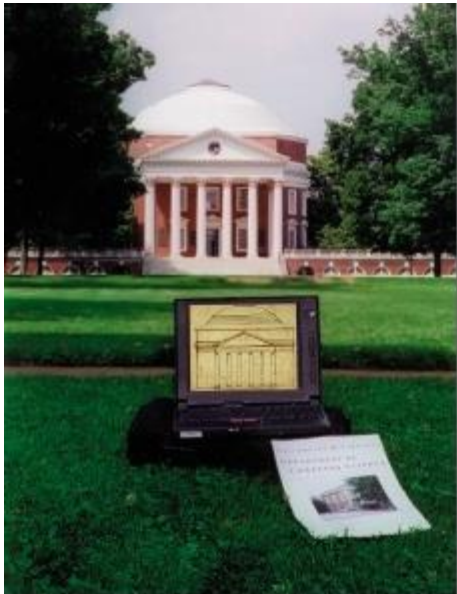


Theory of Computation
CS3102

Gabriel Robins

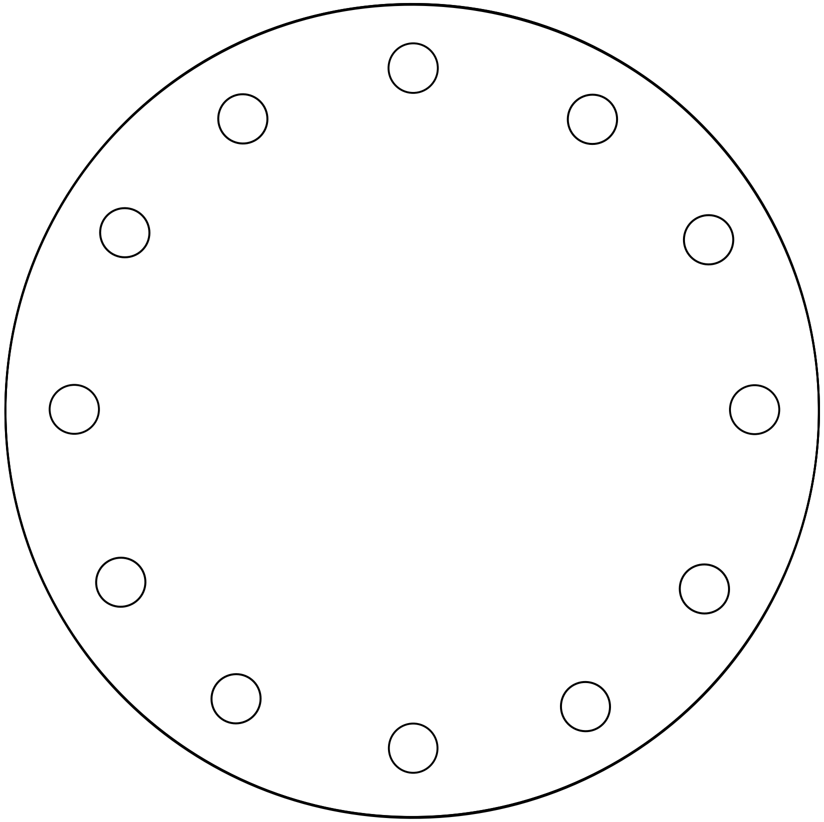
Department of

Computer Science

University of Virginia

www.cs.virginia.edu/robins/theory

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?

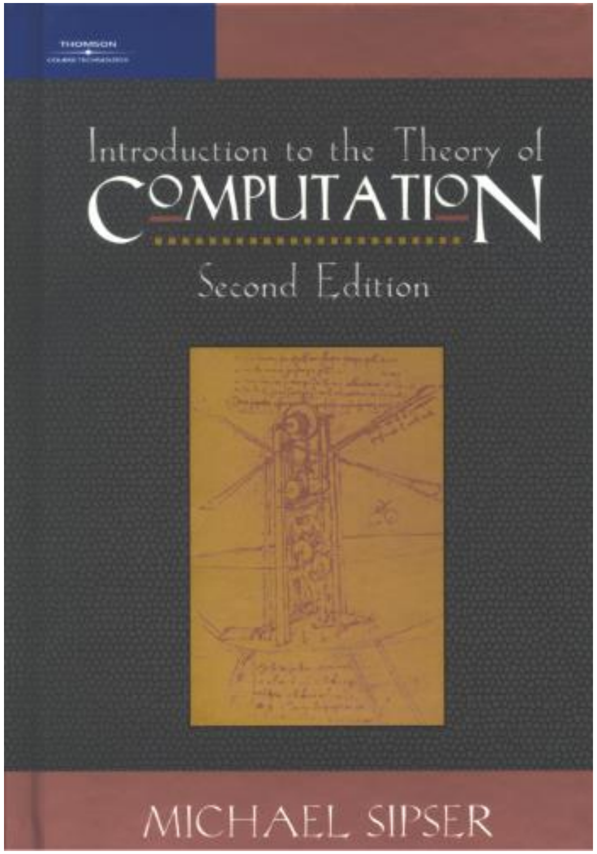


- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Theory of Computation (CS3102) - Textbook

Textbook:

Introduction to the Theory of Computation, by Michael Sipser (MIT), 2nd Edition, 2005



Good Articles / videos:

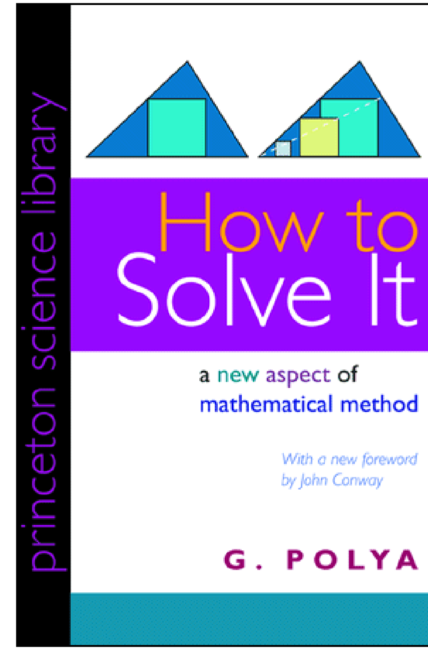
www.cs.virginia.edu/~robins/CS_readings.html

Theory of Computation (CS3102)

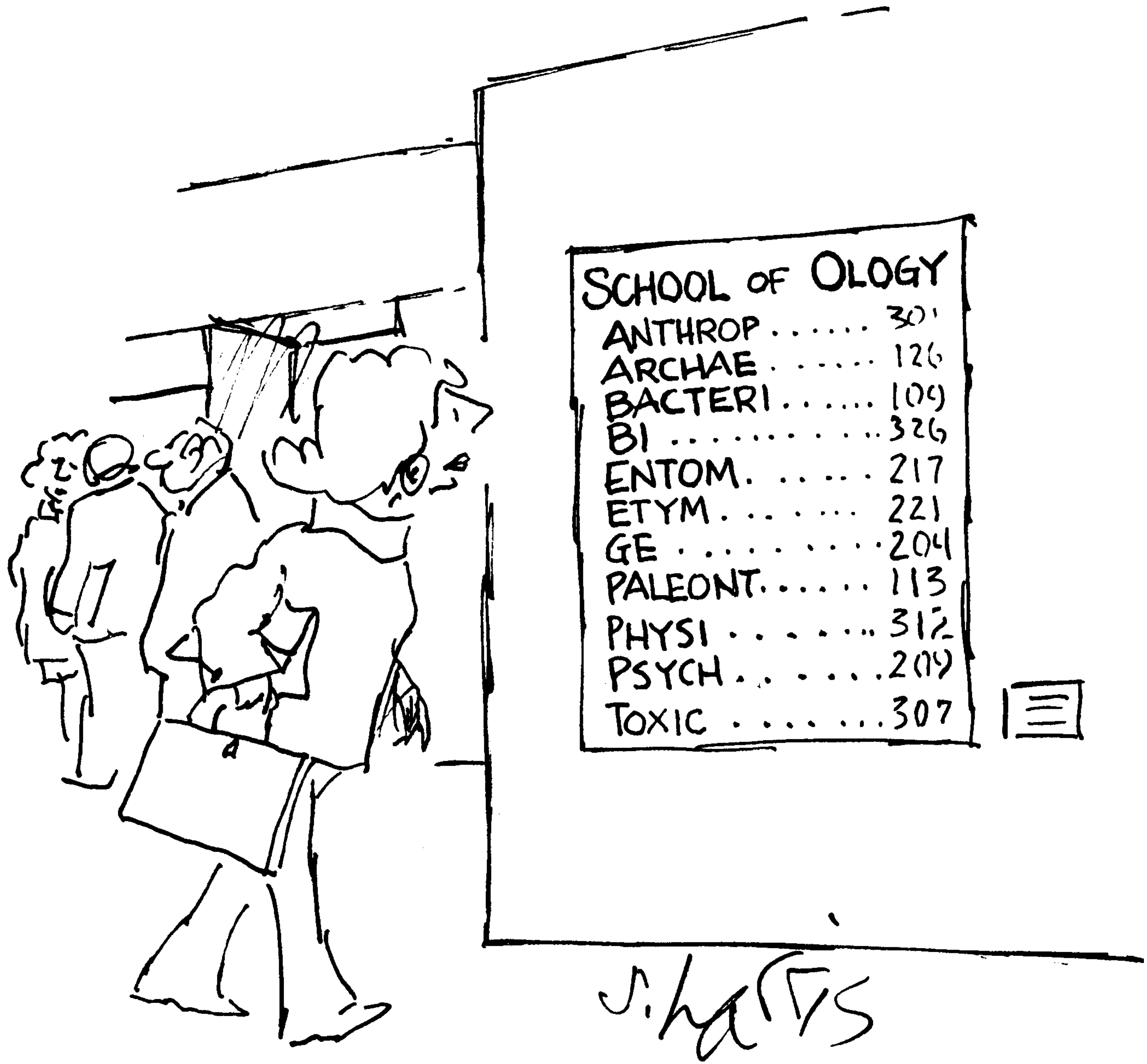
Required reading:

How to Solve It, by George Polya (MIT), Princeton University Press, 1945

- A classic on problem solving

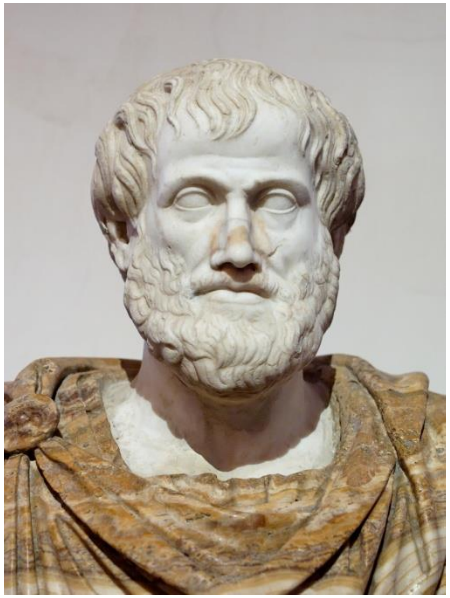


George Polya (1887-1985)

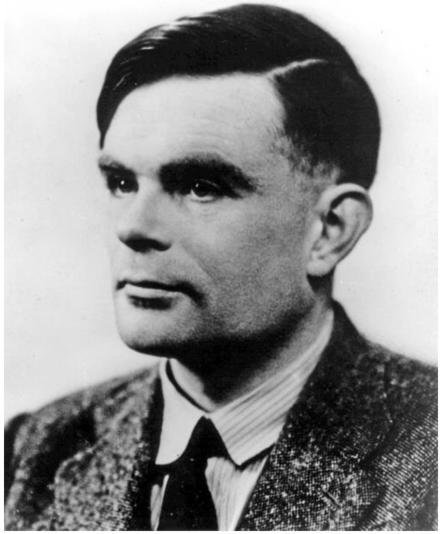

SCHOOL OF OLOGY
ANTHROP 301
ARCHAE 126
BACTERI 109
BI 326
ENTOM 217
ETYM 221
GE 204
PALEONT 113
PHYSI 312
PSYCH 209
TOXIC 307
J. Harris

Theory of Computation (CS3102) - Syllabus

A brief history of computing:



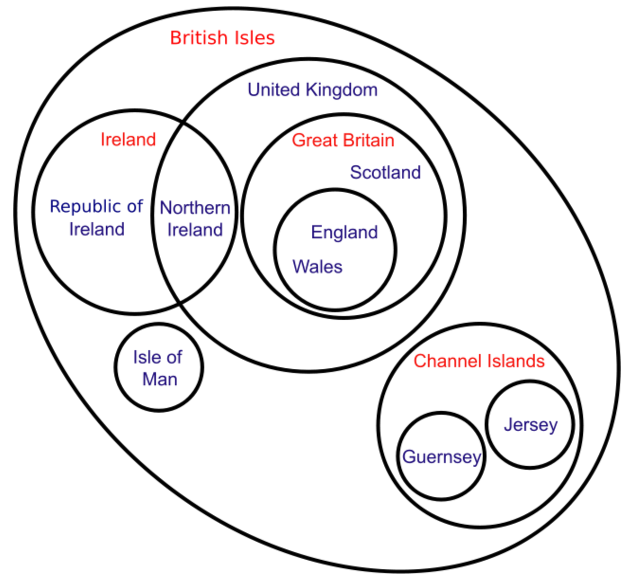
- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan, Babbage, Ada Agusta
- Venn, Carroll, Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Godel, Church, Turing, von Neumann
- Shannon, Kleene, Chomsky

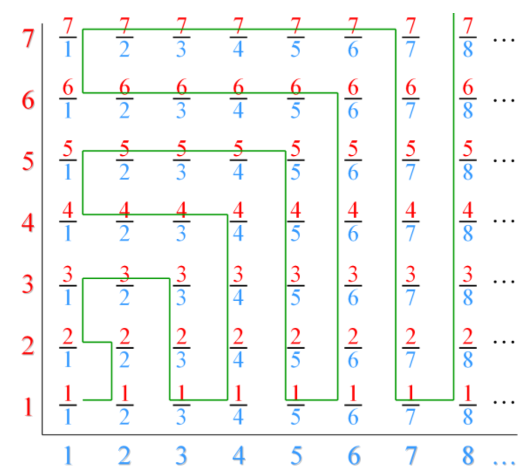


Theory of Computation Syllabus (continued)

Fundamentals:

- Set theory
- Predicate logic
- Formalisms and notation
- Infinities and countability
- Dovetailing / diagonalization
- Proof techniques
- Problem solving
- Asymptotic growth
- Review of graph theory

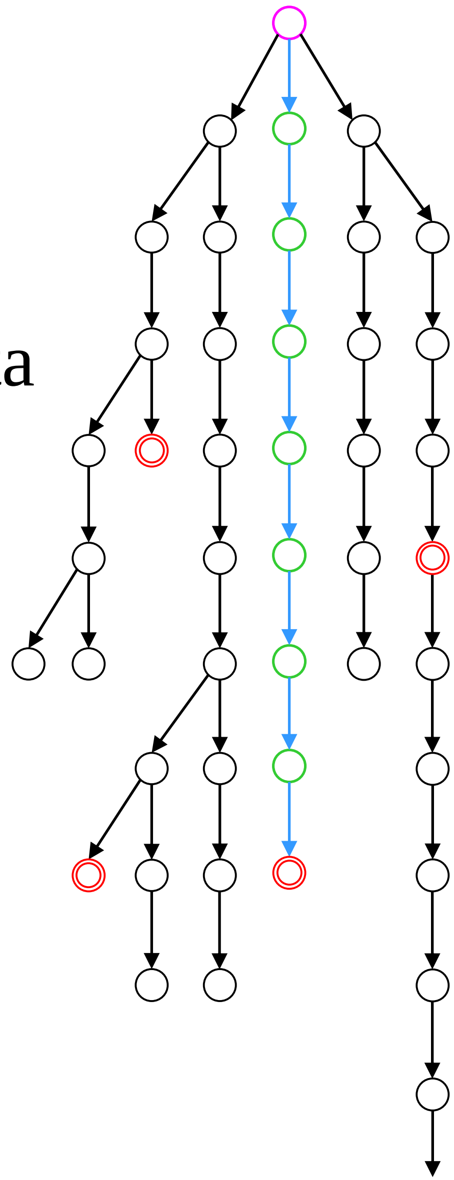




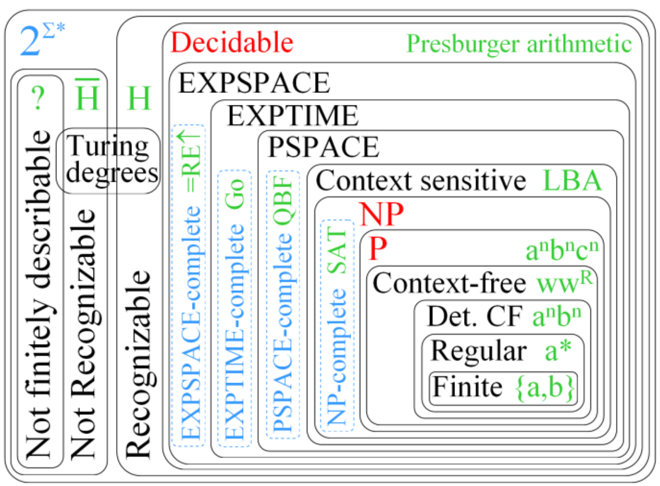
Theory of Computation Syllabus (continued)

Formal languages and machine models:

- The Chomsky hierarchy
- Regular languages / finite automata
- Context-free grammars / pushdown automata
- Unrestricted grammars / Turing machines
- Non-determinism
- Closure operators
- Pumping lemmas
- Non-closures
- Decidable properties



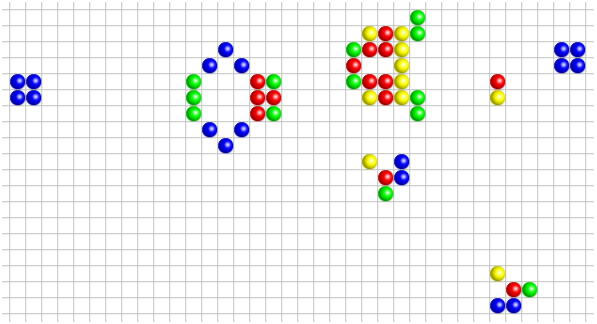
The Extended Chomsky Hierarchy

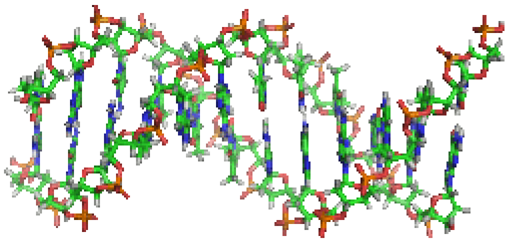


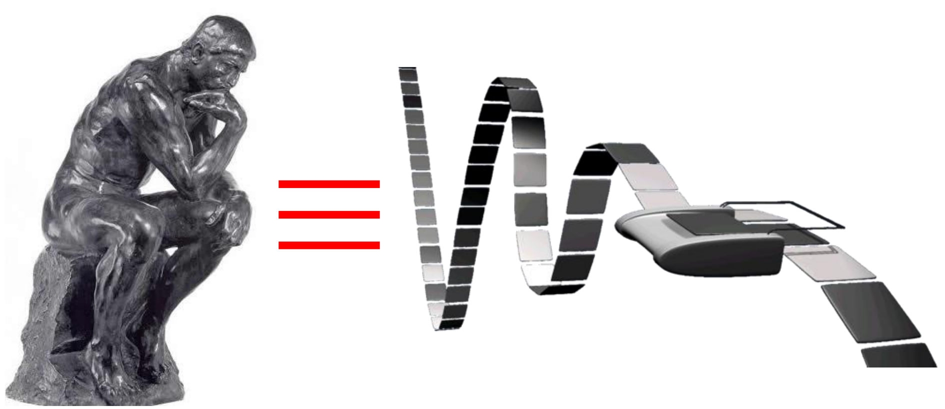
Theory of Computation Syllabus (continued)

Computability and undecidability:

- Basic models
- Modifications and extensions
- Computational universality
- Decidability
- Recognizability
- Undecidability
- Church-Turing thesis
- Rice’s theorem



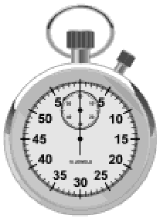


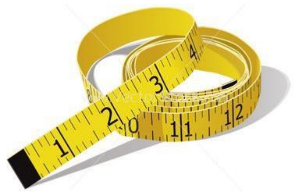


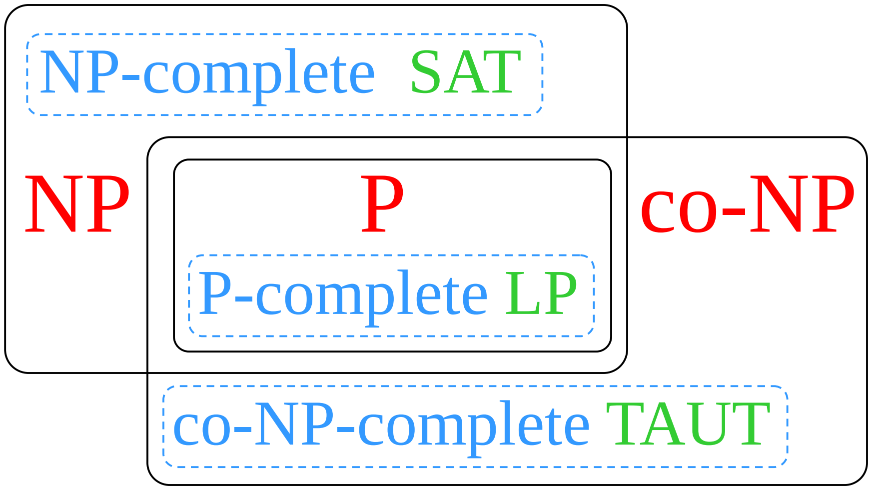
Theory of Computation Syllabus (continued)

NP-completeness:

- Resource-constrained computation
- Complexity classes
- Intractability
- Boolean satisfiability
- Cook-Levin theorem
- Transformations
- Graph clique problem
- Independent sets
- Hamiltonian cycles
- Colorability problems
- Heuristics



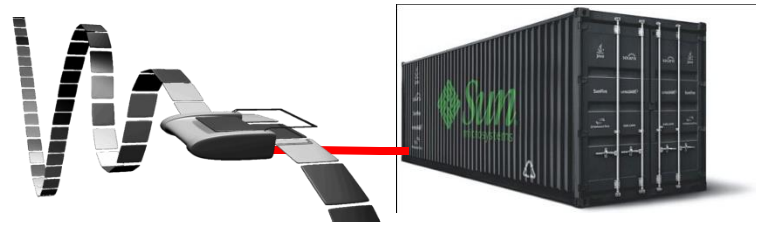


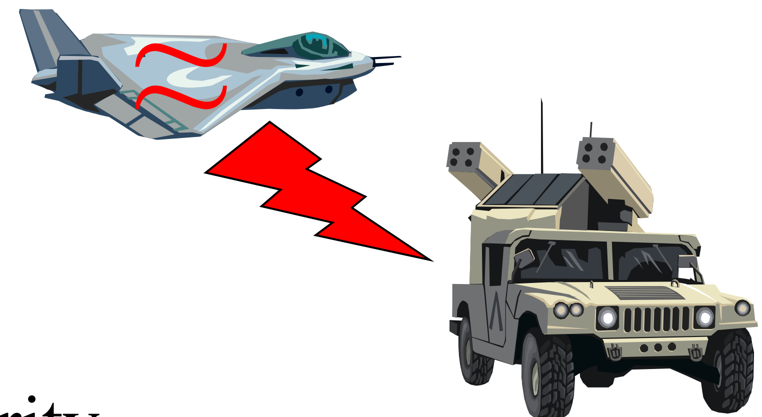


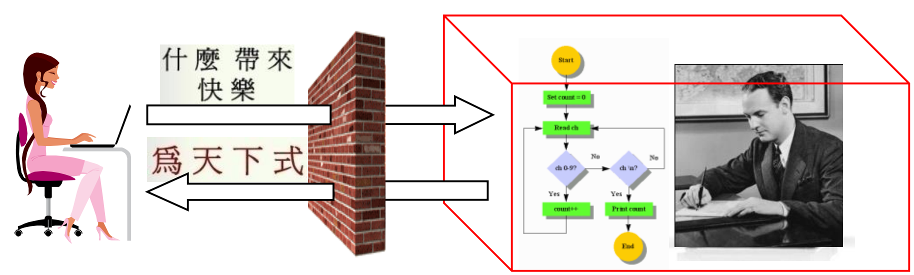
Theory of Computation Syllabus (continued)

Other topics (as time permits):

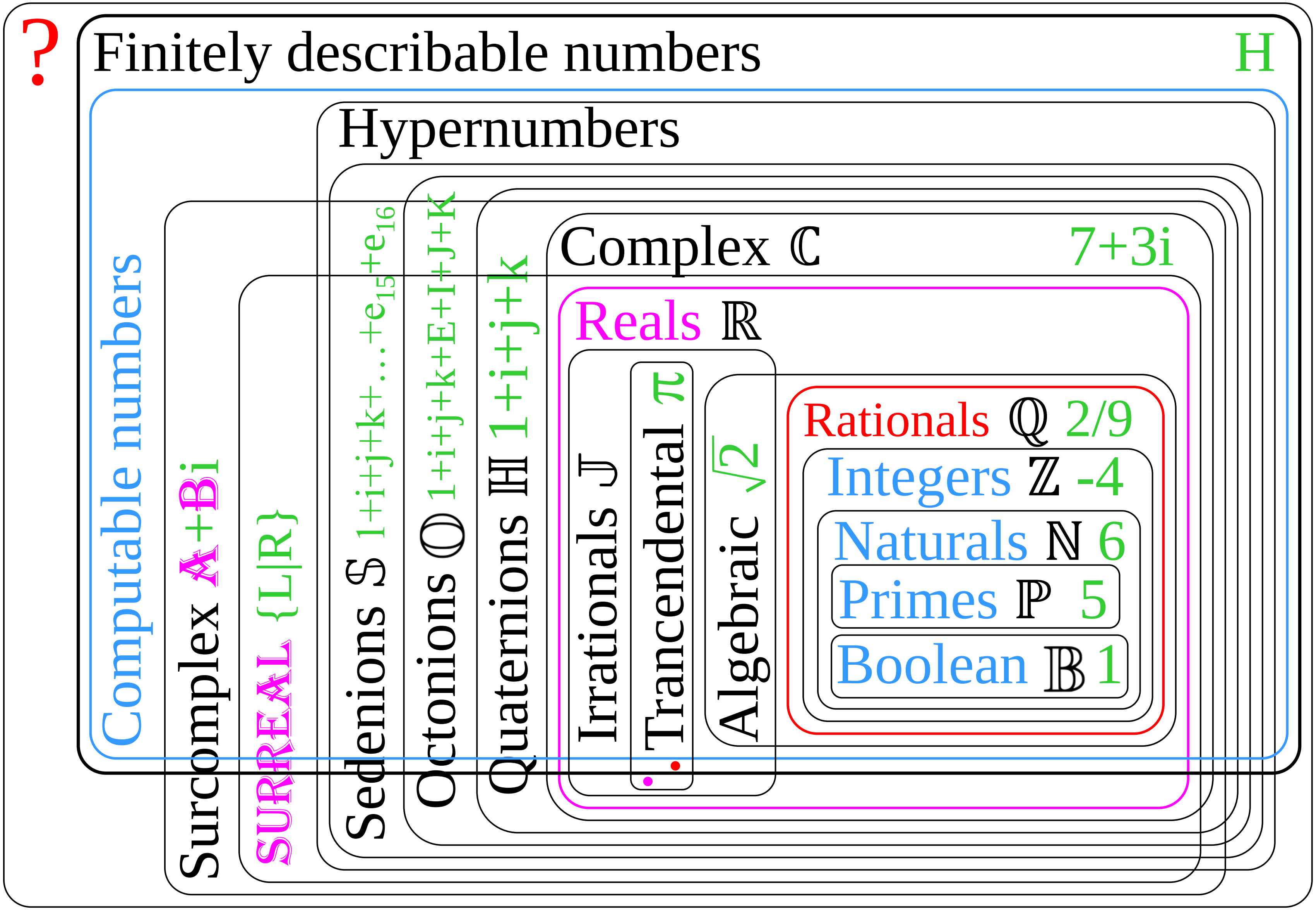
- Generalized number systems
- Oracles and relativization
- Zero-knowledge proofs
- Cryptography & mental poker
- The Busy Beaver problem
- Randomness and compressibility
- The Turing test
- AI and the Technological Singularity







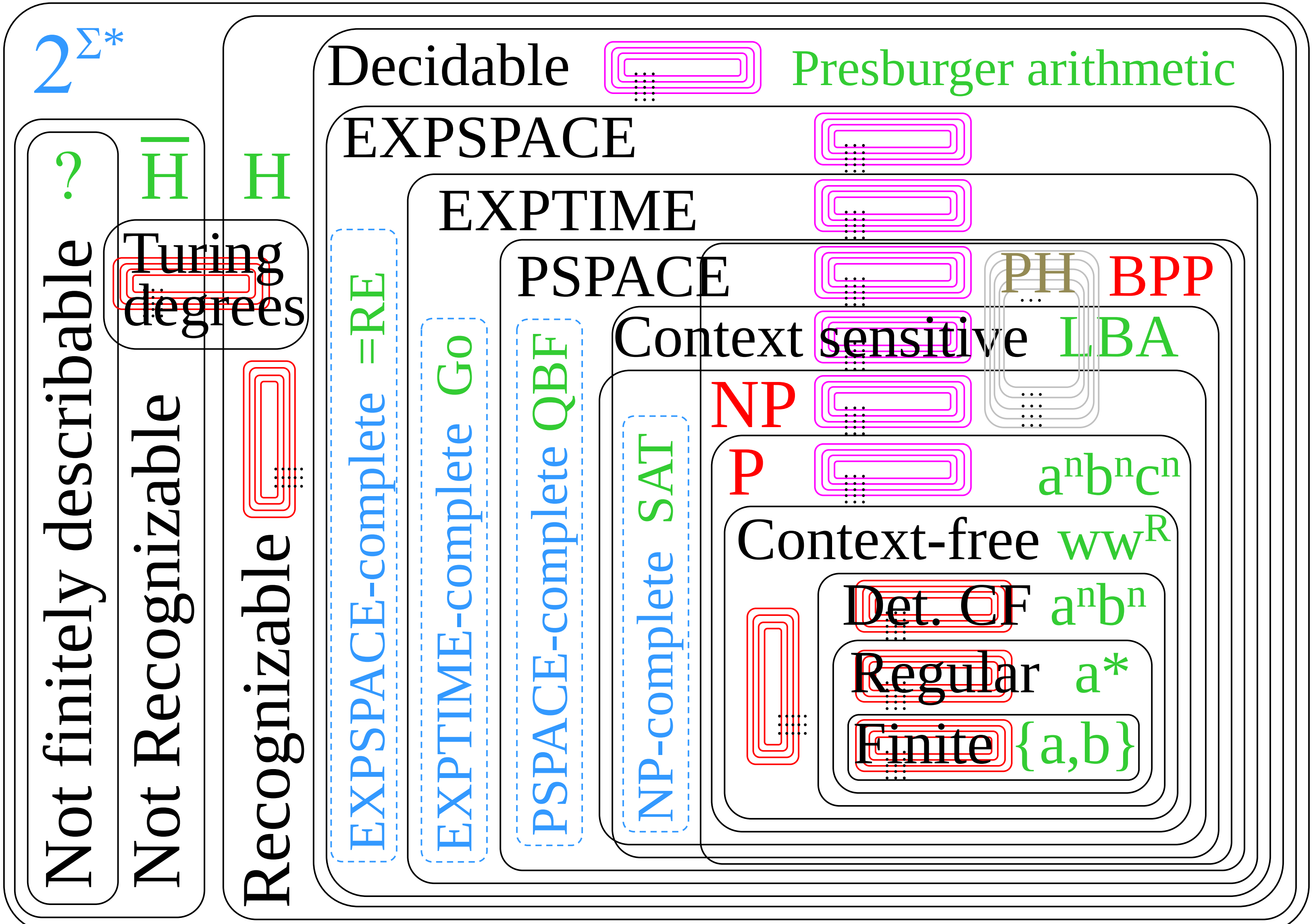
Generalized Numbers



Theorem: some real numbers are not finitely describable!

Theorem: some finitely describable real numbers are not computable!

The Extended Chomsky Hierarchy

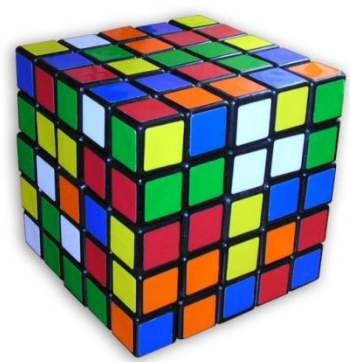


Dense infinite time & space complexity hierarchies

Other infinite complexity & descriptive hierarchies

Overarching Philosophy

- Focus on the "big picture" & "scientific method"
- Emphasis on problem solving & creativity
- Discuss applications & practice
- A primary objective: have fun!

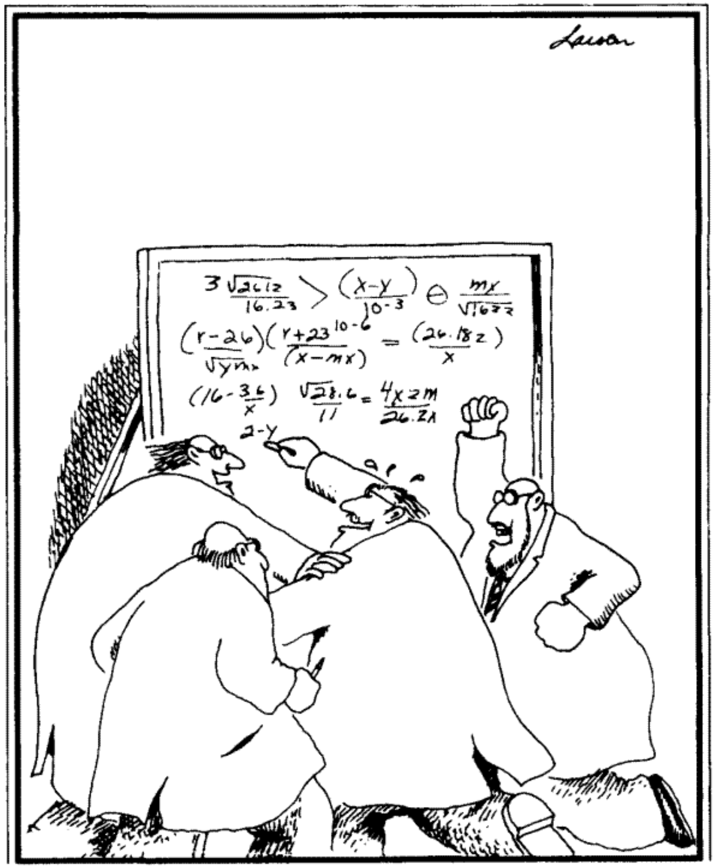


Prerequisites

- Some discrete math & algorithms knowldege
- Ideally, should have taken CS2102
- Course will "bootstrap" (albeit quickly) from first principles
- Critical: Tenacity, patience

Course Organization

- Exams: probably take home
  - Decide by vote
  - Flexible exam schedule
- Problem sets:
  - Lots of problem solving
  - Work in groups!
  - Not formally graded
  - Most exam questions will come from these sets!
- Readings: papers / videos / books
- Extra credit problems
  - In class & take-home
  - Find mistakes in slides, handouts, etc.
- Course materials posted on Web site www.cs.virginia.edu/robins/theory



"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"

Contact Information

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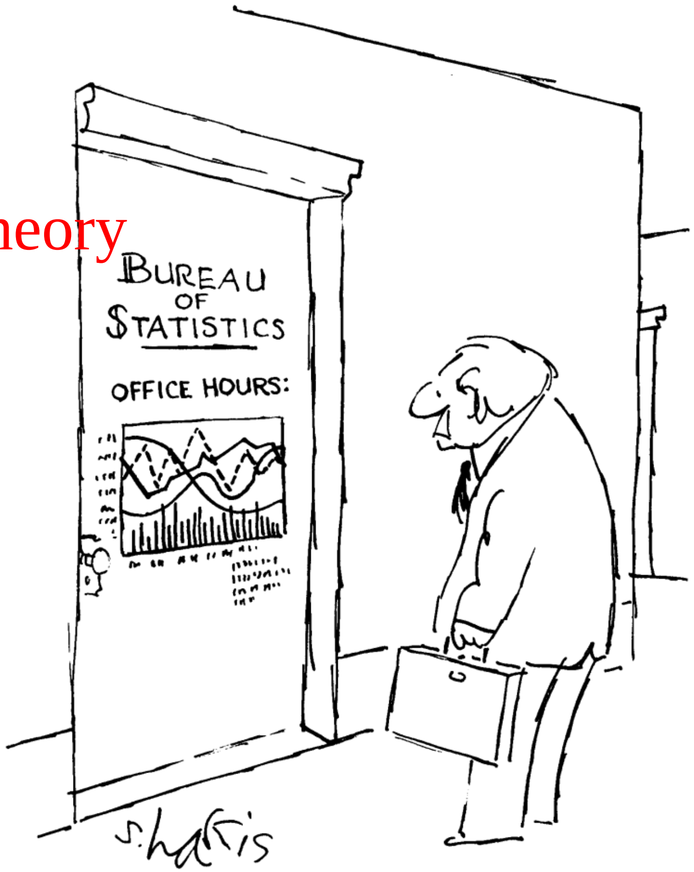
Phone:	(434) 982-2207
Email:	robins@cs.virginia.edu
Web:	www.cs.virginia.edu/robins
	www.cs.virginia.edu/robins/theory

Office hours: after class

- Any other time
- By email (preferred)
- By appointment
- Q&A blog posted on class Web site

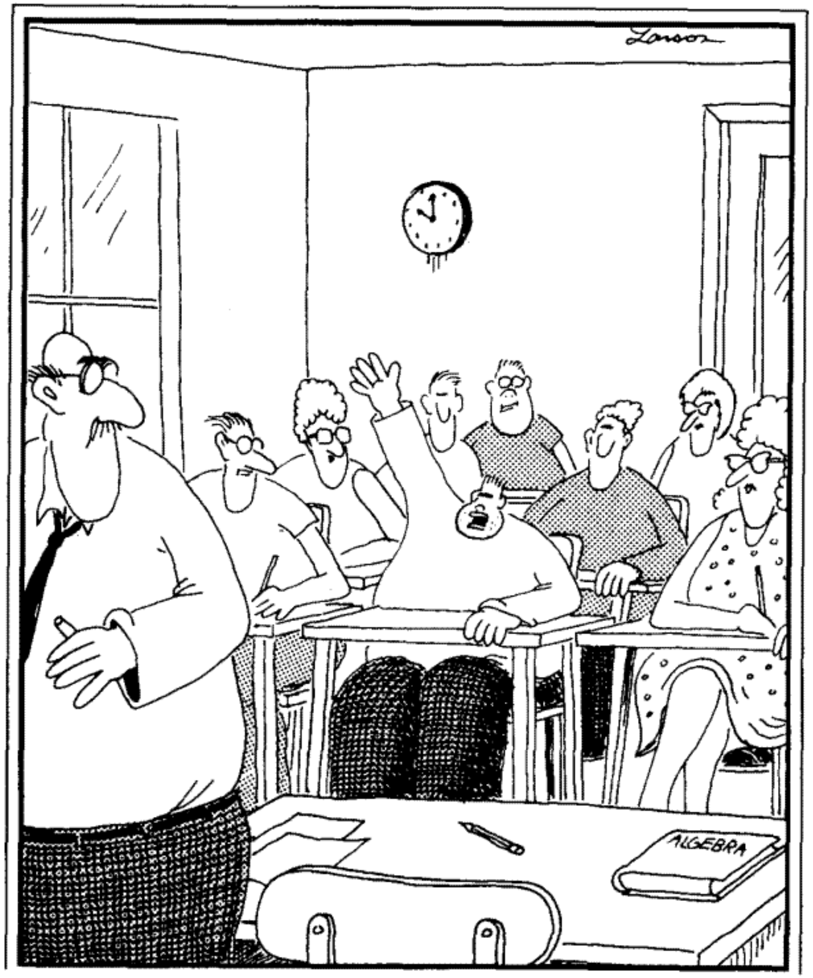


Grading Scheme

• Midterm	35%
• Final	35%
• Readings	30%
• Extra credit	10%

Best strategy:

- Solve lots of problems!
- Do lots of readings / EC!



“Mr. Osborne, may I be excused? My brain is full.”

Course Readings

www.cs.virginia.edu/robins/CS_readings.html

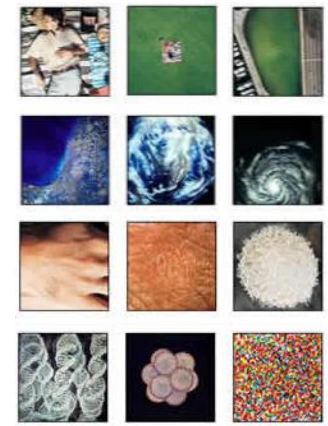
Goal: broad exposure to lots of cool ideas & technologies!

- Required: total of at least 50 items over the semester
- Diversity: minimums in each of 3 categories:
    1. Minimum of 20 videos
    2. Minimum of 20 papers / Web sites
    3. Minimum of 10 books
- More than 50 is even better! (extra credit)
- Some required items in each category
    - Remaining “elective” items should be a diverse mix
- Email all submissions to: homework.cs3102@gmail.com

Required Readings

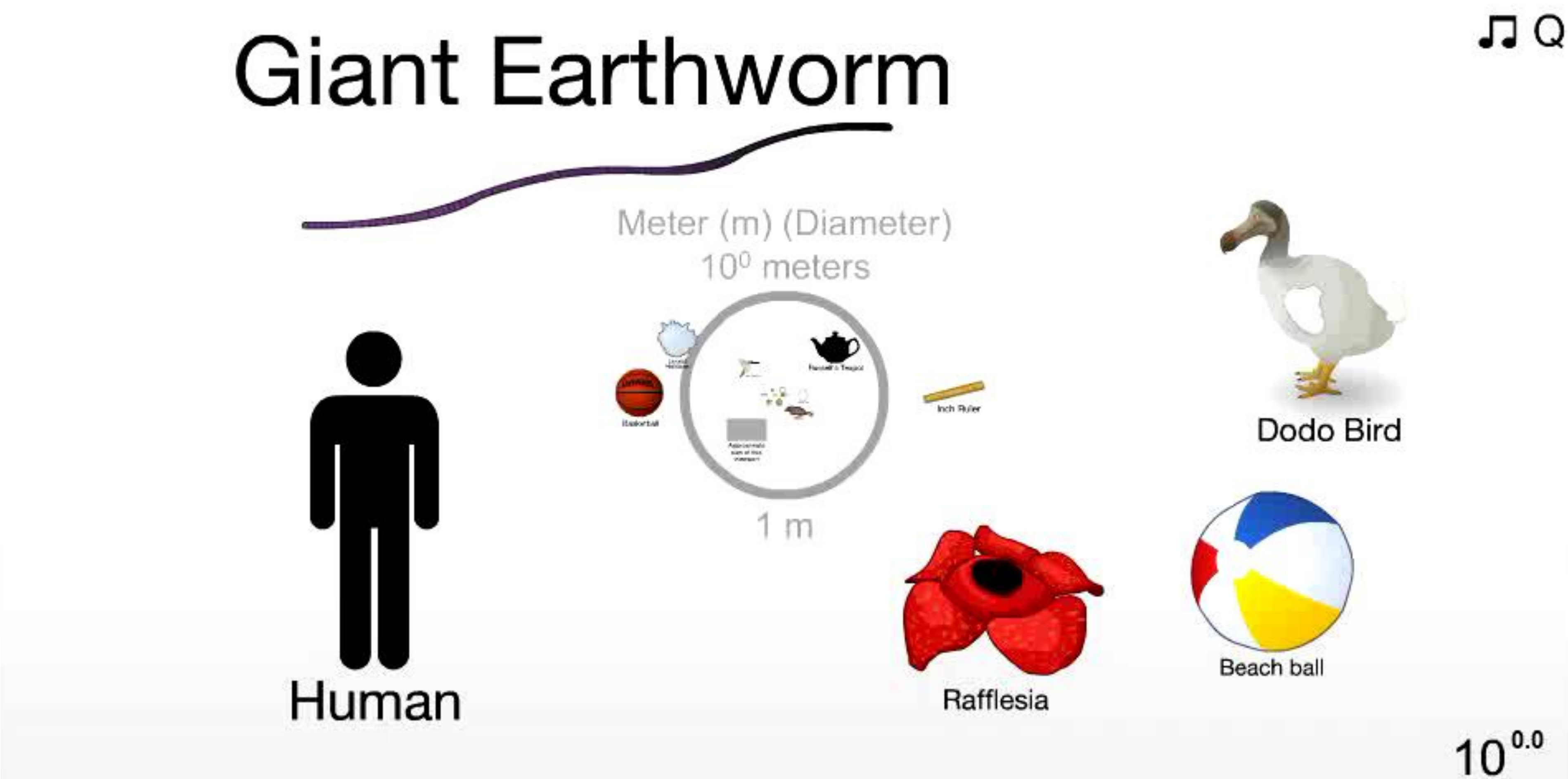
www.cs.virginia.edu/robins/CS_readings.html

- Required videos:
  - Last Lecture, Randy Pausch, 2007
  - Time Management, Randy Pausch, 2007
  - Powers of Ten, Charles and Ray Eames, 1977



Required Reading

- “[Scale of the Universe]”, Cary and Michael Huang, 2012

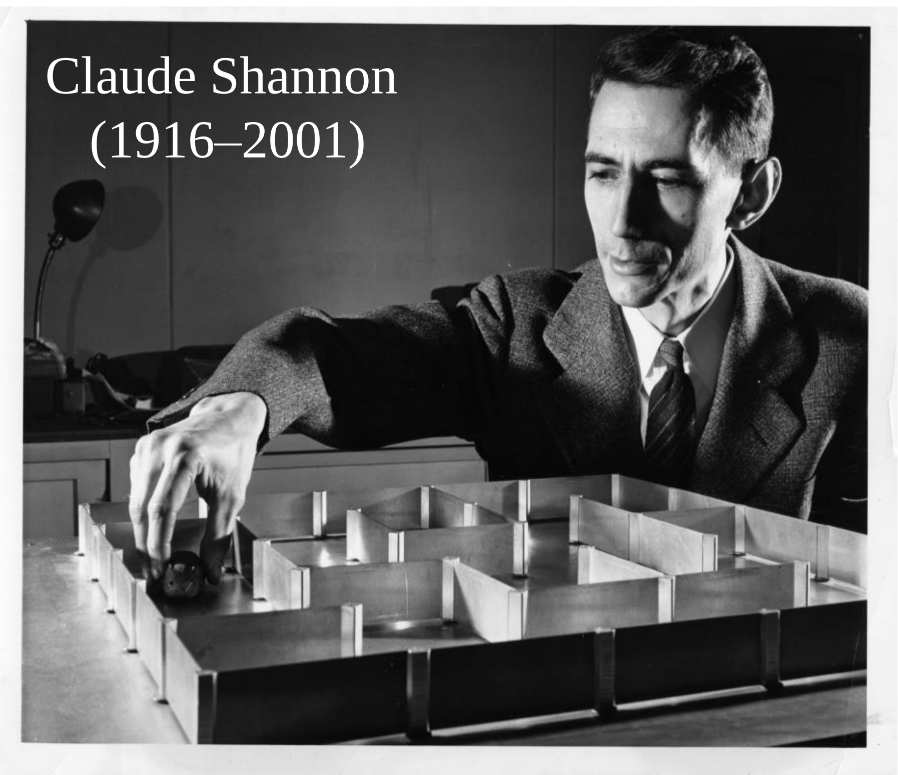


- 10^{-24} to 10^{26} meters ⇒ 50 orders of magnitude!

Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- More required videos:
  - Claude Shannon - Father of the Information Age, UCTV
  - The Pattern Behind Self-Deception, Michael Shermer, 2010



Claude Shannon (1916–2001)

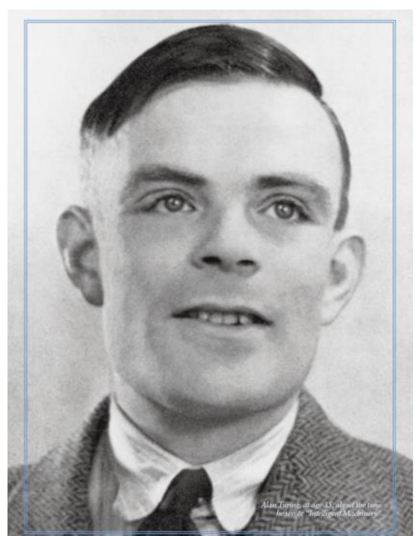
Michael Shermer

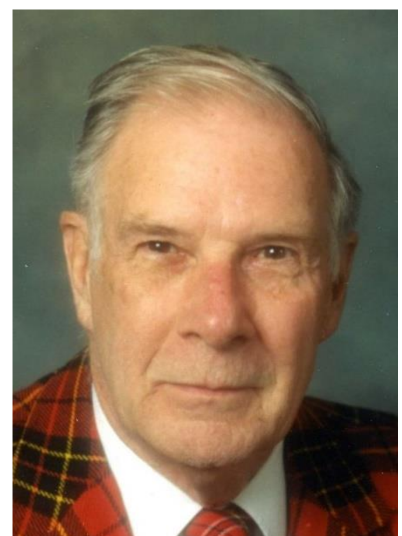
Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Required articles:
  - Decoding an Ancient Computer, Freeth, 2009
  - Alan Turing's Forgotten Ideas, Copeland and Proudfoot, 1999
  - You and Your Research, Richard Hamming, 1986
  - Who Can Name the Bigger Number, Scott Aaronson, 1999


Antikythera computer, 200BC


Alan Turing


Richard Hamming


Scott Aaronson

Extra credit!

Basic Concepts and Notation

Gabriel Robins

Required reading

"When I use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean -- neither more nor less."

A **set** is formally an undefined term, but intuitively it is a (possibly empty) collection of arbitrary objects. A set is usually denoted by curly braces and some (optional) restrictions. Examples of sets are {1,2,3}, {hi, there}, and {k | k is a perfect square}. The symbol $\in$ denotes set **membership**, while the symbol $\notin$ denotes set **non-membership**; for example, $7 \in \{p \mid p$ prime$\}$ states that 7 is a prime number, while $q \notin \{0,2,4,6,...\}$ states that q is not an even number. Some **common sets** are denoted by special notation:

The **natural numbers**:	$\mathbb{N} = \{1,2,3,...\}$
The **integers**:	$\mathbb{Z} = \{...,-3,-2,-1,0,1,2,3,...\}$
The **rational numbers**:	$\mathbb{Q} = \{ \frac{a}{b} \mid a,b \in \mathbb{Z}, b \neq 0\}$
The **real numbers**:	$\mathbb{R} = \{x \mid x$ is a real number$\}$
The **empty set**:	$\varnothing = \{\}$

http://www.cs.virginia.edu/robins/cs3102/basics.pdf

JOHANNES KEPLER'S UPHILL BATTLE

Discrete Math Review Slides

Required reading

Symbolic Logic

Def: *proposition* - statement either true (T) or false (F)

Ex: $1+1=2$

$2+2=3$

$3<7$

$x+4=5$

"today is Monday"

Boolean Functions

- "and" $\wedge$
- "or" $\vee$
- "not" $\neg$
- "xor" $\oplus$
- "nand"
- "nor"
- "implication" $\Rightarrow$
- "equivalence" $\Leftrightarrow$

Logical Implication

- "implies" $\Rightarrow$

Truth table:

p	q	p⇒q
T	T	T
T	F	F
F	T	T
F	F	T

Ex: $(x<0) \wedge (x>0) \Rightarrow (x=0)$

$1<x<y \Rightarrow x^3<y^3$

"today is Sunday" $\Rightarrow 1+1=3$

Logical Equivalence

- "biconditional" $\Leftrightarrow$

or "if and only if" ("iff")

or "necessary and sufficient"

or "logically equivalent" $\equiv$

Truth table:

p	q	p⇔q
T	T	T
T	F	F
F	T	F
F	F	T

Ex: $p \Leftrightarrow p$

$[(x=0) \vee (y=0)] \Leftrightarrow (xy=0)$

$\min(x,y)=\max(x,y) \Leftrightarrow x=y$

Predicates

Def: *predicate* - a function or formula involving some variables

Ex: let P(x) = "x > 3"

x is the variable

"x>3" is the predicate

P(5)

P(1)

Ex: Q(x,y,z) = "$x^2+y^2=z^2$"

Q(2,3,4)

Q(3,4,5)

Quantifiers

- Universal: "for all" $\forall$

$\forall x\ P(x)$

$\Leftrightarrow P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots$

Ex: $\forall x \quad x<x+1$

$\forall x \quad x<x^3$

- Existential: "there exists" $\exists$

$\exists x\ P(x)$

$\Leftrightarrow P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots$

Ex: $\exists x \quad x=x^2$

$\exists x \quad x<x-1$

Combinations:

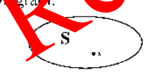
$\forall x \exists y \quad y>x$

Sets

Def: *set* - an unordered collection of elements

Ex: {1, 2, 3} or {hi, there}

Venn Diagram:



Def: two sets are *equal* iff they contain the same elements

Ex: $\{1,2,3\} = \{2,3,1\}$

$\{0\} \neq \{1\}$

$\{3,5\} = \{3,5,3,3,5\}$

Common Sets

Naturals: $N = \{1, 2, 3, 4, \ldots\}$

Integers: $Z = \{\ldots,-2,-1,0,1,2,\ldots\}$

Rationals: $Q = \{\frac{a}{b} \mid a,b \in Z,\ b\neq 0\}$

Reals: $\Re = \{x \mid x \text{ a real } \#\}$

Empty set: $\emptyset = \{\}$

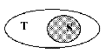
Z^+ = non-negative integers

$\Re^-$ = non-positive reals, etc.

Subsets

- Subset notation: $\subseteq$

$S \subseteq T \Leftrightarrow (x\in S \Rightarrow x\in T)$



- Proper subset: $\subset$

$S \subset T \Leftrightarrow ((S \subseteq T) \wedge (S\neq T))$

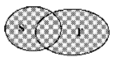
$S=T \Leftrightarrow ((T \subseteq S) \wedge (S \subseteq T))$

$\forall S \quad \emptyset \subseteq S$

$\forall S \quad S \subseteq S$

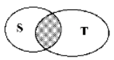
- Union: $\cup$

$S\cup T=\{x \mid x\in S \vee x\in T\}$



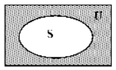
- Intersection: $\cap$

$S\cap T=\{x \mid x\in S \wedge x\in T\}$



- Universal set: U (everything)
- Set complement: S' or $\bar{S}$

$S' = \{x \mid x\notin S\} = U - S$



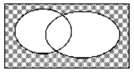
- Disjoint sets: $S\cap T=\emptyset$

$S - T = S \cap T'$

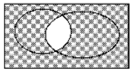
$S - S = \emptyset$

DeMorgan's Laws

$(S\cup T)' = S'\cap T'$



$(S\cap T)' = S'\cup T'$



Boolean logic version:

$(X\wedge Y)'=X'\vee Y'$

$(X\vee Y)'=X'\wedge Y'$

Function Types

- One-to-one function: "1-1"

$a,b\in S \wedge a\neq b \Rightarrow f(a)\neq f(b)$

Ex: $f: \Re\to\Re$, $f(x)=2x$ is 1-1

$g(x)=x^2$ is not 1-1

- Onto function:

$\forall t\in T\ \exists s\in S \ni f(s)=t$

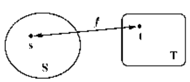
Ex: $f: Z\to Z$, $f(x)=13-x$ is onto

$g(x)=x^2$ is not onto

1-to-1 Correspondence

- 1-to-1 correspondence: $f: S\leftrightarrow T$

f is both 1-1 and onto



Ex: $f: \Re\leftrightarrow\Re \ni f(x)=x$ (identity)

$h: N\leftrightarrow Z \ni h(x)= \frac{x-1}{2}$, x odd, $\frac{-x}{2}$, x even.

Generalized Cardinality

- S is at least as large as T:

$|S \geq T| \Rightarrow \exists f: S\to T$, f onto

i.e., "S covers T"

Ex: $r: \Re\to Z$, $r(x)=\text{round}(x)$

$\Rightarrow |\Re|\geq|Z|$

- S and T have same cardinality:

$|S|=|T| \Rightarrow |S|\geq|T| \wedge |T|\geq|S|$

or

$\exists$ 1-1 correspondence $S\leftrightarrow T$

- Generalizes finite cardinality:

$\{1,2,3,4,5\} \geq \{a,b,c\}$

Infinite Sets

- Infinite set: $|S| > k \ \forall k\in Z$

or

$\exists$ 1-1 corres. $f: S\leftrightarrow T$, $S\subset T$

Ex: {p | p prime}, $\Re$

- Countable set: $|S| \leq |N|$

Ex: $\emptyset$, {p | p prime}, N, Z

- S is strictly smaller than T:

$|S| < |T| \Rightarrow |S|\leq|T| \wedge |S|\neq|T|$

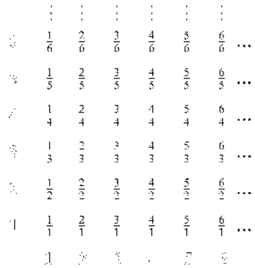
- Uncountable set: $|N| < |S|$

Ex: $|N| < |\Re|$

$|N| < [0,1] = \{x \mid x\in\Re, 0\leq x\leq 1\}$

Thm: $\exists$ 1-1 correspondence $Q\leftrightarrow N$

Pf (dove-tailing):

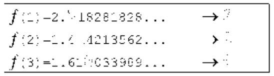


Thm: $|\Re| > |N|$

Pf (diagonalization):

Assume $\exists$ 1-1 corres. $f: \Re\leftrightarrow N$

Construct $X \in \Re$:



$X = 0.\ldots \neq f(k) \ \forall k\in N$

$\Rightarrow f$ not a 1-1 correspondence

$\Rightarrow$ contradiction

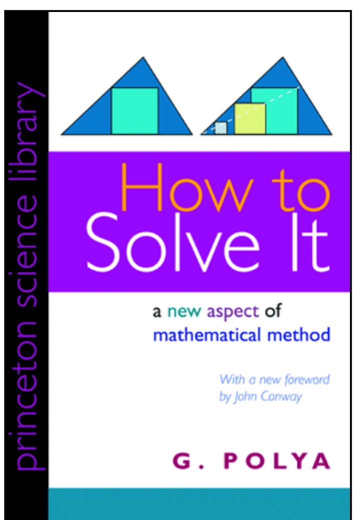
$\Rightarrow \Re$ is uncountable

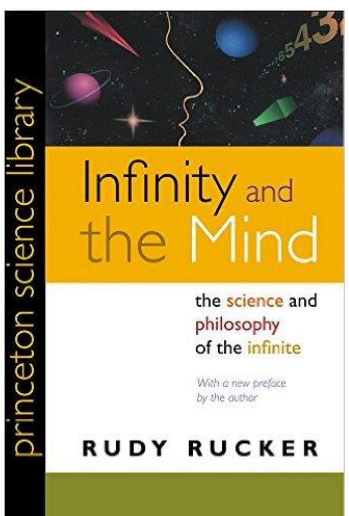
http://www.cs.virginia.edu/robins/cs3102/discrete_math_review_slides.pdf

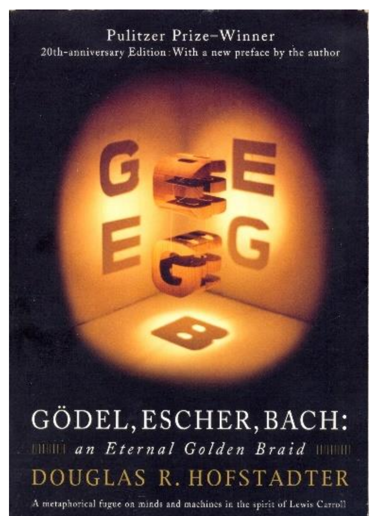
Required Readings

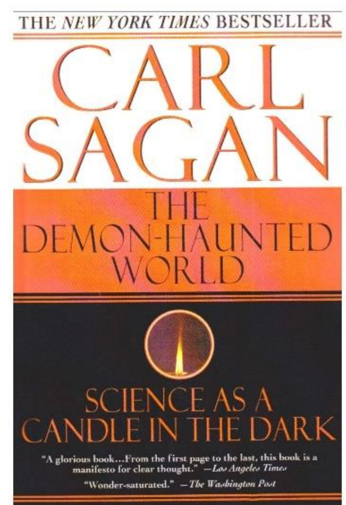
www.cs.virginia.edu/robins/CS_readings.html

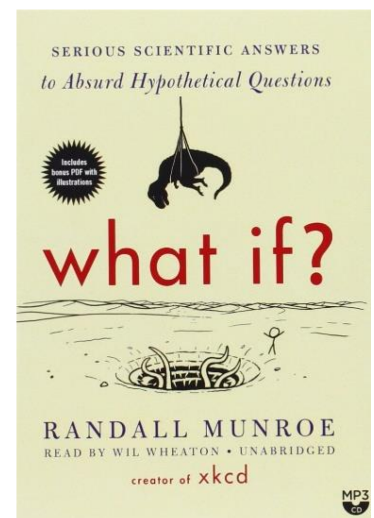
- Required books:
  - "How to Solve It", Polya, 1957
  - "Infinity and the Mind", Rucker, 1995
  - "Godel, Escher, Bach", Hofstadter, 1979
  - "The Demon-Haunted World", Sagan, 2009
  - "What If", Munroe, 2014











Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Remaining videos / articles / books are "electives"
- Pacing: at least 3 submissions per week (due 5pm Monday)
  - Policy intended to help you avoid "cramming"
- Length: 1-2 paragraphs per article / video
  1-2 pages per book
- Books are worth more credit than articles / videos
- Email all submissions to: homework.cs3102@gmail.com
- Additional readings beyond 50 are welcome! (extra credit)

Other "Elective" Readings
www.cs.virginia.edu/robins/CS_readings.html

- Theory and Algorithms:
  - Who Can Name the Bigger Number, Scott Aaronson, 1999
  - The Limits of Reason, Gregory Chaitin, Scientific American, March 2006, pp. 74-81.
  - Breaking Intractability, Joseph Traub and Henryk Wozniakowski, Scientific American, January 1994, pp. 102-107.
  - Confronting Science's Logical Limits, John Casti, Scientific American, October 1996, pp. 102-105.
  - Go Forth and Replicate, Moshe Sipper and James Reggia, Scientific American, August 2001, pp. 34-43.
  - The Science Behind Sudoku, Jean-Paul Delahaye, Scientific American, June 2006, pp. 80-87.
  - The Traveler's Dilemma, Kaushik Basu, Scientific American, June 2007, pp. 90-95.

Other "Elective" Readings
www.cs.virginia.edu/robins/CS_readings.html

- Biological Computing:
  - Computing with DNA, Leonard Adleman, Scientific American, August 1998, pp. 54-61.
  - Bringing DNA Computing to Life, Ehud Shapiro and Yaakov Benenson, Scientific American, May 2006, pp. 44-51.
  - Engineering Life: Building a FAB for Biology, David Baker et al., Scientific American, June 2006, pp. 44-51.
  - Big Lab on a Tiny Chip, Charles Choi, Scientific American, October 2007, pp. 100-103.
  - DNA Computers for Work and Play, Macdonald et al, Scientific American, November 2007, pp. 84-91.

Other "Elective" Readings
www.cs.virginia.edu/robins/CS_readings.html

- Quantum Computing:
  - Quantum Mechanical Computers, Seth Lloyd, Scientific American, 1997, pp. 98-104.
  - Quantum Computing with Molecules, Gershenfeld and Chuang, Scientific American, June 1998, pp. 66-71.
  - Black Hole Computers, Seth Lloyd and Jack Ng, Scientific American, November 2004, pp. 52-61.
  - Computing with Quantum Knots, Graham Collins, Scientific American, April 2006, pp. 56-63.
  - The Limits of Quantum Computers, Scott Aaronson, Scientific American, March 2008, pp. 62-69.
  - Quantum Computing with Ions, Monroe and Wineland, Scientific American, August 2008, pp. 64-71.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- History of Computing:
  - The Origins of Computing, Campbell-Kelly, Scientific American, September 2009, pp. 62-69.
  - Ada and the First Computer, Eugene Kim and Betty Toole, Scientific American, April 1999, pp. 76-81.
- Security and Privacy:
  - Malware Goes Mobile, Mikko Hypponen, Scientific American, November 2006, pp. 70-77.
  - RFID Powder, Tim Hornyak, Scientific American, February 2008, pp. 68-71.
  - Can Phishing be Foiled, Lorrie Cranor, Scientific American, December 2008, pp. 104-110.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- Future of Computing:
  - Microprocessors in 2020, David Patterson, Scientific American, September 1995, pp. 62-67.
  - Computing Without Clocks, Ivan Sutherland and Jo Ebergen, Scientific American, August 2002, pp. 62-69.
  - Making Silicon Lase, Bahram Jalali, Scientific American, February 2007, pp. 58-65.
  - A Robot in Every Home, Bill Gates, Scientific Am, January 2007, pp. 58-65.
  - Ballbots, Ralph Hollis, Scientific American, October 2006, pp. 72-77.
  - Dependable Software by Design, Daniel Jackson, Scientific American, June 2006, pp. 68-75.
  - Not Tonight Dear - I Have to Reboot, Charles Choi, Scientific American, March 2008, pp. 94-97.
  - Self-Powered Nanotech, Zhong Lin Wang, Scientific American, January 2008, pp. 82-87.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- The Web:
  - The Semantic Web in Action, Lee Feigenbaum et al., Scientific American, December 2007, pp. 90-97.
  - Web Science Emerges, Nigel Shadbolt and Tim Berners-Lee, Scientific American, October 2008, pp. 76-81.
- The Wikipedia Computer Science Portal:
  - Theory of computation and Automata theory
  - Formal languages and grammars
  - Chomsky hierarchy and the Complexity Zoo
  - Regular, context-free &Turing-decidable languages
  - Finite & pushdown automata; Turing machines
  - Computational complexity
  - List of data structures and algorithms

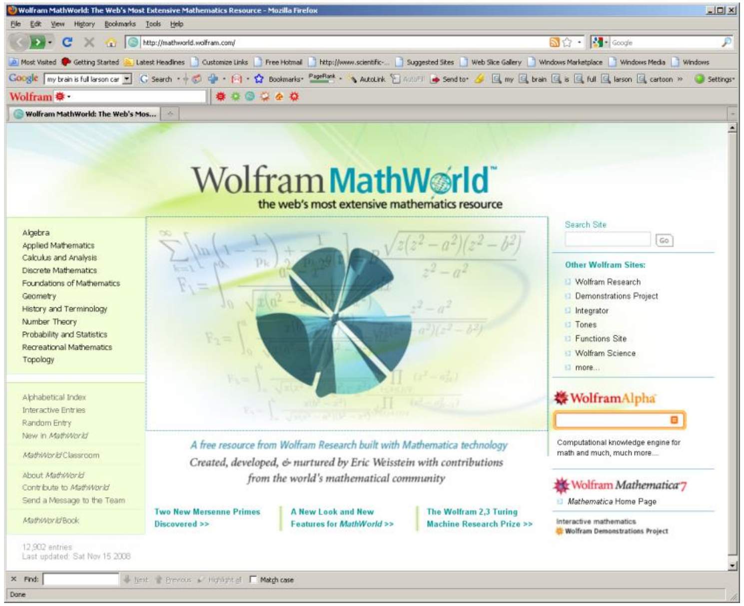
Email all submissions to: homework.cs3102@gmail.com

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- The Wikipedia Math Portal:
  - Problem solving
  - List of Mathematical lists
  - Sets and Infinity
  - Discrete mathematics
  - Proof techniques and list of proofs
  - Information theory & randomness
  - Game theory
- Mathematica's "Math World"

WIKIPEDIA
The Free Encyclopedia



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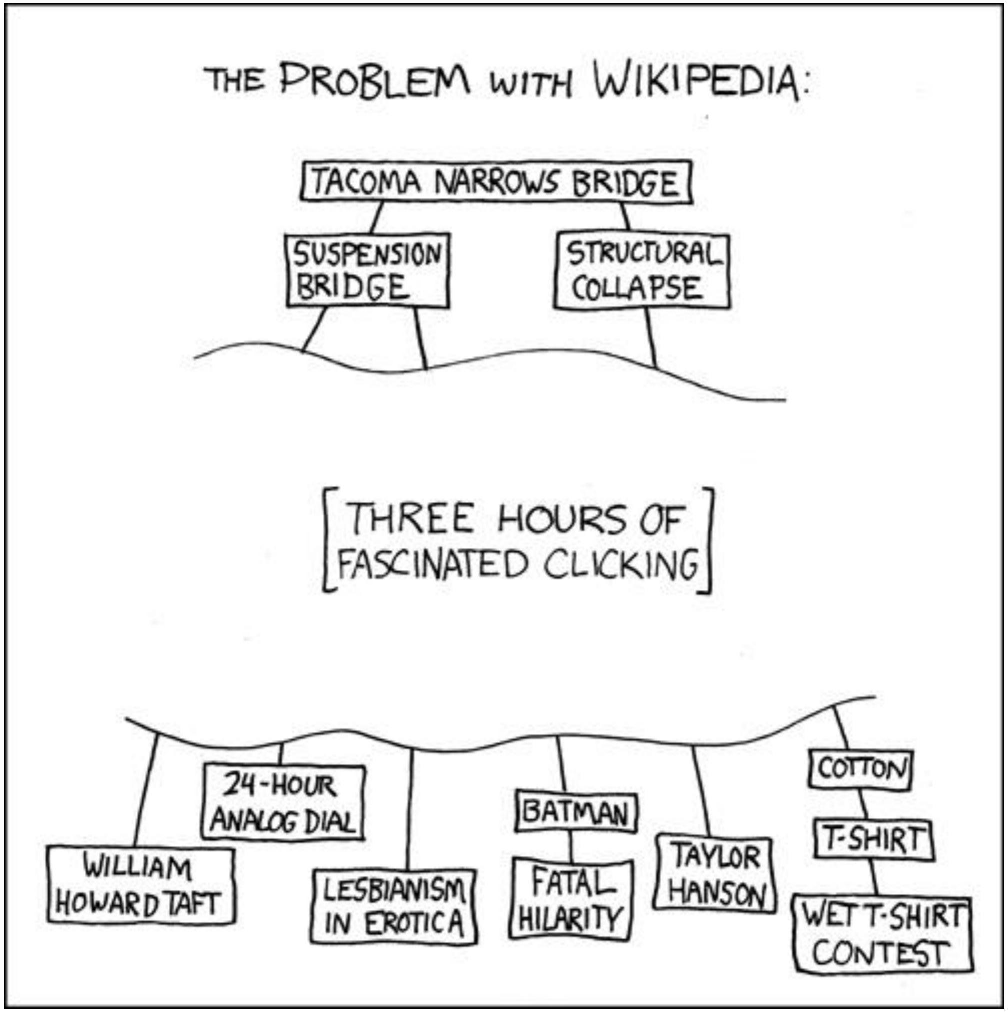
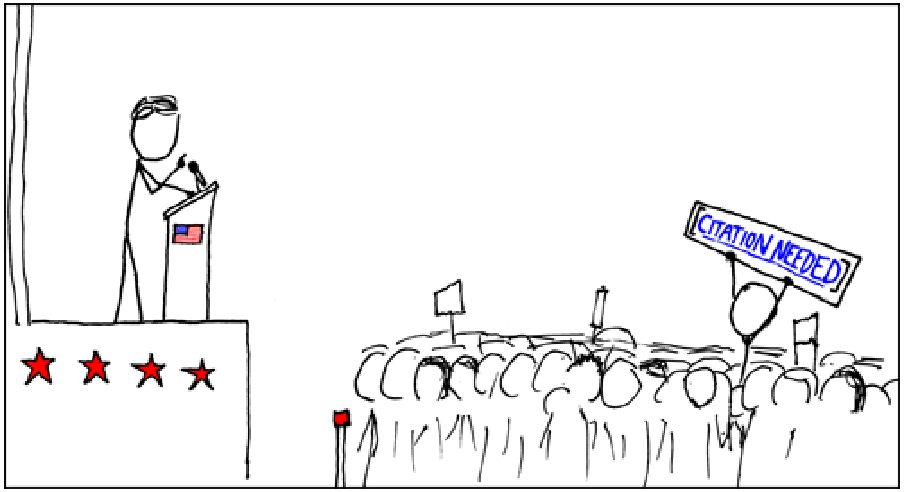
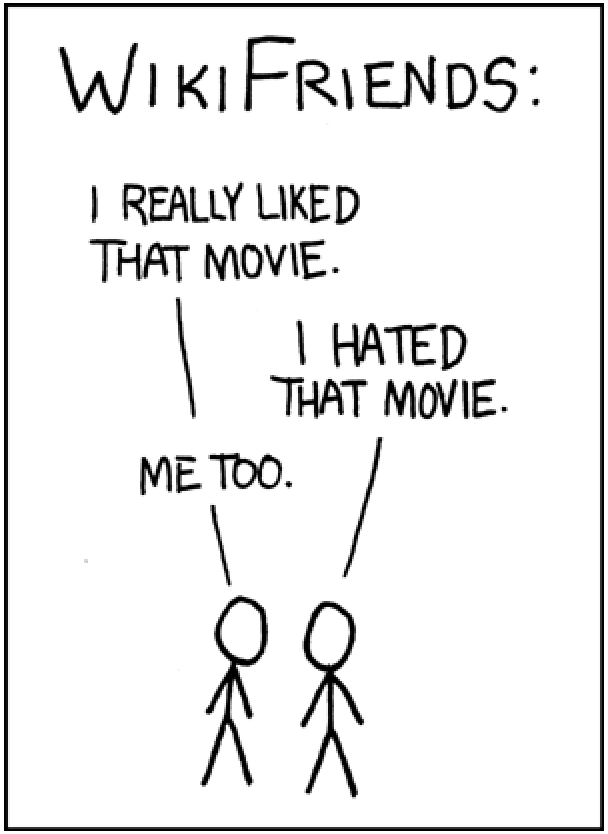
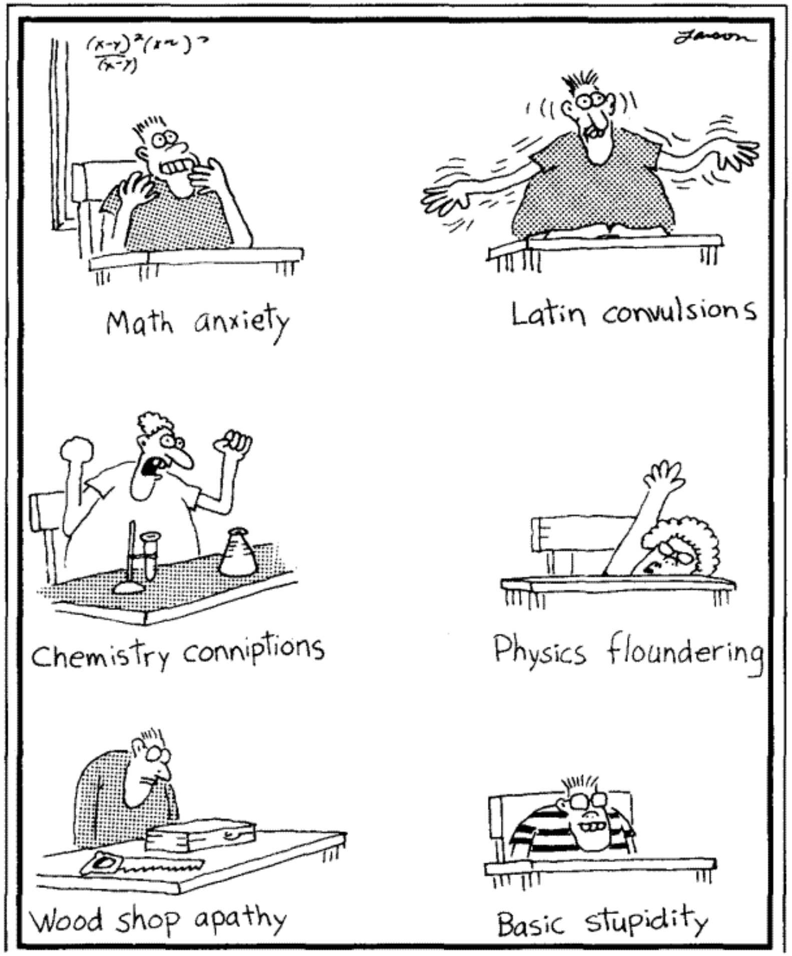

THE PROBLEM WITH WIKIPEDIA:
TACOMA NARROWS BRIDGE
SUSPENSION BRIDGE
STRUCTURAL COLLAPSE
[THREE HOURS OF FASCINATED CLICKING]
WILLIAM HOWARD TAFT
24-HOUR ANALOG DIAL
LESBIANISM IN EROTICA
BATMAN
FATAL HILARITY
TAYLOR HANSON
COTTON
T-SHIRT
WET T-SHIRT CONTEST


[CITATION NEEDED]


WIKIFRIENDS:
I REALLY LIKED THAT MOVIE.
I HATED THAT MOVIE.
ME TOO.

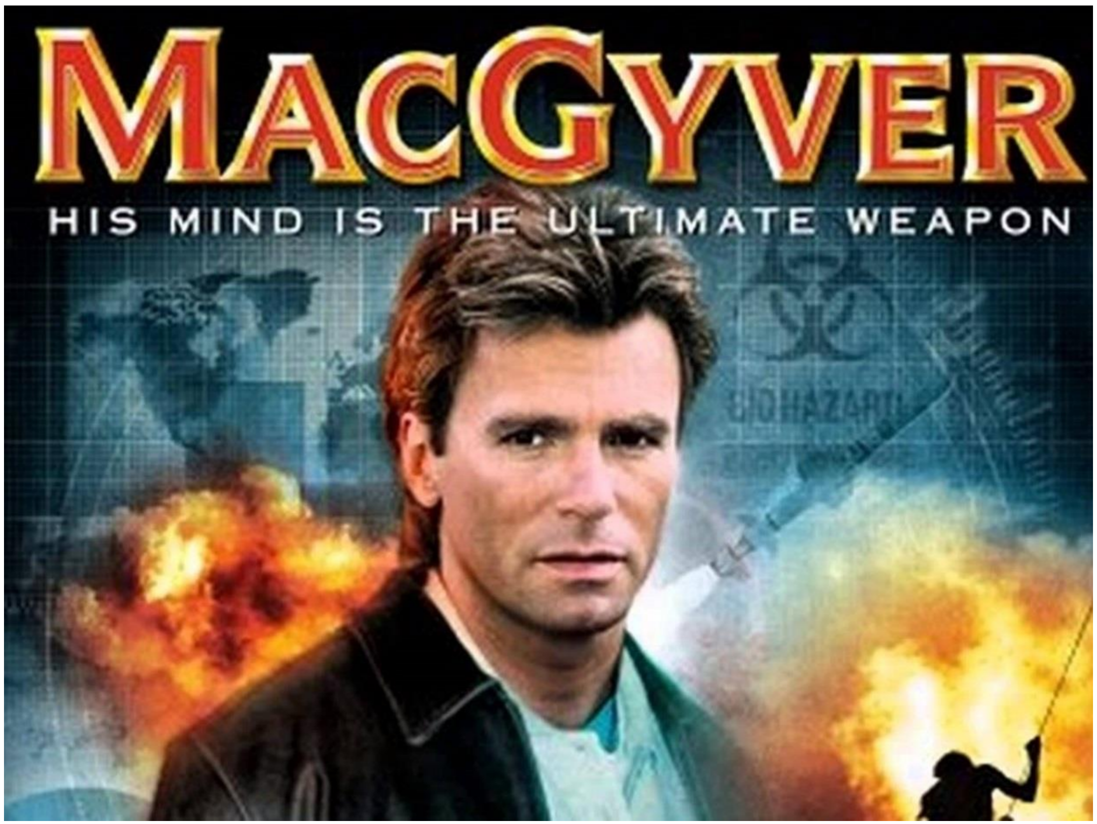
Good Advice

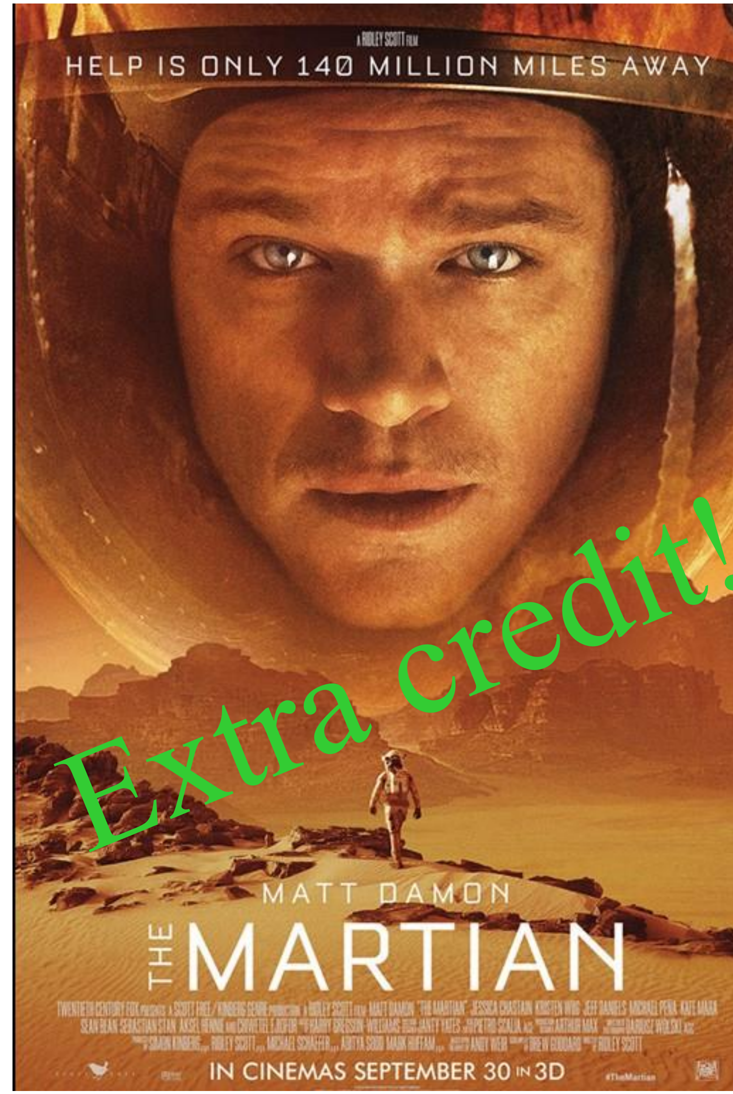
- Ask questions ASAP
- Solve problems ASAP
- Work in study groups
- <u>Do not</u> fall behind
- "Cramming" won't work
- Do lots of extra credit
- Attend every lecture
- Visit class Website often
- Solve lots of problems



Classroom afflictions

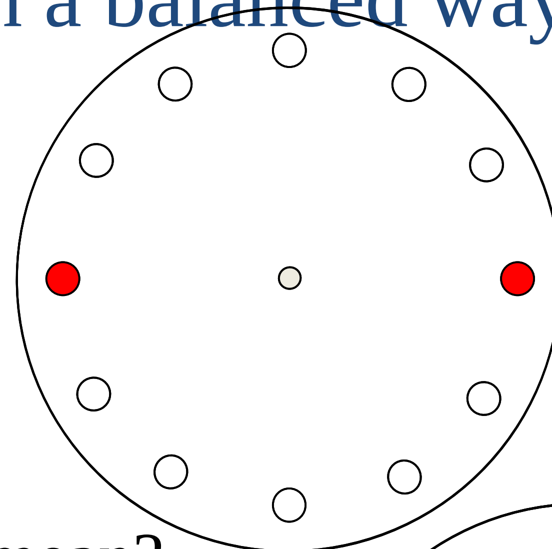
Goal: Become a more effective problem solver!

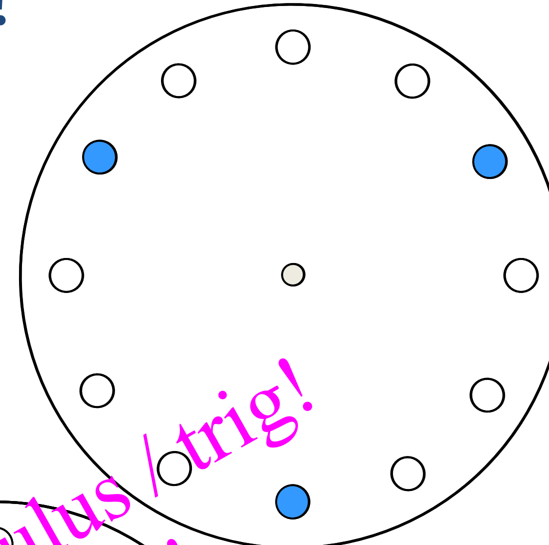


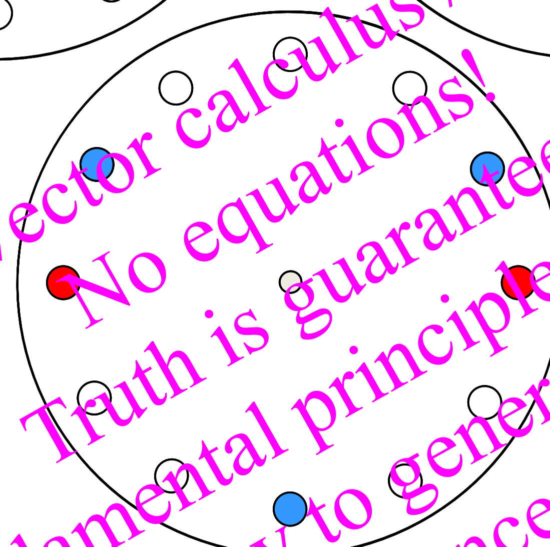


Email all submissions to: homework.cs3102@gmail.com

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?







- What does "balanced" mean?
- Why are 3 test tubes balanced?
- Symmetry!
- Can you merge solutions?
- Superposition!
- Linearity! $f(x + y) = f(x) + f(y)$
- Can you spin 7 test tubes?
- Complementarity!
- Empirical testing…

No vector calculus / trig!
No equations!
Truth is guaranteed!
Fundamental principles exposed!
Easy to generalize!
High elegance / beauty!

Problem: 1 + 2 + 3 + 4 + …+ 100 = ?

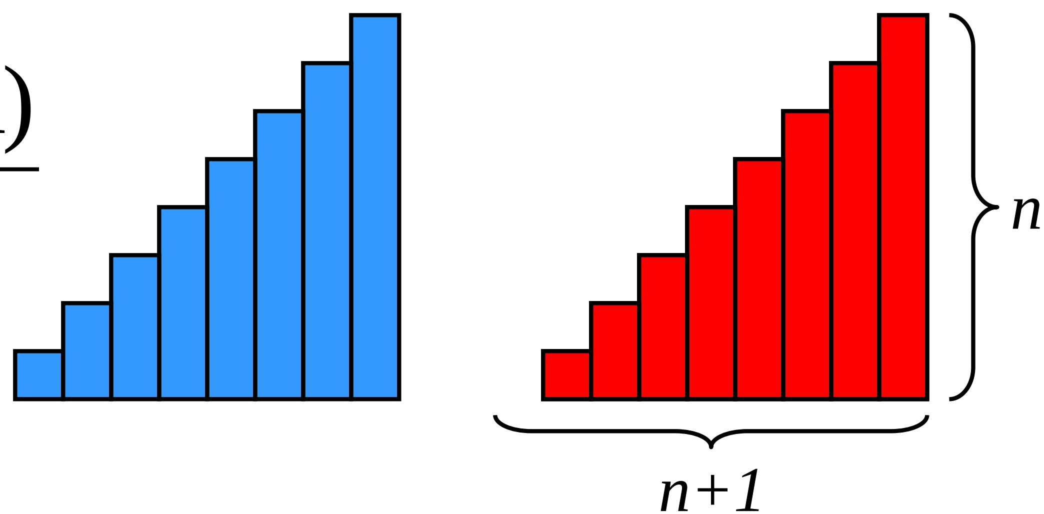
= (100*101)/2

= 5050

Proof: ~~Induction~~…

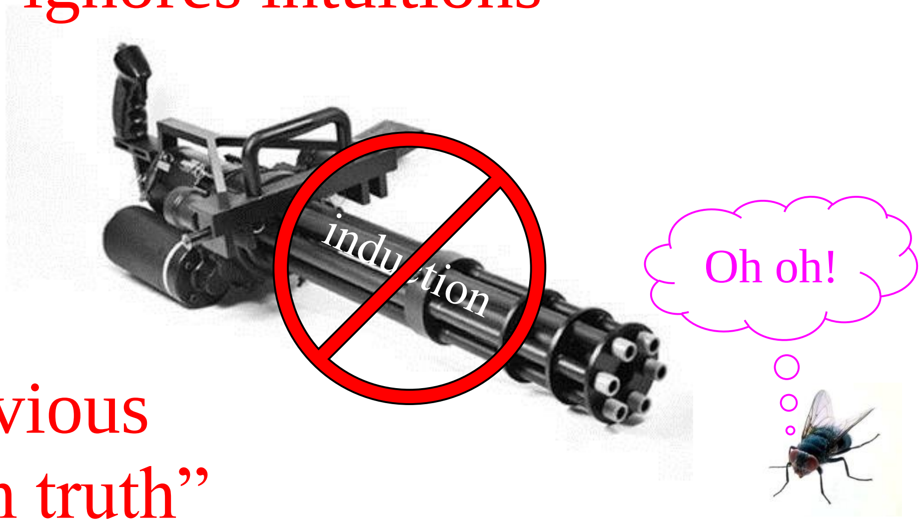
$$\begin{array}{ccccccccccccc} 1 & + & 2 & + & 3 & + & \ldots & + & 99 & + & 100 \\ 100 & + & 99 & + & 98 & + & \ldots & + & 2 & + & 1 \\ \hline 101 & + & 101 & + & 101 & + & \ldots & + & 101 & + & 101 & = & 100*101 \end{array}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$



Drawbacks of Induction

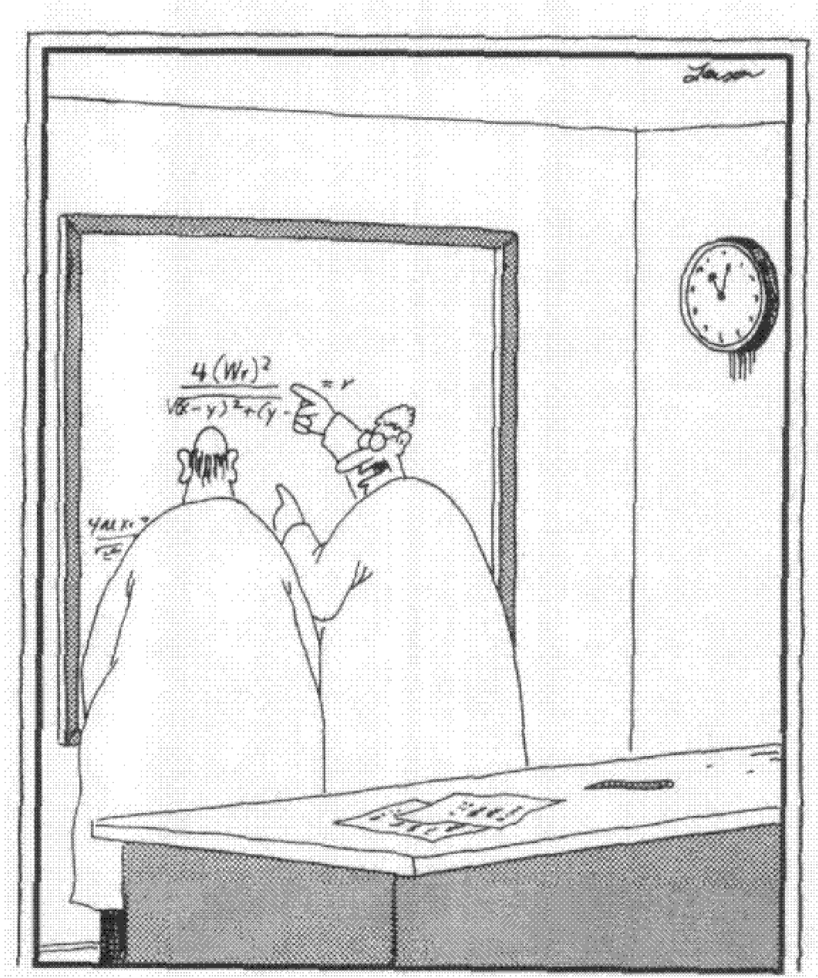
- You must a priori know the formula / result
- Easy to make mistakes in inductive proof
- Mostly "mechanical" – ignores intuitions
- Tedious to construct
- Difficult to check
- Hard to understand
- Not very convincing
- Generalizations not obvious
- Does not "shed light on truth"
- Obfuscates connections



Conclusion: only use induction as a last resort! (i.e., rarely)

Problem: $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = ?$

$$\sum_{i=1}^{n} i^3 = ?$$


"Yes, yes, I *know* that, Sidney ... everybody knows *that!* ... But look: Four wrongs *squared*, minus two wrongs to the fourth power, divided by this formula, *do* make a right."

Extra Credit:

find a short, geometric, induction-free proof.

Email all submissions to: homework.cs3102@gmail.com

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

$$\sum_{i=1}^{\infty} \frac{1}{4^i} = ?$$

Extra Credit:

Find a short, geometric, induction-free proof.

Email all submissions to: homework.cs3102@gmail.com

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = ?$$

Extra Credit:

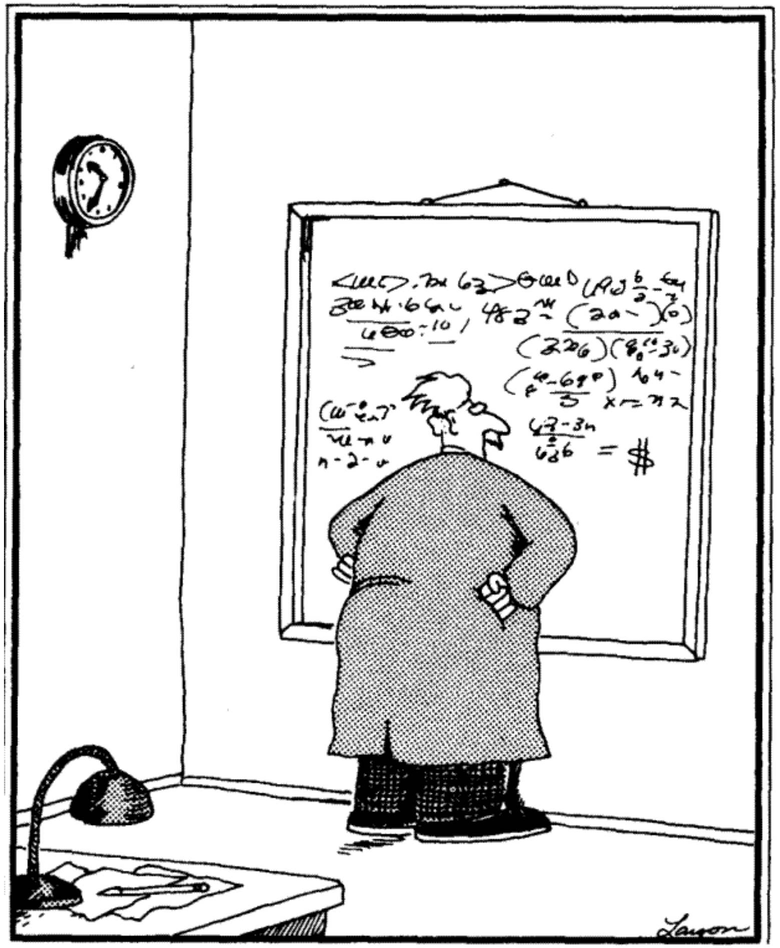
Find a short, geometric, induction-free proof.

Email all submissions to: homework.cs3102@gmail.com

Problem: Prove that $\sqrt{2}$ is irrational.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Einstein discovers that time is actually money.

Problem: Prove that there are an infinity of primes.

Extra Credit: Find a short, induction-free proof.

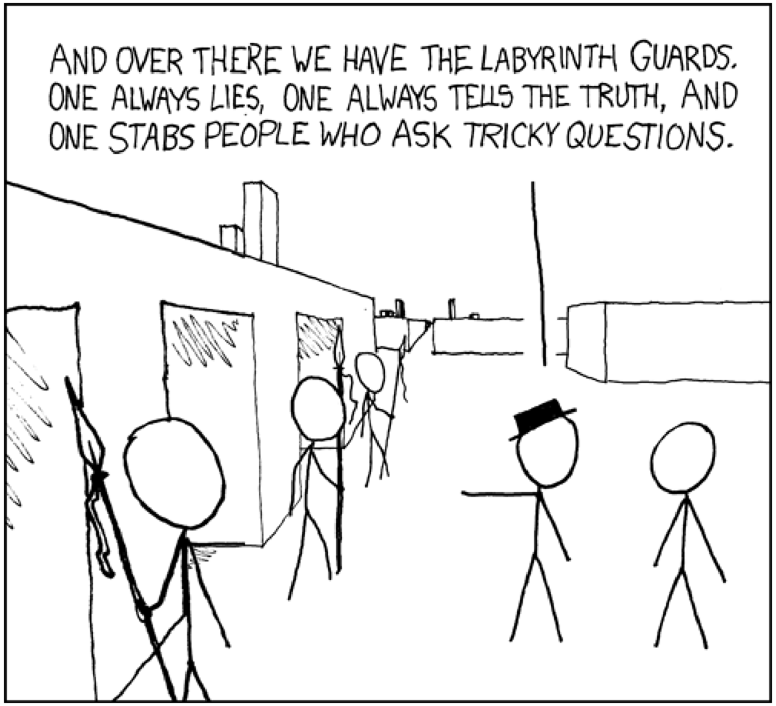
- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Email all submissions to: homework.cs3102@gmail.com

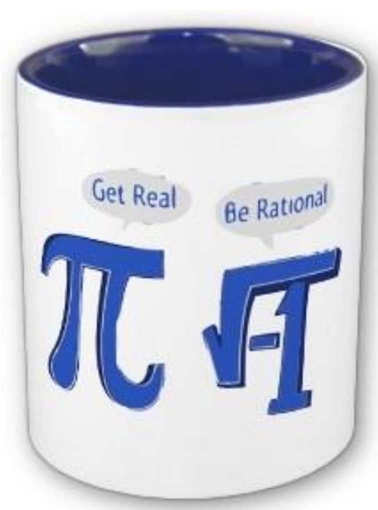
Problem: True or false: there arbitrary long blocks of consecutive composite integers.

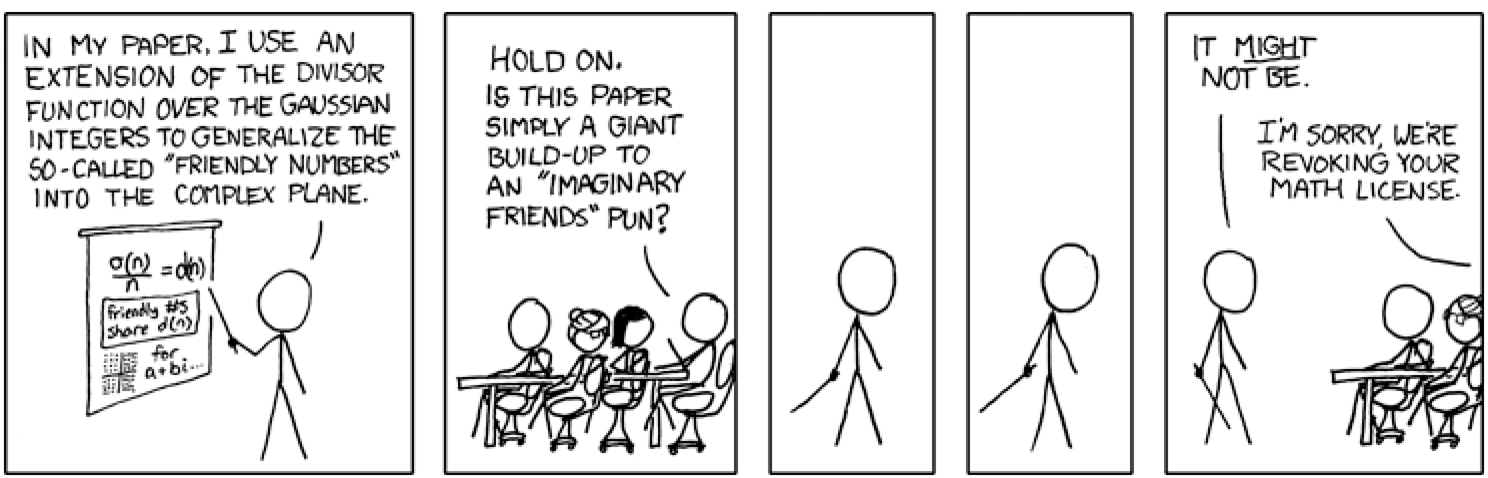
Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i?

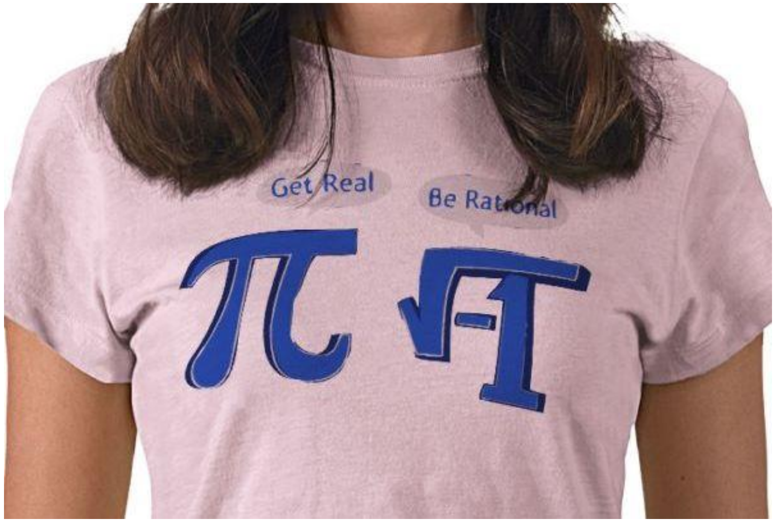




Problem: Does exponentiation preserve irrationality? i.e., are there two irrational numbers x and y such that x^y is rational?

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Historical Perspectives

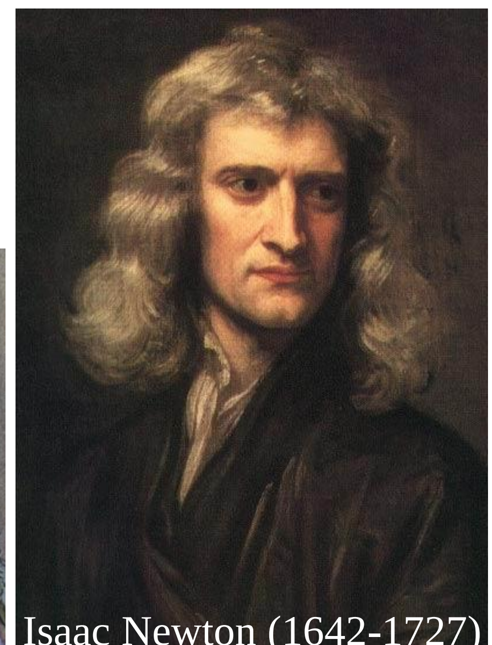
Historical Perspectives

- Knowing the "big picture" is empowering
- Science and mathematics builds heavily on past
- Often the simplest ideas are the most subtle
- Most fundamental progress was done by a few
- We learn much by observing the best minds
- Research benefits from seeing connections
- The field of computer science has many "parents"
- We get inspired and motivated by excellence
- The giants can show us what is possible to achieve
- It is fun to know these things!

“Standing on the Shoulders of Giants”

- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan
- Babbage, Ada Lovelace
- Venn, Carroll

Ada Lovelace (1815-1852)



Isaac Newton (1642-1727)

Euclid (300 BC)

“Standing on the Shoulders of Giants”

- Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Gödel, Church, Turing
- von Neumann, Shannon
- Kleene, Chomsky
- Hoare, McCarthy, Erdos
- Knuth, Backus, Dijkstra

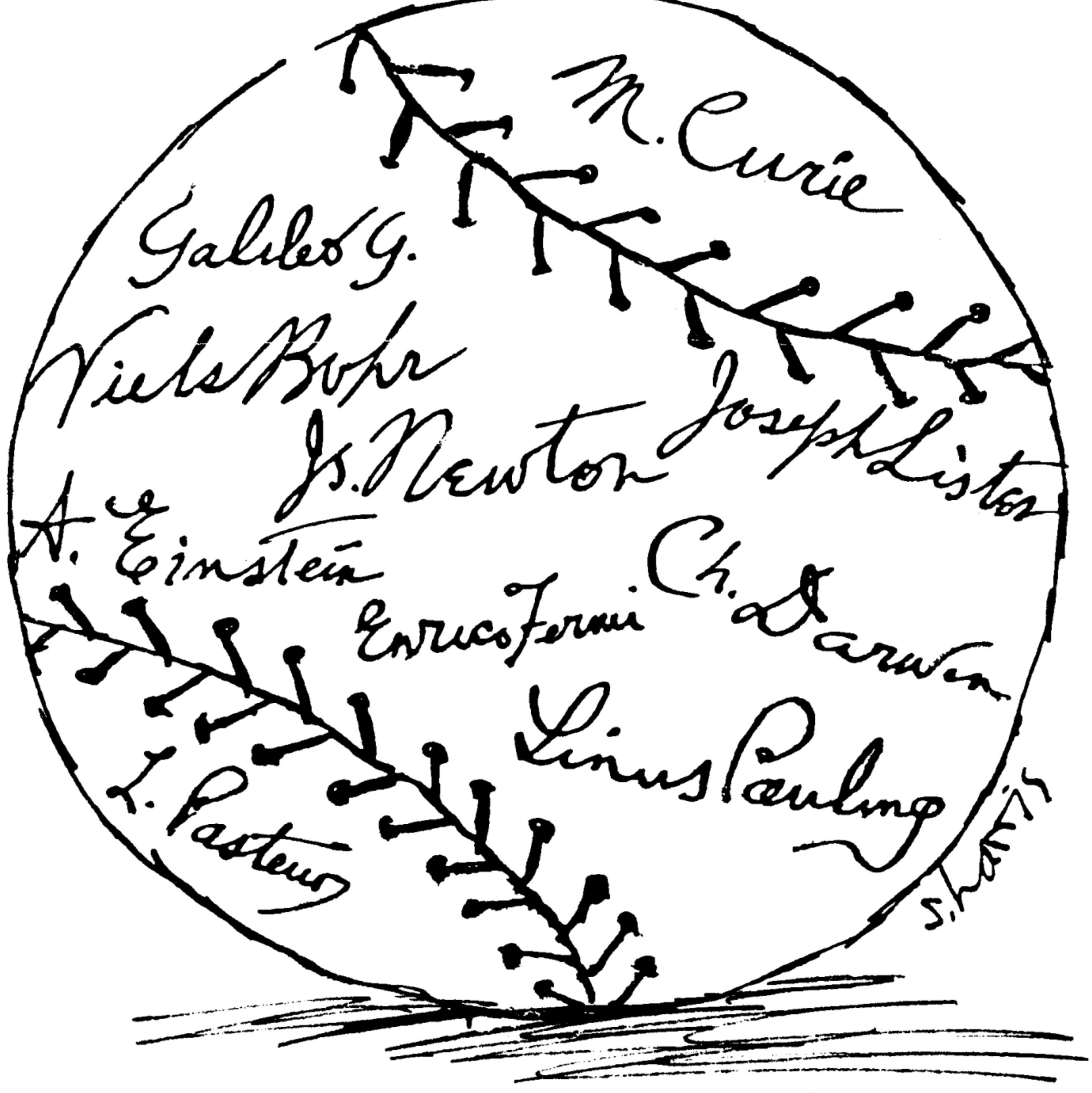
Many others…


Georg Cantor (1845-1918)


Bertrand Russell (1872-1970)


David Hilbert (1862-1943)


Kurt Gödel (1906-1978)

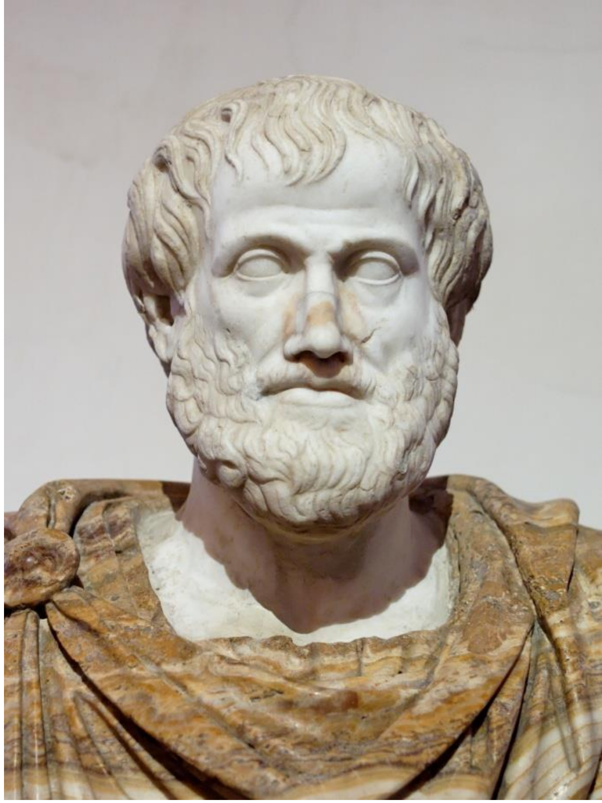


M. Curie
Galileo G.
Niels Bohr
Js. Newton
Joseph Lister
A. Einstein
Enrico Fermi
Ch. Darwin
Linus Pauling
L. Pasteur
s.harris

Gauss
Newton
Archimedes
Euler
Cauchy
Poincare
Riemann
Cantor
Cayley
Hamilton
Eisenstein
Pascal
Abel
Hilbert
Klein
Leibniz
Descartes
Galois
Mobius
Jacob
Johann Bernoulli
Daniel Bernoulli
Dirichlet
Fermat
Pythagoras
Laplace
Lagrange
Kronecker
Jacobi
Bolyai
Lobatchewsky
Noether
Germain
Euclid
Legendre

MAKING PHILOSOPHY ACCESSIBLE: POP-UP PLATO

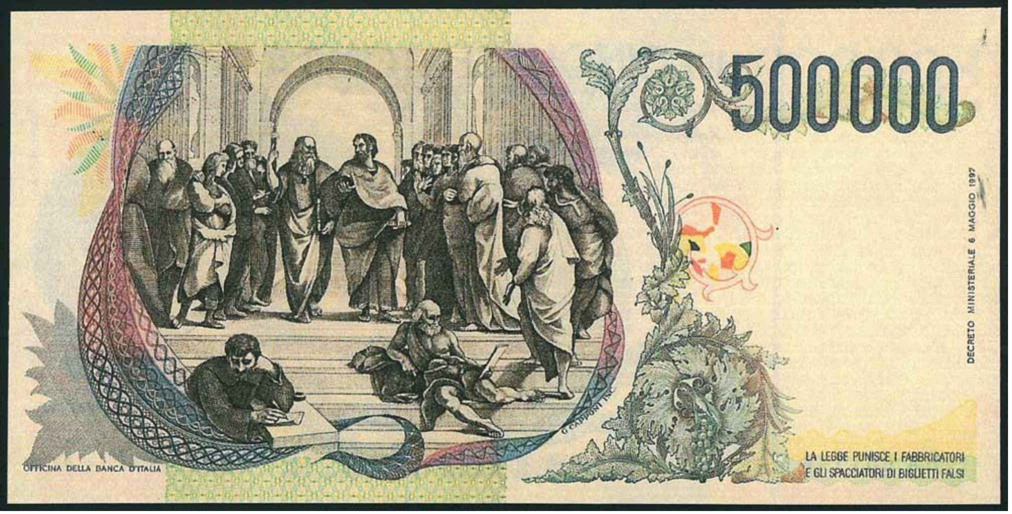
Historical Perspectives

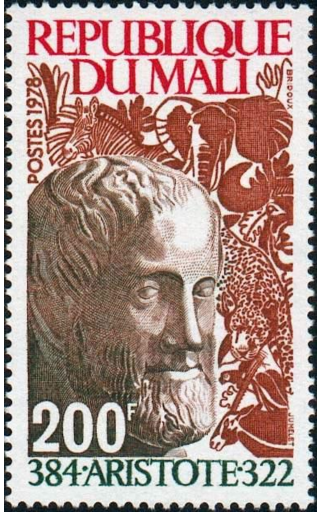


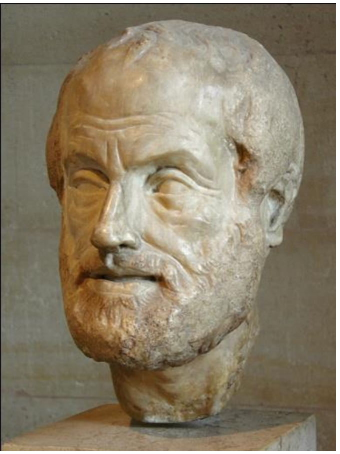
Aristotle (384BC-322BC)

- Founded Western philosophy
- Student of Plato
- Taught Alexander the Great
- "Aristotelianism"
- Developed the "scientific method"
- One of the most influential people ever
- Wrote on physics, theatre, poetry, music, logic, rhetoric, politics, government, ethics, biology, zoology, morality, optics, science, aesthetics, psychology, metaphysics, …
- Last person to know everything known in his own time!

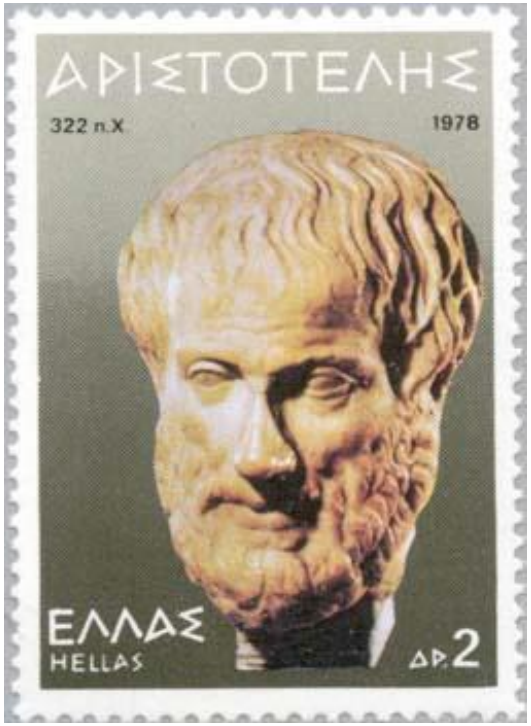
"Almost every serious intellectual advance has had to begin with an attack on some Aristotelian doctrine." – Bertrand Russell

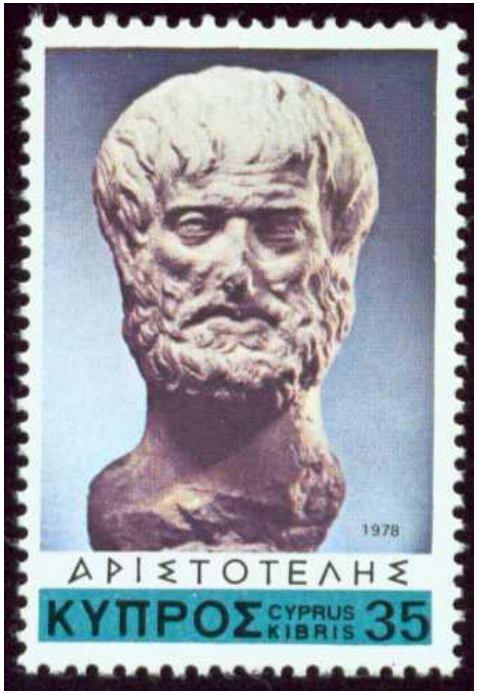






"Wit is educated insolence."
- Aristotle (384-322 B.C.)





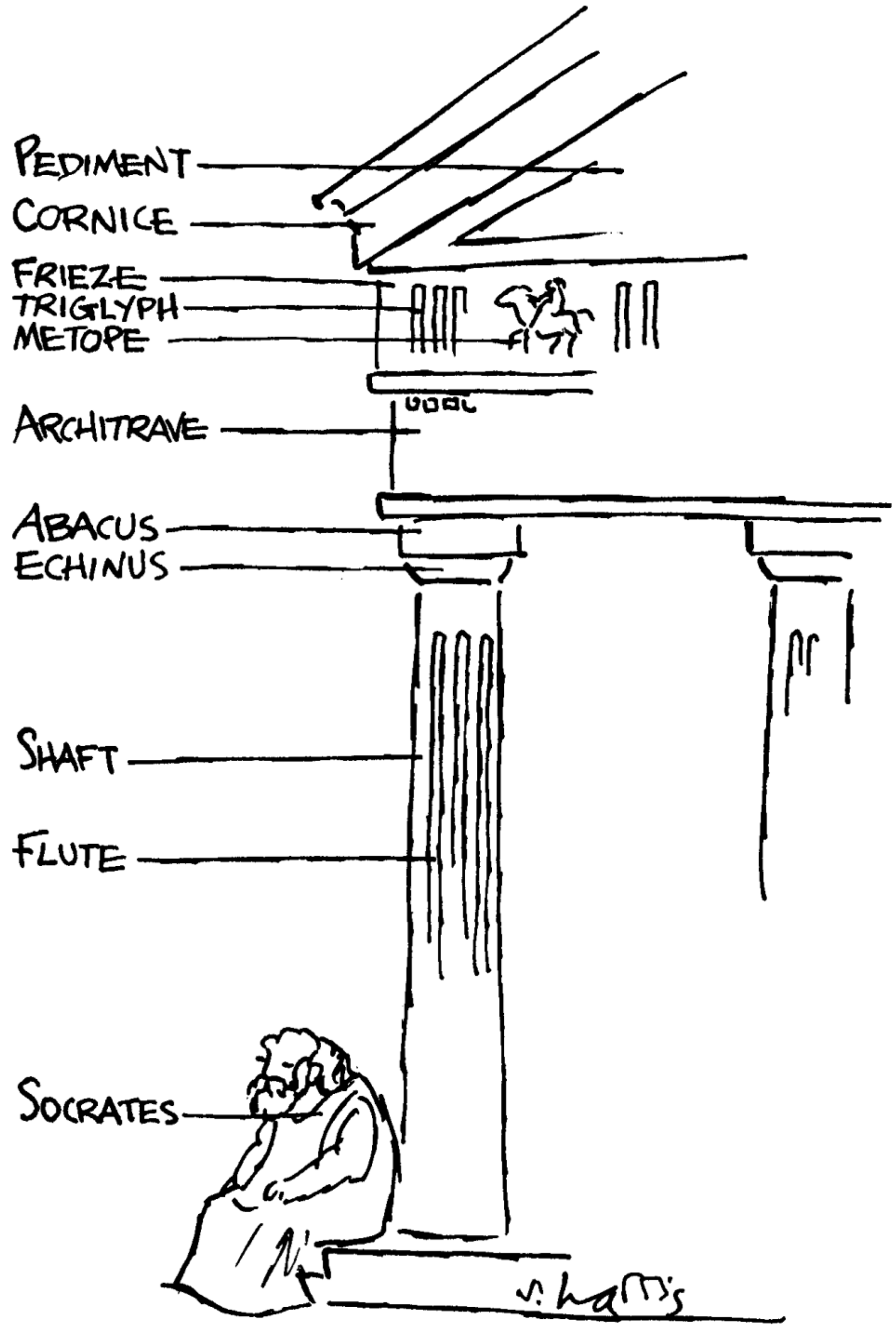
“The School of Athens” (by Raphael, 1483-1520)


ARISTOTELES
87 km
97 / 10 / 09
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
8
B/W QuickCam
a.cidadao@mail.telepac.pt


THE UNKNOWN PHILOSOPHER
WHO FIRST REALIZED LIFE IS NO PICNIC


Birds fly because they're lighter than air.
Some trees have different fruits each year.
At night, clouds rest on the ground.
Are you sure he's Aristotle?
S. Harris

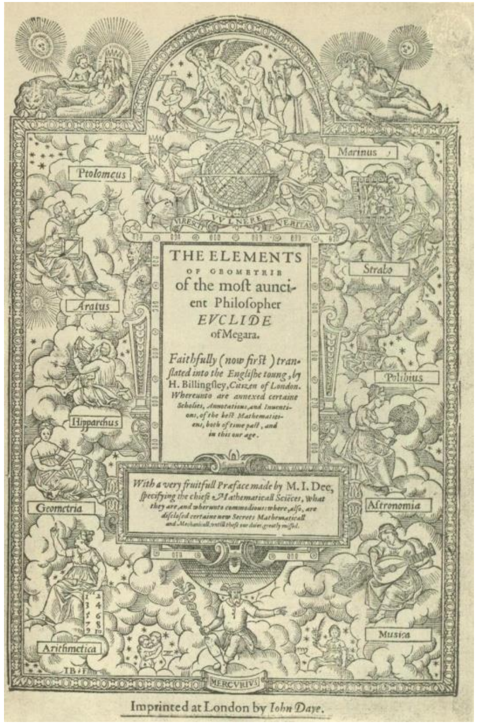
"What I especially like about being a philosopher-scientist is that I don't have to get my hands dirty."


PEDIMENT
CORNICE
FRIEZE
TRIGLYPH
METOPE
ARCHITRAVE
ABACUS
ECHINUS
SHAFT
FLUTE
SOCRATES
S. Harris

Historical Perspectives

Euclid (325BC-265BC)

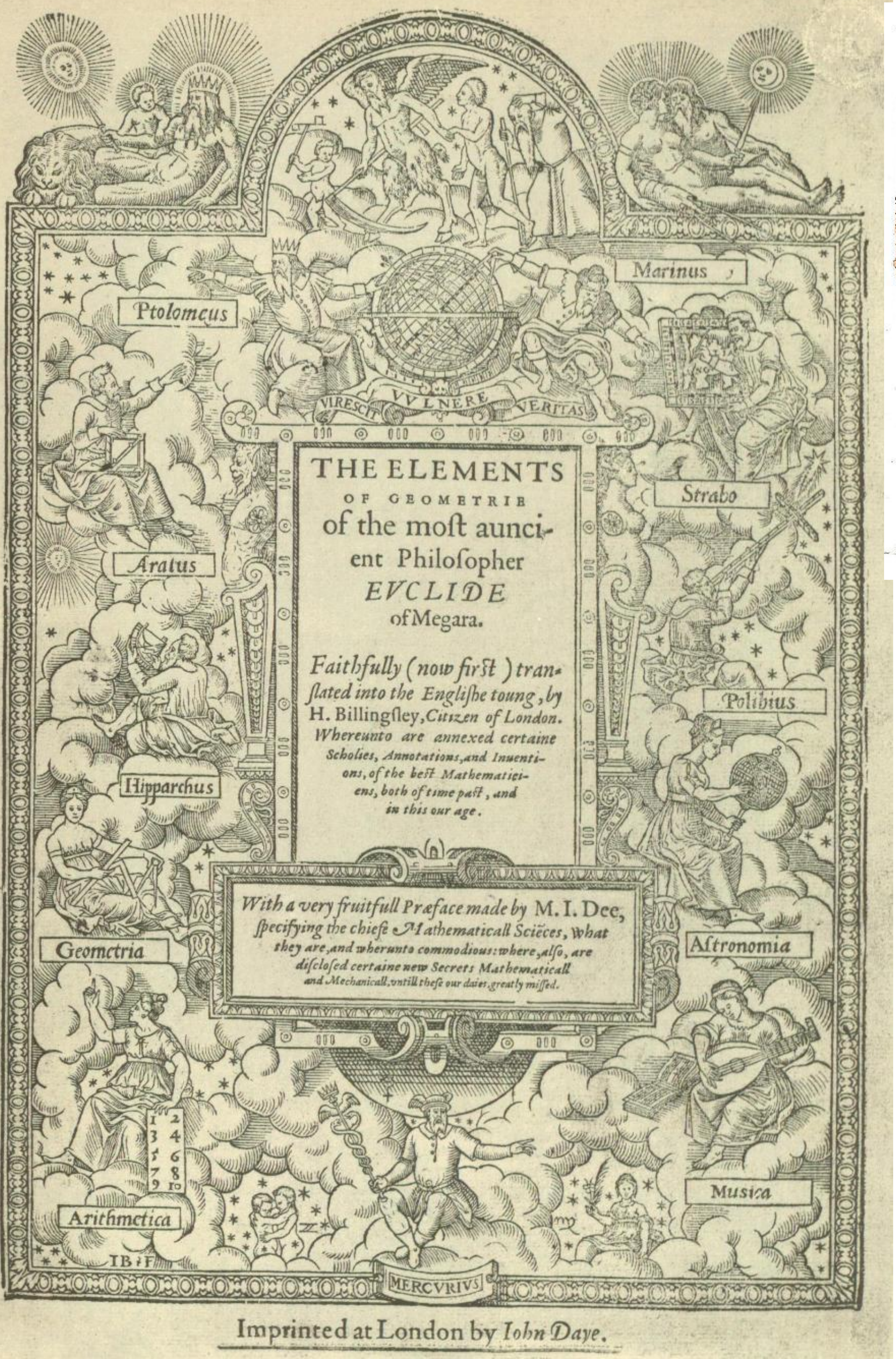
- Founder of geometry & the axiomatic method
- "Elements" – oldest and most impactful textbook
- Unified logic & math
- Introduced rigor and "Euclidean" geometry
- Influenced all other fields of science: Copernicus, Kepler, Galileo, Newton, Russell, Lincoln, Einstein & many others


THE ELEMENTS
OF GEOMETRIE
of the most aunci-
ent Philosopher
EVCLIDE
of Megara.

Faithfully (now first) translated into the Englishe toung, by H. Billingsley, *Citizen of London. Whereunto are annexed certaine Scholies, Annotations, and Inuentions, of the best Mathematiciens, both of time past, and in this our age.*

With a very fruitfull Præface made by M. I. Dee, *specifying the chiefe Mathematicall Sciēces, what they are, and wherunto commodious: where, also, are disclosed certaine new Secrets Mathematicall and Mechanicall, vntill these our daies, greatly missed.*

Imprinted at London by *Iohn Daye.*



THE ELEMENTS
OF GEOMETRIE
of the most aunci-
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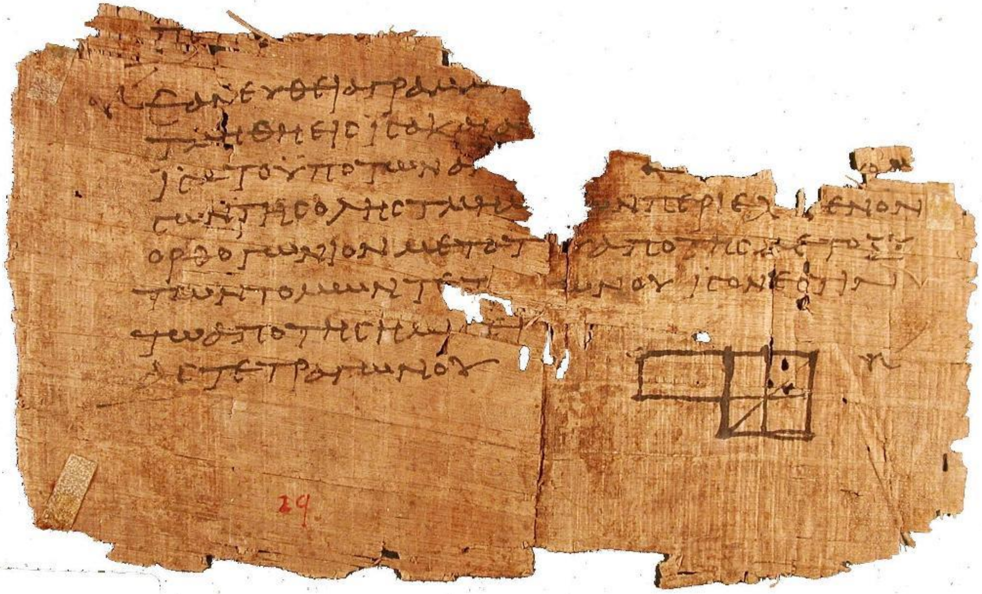
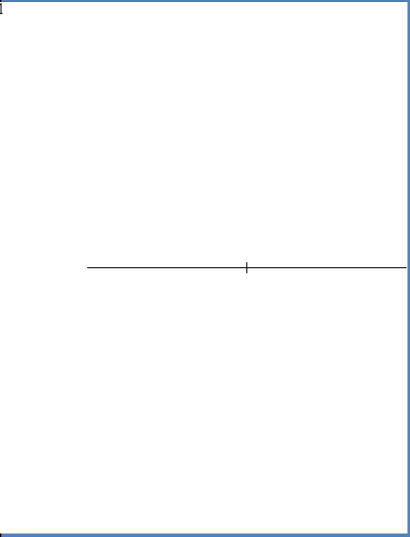
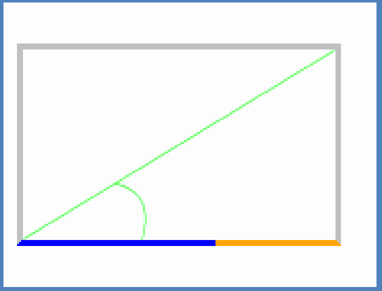
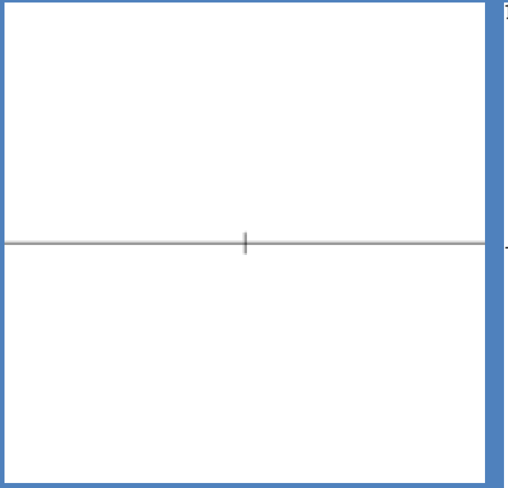
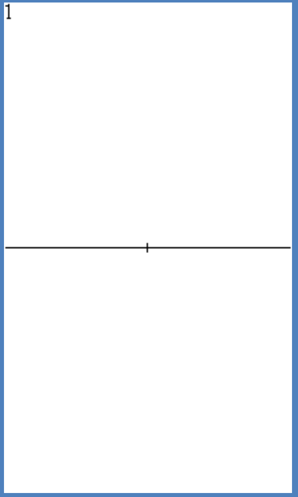


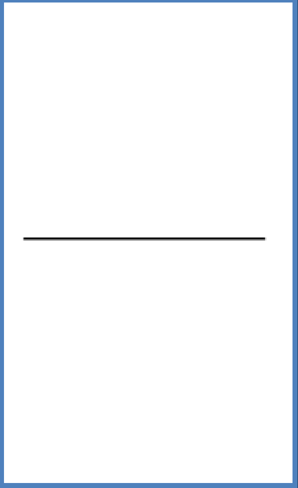
Euclid's Straight-Edge and Compass Geometric Constructions

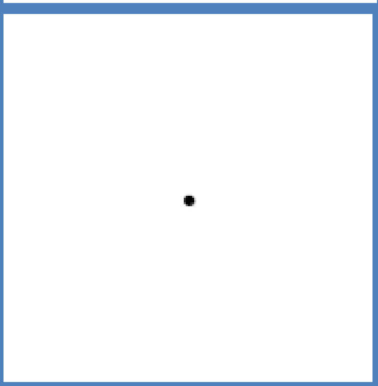


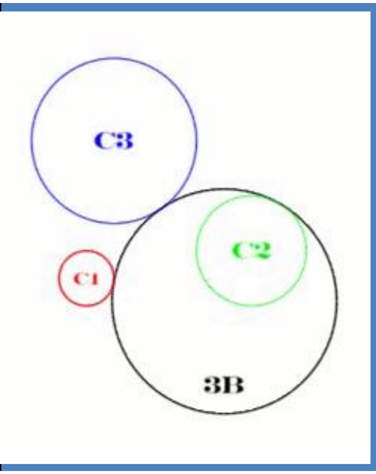


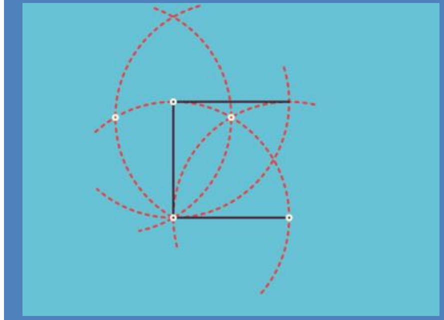


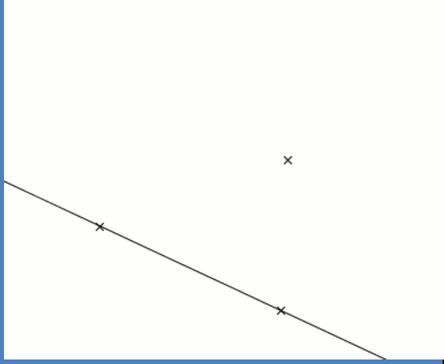




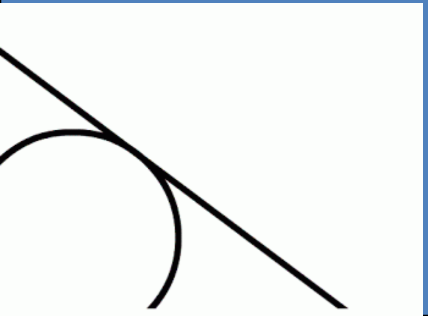


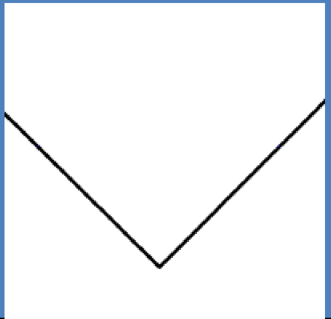


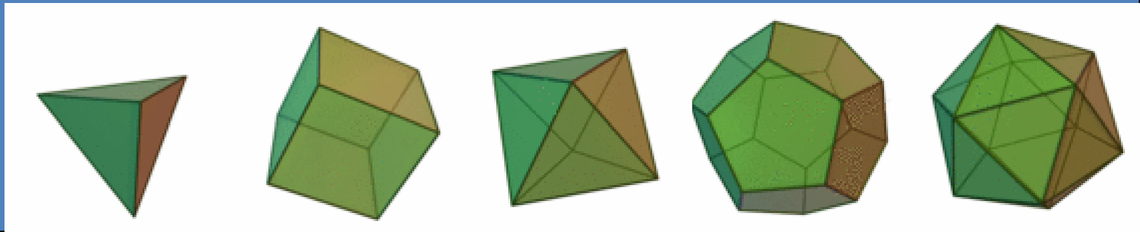




Extra credit!





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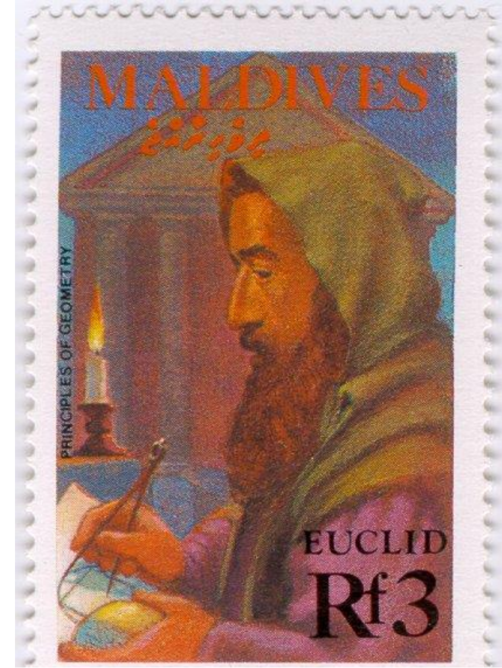
(detail) Euclid and his pupils
1983
SCALA NEW YORK/FLORENCE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PRINCIPLES OF GEOMETRY
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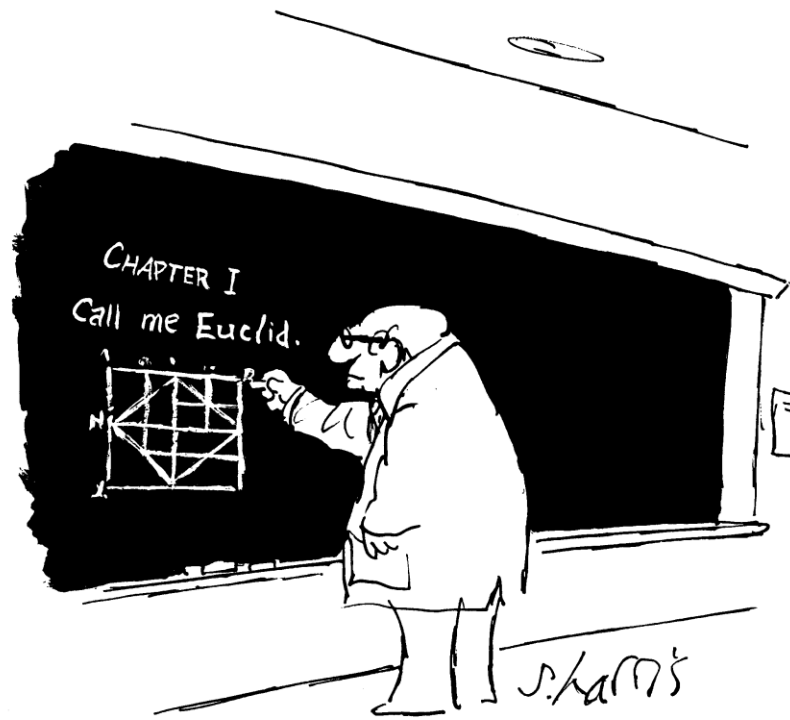

CHAPTER I
Call me Euclid.
S.Harris


EUCLIDES
12 km
© António J. Cidadão
98 / 04 / 07
D=254mm f/D=10
Moon "LIGHT"
11
B/W QuickCam
a.cidadao@mail.telepac.pt

Euclid's Axioms

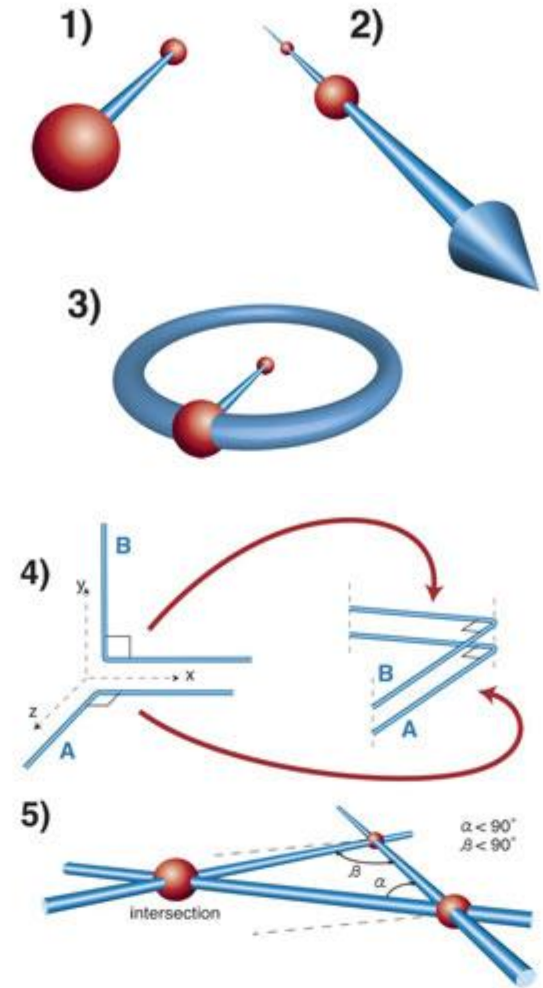
1: Any two points can be connected by exactly one straight line.

2: Any segment can be extended indefinitely into a straight line.

3: A circle exists for any given center and radius.

4: All right angles are equal to each other.

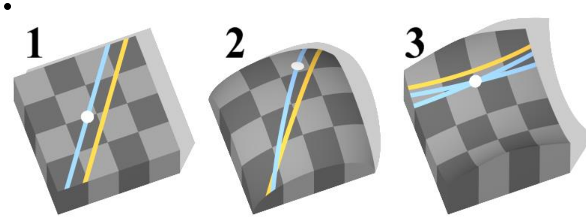
5: The parallel postulate: Given a line and a point off that line, there is exactly one line passing through the point, which does not intersect the first line.

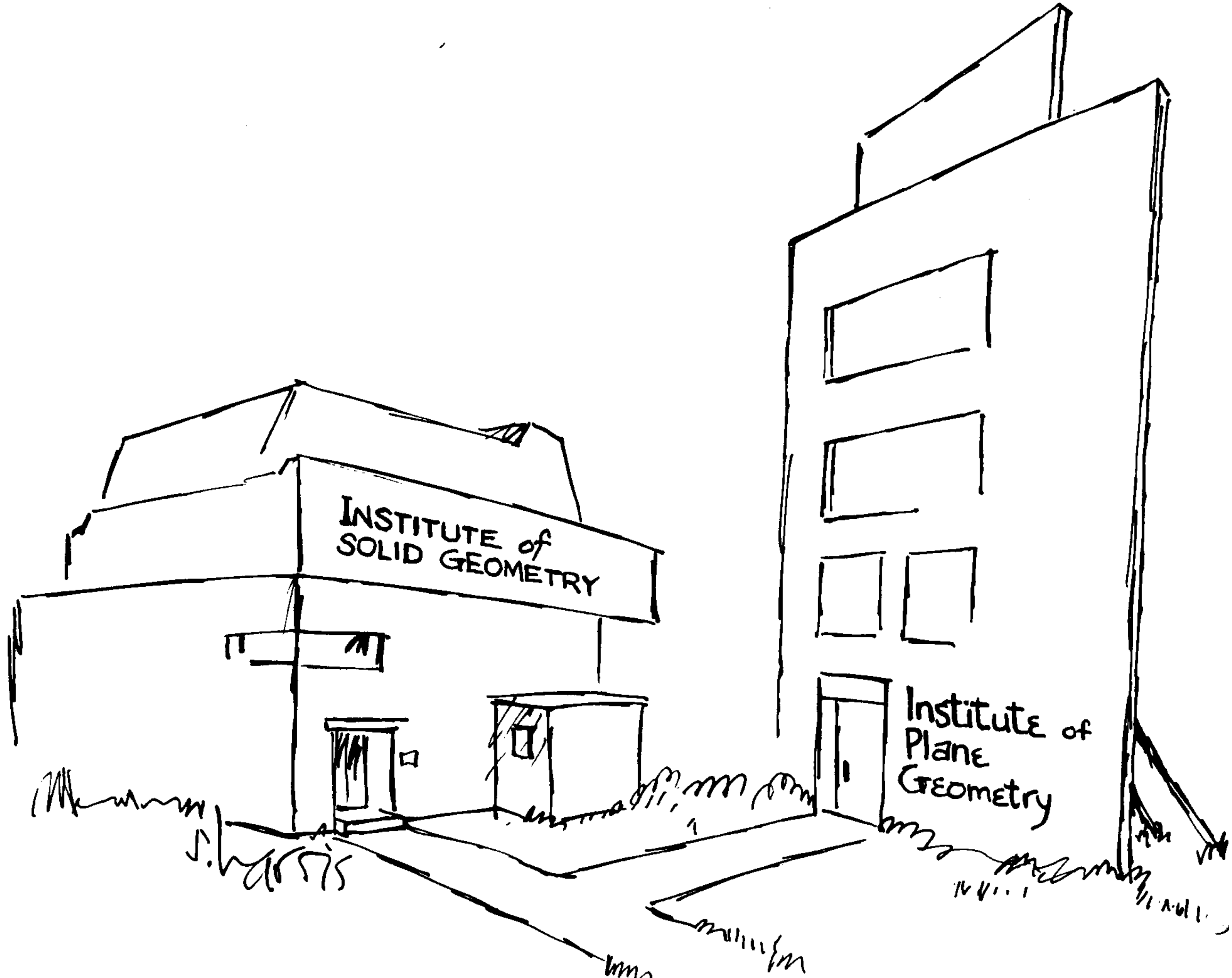


The first 28 propositions of Euclid's Elements were proven without using the parallel postulate!

Theorem [Beltrami, 1868]: The parallel postulate is independent of the other axioms of Euclidean geometry.

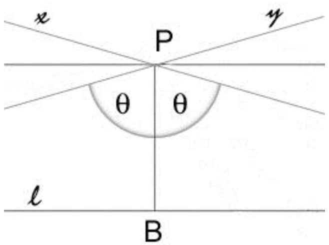
The parallel postulate can be modified to yield non-Euclidean geometries!



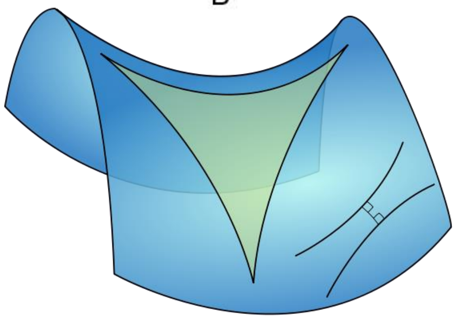

INSTITUTE of SOLID GEOMETRY
Institute of Plane Geometry

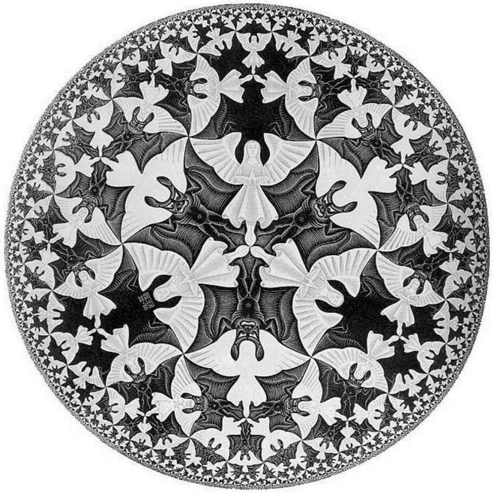
Non-Euclidean Geometries

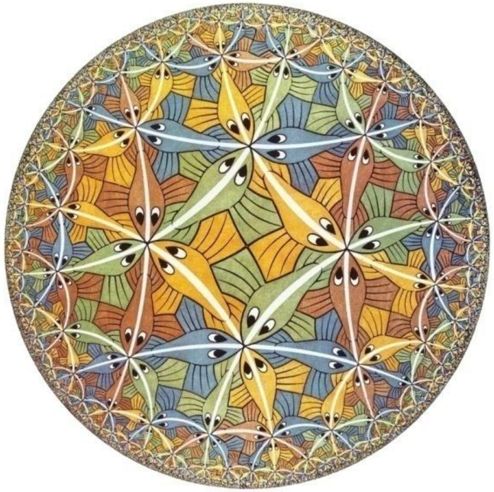
Hyperbolic geometry: Given a line and a point off that line, there are an infinity of lines passing through that point that do not intersect the first line.

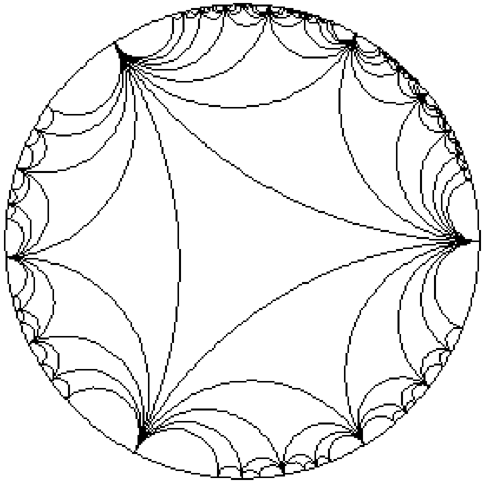


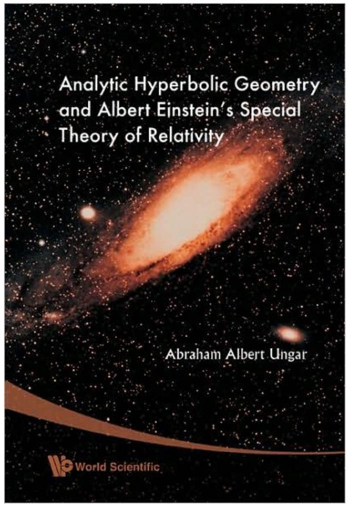
- Sum of triangle angles is less than 180°
- Different triangles have different angle sum
- Triangles with same angles have same area
- There are no similar triangles
- Used in relativity theory





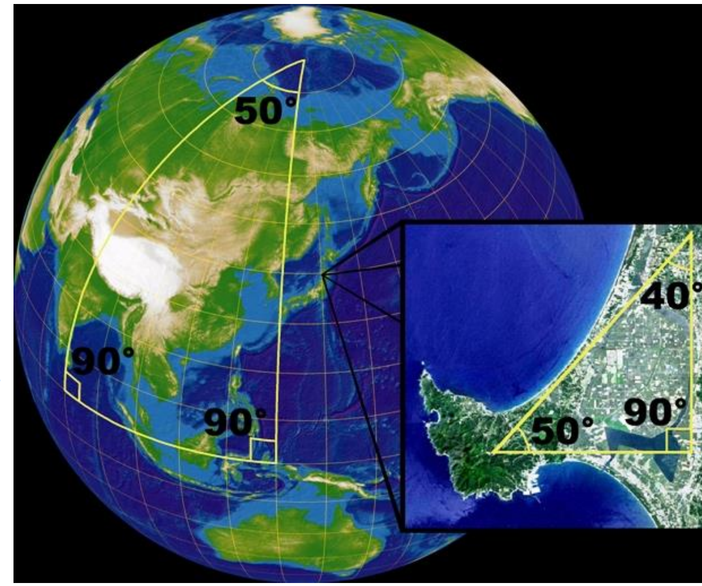




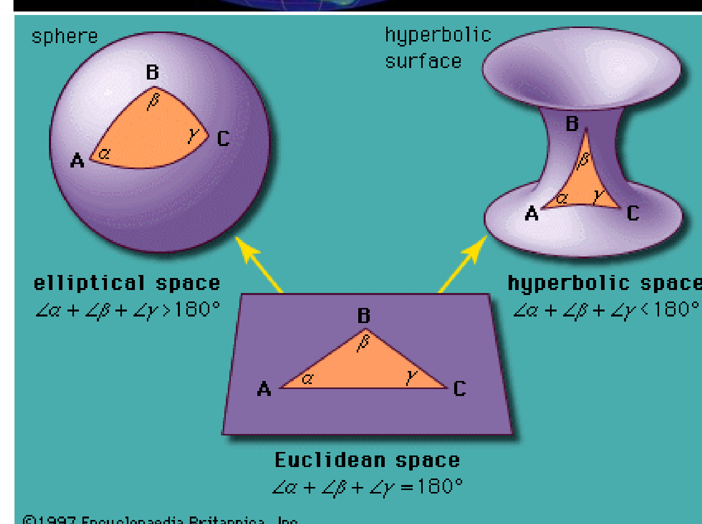


Non-Euclidean Geometries

Spherical / Elliptic geometry: Given a line and a point off that line, there are no lines passing through that point that do not intersect the first line.



- Lines are geodesics - “great circles”
- Sum of triangle angles is > 180°
- Not all triangles have same angle sum
- Figures can not scale up indefinitely
- Area does not scale as the square
- Volume does not scale as the cube
- The Pythagorean theorem fails
- Self-consistent, and complete

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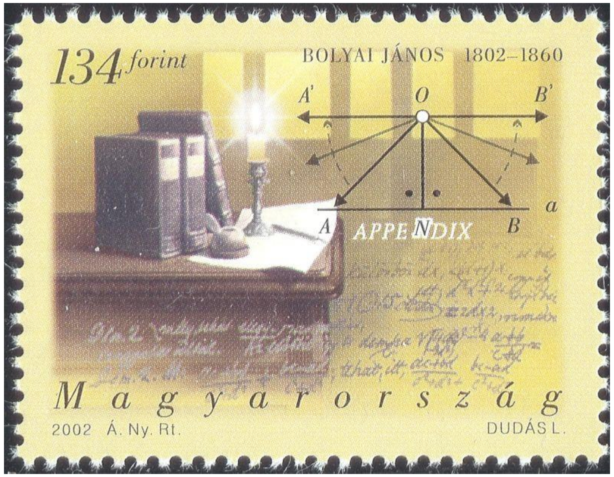
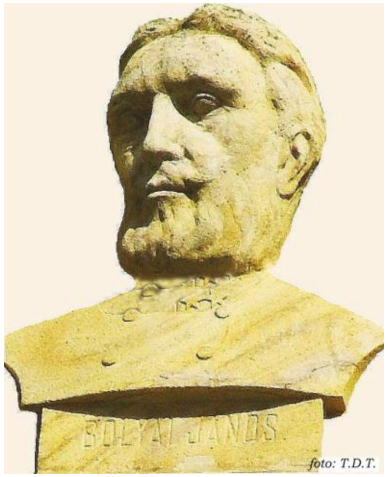
Founders of Non-Euclidean Geometry

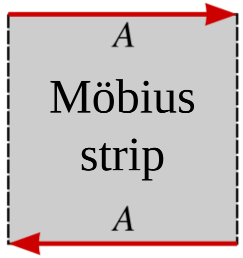
János Bolyai (1802-1860)

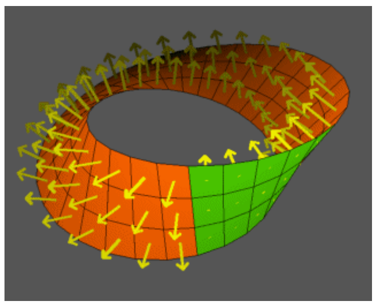


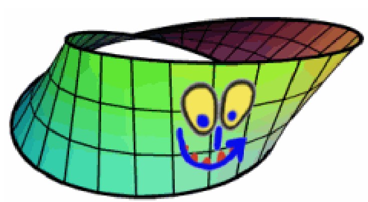


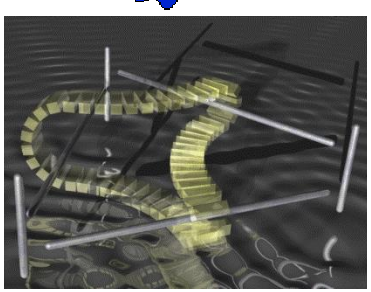
Nikolai Ivanovich Lobachevsky (1792-1856)

Non-Euclidean Non-Orientable Surfaces

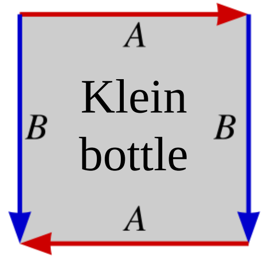


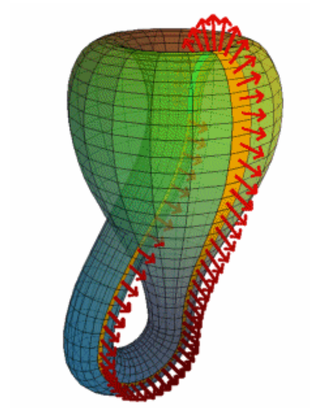


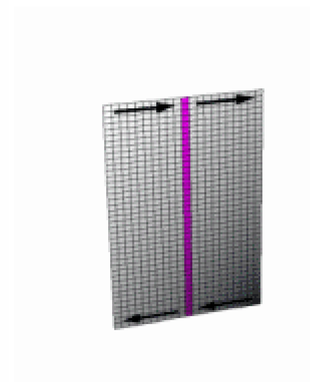


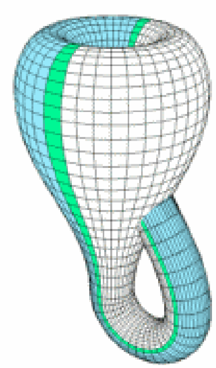


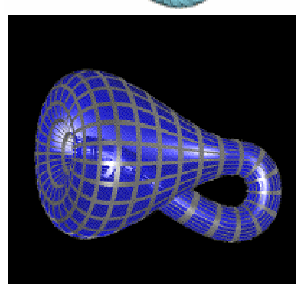
one side,
one boundary!











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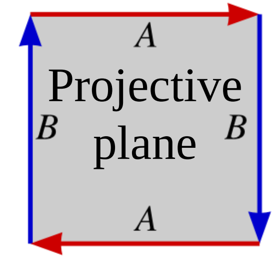

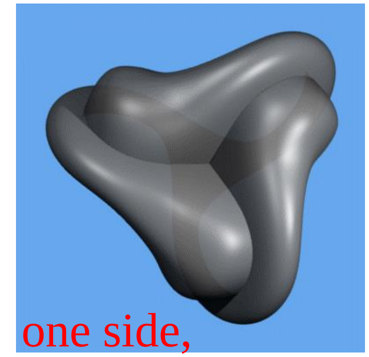

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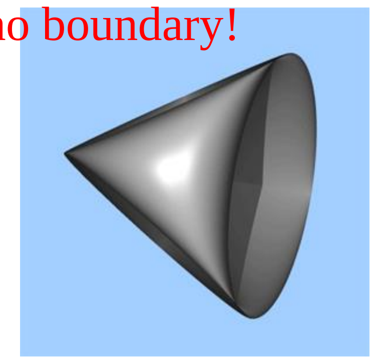
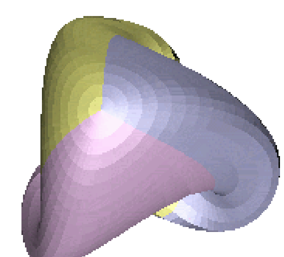
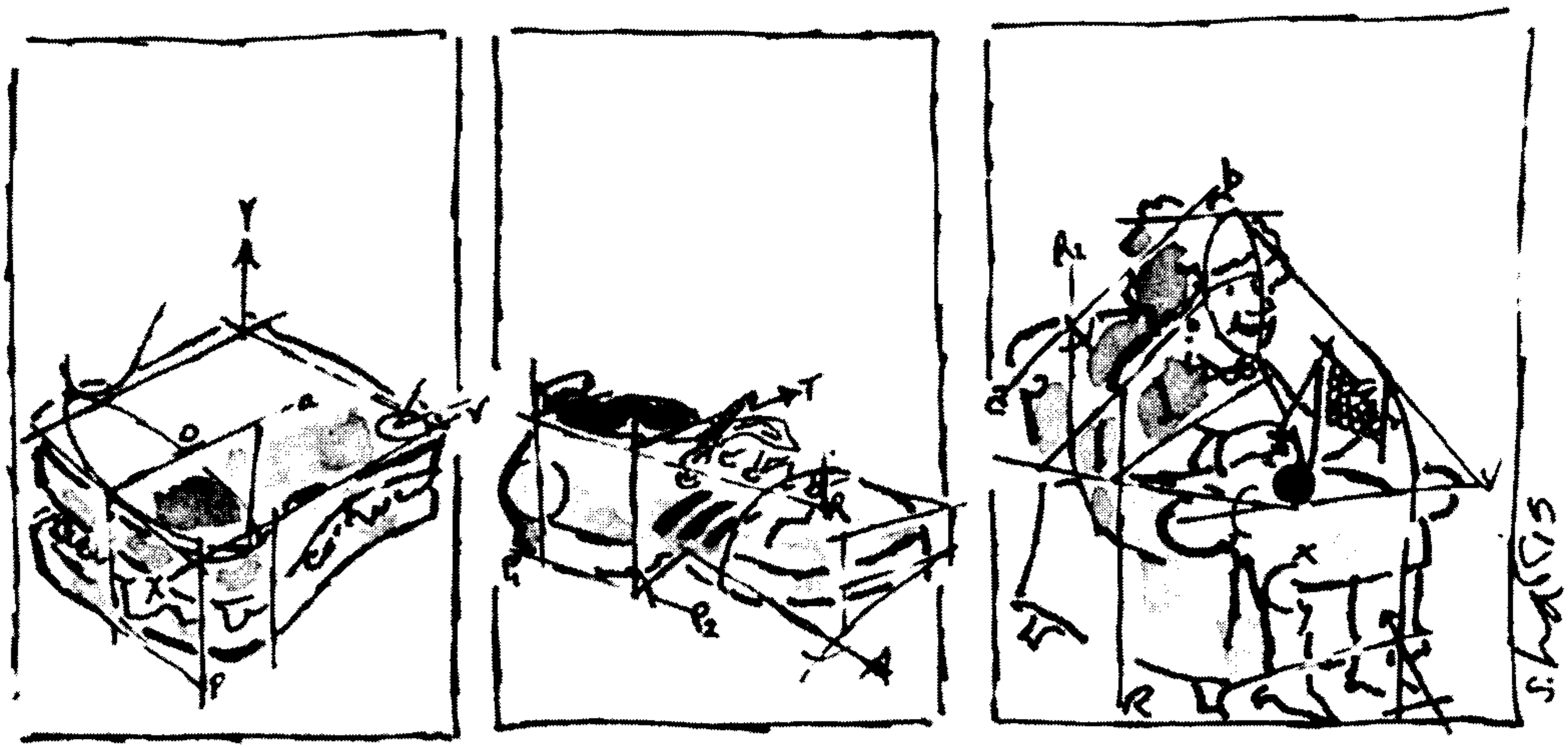


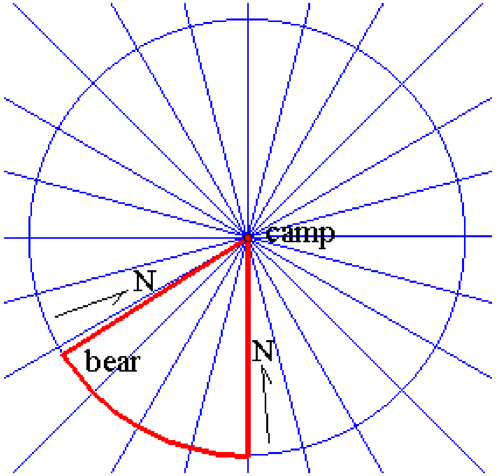


THE GEOMETRY OF EVERYDAY LIFE



TUNA SANDWICH SNEAKER GRANDMA

Problem: A man leaves his house and walks one mile south. He then walks one mile west and sees a Bear. Then he walks one mile north back to his house. What color was the bear?




Problem: Is the house location unique?