babbage, Ada Agusta
- Venn, Carroll, Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Godel, Church, Turing, von Neumann
- Shannon, Kleene, Chomsky

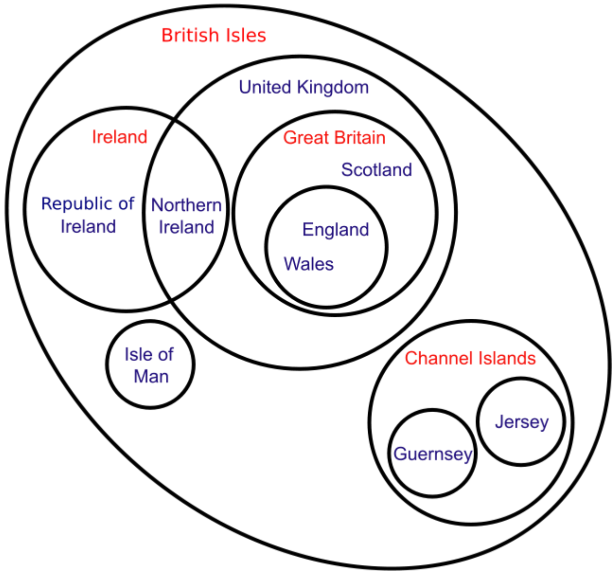


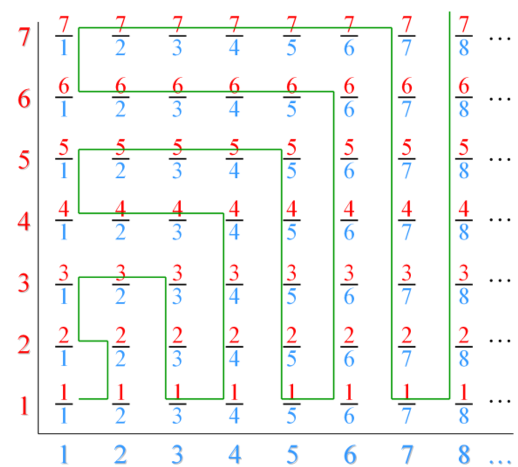


Theory of Computation Syllabus (continued)

Fundamentals:

- Set theory
- Predicate logic
- Formalisms and notation
- Infinities and countability
- Dovetailing / diagonalization
- Proof techniques
- Problem solving
- Asymptotic growth
- Review of graph theory

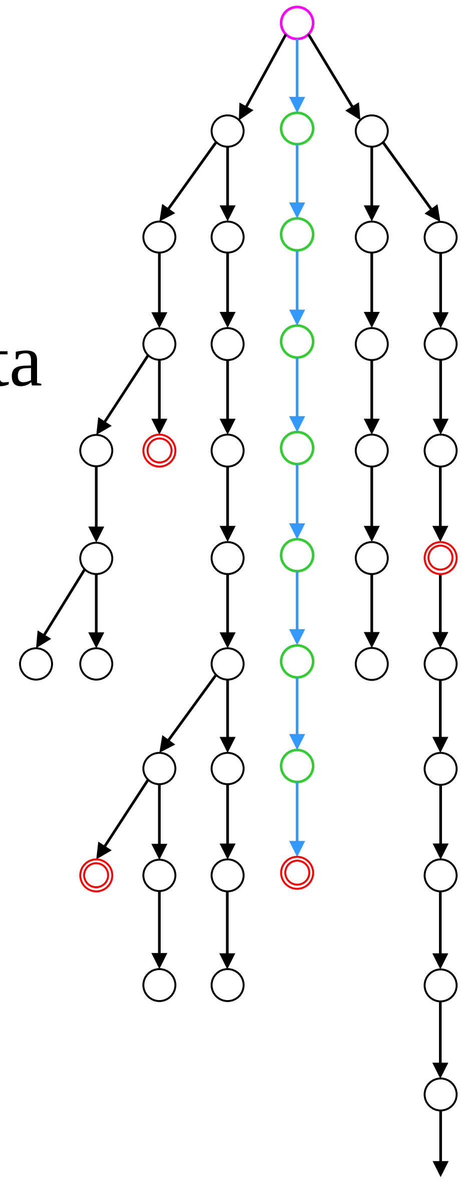




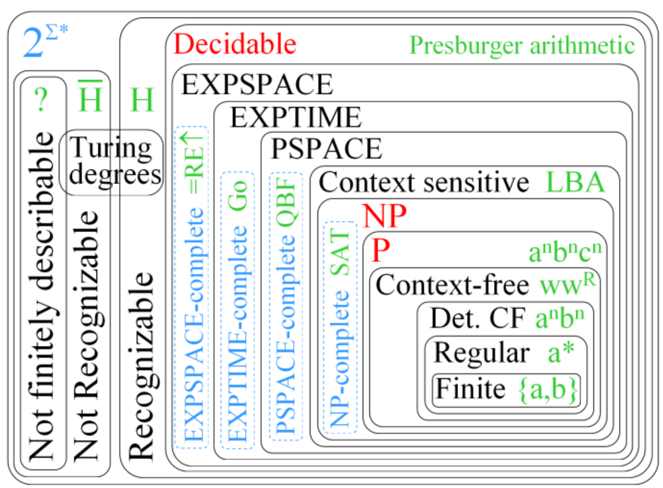
Theory of Computation Syllabus (continued)

Formal languages and machine models:

- The Chomsky hierarchy
- Regular languages / finite automata
- Context-free grammars / pushdown automata
- Unrestricted grammars / Turing machines
- Non-determinism
- Closure operators
- Pumping lemmas
- Non-closures
- Decidable properties



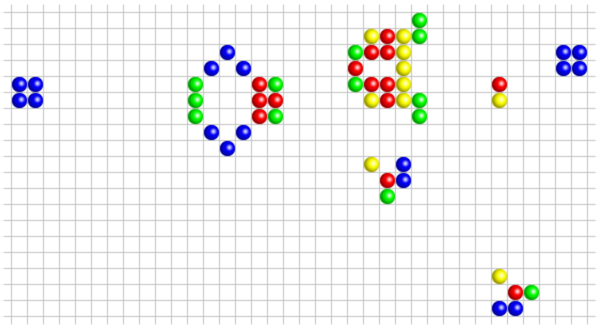
The Extended Chomsky Hierarchy

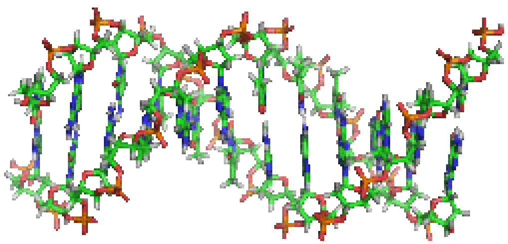


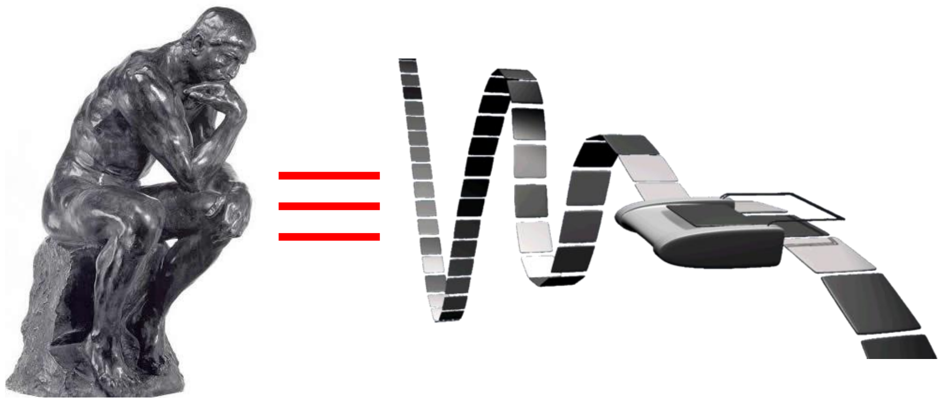
Theory of Computation Syllabus (continued)

Computability and undecidability:

- Basic models
- Modifications and extensions
- Computational universality
- Decidability
- Recognizability
- Undecidability
- Church-Turing thesis
- Rice’s theorem



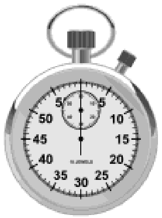


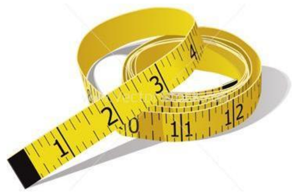


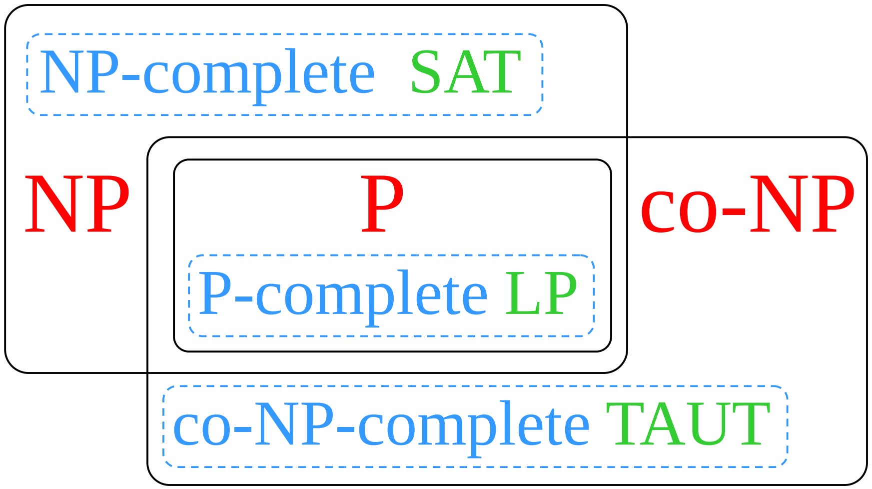
Theory of Computation Syllabus (continued)

NP-completeness:

- Resource-constrained computation
- Complexity classes
- Intractability
- Boolean satisfiability
- Cook-Levin theorem
- Transformations
- Graph clique problem
- Independent sets
- Hamiltonian cycles
- Colorability problems
- Heuristics

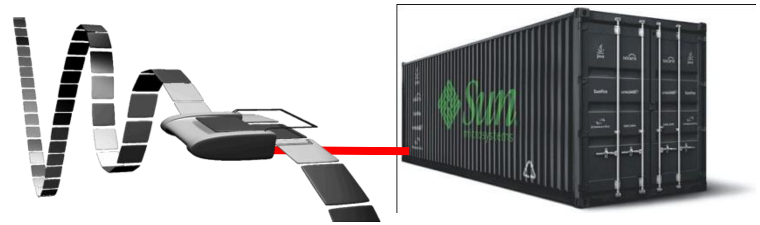




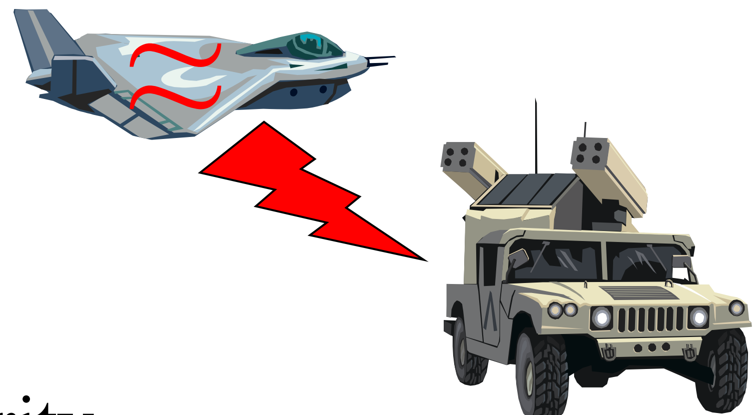


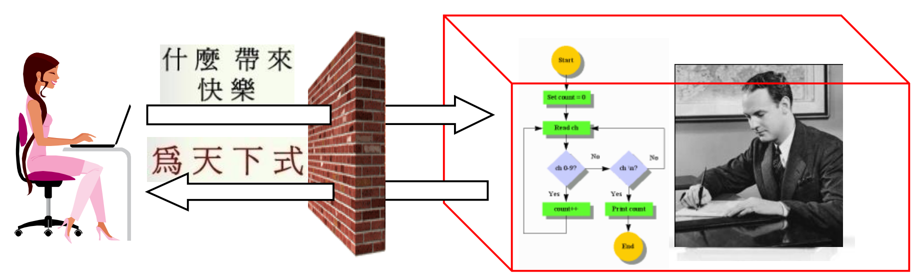
Theory of Computation Syllabus (continued)

Other topics (as time permits):

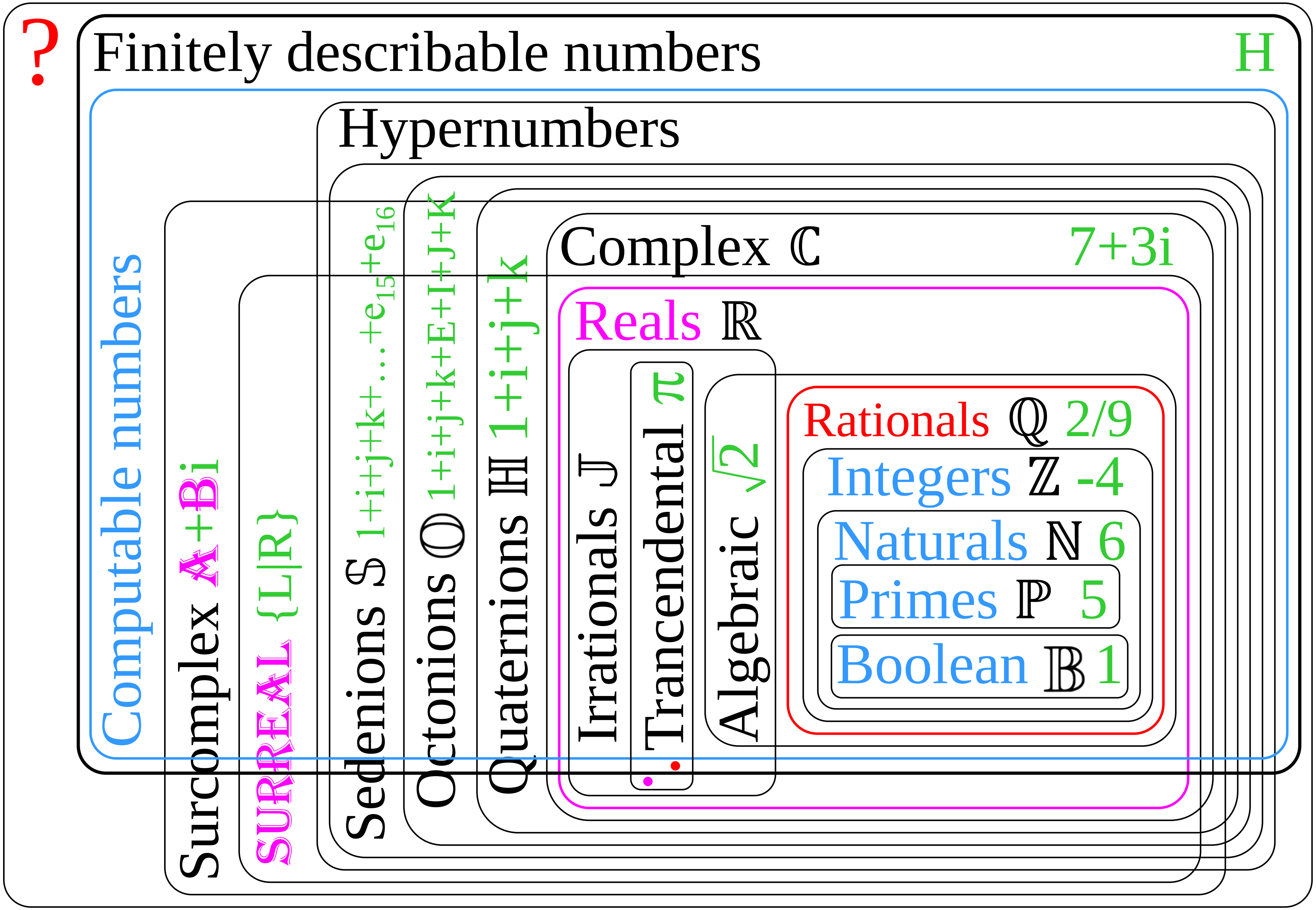


- Generalized number systems
- Oracles and relativization
- Zero-knowledge proofs
- Cryptography & mental poker
- The Busy Beaver problem
- Randomness and compressibility
- The Turing test
- AI and the Technological Singularity





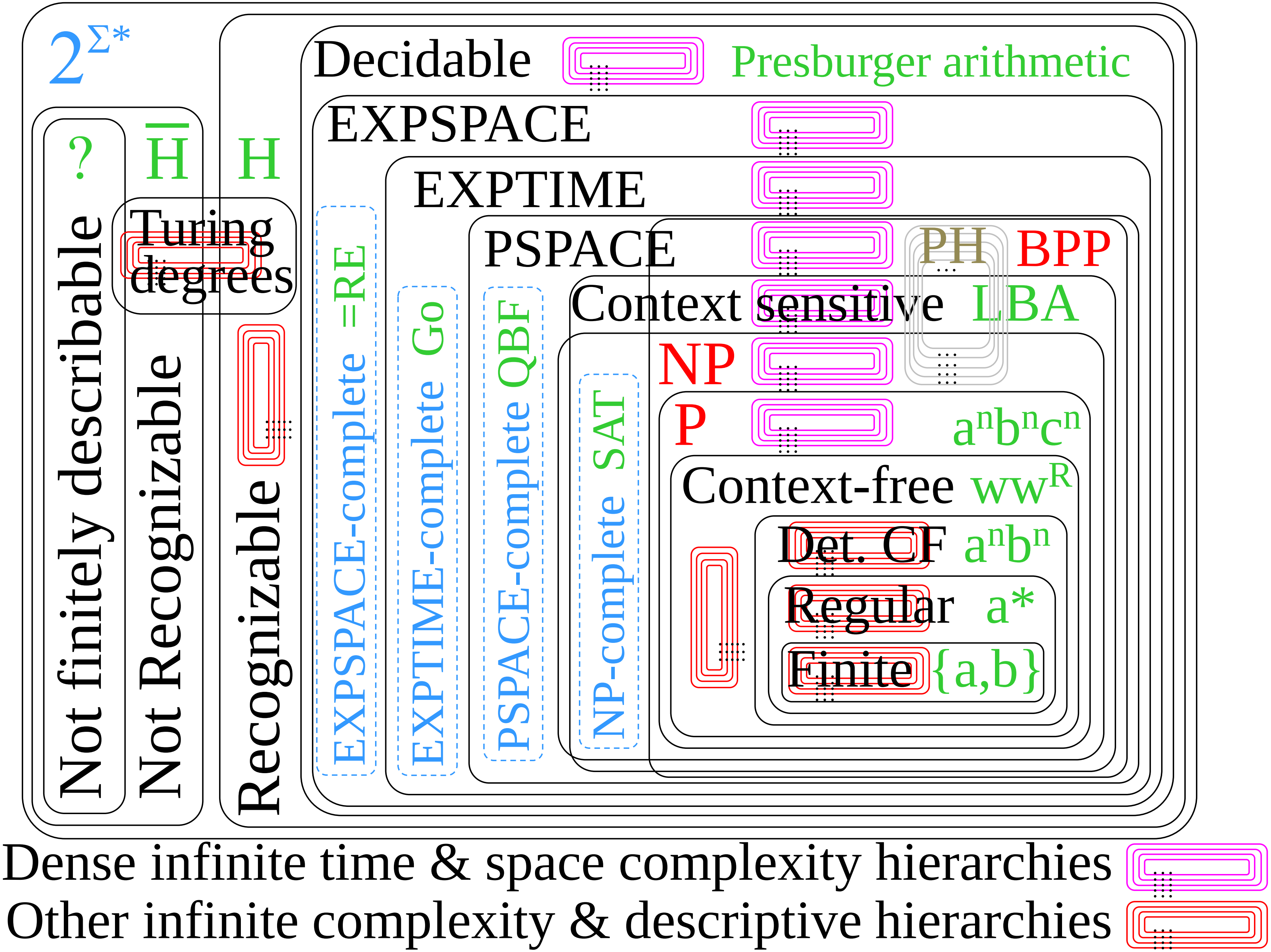
Generalized Numbers



Theorem: some real numbers are not finitely describable!

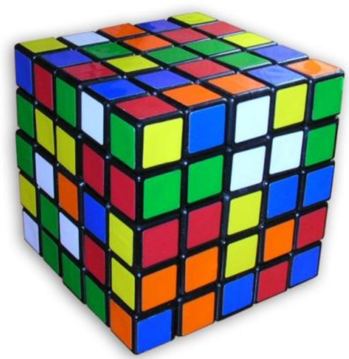
Theorem: some finitely describable real numbers are not computable!

The Extended Chomsky Hierarchy



Overarching Philosophy

- Focus on the "big picture" & "scientific method"
- Emphasis on problem solving & creativity
- Discuss applications & practice
- A primary objective: have fun!

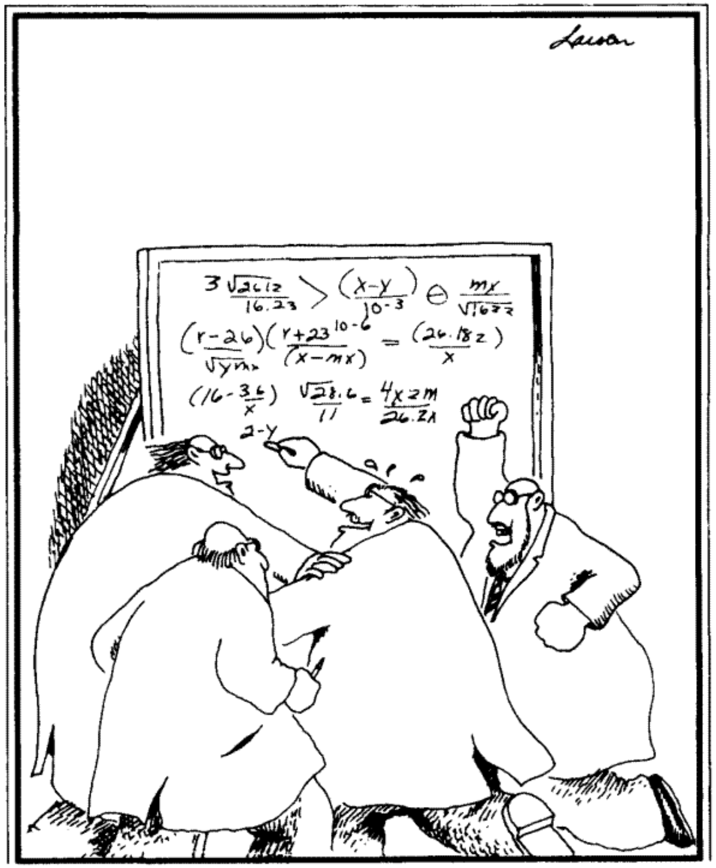


Prerequisites

- Some discrete math & algorithms knowldege
- Ideally, should have taken CS2102
- Course will "bootstrap" (albeit quickly) from first principles
- Critical: Tenacity, patience

Course Organization

- Exams: probably take home
  - Decide by vote
  - Flexible exam schedule
- Problem sets:
  - Lots of problem solving
  - Work in groups!
  - Not formally graded
  - Most exam questions will come from these sets!
- Readings: papers / videos / books
- Extra credit problems
  - In class & take-home
  - Find mistakes in slides, handouts, etc.
- Course materials posted on Web site
  www.cs.virginia.edu/robins/theory



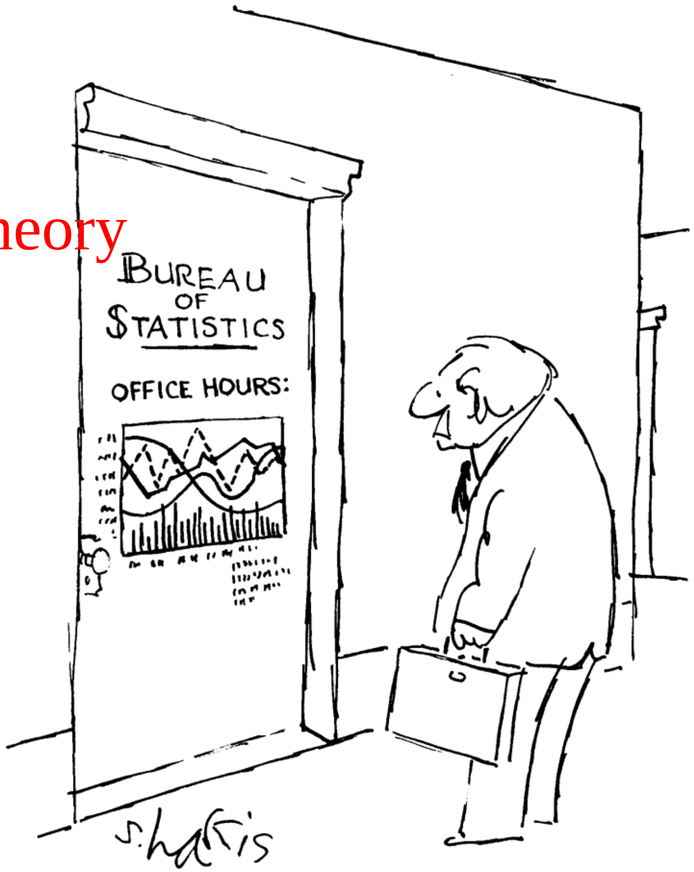
"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"

Contact Information

Professor	Gabriel Robins
Office:	409 Rice Hall
Phone:	(434) 982-2207
Email:	robins@cs.virginia.edu
Web:	www.cs.virginia.edu/robins
	www.cs.virginia.edu/robins/theory

Office hours: after class

- Any other time
- By email (preferred)
- By appointment
- Q&A blog posted on class Web site

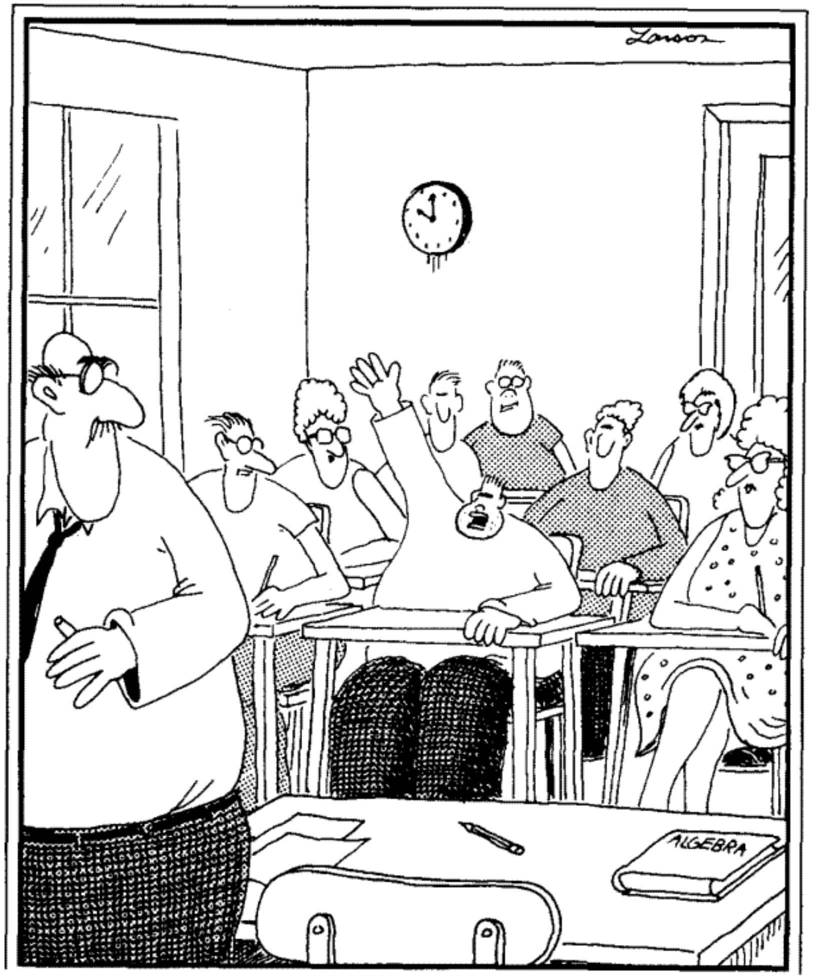


Grading Scheme

- Midterm 35%
- Final 35%
- Readings 30%
- Extra credit 10%

Best strategy:

- Solve lots of problems!
- Do lots of readings / EC!



"Mr. Osborne, may I be excused? My brain is full."

Course Readings

www.cs.virginia.edu/robins/CS_readings.html

Goal: broad exposure to lots of cool ideas & technologies!

- Required: total of at least 50 items over the semester
- Diversity: minimums in each of 3 categories:
    1. Minimum of 20 videos
    2. Minimum of 20 papers / Web sites
    3. Minimum of 10 books
- More than 50 is even better! (extra credit)
- Some required items in each category
    - Remaining "elective" items should be a diverse mix
- Email all submissions to: homework.cs3102@gmail.com

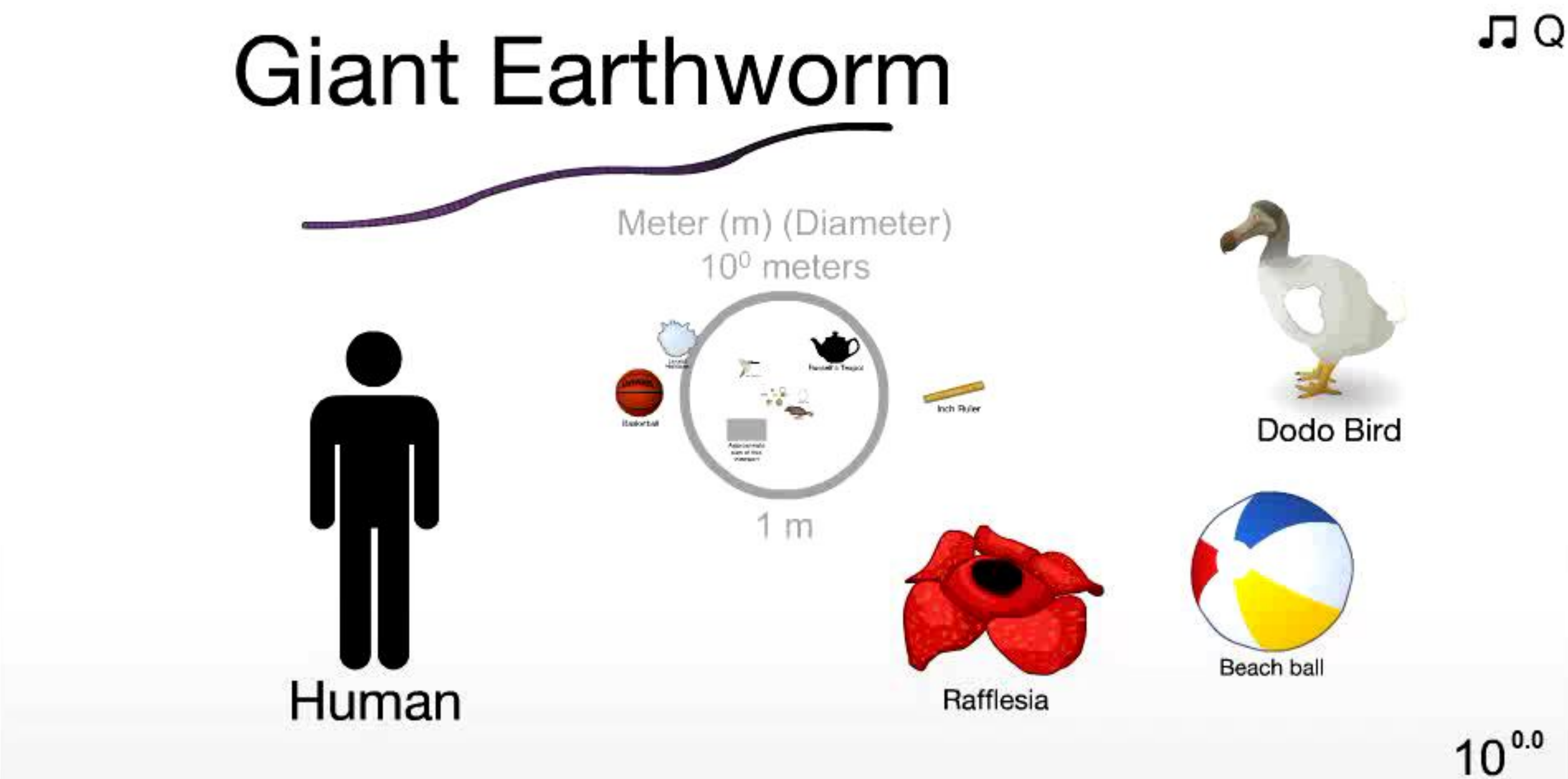
Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Required videos:
  - Last Lecture, Randy Pausch, 2007
  - Time Management, Randy Pausch, 2007
  - Powers of Ten, Charles and Ray Eames, 1977

Required Reading

- "Scale of the Universe", Cary and Michael Huang, 2012

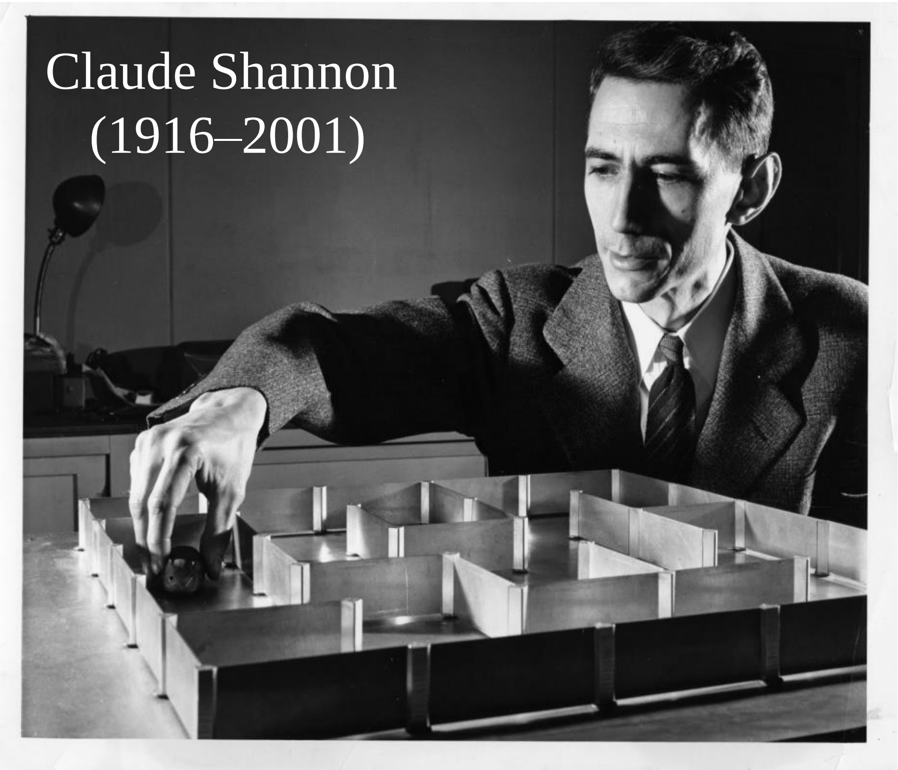


- 10^{-24} to 10^{26} meters $\Rightarrow$ 50 orders of magnitude!

Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- More required videos:
  - Claude Shannon - Father of the Information Age, UCTV
  - The Pattern Behind Self-Deception, Michael Shermer, 2010

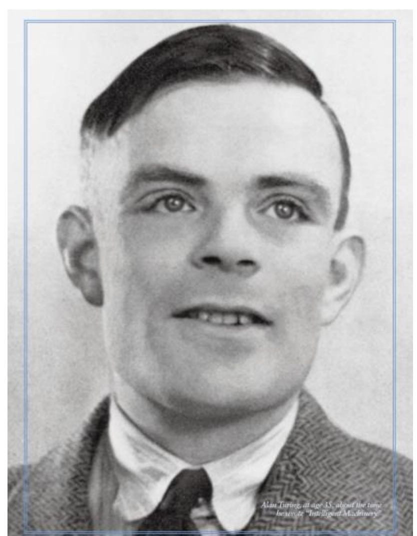


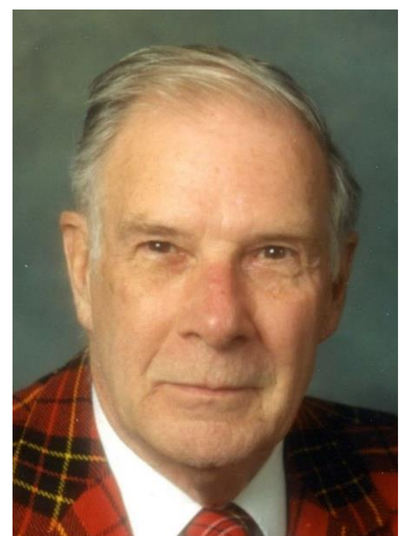
Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Required articles:
  - Decoding an Ancient Computer, Freeth, 2009
  - Alan Turing’s Forgotten Ideas, Copeland and Proudfoot, 1999
  - You and Your Research, Richard Hamming, 1986
  - Who Can Name the Bigger Number, Scott Aaronson, 1999


Antikythera computer, 200BC


Alan Turing


Richard Hamming


Scott Aaronson

Extra credit!

Basic Concepts and Notation

Gabriel Robins

Required reading

"When I use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean -- neither more nor less."

A **set** is formally an undefined term, but intuitively it is a (possibly empty) collection of arbitrary objects. A set is usually denoted by curly braces and some (optional) restrictions. Examples of sets are {1,2,3}, {hi, there}, and {k | k is a perfect square}. The symbol $\in$ denotes set **membership**, while the symbol $\notin$ denotes set **non-membership**; for example, $7 \in \{p \mid p \text{ prime}\}$ states that 7 is a prime number, while $q \notin \{0,2,4,6,\ldots\}$ states that q is not an even number. Some **common sets** are denoted by special notation:

The **natural numbers**: $\mathbb{N} = \{1,2,3,\ldots\}$

The **integers**: $\mathbb{Z} = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$

The **rational numbers**: $\mathbb{Q} = \{\frac{a}{b} \mid a,b \in \mathbb{Z},\ b \neq 0\}$

The **real numbers**: $\mathbb{R} = \{x \mid x \text{ is a real number}\}$

The **empty set**: $\varnothing = \{\}$

http://www.cs.virginia.edu/robins/cs3102/basics.pdf

JOHANNES KEPLER'S UPHILL BATTLE

Discrete Math Review Slides

Required reading

Symbolic Logic

Def: *proposition* - statement either true (T) or false (F)

Ex: $1+1=2$

$2+2=3$

$3<7$

$x+4=5$

"today is Monday"

Boolean Functions

- "and" $\wedge$
- "or" $\vee$
- "not" $\neg$
- "xor" $\oplus$
- "nand"
- "nor"
- "implication" $\Rightarrow$
- "equivalence" $\Leftrightarrow$

Logical Implication

- "implies" $\Rightarrow$

Truth table:

p	q	p⇒q
T	T	T
T	F	F
F	T	T
F	F	T

Ex: $(x<0) \wedge (x>0) \Rightarrow (x=0)$

$1<x<y \Rightarrow x^3<y^3$

"today is Sunday" $\Rightarrow 1+1=3$

Logical Equivalence

- "biconditional" $\Leftrightarrow$

or "if and only if" ("iff")

or "necessary and sufficient"

or "logically equivalent" $\equiv$

Truth table:

p	q	p⇔q
T	T	T
T	F	F
F	T	F
F	F	T

Ex: $p \Leftrightarrow p$

$[(x=0) \vee (y=0)] \Leftrightarrow (xy=0)$

$\min(x,y)=\max(x,y) \Leftrightarrow x=y$

Predicates

Def: *predicate* - a function or formula involving some variables

Ex: let P(x) = "x > 3"

x is the variable

"x>3" is the predicate

P(5)

P(1)

Ex: $Q(x,y,z) =$ "$x^2+y^2=z^2$"

Q(2,3,4)

Q(3,4,5)

Quantifiers

- Universal: "for all" $\forall$

$\forall x\ P(x)$

$\Leftrightarrow P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots$

Ex: $\forall x \quad x<x+1$

$\forall x \quad x<x^3$

- Existential: "there exists" $\exists$

$\exists x\ P(x)$

$\Leftrightarrow P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots$

Ex: $\exists x \quad x=x^2$

$\exists x \quad x<x-1$

Combinations:

$\forall x \exists y \quad y>x$

Sets

Def: *set* - an unordered collection of elements

Ex: {1, 2, 3} or {hi, there}

Venn Diagram:

Def: two sets are *equal* iff they contain the same elements

Ex: {1, 2, 3} = {2, 3, 1}

$\{0\} \neq \{1\}$

{3, 5} = {3, 5, 3, 3, 5}

Common Sets

Naturals: N = {1, 2, 3, 4, ...}

Integers: Z = {...,-2, -1, 0, 1, 2,...}

Rationals: $Q = \{\frac{a}{b} \mid a,b \in Z,\ b\neq 0\}$

Reals: $\Re$ = {x | x a real #}

Empty set: $\emptyset$ = {}

Z^+ = non-negative integers

$\Re^-$ = non-positive reals, etc.

Subsets

- Subset notation: $\subseteq$

$S \subseteq T \Leftrightarrow (x\in S \Rightarrow x\in T)$

- Proper subset: $\subset$

$S \subset T \Leftrightarrow ((S \subseteq T) \wedge (S\neq T))$

$S=T \Leftrightarrow ((T \subseteq S) \wedge (S \subseteq T))$

$\forall S \quad \emptyset \subseteq S$

$\forall S \quad S \subseteq S$

Union and Intersection

- Union: $\cup$

$S\cup T=\{x \mid x\in S \vee x\in T\}$

- Intersection: $\cap$

$S\cap T=\{x \mid x\in S \wedge x\in T\}$

Universal set and complement

- Universal set: U (everything)
- Set complement: S' or $\bar{S}$

$S' = \{x \mid x\notin S\} = U - S$

- Disjoint sets: $S\cap T=\emptyset$

$S - T = S \cap T'$

$S - S = \emptyset$

DeMorgan's Laws

$(S\cup T)' = S'\cap T'$

$(S\cap T)' = S'\cup T'$

Boolean logic version:

$(X\wedge Y)'=X'\vee Y'$

$(X\vee Y)'=X'\wedge Y'$

Function Types

- One-to-one function: "1-1"

$a,b\in S \wedge a\neq b \Rightarrow f(a)\neq f(b)$

Ex: $f: \Re\to\Re$, $f(x)=2x$ is 1-1

$g(x)=x^2$ is not 1-1

- Onto function:

$\forall t \in T\ \exists s\in S \ni f(s)=t$

Ex: $f: Z\to Z$, $f(x)=13-x$ is onto

$g(x)=x^2$ is not onto

1-to-1 Correspondence

- 1-to-1 correspondence: $f: S\leftrightarrow T$

f is both 1-1 and onto

Ex: $f: \Re\leftrightarrow\Re \ni f(x)=x$ (identity)

$h: N\leftrightarrow Z \ni h(x)= \frac{x-1}{2}$, x odd, $\frac{-x}{2}$, x even.

Generalized Cardinality

- S is at least as large as T:

$|S \geq T| \Rightarrow \exists f: S\to T$, f onto

i.e., "S covers T"

Ex: $r: \Re\to Z$, $r(x)=\text{round}(x)$

$\Rightarrow |\Re|\geq|Z|$

- S and T have same cardinality:

$|S=|T| \Rightarrow |S|\geq|T| \wedge |T|\geq|S$

or

$\exists$ 1-1 correspondence $S\leftrightarrow T$

- Generalizes finite cardinality:

$\{1, 2, 3, 4, 5\} \geq \{a, b, c\}$

Infinite Sets

- Infinite set: $|S| > k\ \forall k\in Z$

or

$\exists$ 1-1 corres. $f: S\leftrightarrow T$, $S\subset T$

Ex: {p | p prime}, $\Re$

- Countable set: $|S| \leq |N|$

Ex: $\emptyset$, {p | p prime}, N, Z

- S is strictly smaller than T:

$|S| < |T| \Rightarrow |S|\leq|T| \wedge |S|\neq|T|$

- Uncountable set: $|N| < |S|$

Ex: $|N| < \Re$

$|N| < [0,1] = \{x \mid x\in\Re,\ 0\leq x\leq 1\}$

Countability of Q

Thm: $\exists$ 1-1 correspondence $Q\leftrightarrow N$

Pf (dove-tailing):

$\frac{1}{6}\ \frac{2}{6}\ \frac{3}{6}\ \frac{4}{6}\ \frac{5}{6}\ \frac{6}{6}\ \ldots$

$\frac{1}{5}\ \frac{2}{5}\ \frac{3}{5}\ \frac{4}{5}\ \frac{5}{5}\ \frac{6}{5}\ \ldots$

$\frac{1}{4}\ \frac{2}{4}\ \frac{3}{4}\ \frac{4}{4}\ \frac{5}{4}\ \frac{6}{4}\ \ldots$

$\frac{1}{3}\ \frac{2}{3}\ \frac{3}{3}\ \frac{4}{3}\ \frac{5}{3}\ \frac{6}{3}\ \ldots$

$\frac{1}{2}\ \frac{2}{2}\ \frac{3}{2}\ \frac{4}{2}\ \frac{5}{2}\ \frac{6}{2}\ \ldots$

$\frac{1}{1}\ \frac{2}{1}\ \frac{3}{1}\ \frac{4}{1}\ \frac{5}{1}\ \frac{6}{1}\ \ldots$

Uncountability of the Reals

Thm: $|\Re| > |N|$

Pf (diagonalization):

Assume $\exists$ 1-1 corres. $f: \Re\leftrightarrow N$

Construct $X \in \Re$:

$f(1) = 2.718281828\ldots$

$f(2) = 1.414213562\ldots$

$f(3) = 1.618033989\ldots$

$X = 0.\ldots \neq f(K)\ \forall K\in N$

$\Rightarrow f$ not a 1-1 correspondence

$\Rightarrow$ contradiction

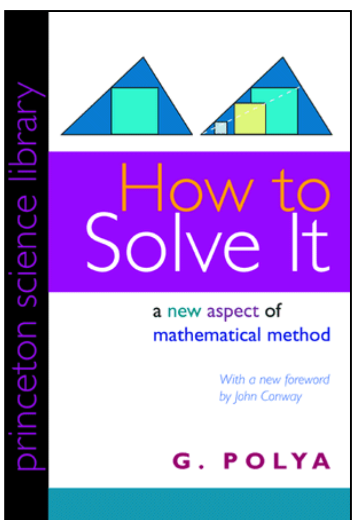
$\Rightarrow \Re$ is uncountable

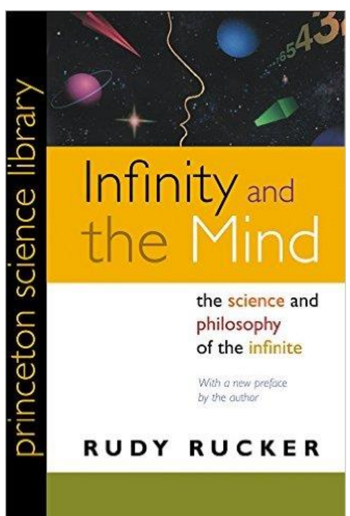
http://www.cs.virginia.edu/robins/cs3102/discrete_math_review_slides.pdf

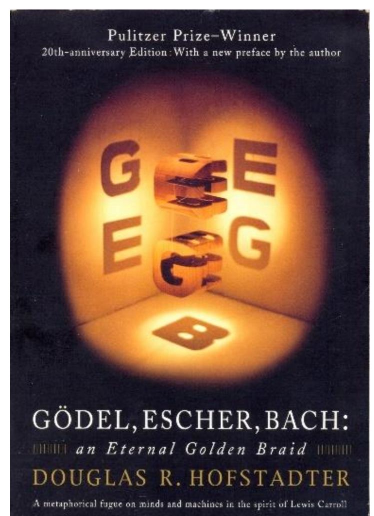
Required Readings

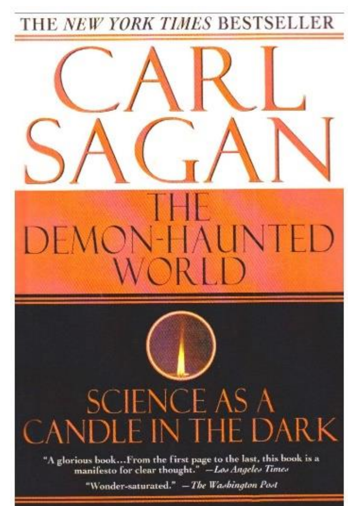
www.cs.virginia.edu/robins/CS_readings.html

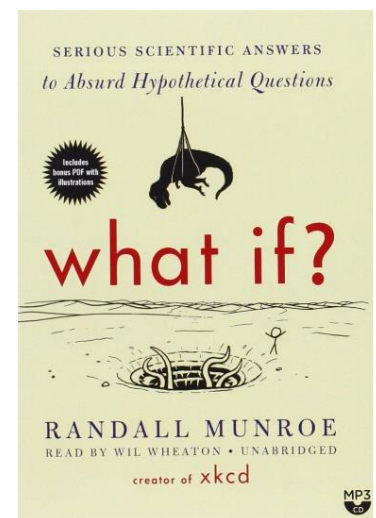
- Required books:
  - "How to Solve It", Polya, 1957
  - "Infinity and the Mind", Rucker, 1995
  - "Godel, Escher, Bach", Hofstadter, 1979
  - "The Demon-Haunted World", Sagan, 2009
  - "What If", Munroe, 2014











Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Remaining videos / articles / books are “electives”
- Pacing: at least 3 submissions per week (due 5pm Monday)
  - Policy intended to help you avoid “cramming”
- Length: 1-2 paragraphs per article / video
  1-2 pages per book
- Books are worth more credit than articles / videos
- Email all submissions to: homework.cs3102@gmail.com
- Additional readings beyond 50 are welcome! (extra credit)

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- Theory and Algorithms:
  - Who Can Name the Bigger Number, Scott Aaronson, 1999
  - The Limits of Reason, Gregory Chaitin, Scientific American, March 2006, pp. 74-81.
  - Breaking Intractability, Joseph Traub and Henryk Wozniakowski, Scientific American, January 1994, pp. 102-107.
  - Confronting Science's Logical Limits, John Casti, Scientific American, October 1996, pp. 102-105.
  - Go Forth and Replicate, Moshe Sipper and James Reggia, Scientific American, August 2001, pp. 34-43.
  - The Science Behind Sudoku, Jean-Paul Delahaye, Scientific American, June 2006, pp. 80-87.
  - The Traveler's Dilemma, Kaushik Basu, Scientific American, June 2007, pp. 90-95.

Other "Elective" Readings
www.cs.virginia.edu/robins/CS_readings.html

- Biological Computing:
  - Computing with DNA, Leonard Adleman, Scientific American, August 1998, pp. 54-61.
  - Bringing DNA Computing to Life, Ehud Shapiro and Yaakov Benenson, Scientific American, May 2006, pp. 44-51.
  - Engineering Life: Building a FAB for Biology, David Baker et al., Scientific American, June 2006, pp. 44-51.
  - Big Lab on a Tiny Chip, Charles Choi, Scientific American, October 2007, pp. 100-103.
  - DNA Computers for Work and Play, Macdonald et al, Scientific American, November 2007, pp. 84-91.

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- Quantum Computing:
  - Quantum Mechanical Computers, Seth Lloyd, Scientific American, 1997, pp. 98-104.
  - Quantum Computing with Molecules, Gershenfeld and Chuang, Scientific American, June 1998, pp. 66-71.
  - Black Hole Computers, Seth Lloyd and Jack Ng, Scientific American, November 2004, pp. 52-61.
  - Computing with Quantum Knots, Graham Collins, Scientific American, April 2006, pp. 56-63.
  - The Limits of Quantum Computers, Scott Aaronson, Scientific American, March 2008, pp. 62-69.
  - Quantum Computing with Ions, Monroe and Wineland, Scientific American, August 2008, pp. 64-71.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- History of Computing:
  - The Origins of Computing, Campbell-Kelly, Scientific American, September 2009, pp. 62-69.
  - Ada and the First Computer, Eugene Kim and Betty Toole, Scientific American, April 1999, pp. 76-81.
- Security and Privacy:
  - Malware Goes Mobile, Mikko Hypponen, Scientific American, November 2006, pp. 70-77.
  - RFID Powder, Tim Hornyak, Scientific American, February 2008, pp. 68-71.
  - Can Phishing be Foiled, Lorrie Cranor, Scientific American, December 2008, pp. 104-110.

Other “Elective” Readings
www.cs.virginia.edu/robins/CS_readings.html

- Future of Computing:
  - Microprocessors in 2020, David Patterson, Scientific American, September 1995, pp. 62-67.
  - Computing Without Clocks, Ivan Sutherland and Jo Ebergen, Scientific American, August 2002, pp. 62-69.
  - Making Silicon Lase, Bahram Jalali, Scientific American, February 2007, pp. 58-65.
  - A Robot in Every Home, Bill Gates, Scientific Am, January 2007, pp. 58-65.
  - Ballbots, Ralph Hollis, Scientific American, October 2006, pp. 72-77.
  - Dependable Software by Design, Daniel Jackson, Scientific American, June 2006, pp. 68-75.
  - Not Tonight Dear - I Have to Reboot, Charles Choi, Scientific American, March 2008, pp. 94-97.
  - Self-Powered Nanotech, Zhong Lin Wang, Scientific American, January 2008, pp. 82-87.

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

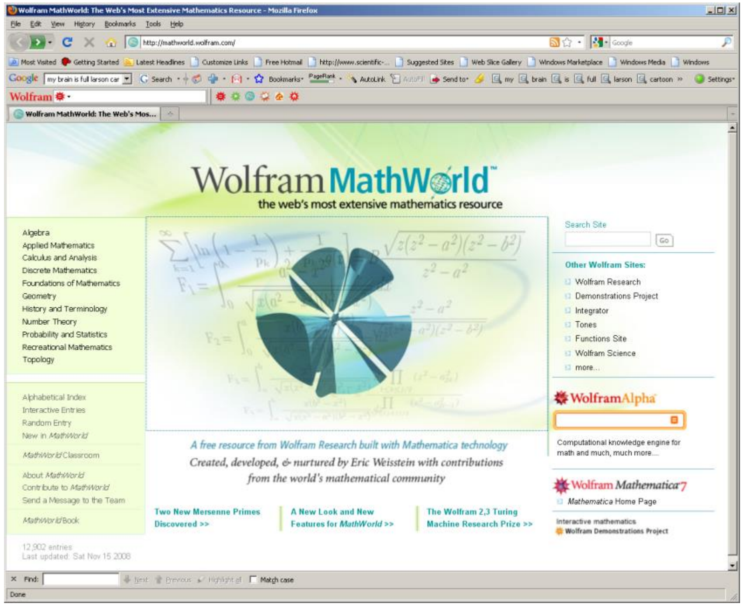
- The Web:
  - The Semantic Web in Action, Lee Feigenbaum et al., Scientific American, December 2007, pp. 90-97.
  - Web Science Emerges, Nigel Shadbolt and Tim Berners-Lee, Scientific American, October 2008, pp. 76-81.
- The Wikipedia Computer Science Portal:
  - Theory of computation and Automata theory
  - Formal languages and grammars
  - Chomsky hierarchy and the Complexity Zoo
  - Regular, context-free &Turing-decidable languages
  - Finite & pushdown automata; Turing machines
  - Computational complexity
  - List of data structures and algorithms

Email all submissions to: homework.cs3102@gmail.com

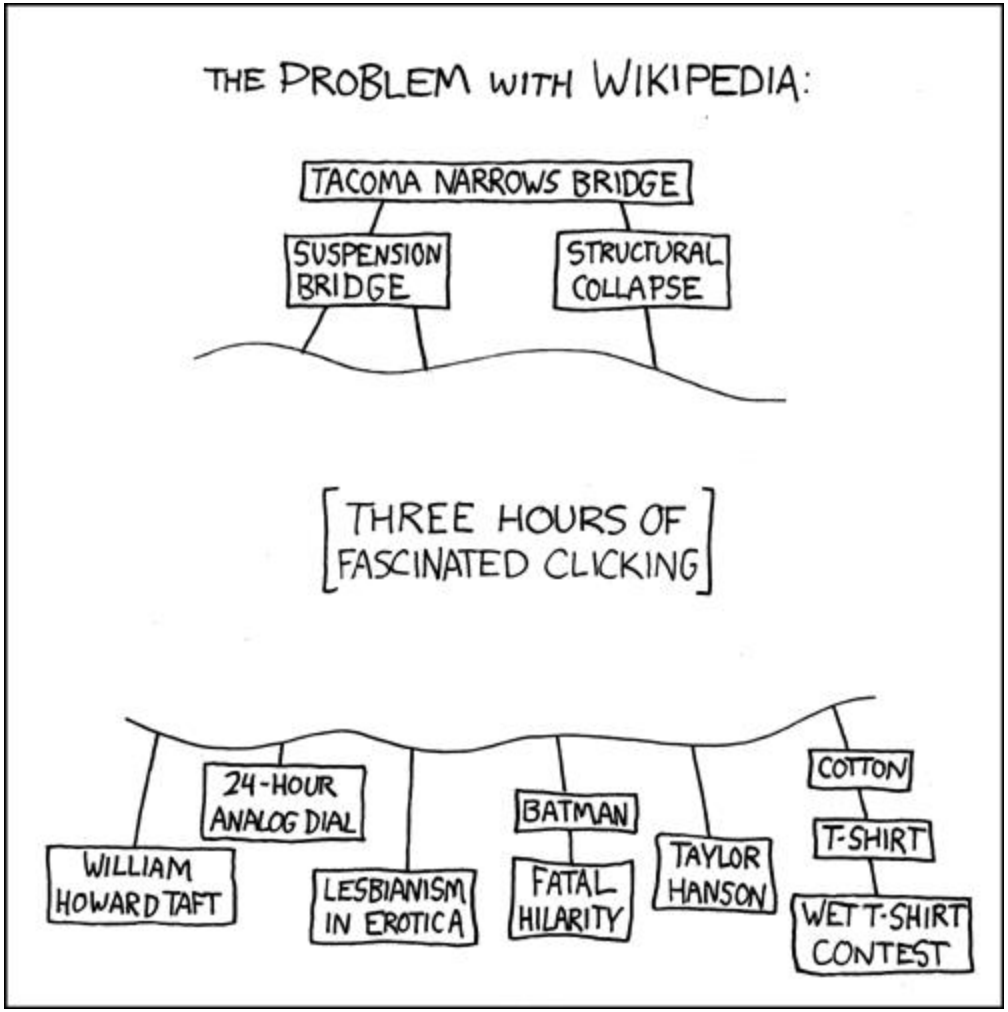
Other “Elective” Readings

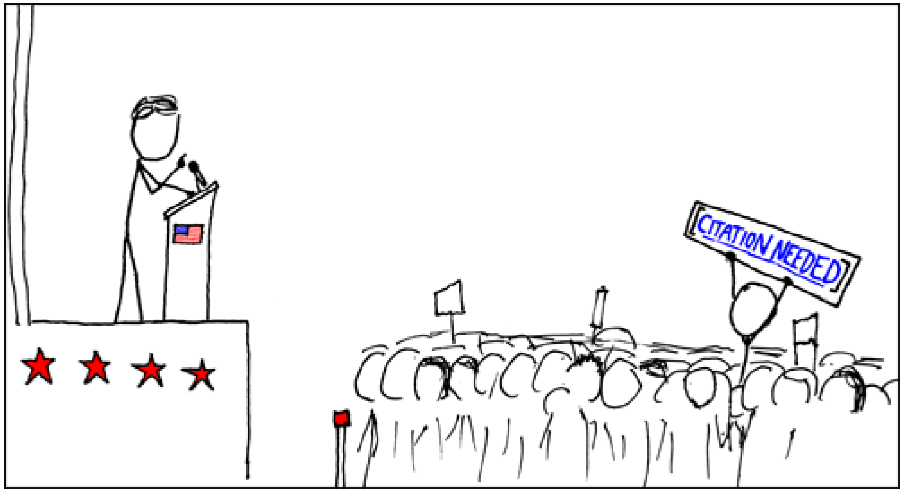
www.cs.virginia.edu/robins/CS_readings.html

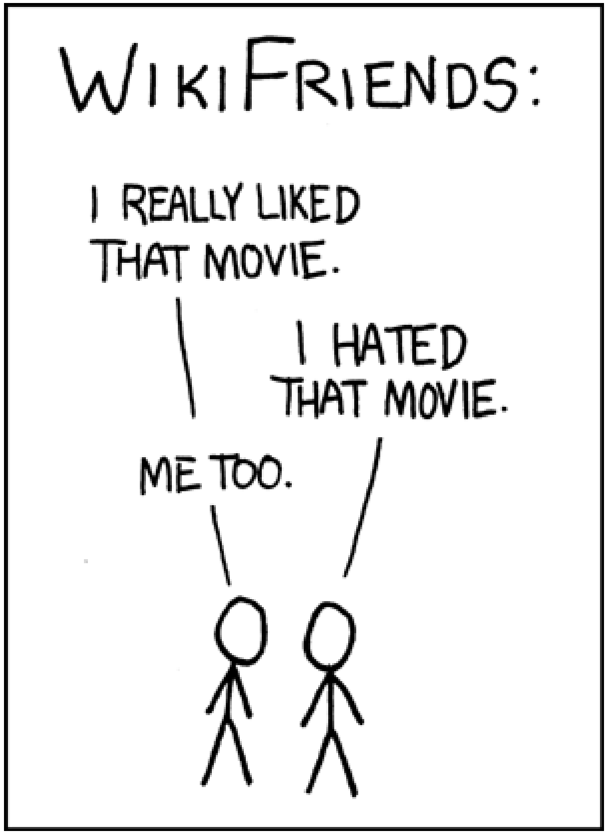
- The Wikipedia Math Portal:
  - Problem solving
  - List of Mathematical lists
  - Sets and Infinity
  - Discrete mathematics
  - Proof techniques and list of proofs
  - Information theory & randomness
  - Game theory
- Mathematica's “Math World”



Email all submissions to: homework.cs3102@gmail.com

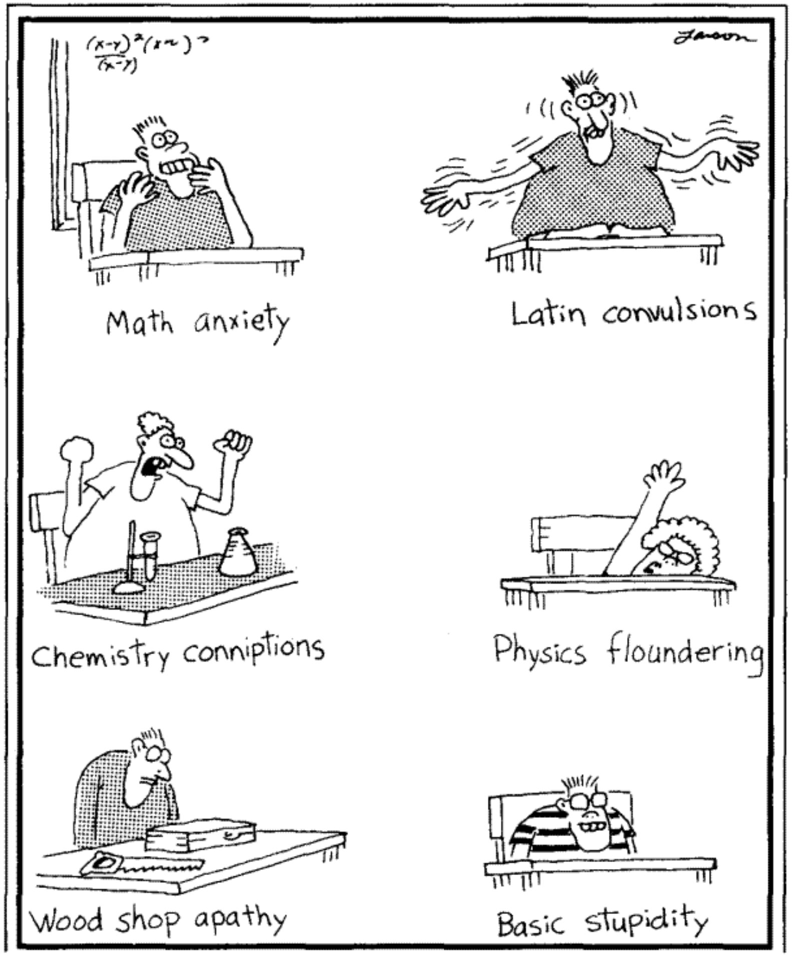

THE PROBLEM WITH WIKIPEDIA:
TACOMA NARROWS BRIDGE
SUSPENSION BRIDGE
STRUCTURAL COLLAPSE
[THREE HOURS OF FASCINATED CLICKING]
24-HOUR ANALOG DIAL
WILLIAM HOWARD TAFT
LESBIANISM IN EROTICA
BATMAN
FATAL HILARITY
TAYLOR HANSON
COTTON
T-SHIRT
WET T-SHIRT CONTEST


[CITATION NEEDED]


WIKIFRIENDS:
I REALLY LIKED THAT MOVIE.
I HATED THAT MOVIE.
ME TOO.

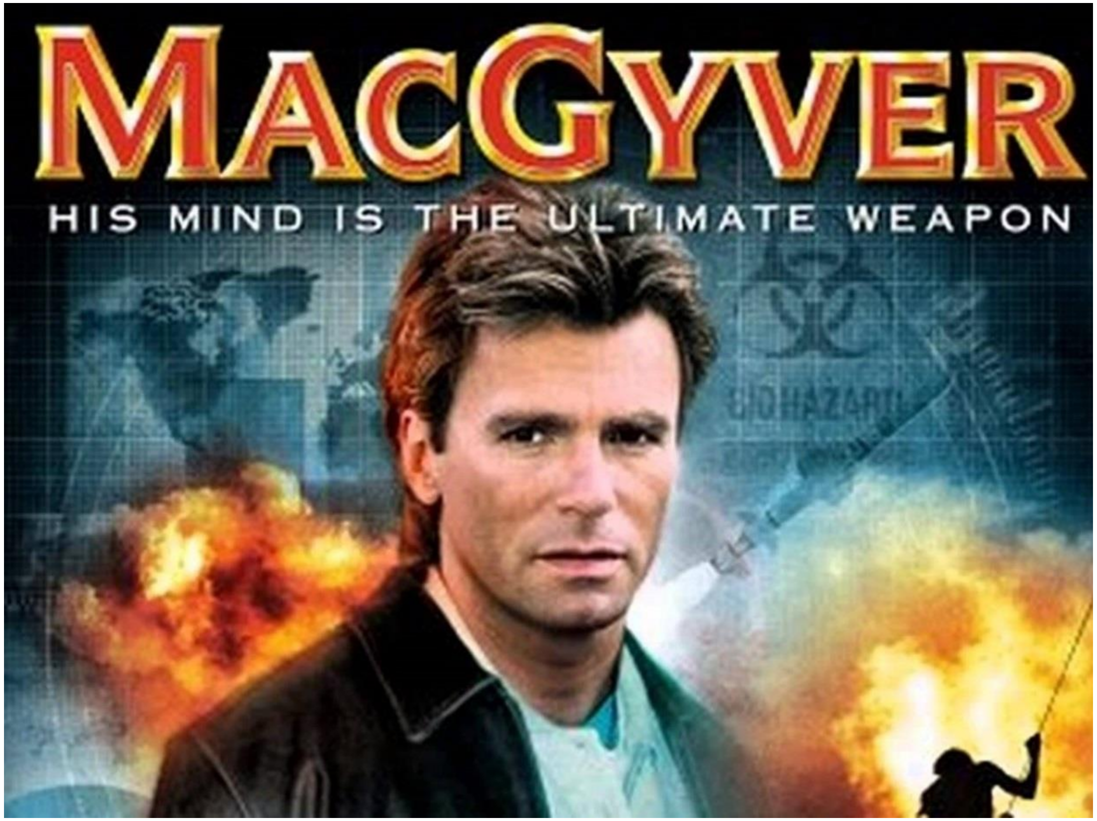
Good Advice

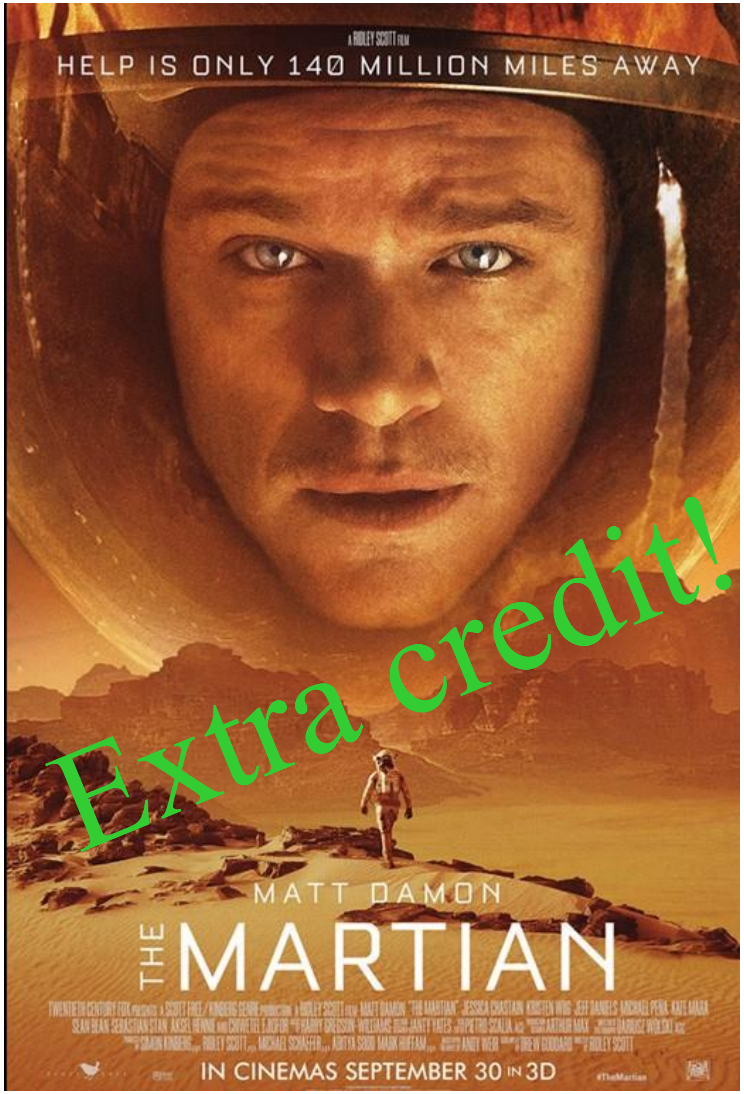
- Ask questions ASAP
- Solve problems ASAP
- Work in study groups
- Do not fall behind
- "Cramming" won't work
- Do lots of extra credit
- Attend every lecture
- Visit class Website often
- Solve lots of problems



Classroom afflictions

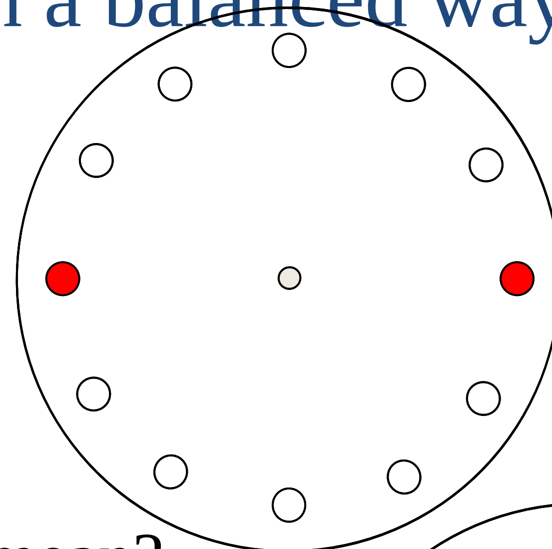
Goal: Become a more effective problem solver!

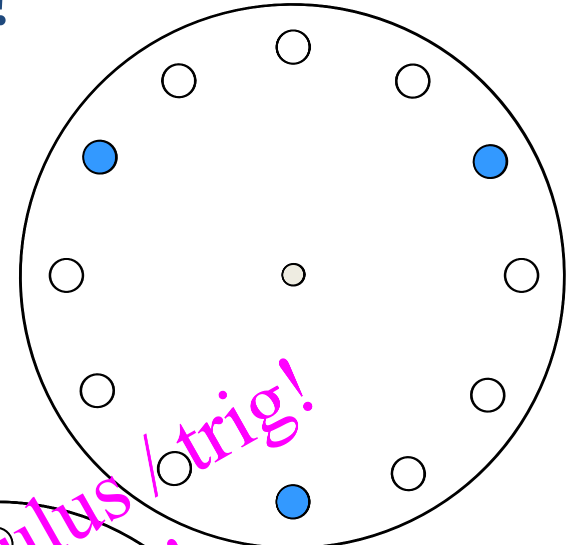


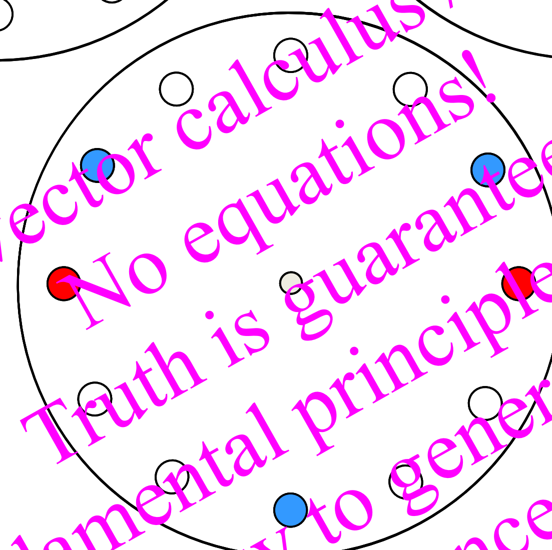


Email all submissions to: homework.cs3102@gmail.com

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?







- What does "balanced" mean?
- Why are 3 test tubes balanced?
- Symmetry!
- Can you merge solutions?
- Superposition!
- Linearity! $f(x + y) = f(x) + f(y)$
- Can you spin 7 test tubes?
- Complementarity!
- Empirical testing…

No vector calculus / trig!
No equations!
Truth is guaranteed!
Fundamental principles exposed!
Easy to generalize!
High elegance / beauty!

Problem: 1 + 2 + 3 + 4 + …+ 100 = ?

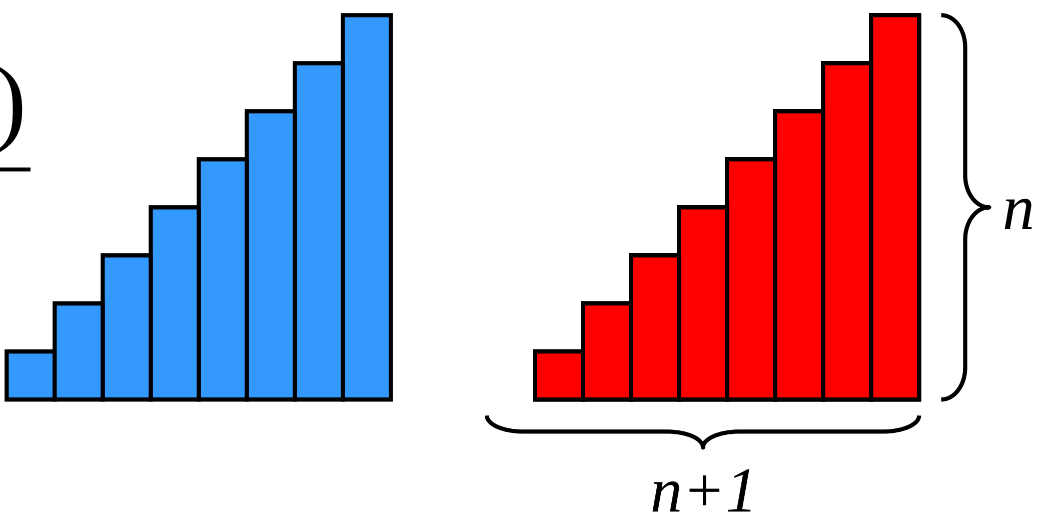
= (100*101)/2

= 5050

Proof: ~~Induction…~~

$$
\begin{array}{ccccccccccccc}
1 & + & 2 & + & 3 & + & \ldots & + & 99 & + & 100 \\
100 & + & 99 & + & 98 & + & \ldots & + & 2 & + & 1 \\
\hline
101 & + & 101 & + & 101 & + & \ldots & + & 101 & + & 101 & = & 100*101
\end{array}
$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$



Drawbacks of Induction

- You must a priori know the formula / result
- Easy to make mistakes in inductive proof
- Mostly "mechanical" – ignores intuitions
- Tedious to construct
- Difficult to check
- Hard to understand
- Not very convincing
- Generalizations not obvious
- Does not "shed light on truth"
- Obfuscates connections

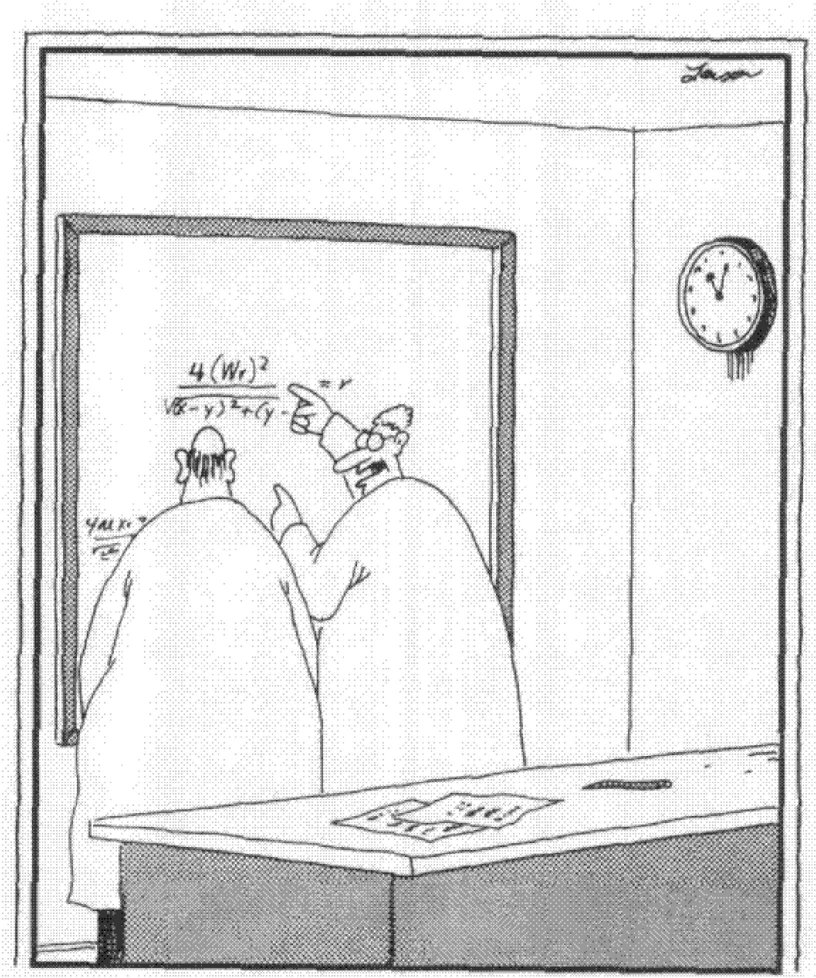
Conclusion: only use induction as a last resort! (i.e., rarely)

Problem: $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = ?$

$$\sum_{i=1}^{n} i^3 = ?$$

Extra Credit:

find a short, geometric, induction-free proof.



"Yes, yes, I *know* that, Sidney ... everybody knows *that!* ... But look: Four wrongs *squared*, minus two wrongs to the fourth power, divided by this formula, *do* make a right."

Email all submissions to: homework.cs3102@gmail.com

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

$$\sum_{i=1}^{\infty} \frac{1}{4^i} = ?$$

Extra Credit:

Find a short, geometric, induction-free proof.

Email all submissions to: homework.cs3102@gmail.com

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = ?$$

Extra Credit:

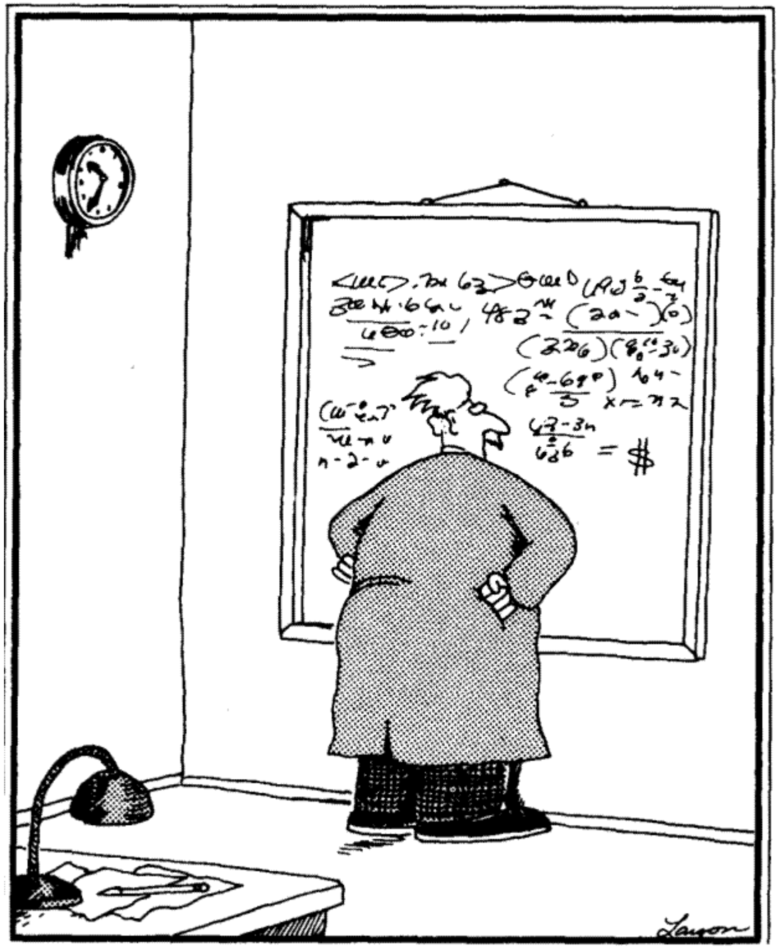
Find a short, geometric, induction-free proof.

Email all submissions to: homework.cs3102@gmail.com

Problem: Prove that $\sqrt{2}$ is irrational.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Einstein discovers that time is actually money.

Problem: Prove that there are an infinity of primes.

Extra Credit: Find a short, induction-free proof.

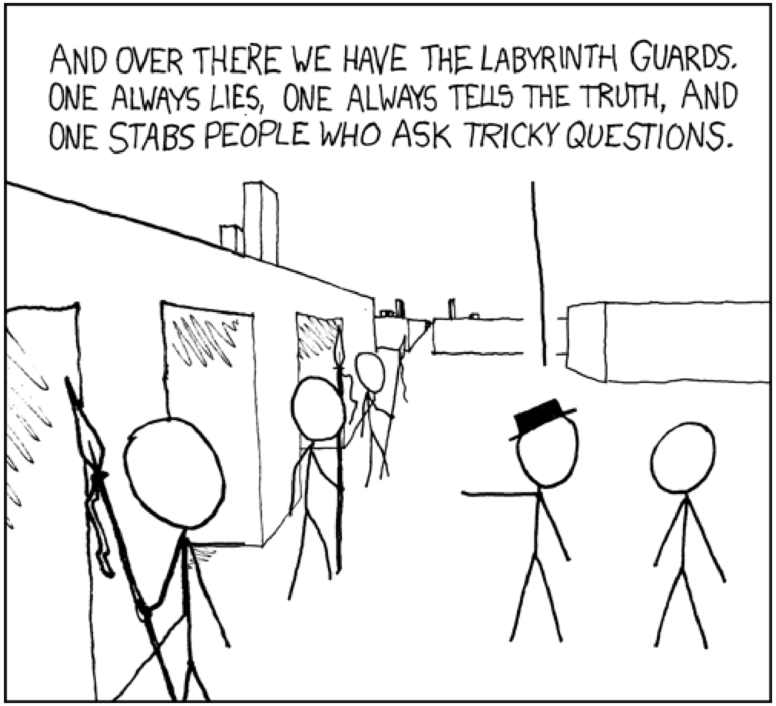
- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Email all submissions to: homework.cs3102@gmail.com

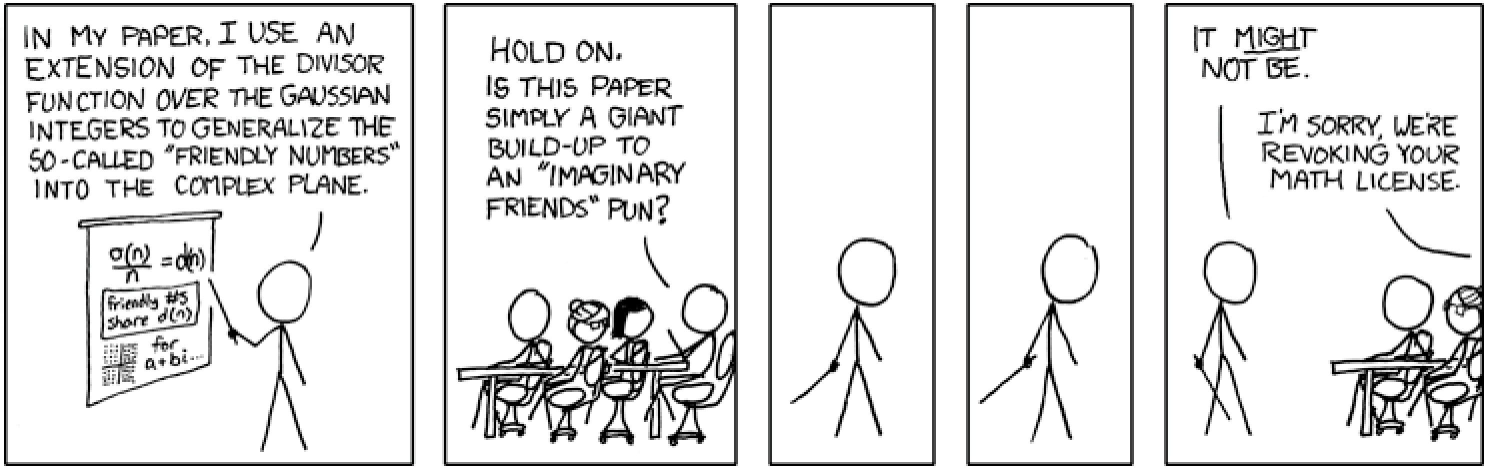
Problem: True or false: there arbitrary long blocks of consecutive composite integers.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



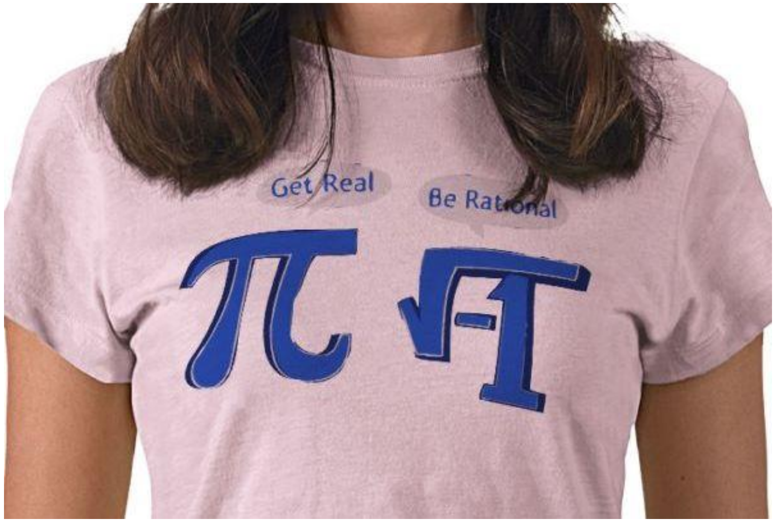
Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i?



Problem: Does exponentiation preserve irrationality?
i.e., are there two irrational numbers x and y such that x^y is rational?

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Historical Perspectives

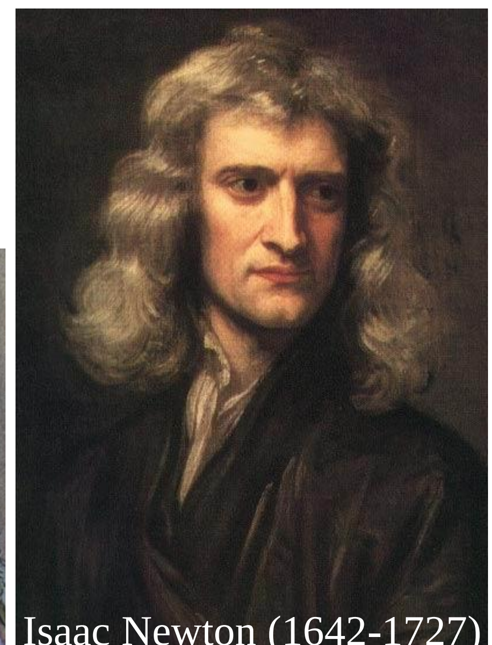
Historical Perspectives

- Knowing the "big picture" is empowering
- Science and mathematics builds heavily on past
- Often the simplest ideas are the most subtle
- Most fundamental progress was done by a few
- We learn much by observing the best minds
- Research benefits from seeing connections
- The field of computer science has many "parents"
- We get inspired and motivated by excellence
- The giants can show us what is possible to achieve
- It is fun to know these things!

"Standing on the Shoulders of Giants"

- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan
- Babbage, Ada Lovelace
- Venn, Carroll

Ada Lovelace (1815-1852)



Isaac Newton (1642-1727)

Euclid (300 BC)

“Standing on the Shoulders of Giants”

- Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Gödel, Church, Turing
- von Neumann, Shannon
- Kleene, Chomsky
- Hoare, McCarthy, Erdos
- Knuth, Backus, Dijkstra

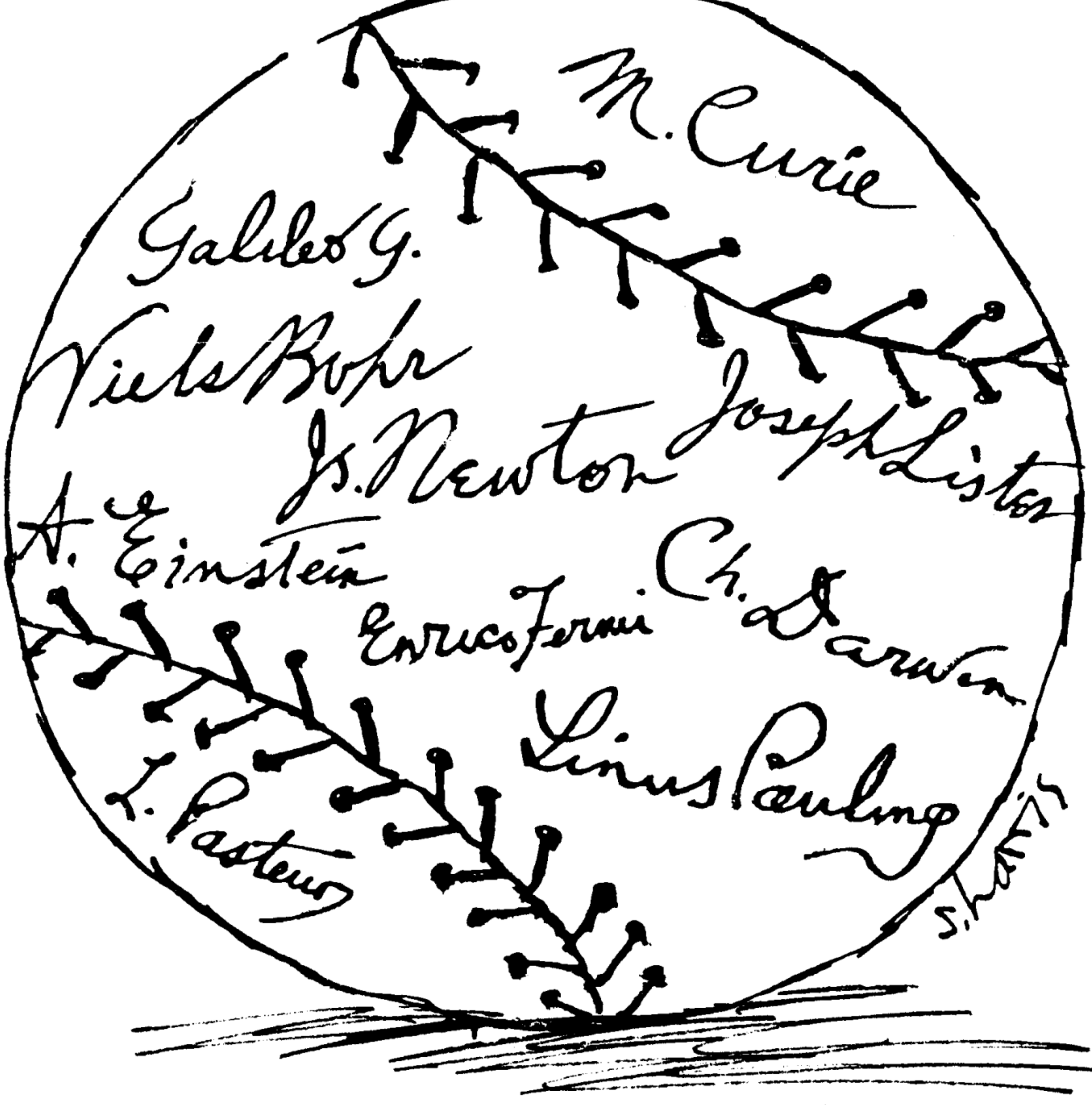
Many others…

Georg Cantor (1845-1918)

Bertrand Russell (1872-1970)

David Hilbert (1862-1943)

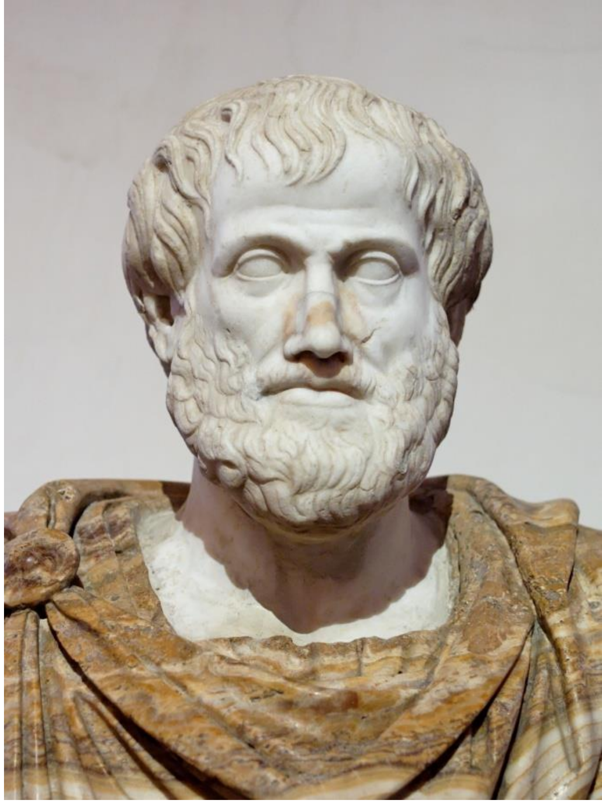
Kurt Gödel (1906-1978)


M. Curie
Galileo G.
Niels Bohr
Js. Newton
Joseph Lister
A. Einstein
Ch. Darwin
Enrico Fermi
Linus Pauling
L. Pasteur

Gauss
Newton
Archimedes
Euler
Cauchy
Poincare
Riemann
Cantor
Cayley
Hamilton
Eisenstein
Pascal
Abel
Hilbert
Klein
Leibniz
Descartes
Galois
Mobius
Jacob
Johann Bernoulli
Daniel Bernoulli
Dirichlet
Fermat
Pythagoras
Laplace
Lagrange
Kronecker
Jacobi
Bolyai
Lobatchewsky
Noether
Germain
Euclid
Legendre

MAKING PHILOSOPHY ACCESSIBLE: POP-UP PLATO

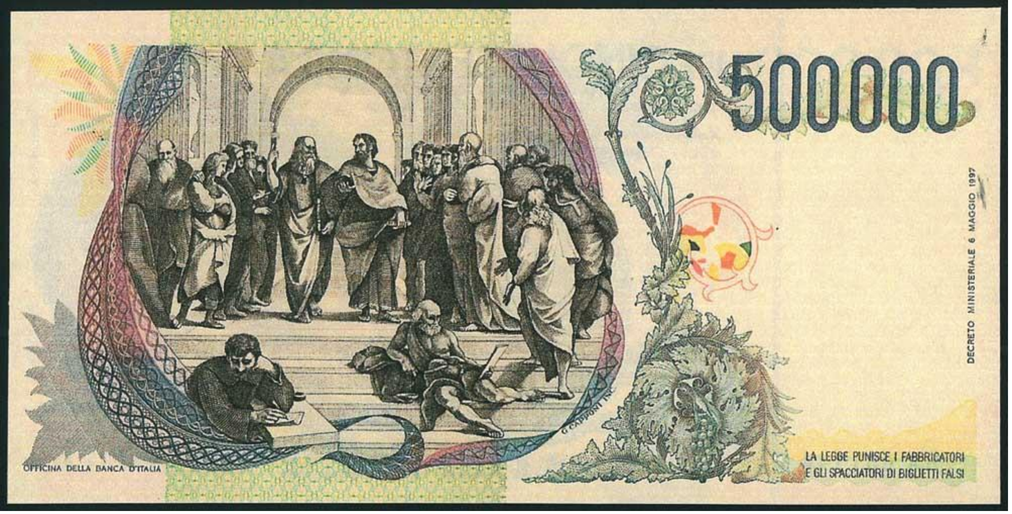
Historical Perspectives

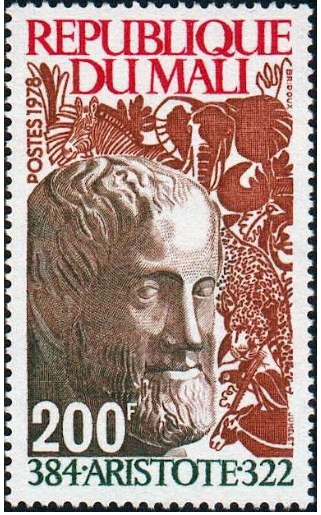


Aristotle (384BC-322BC)

- Founded Western philosophy
- Student of Plato
- Taught Alexander the Great
- "Aristotelianism"
- Developed the "scientific method"
- One of the most influential people ever
- Wrote on physics, theatre, poetry, music, logic, rhetoric, politics, government, ethics, biology, zoology, morality, optics, science, aesthetics, psychology, metaphysics, …
- Last person to know everything known in his own time!

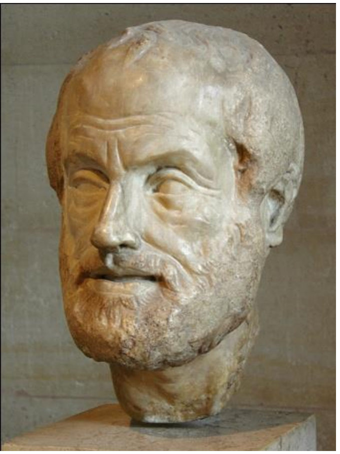
"Almost every serious intellectual advance has had to begin with an attack on some Aristotelian doctrine." – Bertrand Russell

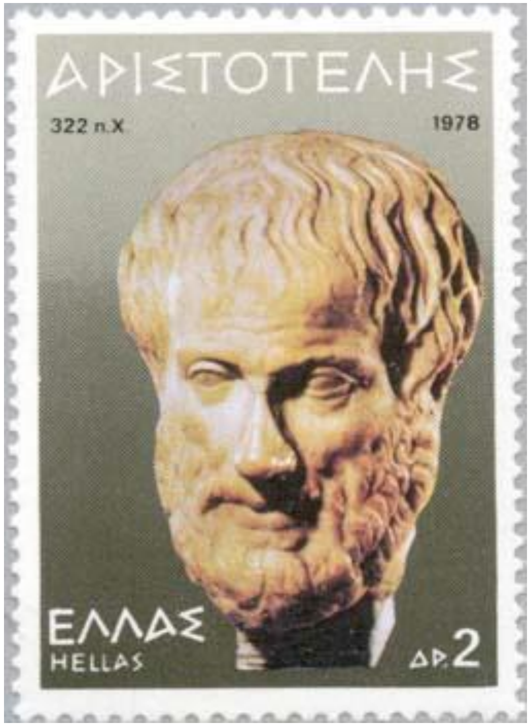


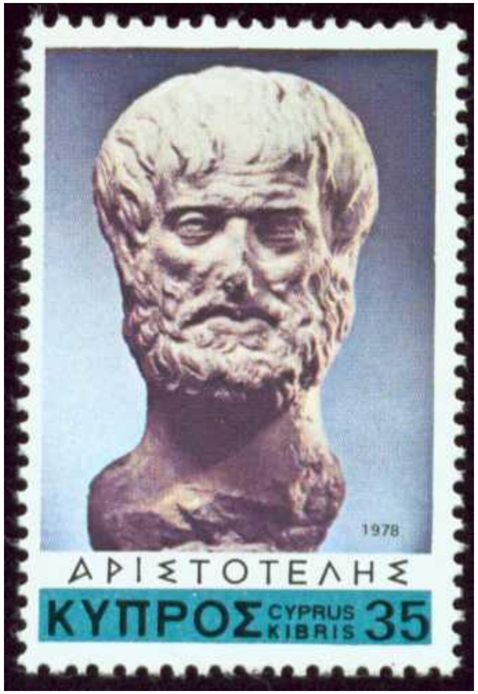


"Wit is educated insolence."

- Aristotle (384-322 B.C.)







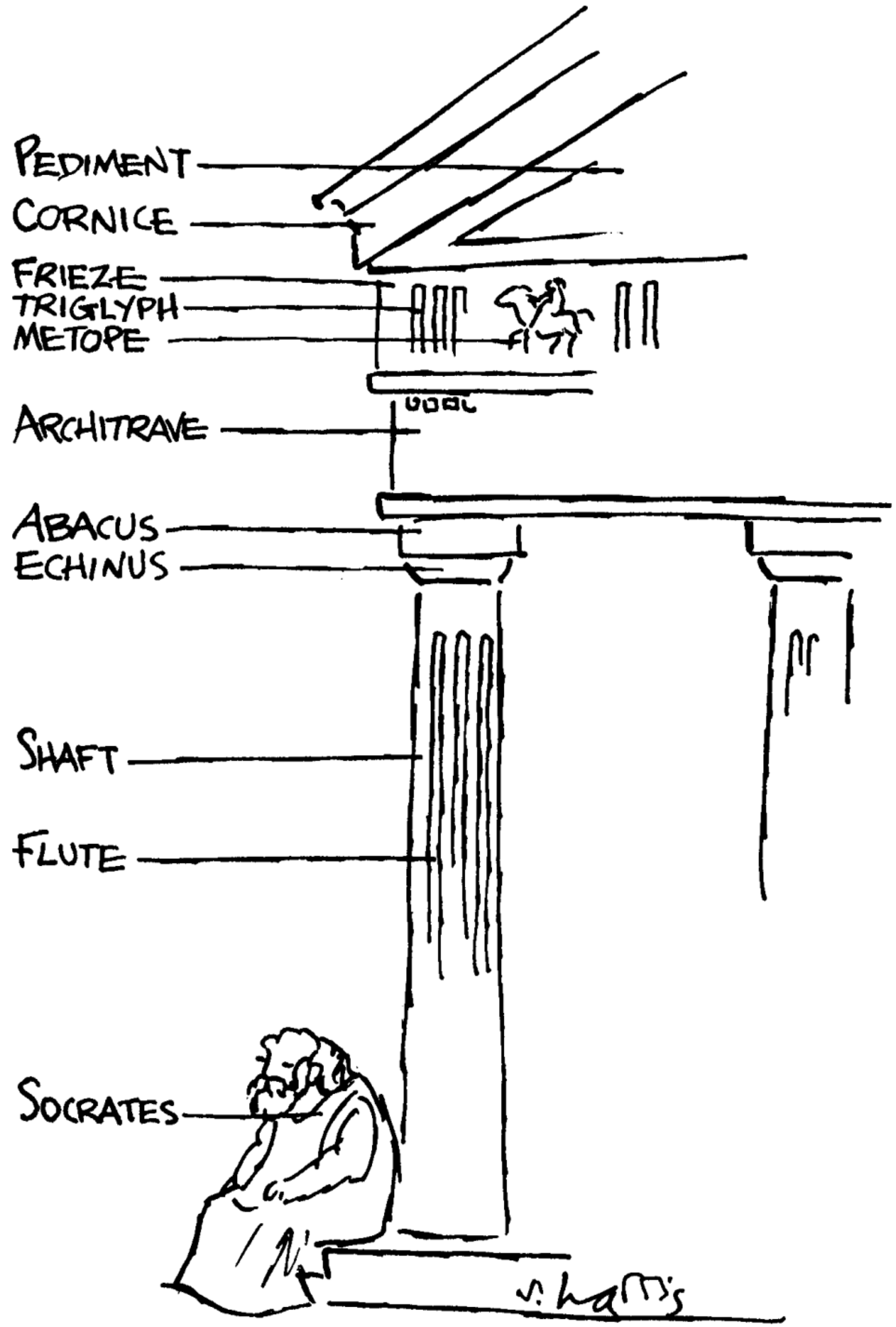
“The School of Athens” (by Raphael, 1483-1520)


ARISTOTELES
87 km
97 / 10 / 09
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
8
B/W QuickCam
a.cidadao@mail.telepac.pt


THE UNKNOWN PHILOSOPHER
WHO FIRST REALIZED LIFE IS NO PICNIC


Birds fly because they're lighter than air.
Some trees have different fruits each year.
At night, clouds rest on the ground.
Are you sure he's Aristotle?
S. Harris

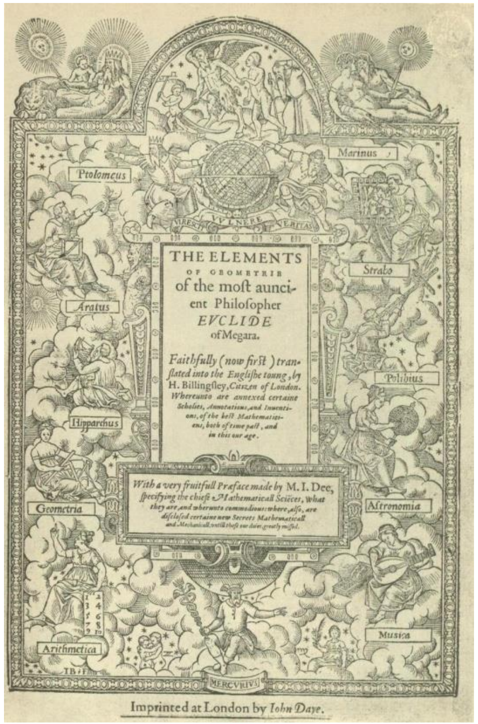
"What I especially like about being a philosopher-scientist is that I don't have to get my hands dirty."


PEDIMENT
CORNICE
FRIEZE
TRIGLYPH
METOPE
ARCHITRAVE
ABACUS
ECHINUS
SHAFT
FLUTE
SOCRATES

Historical Perspectives

Euclid (325BC-265BC)

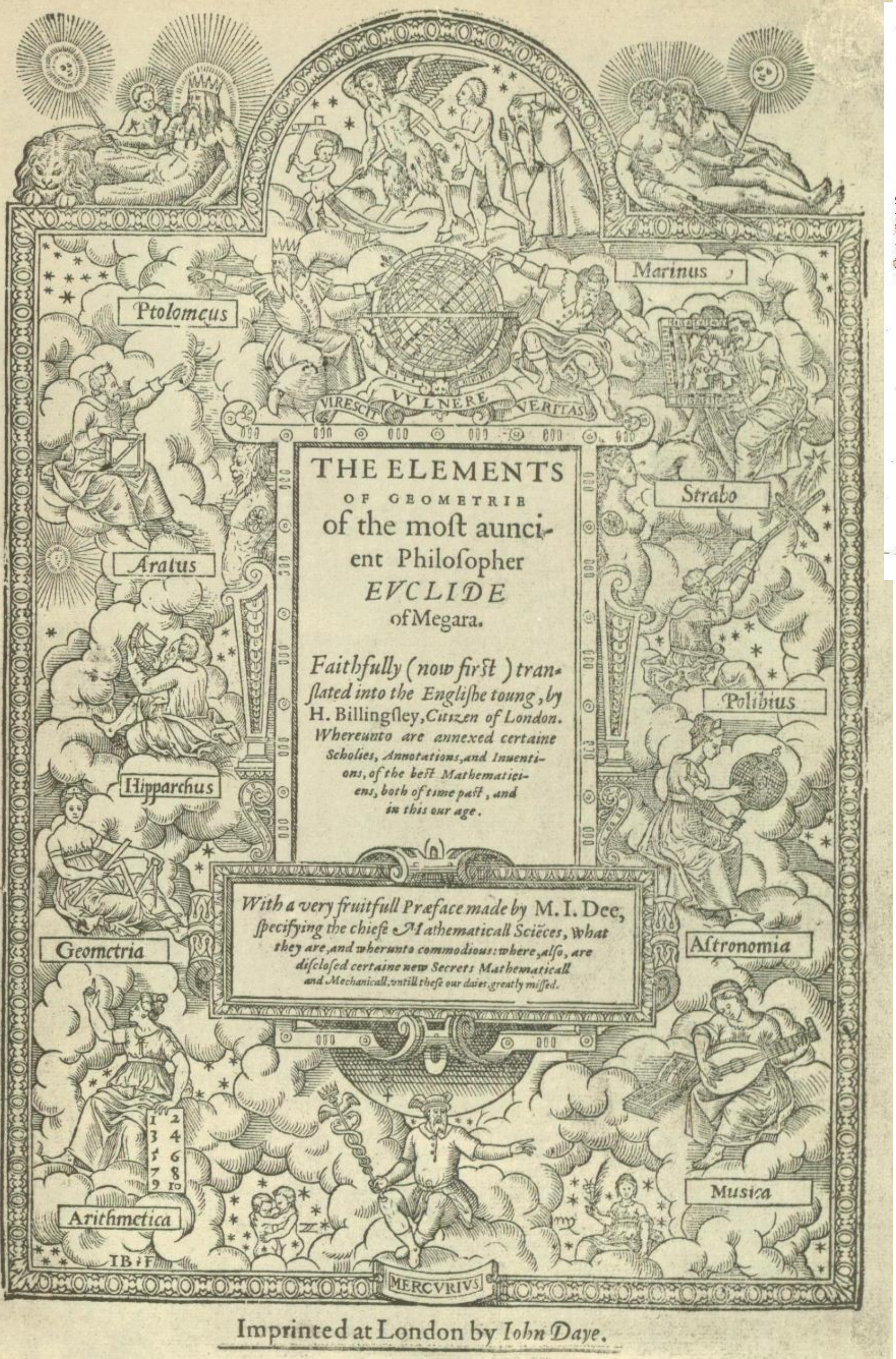
- Founder of geometry & the axiomatic method
- "Elements" – oldest and most impactful textbook
- Unified logic & math
- Introduced rigor and "Euclidean" geometry
- Influenced all other fields of science: Copernicus, Kepler, Galileo, Newton, Russell, Lincoln, Einstein & many others

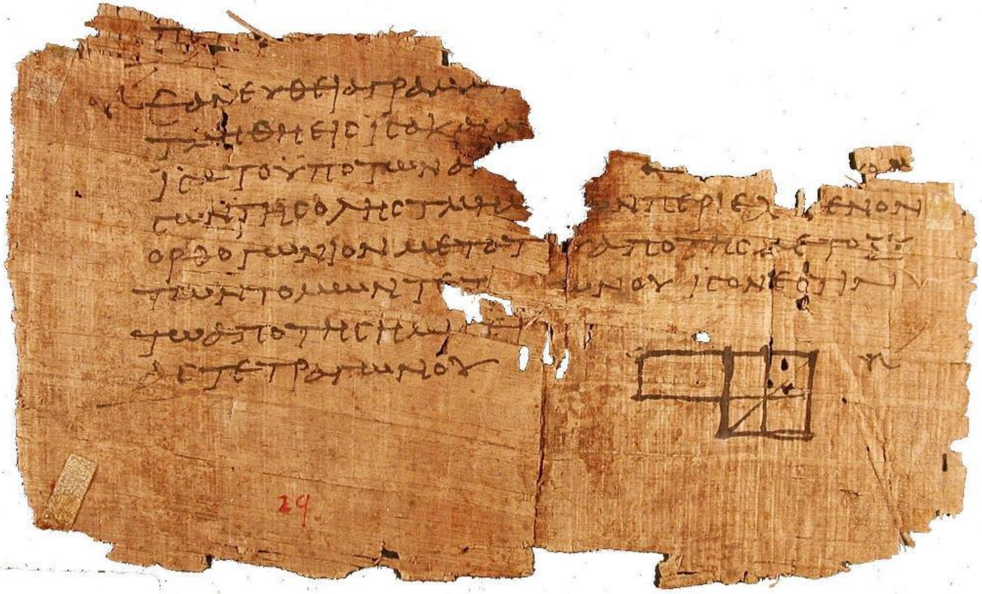

THE ELEMENTS OF GEOMETRIE of the moſt aunciént Philoſopher EVCLIDE of Megara.

Faithfully (now firſt) tranſlated into the Engliſhe toung, by H. Billingſley, Citizen of London. Whereunto are annexed certaine Scholies, Annotations, and Inuentions, of the beſt Mathematiciens, both of time paſt, and in this our age.

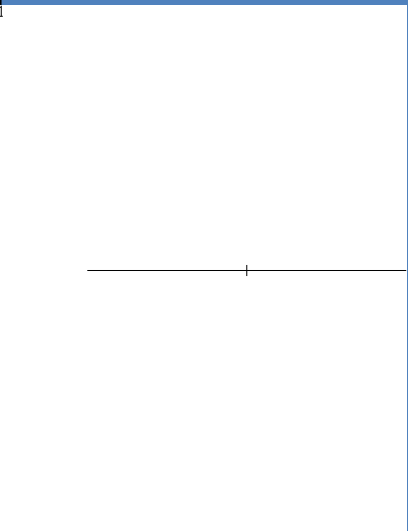
With a very fruitfull Præface made by M. I. Dee, ſpecifying the chiefe Mathematicall Sciẽces, what they are, and wherunto commodious: where, alſo, are diſcloſed certaine new Secrets Mathematicall and Mechanicall, untill theſe our daies greatly miſſed.

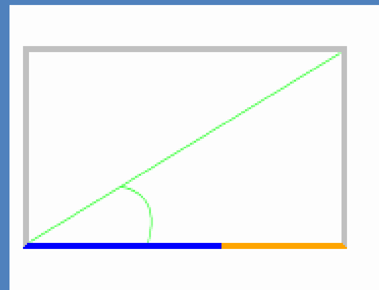
Imprinted at London by Iohn Daye.

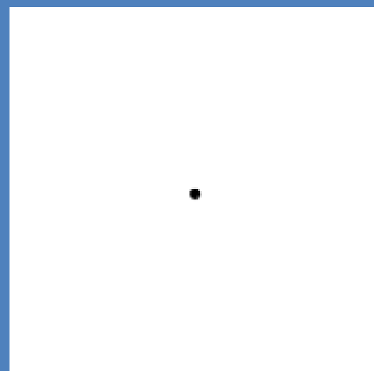

Ptolomeus
Marinus
VIRESCIT VVLNERE VERITAS
Aratus
Strabo
Polibius
Hipparchus
Geometria
Astronomia
Arithmetica
Musica
MERCVRIVS
THE ELEMENTS
OF GEOMETRIE
of the most aunci-
ent Philosopher
EVCLIDE
of Megara.
Faithfully (now first) tran-
slated into the Englishe toung, by
H. Billingsley, Citizen of London.
Whereunto are annexed certaine
Scholies, Annotations, and Inuenti-
ons, of the best Mathematici-
ens, both of time past, and
in this our age.
With a very fruitfull Præface made by M. I. Dee,
specifying the chiefe Mathematicall Sciēces, what
they are, and wherunto commodious: where, also, are
disclosed certaine new Secrets Mathematicall
and Mechanicall, vntill these our daies, greatly missed.
Imprinted at London by Iohn Daye.

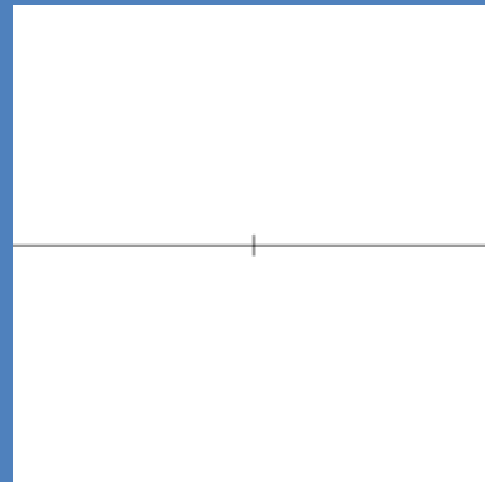


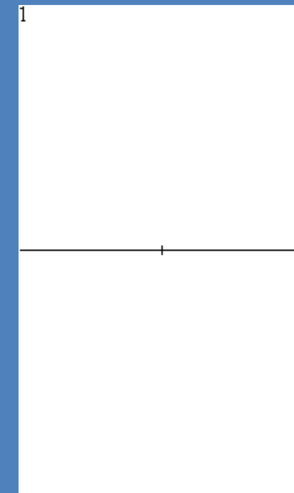
Euclid's Straight-Edge and Compass Geometric Constructions

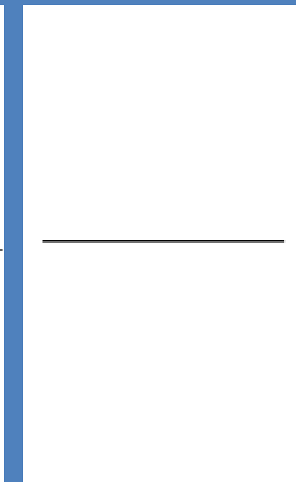


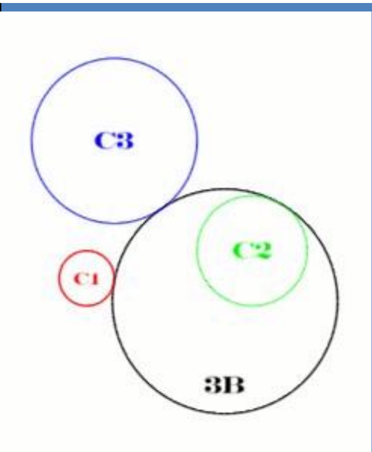


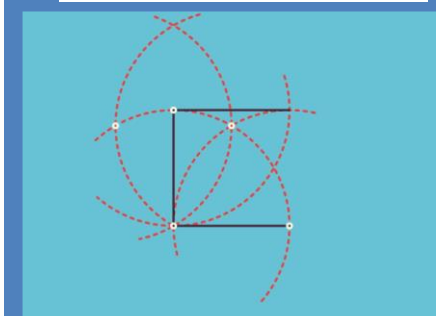


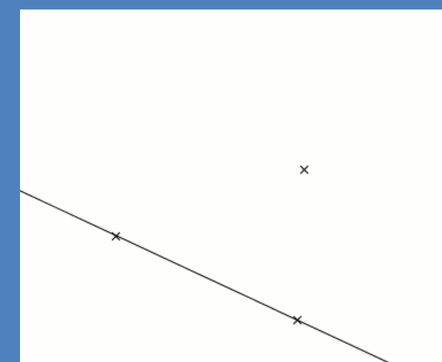




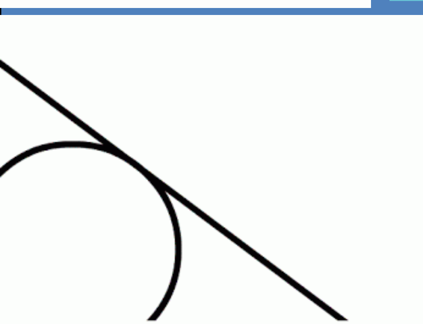


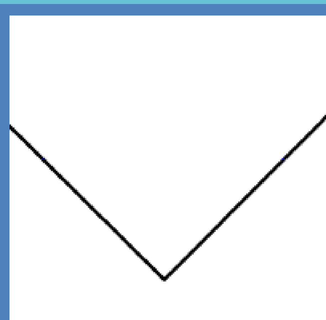


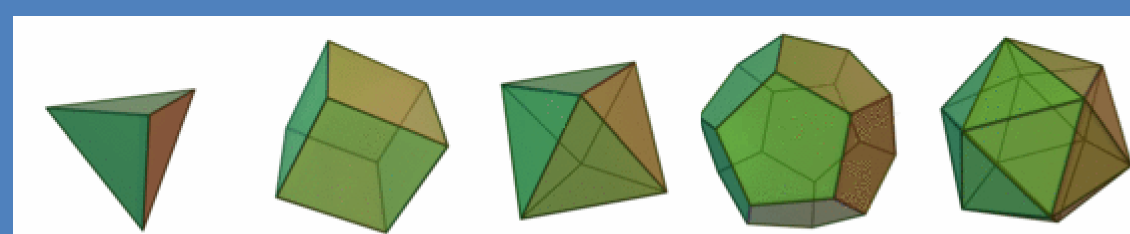




Extra credit!





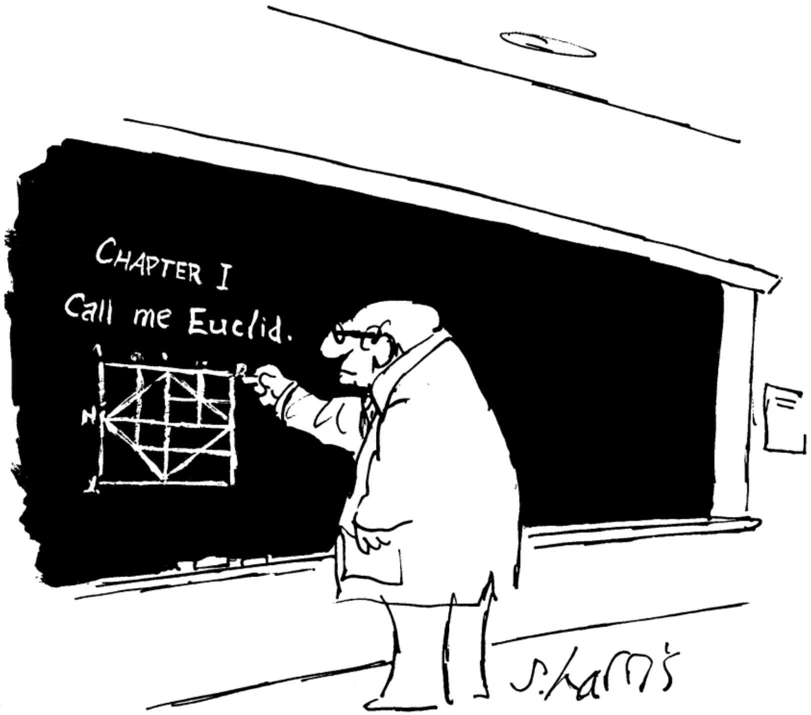



SCHOOL OF ATHENS
50c
SIERRA LEONE
500th ANNIVERSARY
BIRTH OF RAPHAEL
(detail) Euclid and his pupils
1983
SCALA NEW YORK/FLORENCE


POSTE VATICANE
2500
PONTIFICIA
ACADEMIA
SCIENTIARVM
I.P.Z.S.-ROMA-1986
P.N. ARGHITTU INC.


EUCLIDES
12 km
© António J. Cidadão
98 / 04 / 07
D=254mm f/D=10
Moon "LIGHT"
11
B/W QuickCam
a.cidadao@mail.telepac.pt


MALDIVES
PRINCIPLES OF GEOMETRY
EUCLID
Rf3


CHAPTER I
Call me Euclid.
S. Harris

Euclid's Axioms

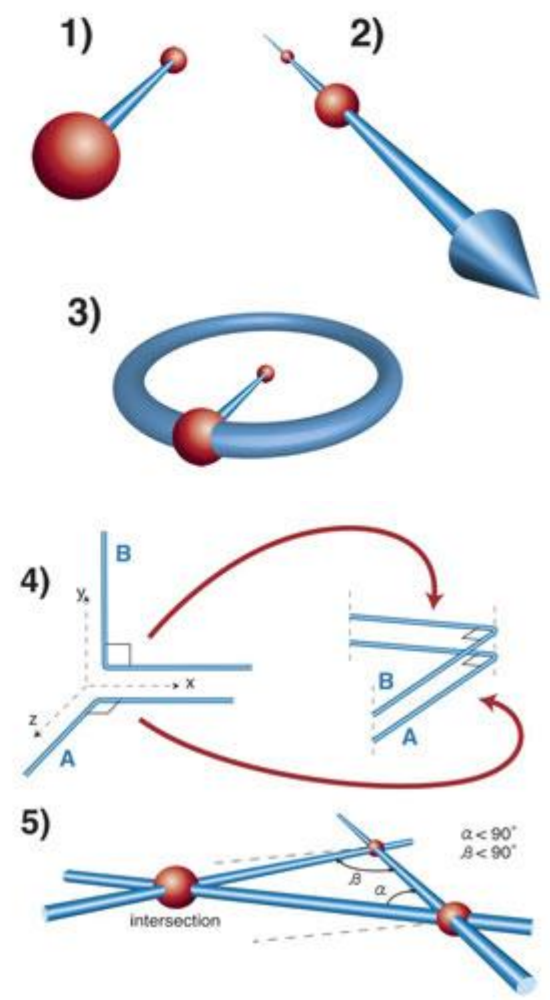
1: Any two points can be connected by exactly one straight line.

2: Any segment can be extended indefinitely into a straight line.

3: A circle exists for any given center and radius.

4: All right angles are equal to each other.

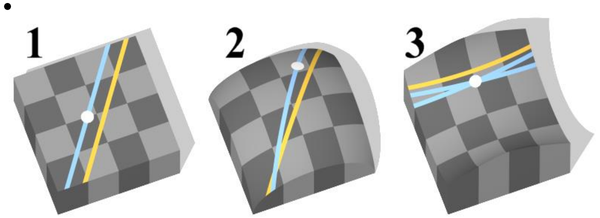
5: The parallel postulate: Given a line and a point off that line, there is exactly one line passing through the point, which does not intersect the first line.

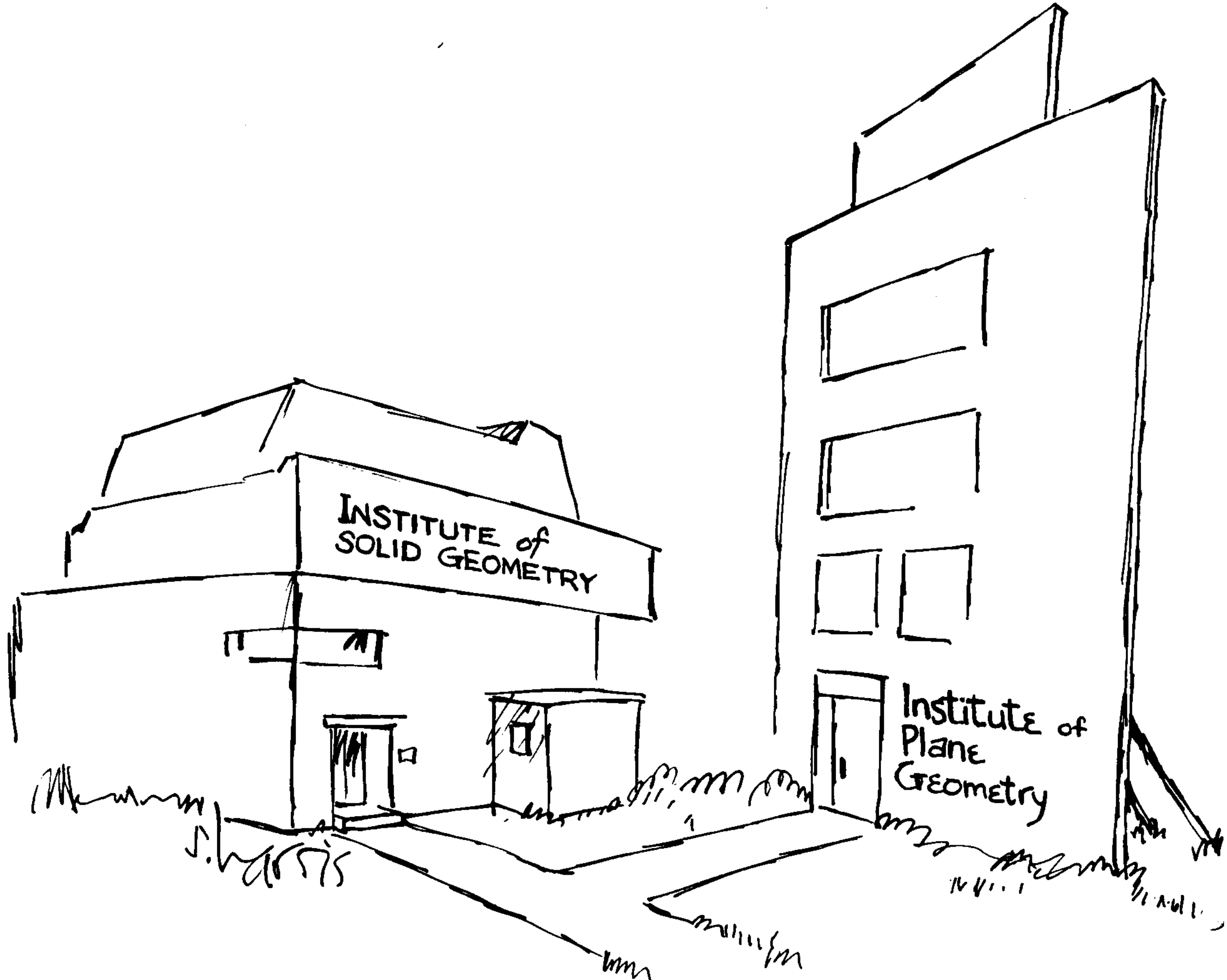


The first 28 propositions of Euclid's Elements were proven without using the parallel postulate!

Theorem [Beltrami, 1868]: The parallel postulate is independent of the other axioms of Euclidean geometry.

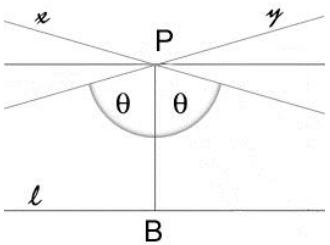
The parallel postulate can be modified to yield non-Euclidean geometries!



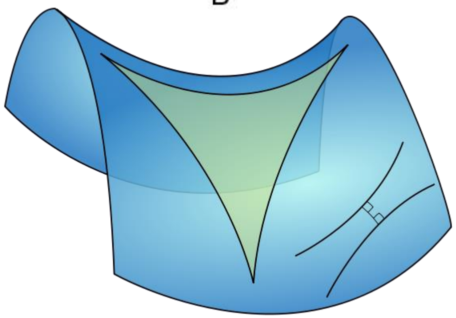

INSTITUTE of
SOLID GEOMETRY
Institute of
Plane
Geometry

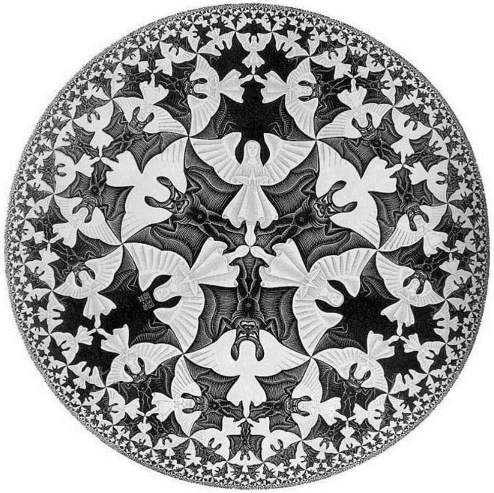
Non-Euclidean Geometries

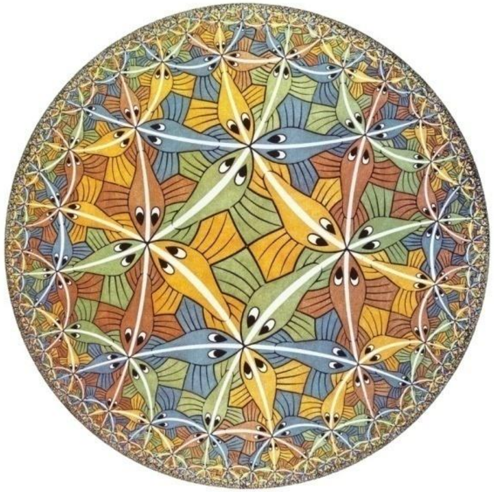
Hyperbolic geometry: Given a line and a point off that line, there are an infinity of lines passing through that point that do not intersect the first line.

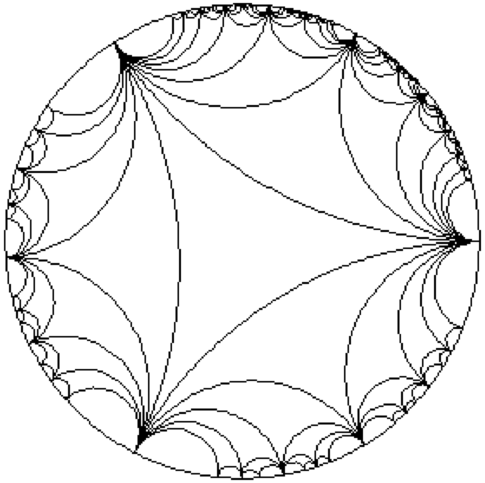


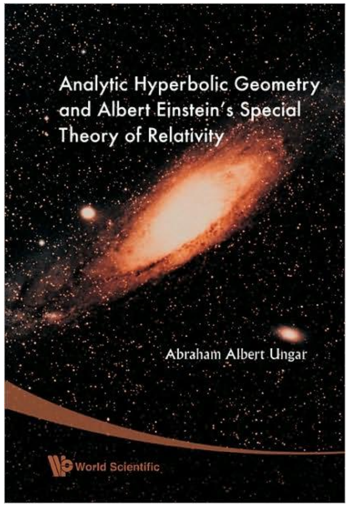
- Sum of triangle angles is less than 180^{o}
- Different triangles have different angle sum
- Triangles with same angles have same area
- There are no similar triangles
- Used in relativity theory







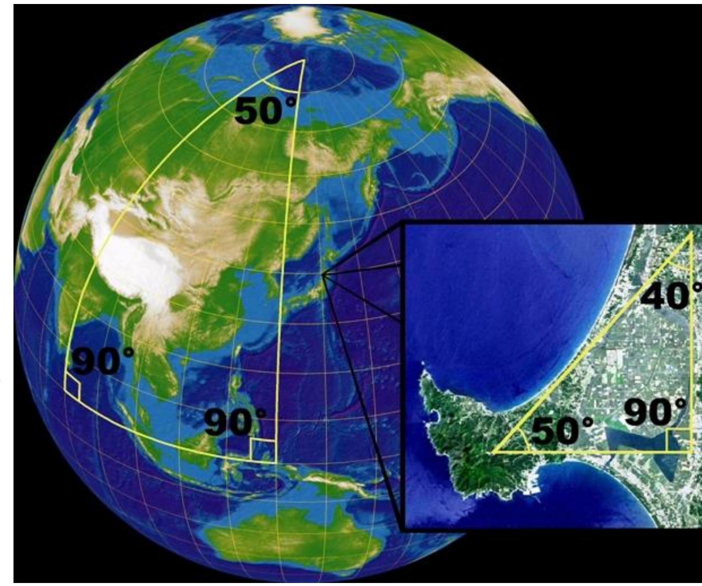


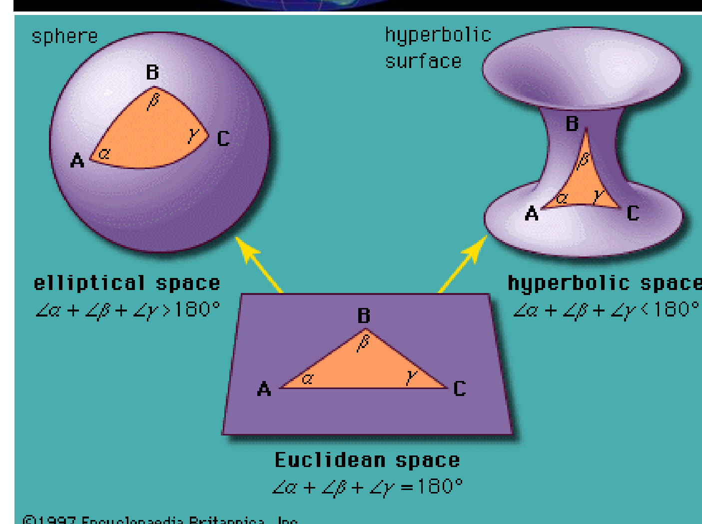


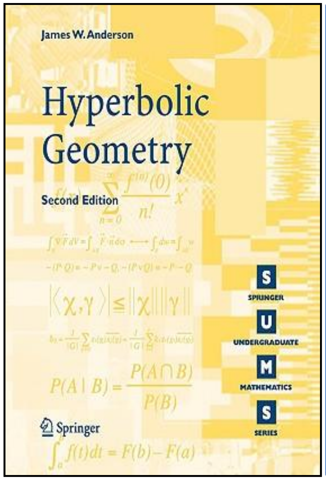
Non-Euclidean Geometries

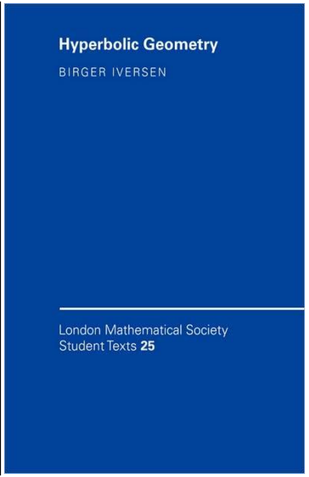
Spherical / Elliptic geometry: Given a line and a point off that line, there are no lines passing through that point that do not intersect the first line.

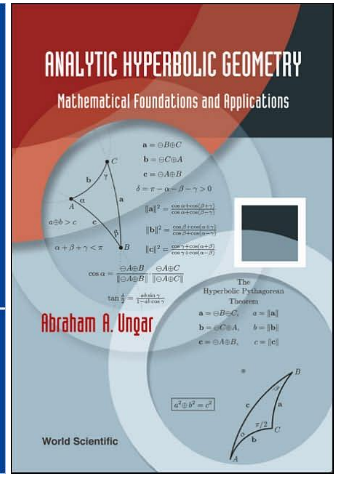
- Lines are geodesics - “great circles”
- Sum of triangle angles is > 180^o
- Not all triangles have same angle sum
- Figures can not scale up indefinitely
- Area does not scale as the square
- Volume does not scale as the cube
- The Pythagorean theorem fails
- Self-consistent, and complete

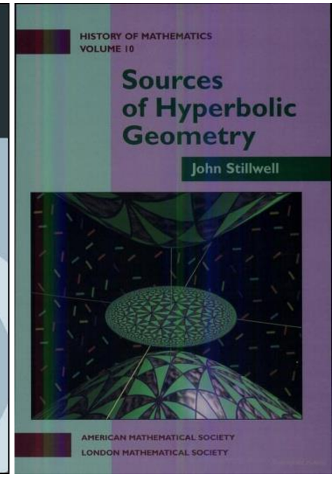


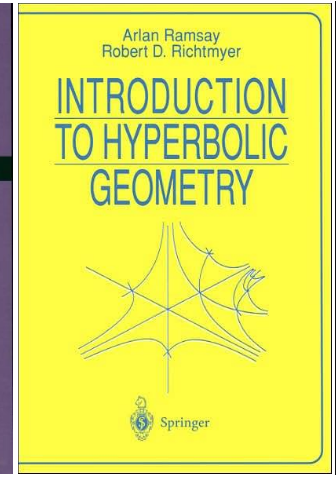


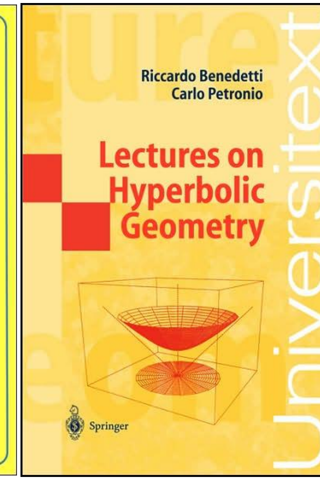

James W. Anderson
Hyperbolic Geometry
Second Edition
Springer
SPRINGER UNDERGRADUATE MATHEMATICS SERIES

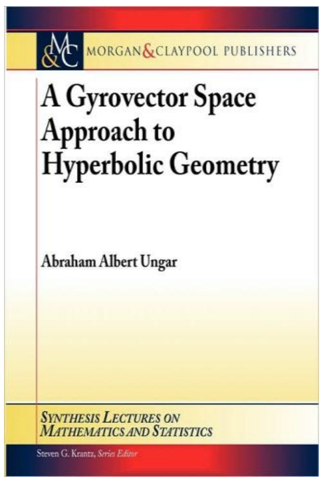

Hyperbolic Geometry
BIRGER IVERSEN
London Mathematical Society Student Texts 25

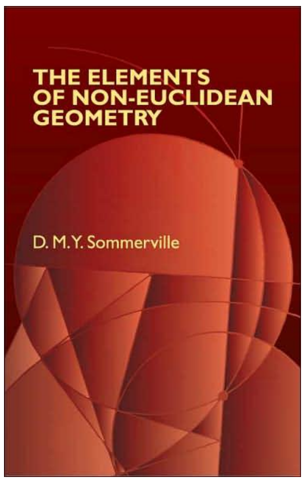

ANALYTIC HYPERBOLIC GEOMETRY
Mathematical Foundations and Applications
Abraham A. Ungar
World Scientific

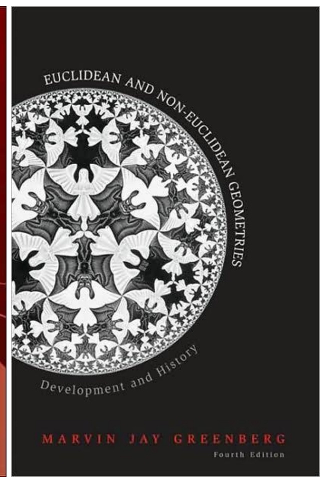

HISTORY OF MATHEMATICS VOLUME 10
Sources of Hyperbolic Geometry
John Stillwell
AMERICAN MATHEMATICAL SOCIETY
LONDON MATHEMATICAL SOCIETY

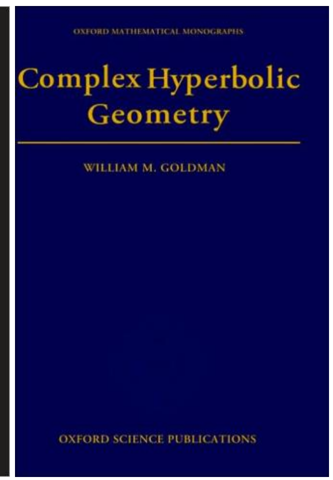

Arlan Ramsay
Robert D. Richtmyer
INTRODUCTION TO HYPERBOLIC GEOMETRY
Springer

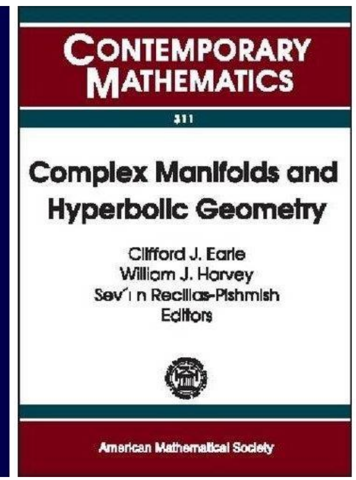

Riccardo Benedetti
Carlo Petronio
Lectures on Hyperbolic Geometry
Springer
Universitext

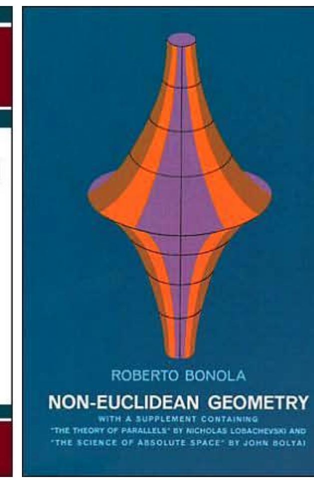

MORGAN&CLAYPOOL PUBLISHERS
A Gyrovector Space Approach to Hyperbolic Geometry
Abraham Albert Ungar
SYNTHESIS LECTURES ON MATHEMATICS AND STATISTICS
Steven G. Krantz, Series Editor

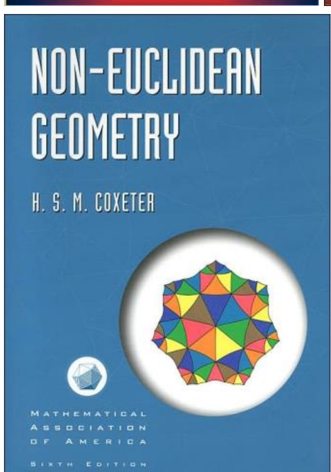

THE ELEMENTS OF NON-EUCLIDEAN GEOMETRY
D. M. Y. Sommerville

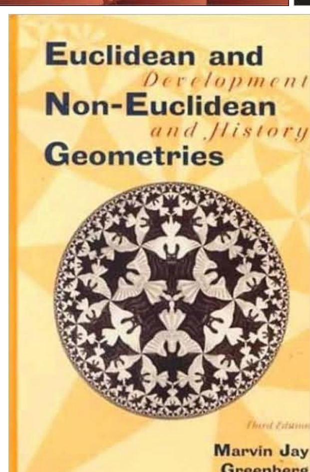

EUCLIDEAN AND NON-EUCLIDEAN GEOMETRIES
Development and History
MARVIN JAY GREENBERG
Fourth Edition

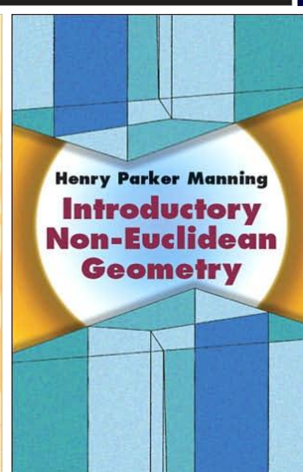

OXFORD MATHEMATICAL MONOGRAPHS
Complex Hyperbolic Geometry
WILLIAM M. GOLDMAN
OXFORD SCIENCE PUBLICATIONS

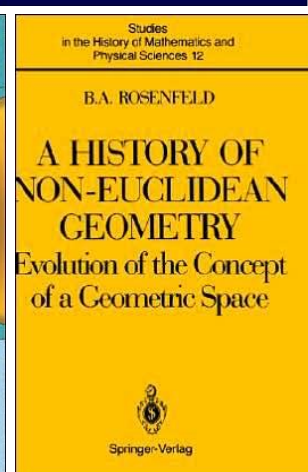

CONTEMPORARY MATHEMATICS
311
Complex Manifolds and Hyperbolic Geometry
Clifford J. Earle
William J. Harvey
Sevín Recillas-Pishmish
Editors
American Mathematical Society

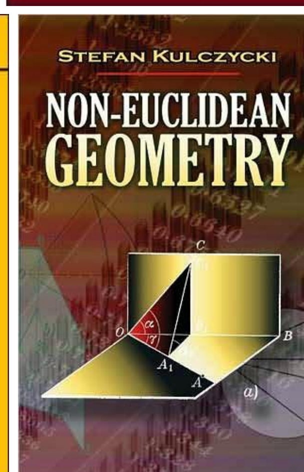

ROBERTO BONOLA
NON-EUCLIDEAN GEOMETRY
WITH A SUPPLEMENT CONTAINING
"THE THEORY OF PARALLELS" BY NICHOLAS LOBACHEVSKI AND
"THE SCIENCE OF ABSOLUTE SPACE" BY JOHN BOLYAI

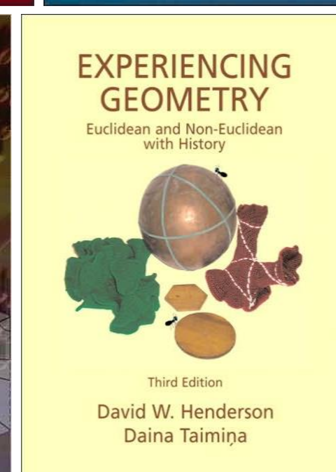

NON-EUCLIDEAN GEOMETRY
H. S. M. COXETER
MATHEMATICAL ASSOCIATION OF AMERICA
SIXTH EDITION

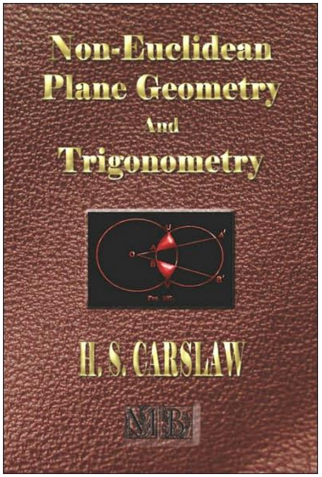
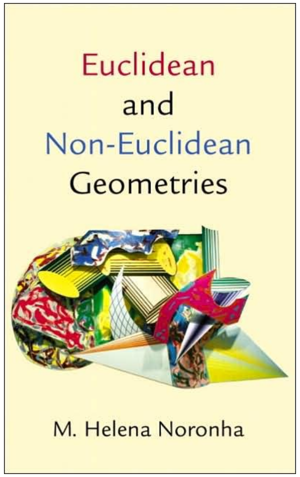
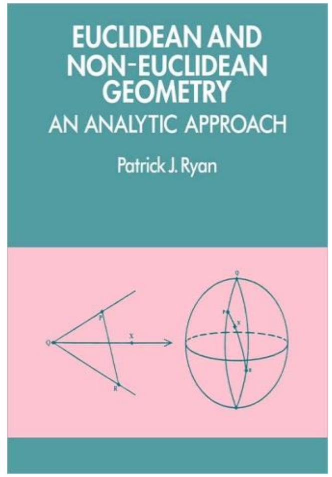
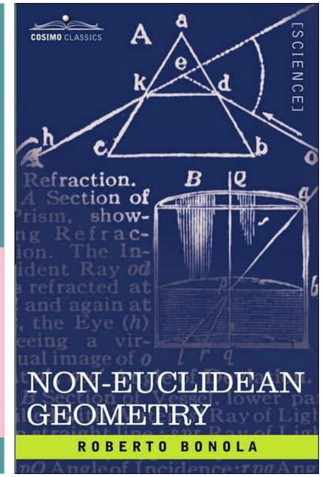
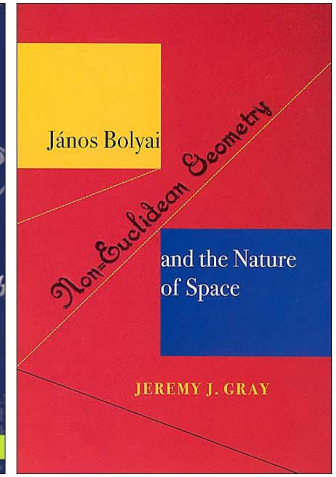
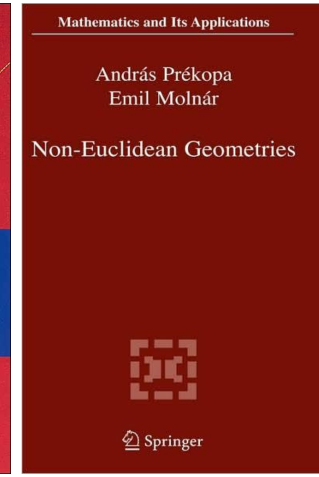
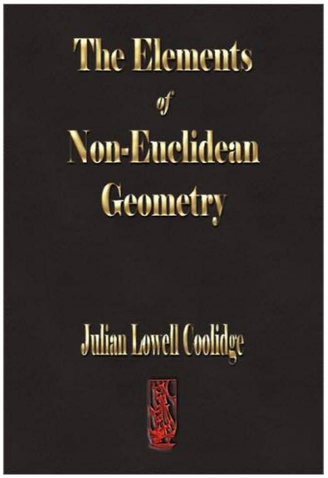
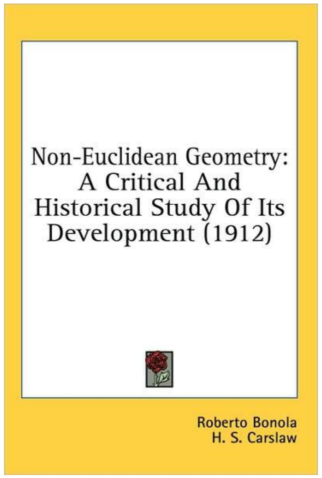

Euclidean and Non-Euclidean Geometries
Development and History
Third Edition
Marvin Jay Greenberg

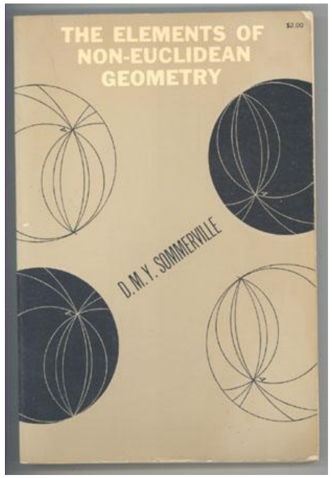
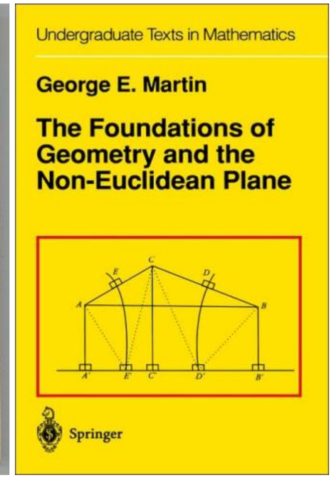
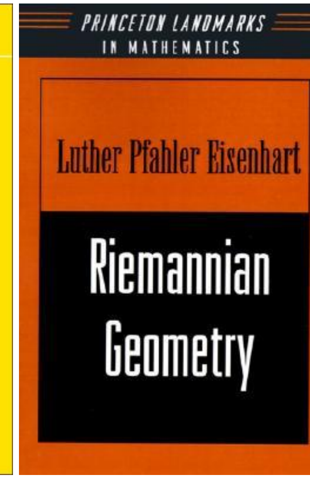
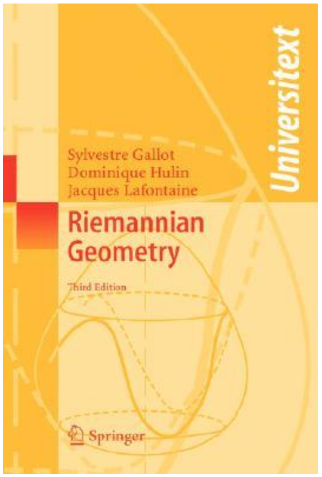
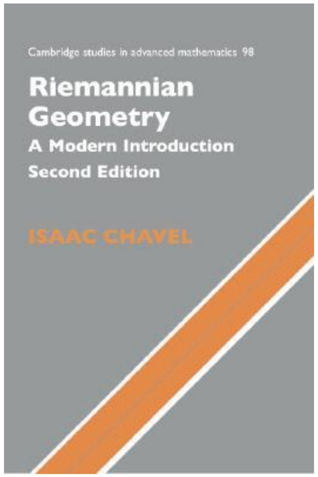
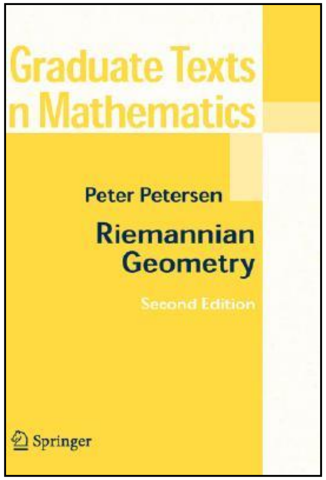
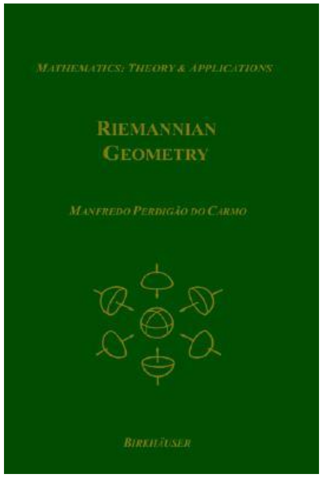
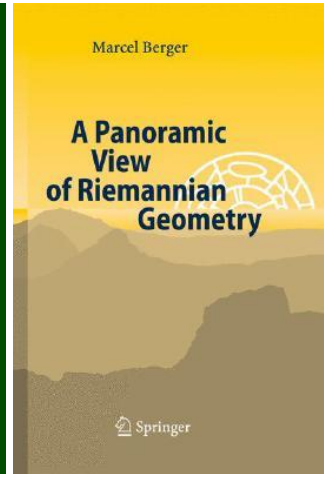
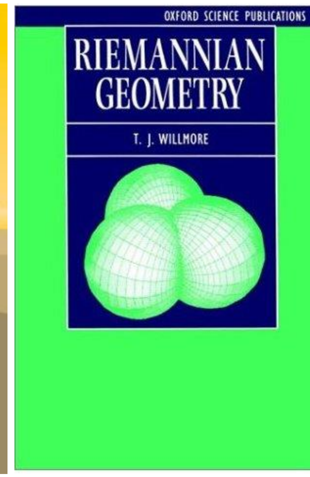

Henry Parker Manning
Introductory Non-Euclidean Geometry


Studies in the History of Mathematics and Physical Sciences 12
B.A. ROSENFELD
A HISTORY OF NON-EUCLIDEAN GEOMETRY
Evolution of the Concept of a Geometric Space
Springer-Verlag


STEFAN KULCZYCKI
NON-EUCLIDEAN GEOMETRY

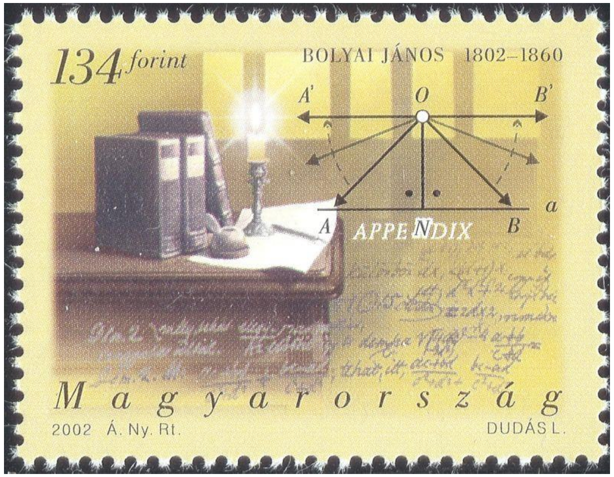

EXPERIENCING GEOMETRY
Euclidean and Non-Euclidean with History
Third Edition
David W. Henderson
Daina Taimiņa

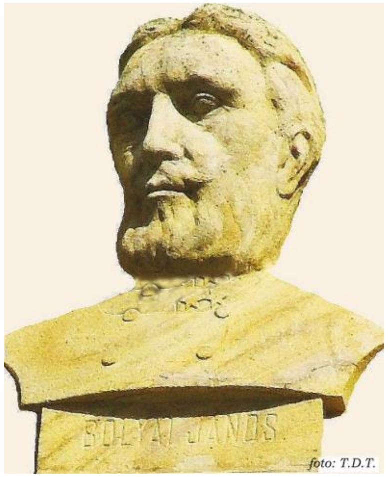

Non-Euclidean
Plane Geometry
And
Trigonometry
H. S. CARSLAW

Euclidean
and
Non-Euclidean
Geometries
M. Helena Noronha

EUCLIDEAN AND
NON-EUCLIDEAN
GEOMETRY
AN ANALYTIC APPROACH
Patrick J. Ryan

COSIMO CLASSICS
[SCIENCE]
NON-EUCLIDEAN
GEOMETRY
ROBERTO BONOLA

János Bolyai
Non-Euclidean Geometry
and the Nature
of Space
JEREMY J. GRAY

Mathematics and Its Applications
András Prékopa
Emil Molnár
Non-Euclidean Geometries
Springer

The Elements
of
Non-Euclidean
Geometry
Julian Lowell Coolidge

Non-Euclidean Geometry:
A Critical And
Historical Study Of Its
Development (1912)
Roberto Bonola
H. S. Carslaw

Introduction To Non-
Euclidean Geometry
Harold E. Wolfe

THE ELEMENTS OF
NON-EUCLIDEAN
GEOMETRY
D. M. Y. SOMMERVILLE

Undergraduate Texts in Mathematics
George E. Martin
The Foundations of
Geometry and the
Non-Euclidean Plane
Springer

PRINCETON LANDMARKS
IN MATHEMATICS
Luther Pfahler Eisenhart
Riemannian
Geometry

Universitext
Sylvestre Gallot
Dominique Hulin
Jacques Lafontaine
Riemannian
Geometry
Third Edition
Springer

Cambridge studies in advanced mathematics 98
Riemannian
Geometry
A Modern Introduction
Second Edition
ISAAC CHAVEL

Graduate Texts
in Mathematics
Peter Petersen
Riemannian
Geometry
Second Edition
Springer

MATHEMATICS: THEORY & APPLICATIONS
RIEMANNIAN
GEOMETRY
MANFREDO PERDIGÃO DO CARMO
BIRKHÄUSER

Marcel Berger
A Panoramic
View
of Riemannian
Geometry
Springer

OXFORD SCIENCE PUBLICATIONS
RIEMANNIAN
GEOMETRY
T. J. WILLMORE

Founders of Non-Euclidean Geometry

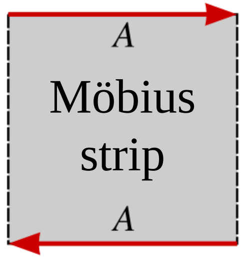
János Bolyai (1802-1860)

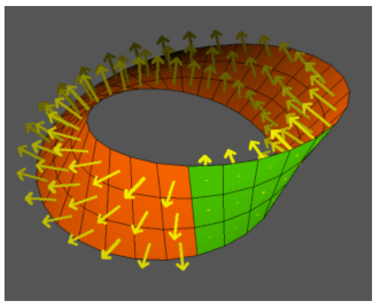


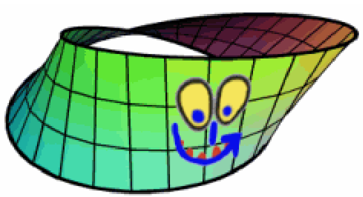


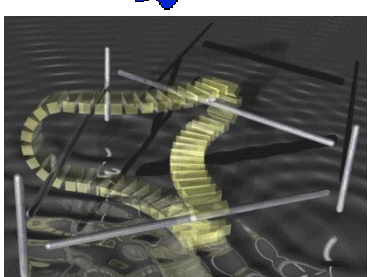
Nikolai Ivanovich Lobachevsky (1792-1856)

Non-Euclidean Non-Orientable Surfaces

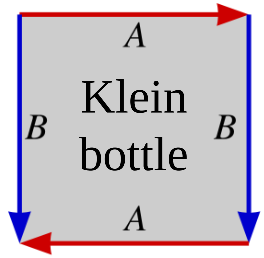


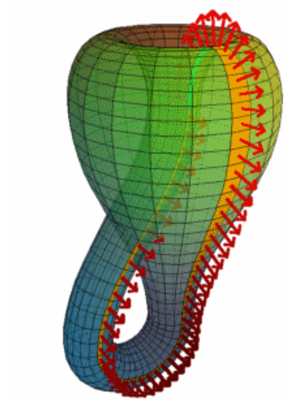


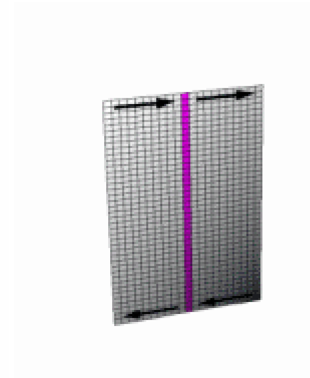


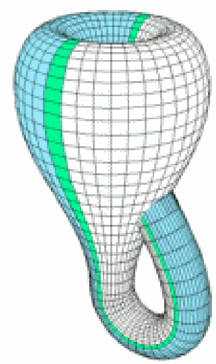


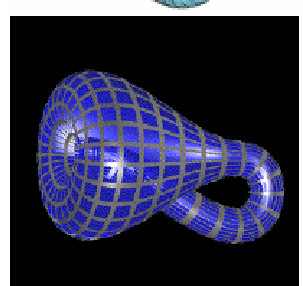
one side,
one boundary!



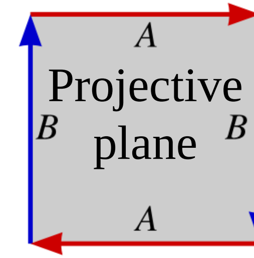


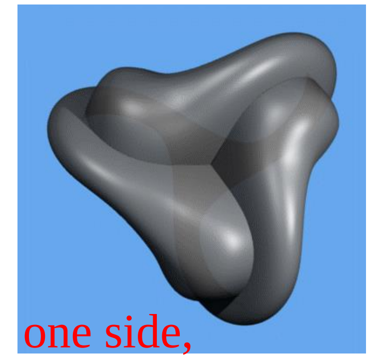




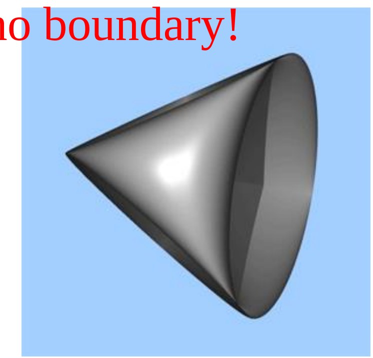


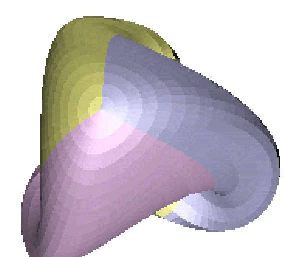
one side,
no boundary!



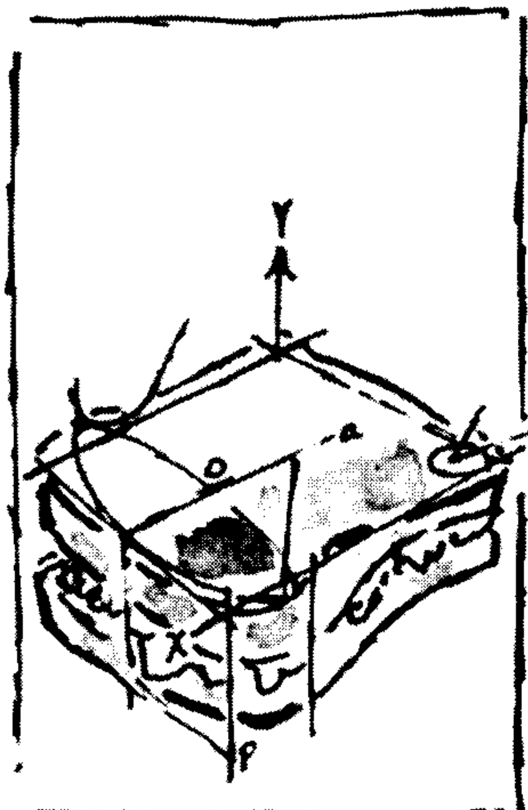


one side,
no boundary!

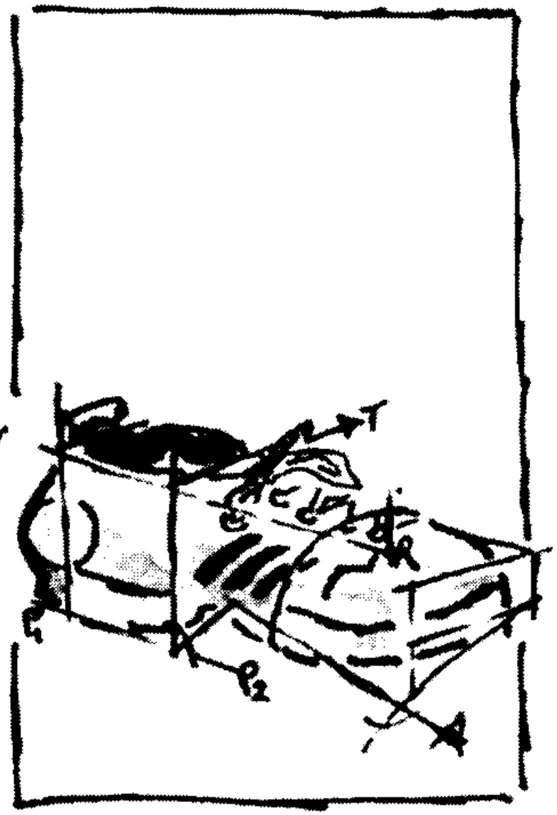




THE GEOMETRY OF EVERYDAY LIFE



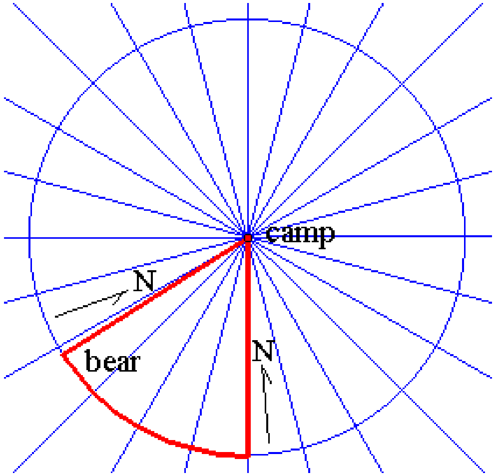
TUNA SANDWICH



SNEAKER

GRANDMA

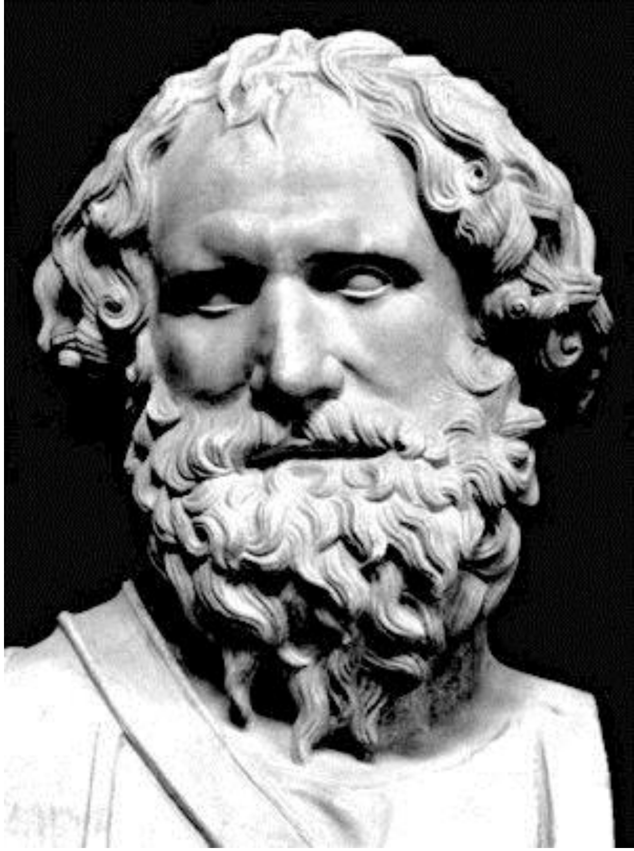
Problem: A man leaves his house and walks one mile south. He then walks one mile west and sees a Bear. Then he walks one mile north back to his house. What color was the bear?



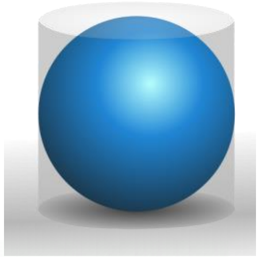

Problem: Is the house location unique?

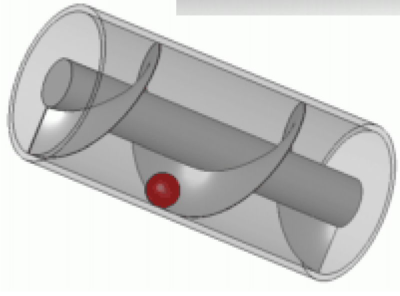
Historical Perspectives

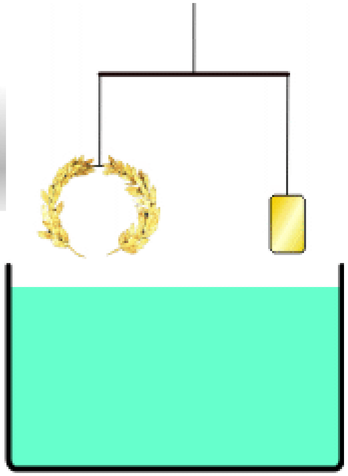
Archimedes of Syracuse (287-212 BC)

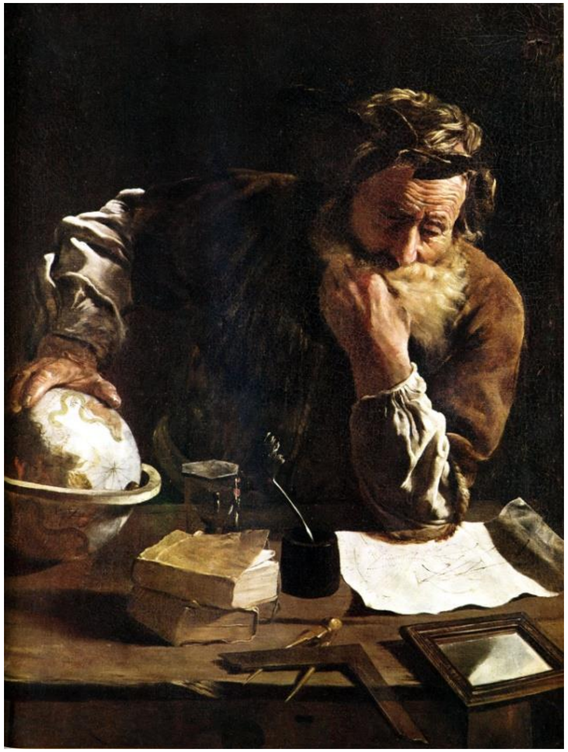


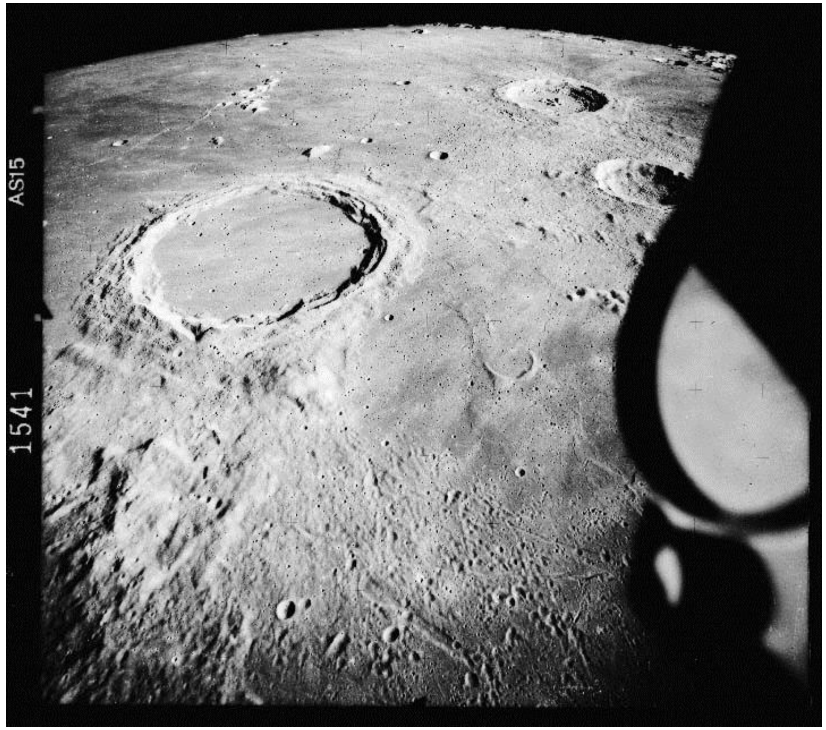
- Mathematician, physicist, engineer, inventor, astronomer
- Leading scientist of classical antiquity
- Originated hydrostatics, mechanics Archimedean screw, spiral, lever
- Discovered Archimedes' principle
- Used infinitesimals, approximated Pi
- Designed siege and naval weapons
- Invented large number notation

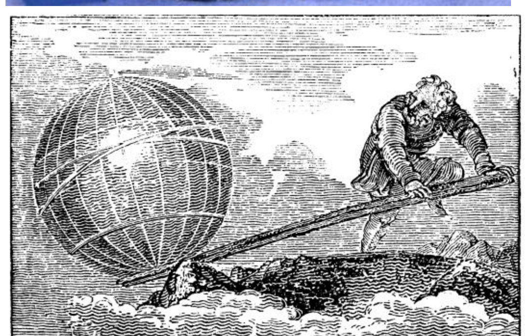


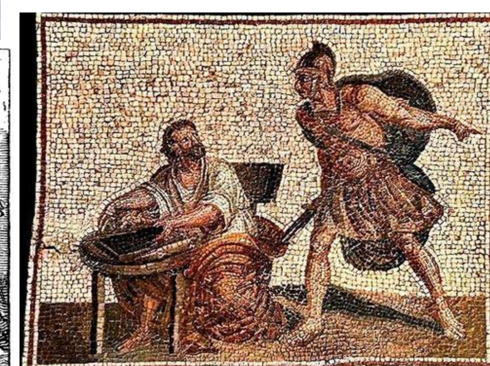


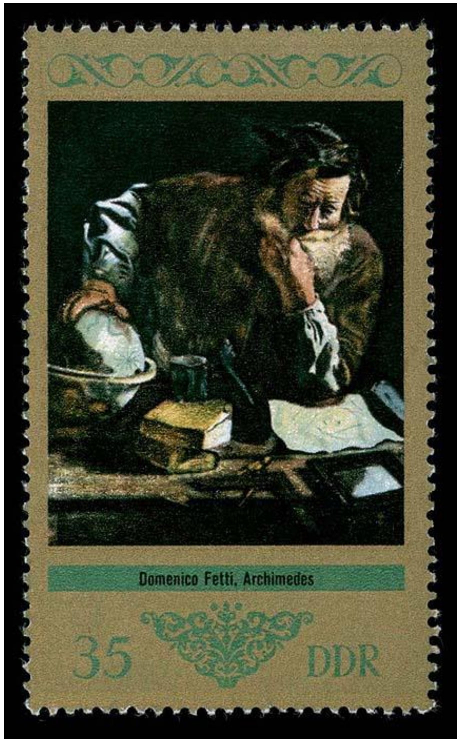


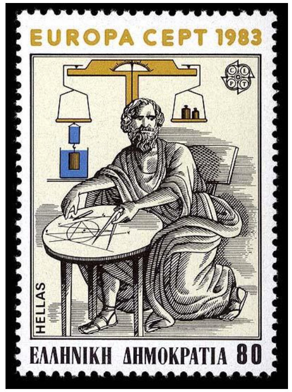


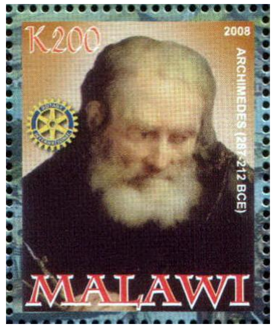

AS15
1541

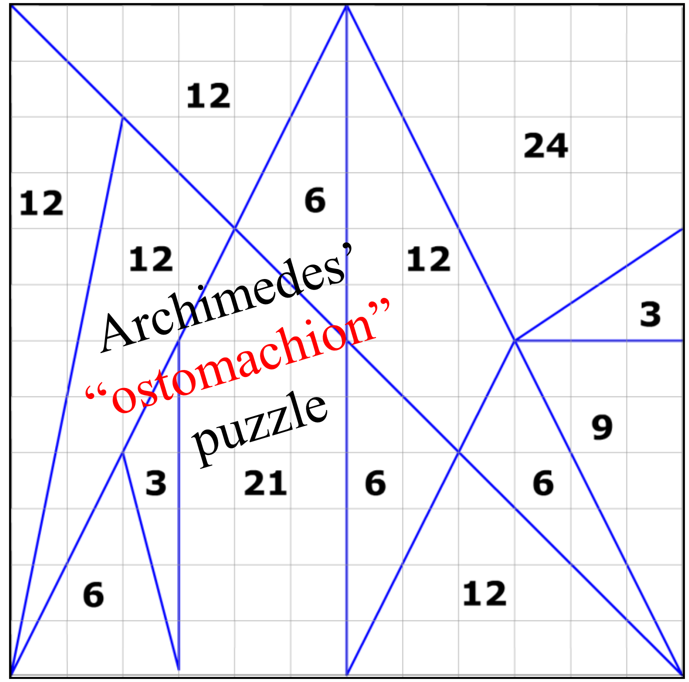


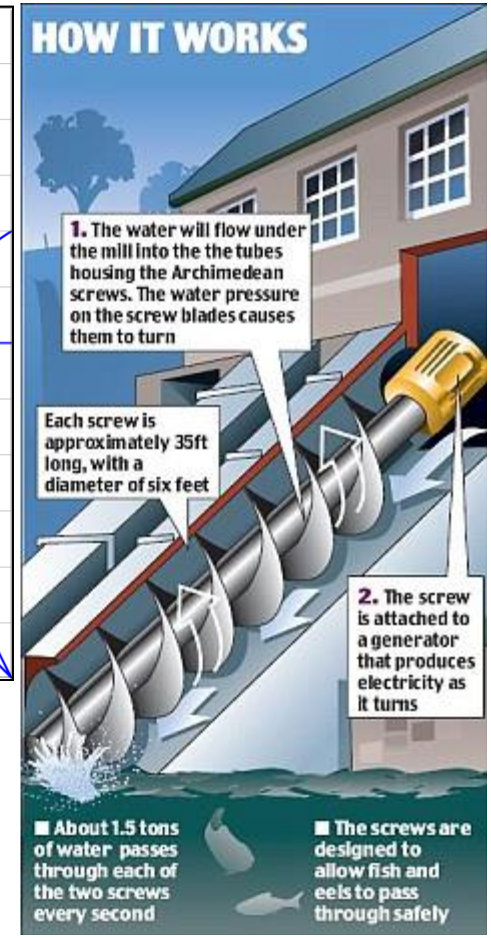



Domenico Fetti, Archimedes
35
DDR

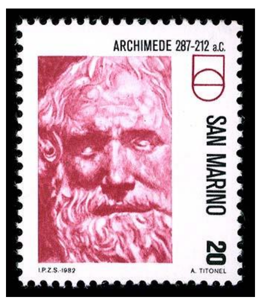

EUROPA CEPT 1983
HELLAS
ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ 80


K200
2008
ARCHIMEDES (287-212 BCE)
MALAWI

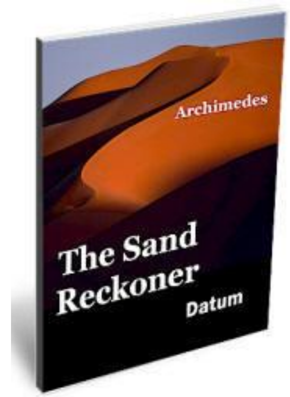

12
24
12
6
12
12
3
Archimedes'
"ostomachion"
puzzle
9
3
21
6
6
6
12


HOW IT WORKS
1. The water will flow under the mill into the the tubes housing the Archimedean screws. The water pressure on the screw blades causes them to turn
Each screw is approximately 35ft long, with a diameter of six feet
2. The screw is attached to a generator that produces electricity as it turns
About 1.5 tons of water passes through each of the two screws every second
The screws are designed to allow fish and eels to pass through safely

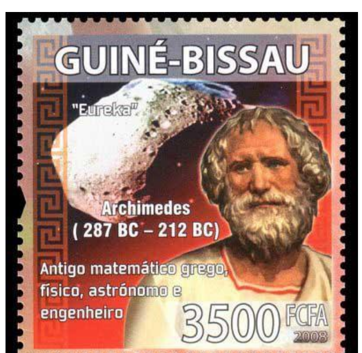

NICARAGUA
LEY DE ARQUIMEDES
AEREO
2
CORDOBAS
DE LA RUE
LAS 10 FORMULAS MATEMATICAS QUE CAMBIARON LA FAZ DE LA TIERRA


ARCHIMEDE 287-212 a.C.
SAN MARINO
20


CORREOS
25
CTS
ESPAÑA


Archimedes
The Sand Reckoner
Datum


EUROPA
ARCHIMEDE
ITALIA
500
I.P.Z.S.-ROMA-1983
E. DONNINI


GUINÉ-BISSAU
"Eureka"
Archimedes
(287 BC – 212 BC)
Antigo matemático grego, físico, astrónomo e engenheiro
3500 FCFA
2008


€ 0,41
800
2000
ITALIA
ANNO MONDIALE DELLA MATEMATICA


ARCHIMEDES
is my homeboy


17.4M/17.4M
zoom. Use Alt and Ctrl for additional options.


EUREKA!

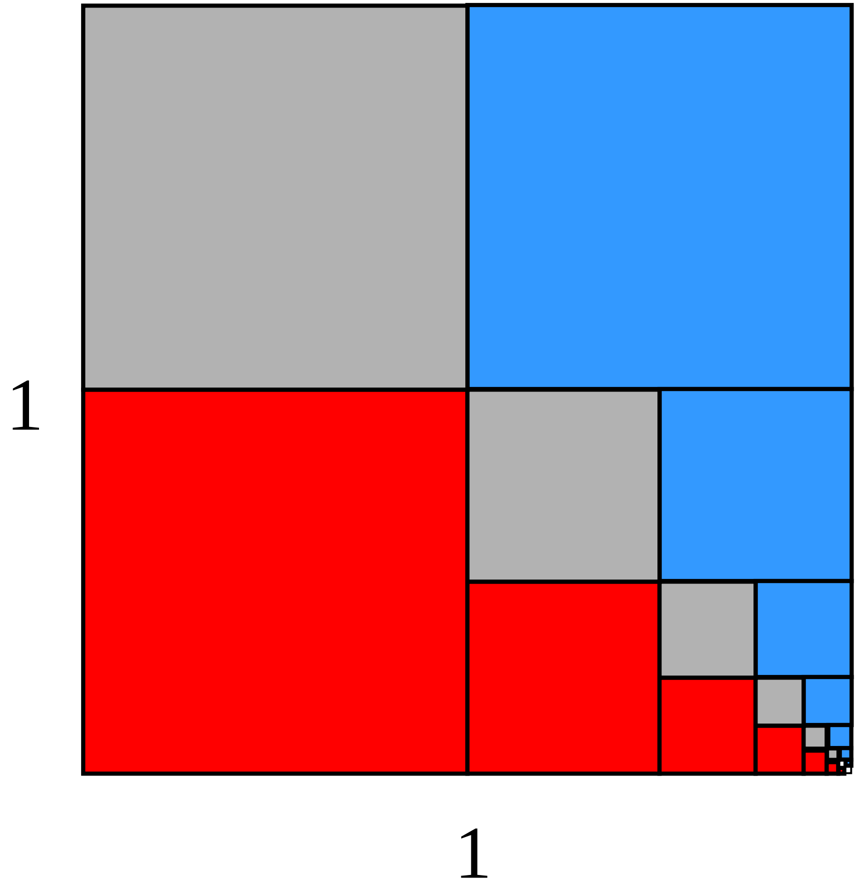

ARCHIMEDES
83 km / 2150 m
97 / 10 / 09
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
8
B/W QuickCam
a.cidadao@mail.telepac.pt


S. Harris
"THERE GOES ARCHIMEDES WITH HIS CONFOUNDED LEVER AGAIN"

"The periodic table."

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.

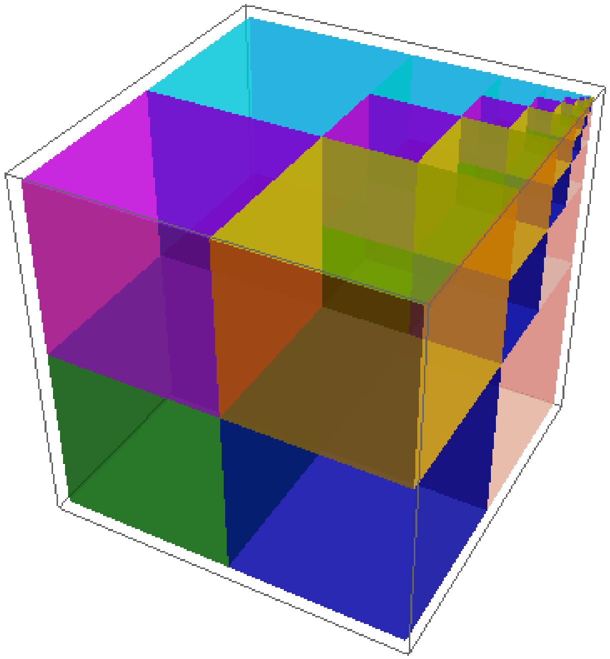


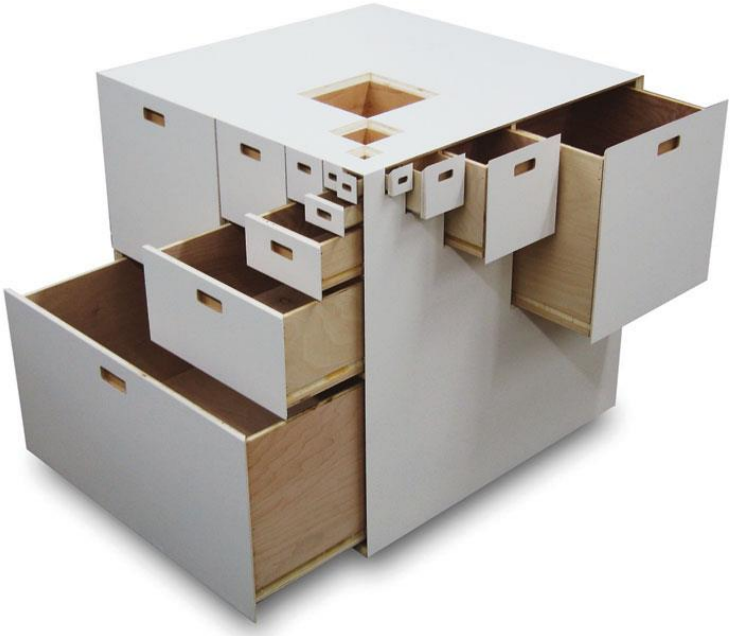
$$\sum_{i=1}^{\infty} \frac{1}{4^i} = \frac{1}{3}$$

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = \frac{1}{7}$$





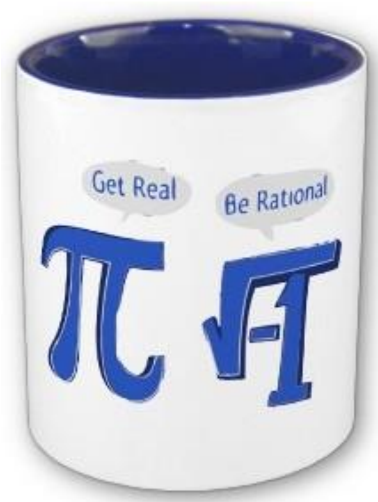
Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i?

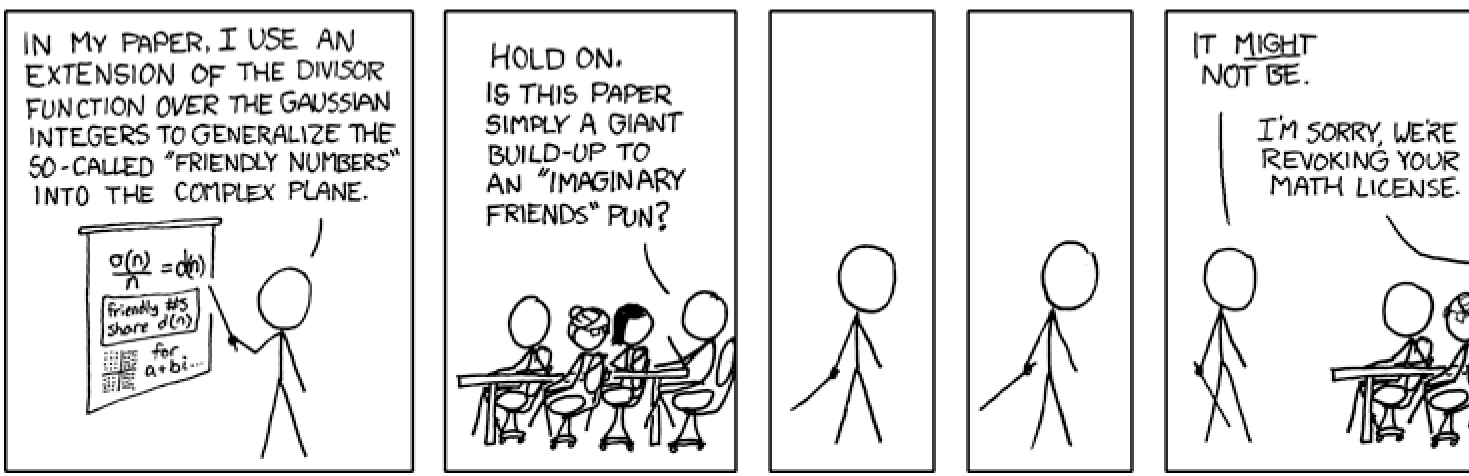
$$i^i = \frac{1}{\sqrt{e^{\pi}}} = 0.207879...$$

$$e^{ix} = \cos(x) + i\sin(x)$$

$$i^i = \frac{1}{\sqrt{e^{\pi+2k\pi}}}$$

i^i is multi-valued!

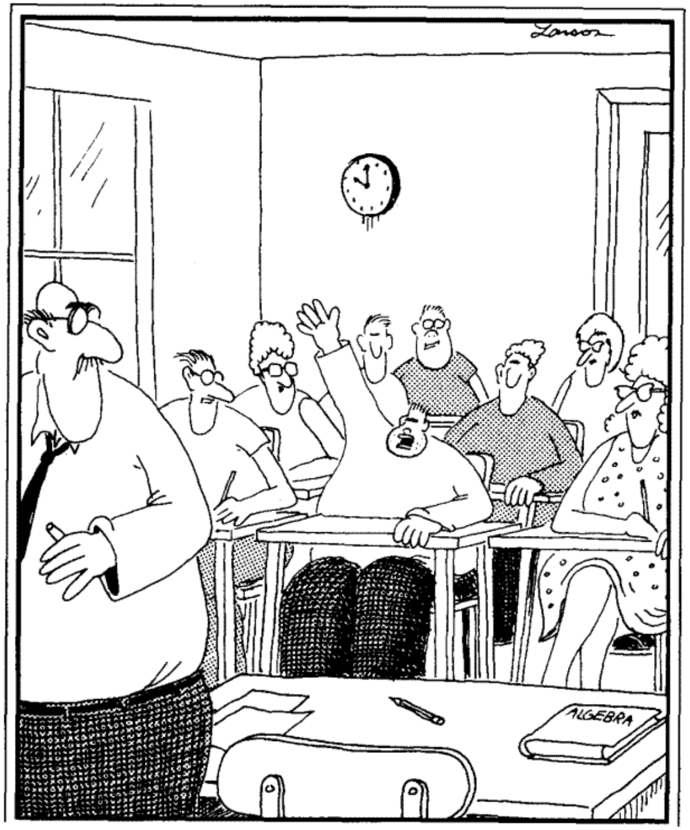




Problem: Solve the following equation for X:

$$X^{X^{X^{X^{X^{\cdot^{\cdot^{\cdot}}}}}}} = 2$$

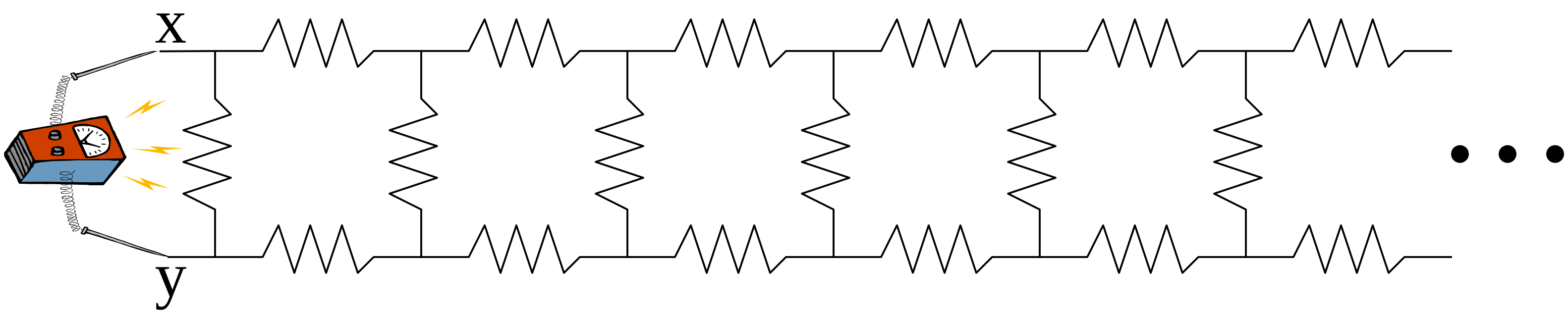
where the stack of exponentiated x's extends forever.

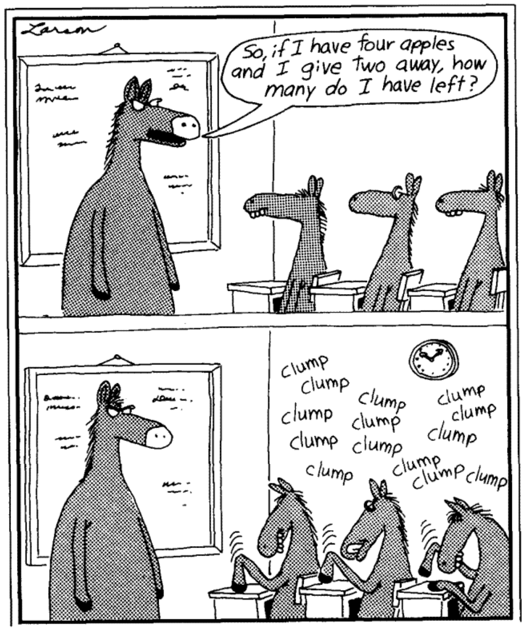


"Mr. Osborne, may I be excused? My brain is full."

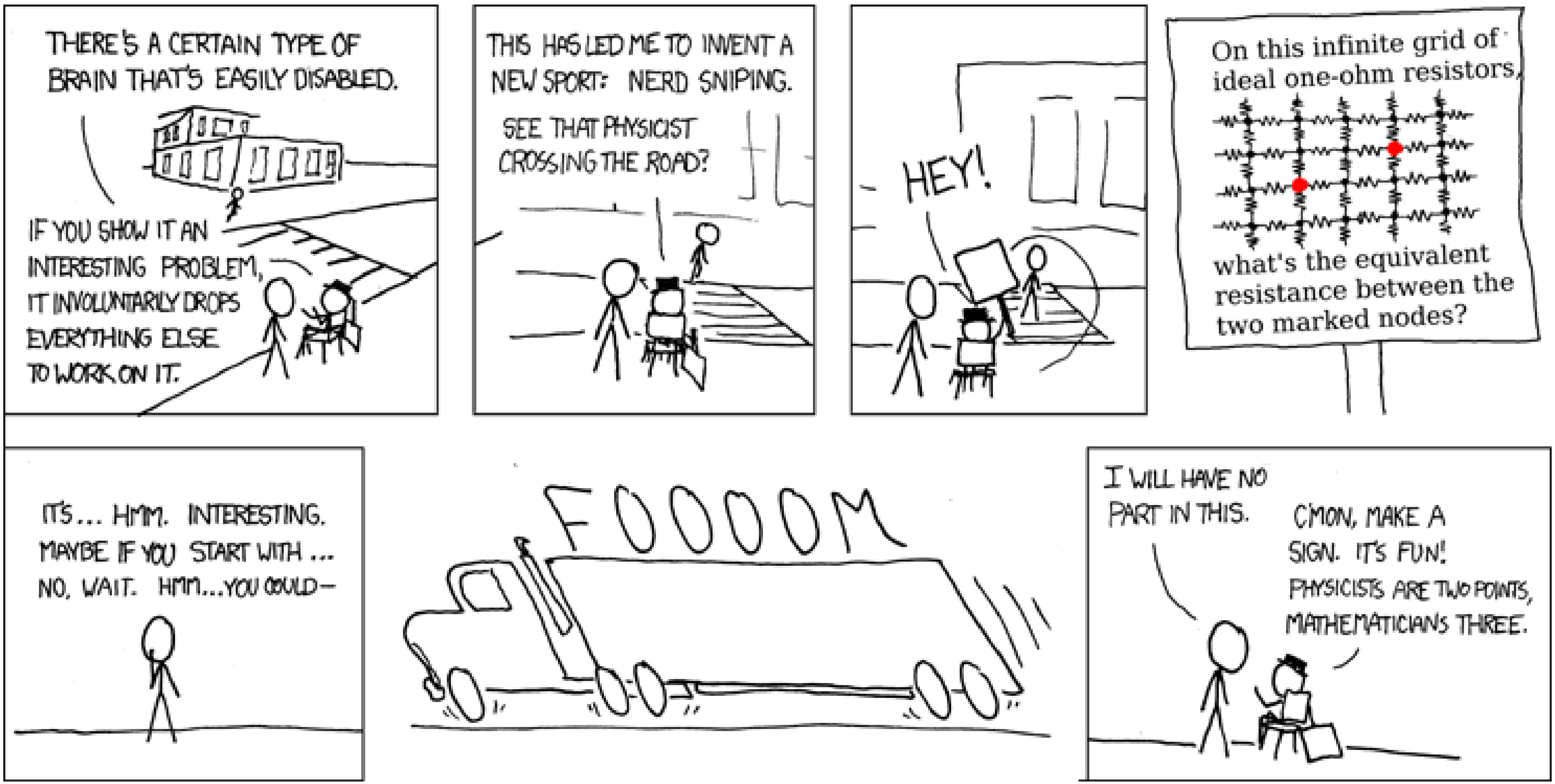
- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Problem: For the given infinite ladder of resistors of resistance R each, what is the resistance measured between points x and y?





- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



THERE'S A CERTAIN TYPE OF BRAIN THAT'S EASILY DISABLED.
IF YOU SHOW IT AN INTERESTING PROBLEM, IT INVOLUNTARILY DROPS EVERYTHING ELSE TO WORK ON IT.
THIS HAS LED ME TO INVENT A NEW SPORT: NERD SNIPING.
SEE THAT PHYSICIST CROSSING THE ROAD?
HEY!
On this infinite grid of ideal one-ohm resistors, what's the equivalent resistance between the two marked nodes?
IT'S... HMM. INTERESTING. MAYBE IF YOU START WITH ... NO, WAIT. HMM...YOU COULD—
FOOOOM
I WILL HAVE NO PART IN THIS.
C'MON, MAKE A SIGN. IT'S FUN! PHYSICISTS ARE TWO POINTS, MATHEMATICIANS THREE.