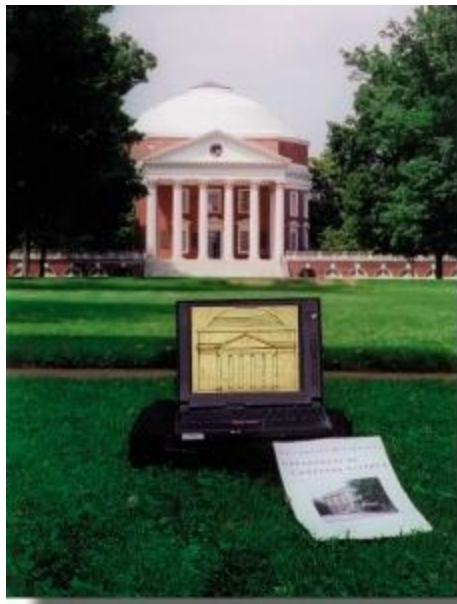


Theory of Computation

CS3102



Gabriel Robins

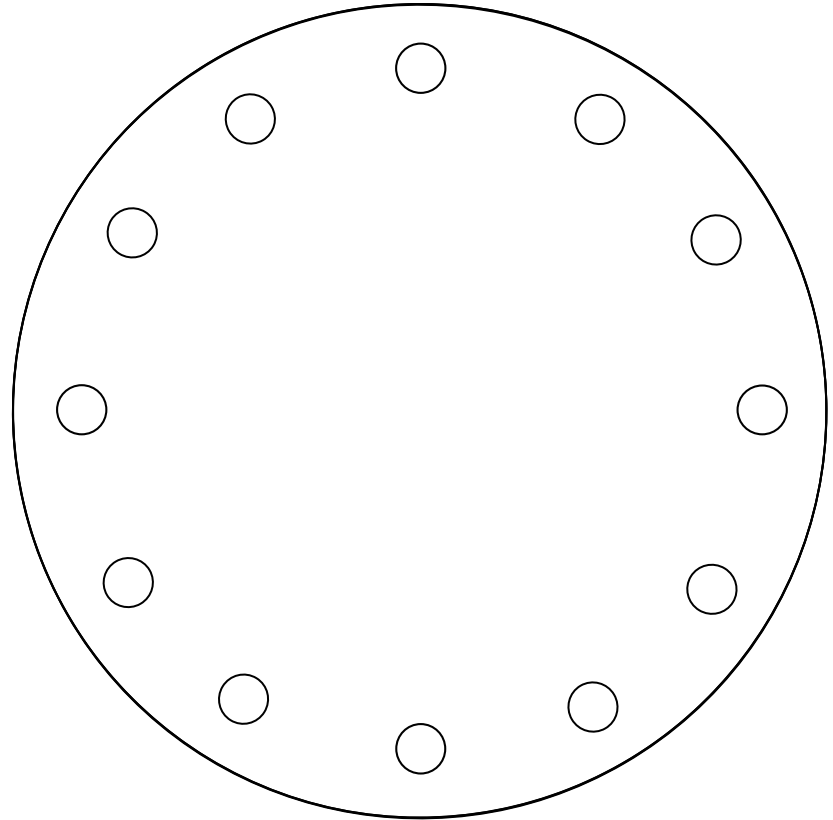
Department of
Computer Science

University of Virginia

www.cs.virginia.edu/robins/theory



Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?



- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

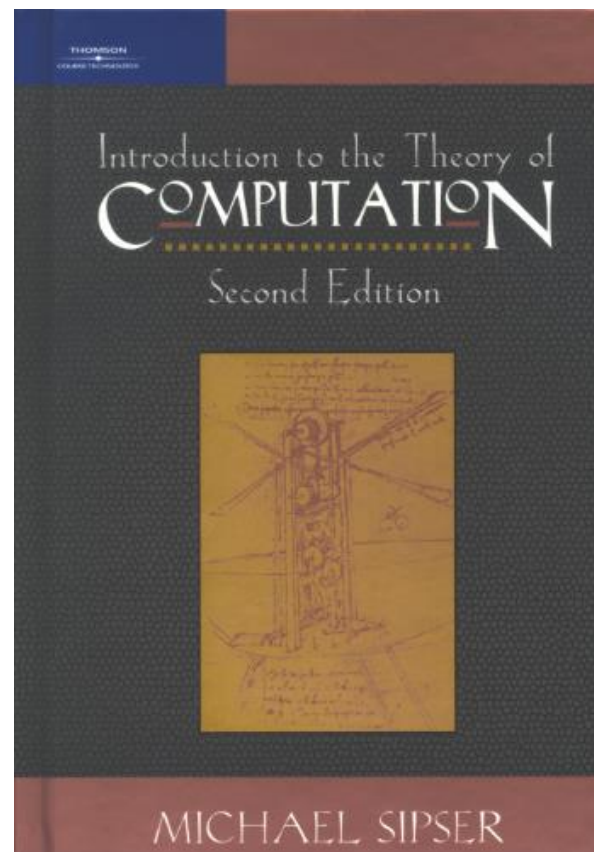
Theory of Computation (CS3102) - Textbook

Textbook:

Introduction to the Theory of
Computation, by Michael Sipser
(MIT), 2nd Edition, 2005

Good Articles / videos:

www.cs.virginia.edu/~robins/CS_readings.html

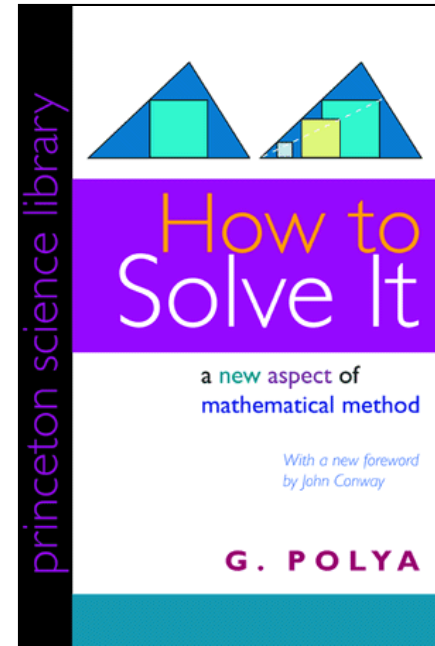


Theory of Computation (CS3102)

Required reading:

How to Solve It, by George Polya
(MIT), Princeton University Press, 1945

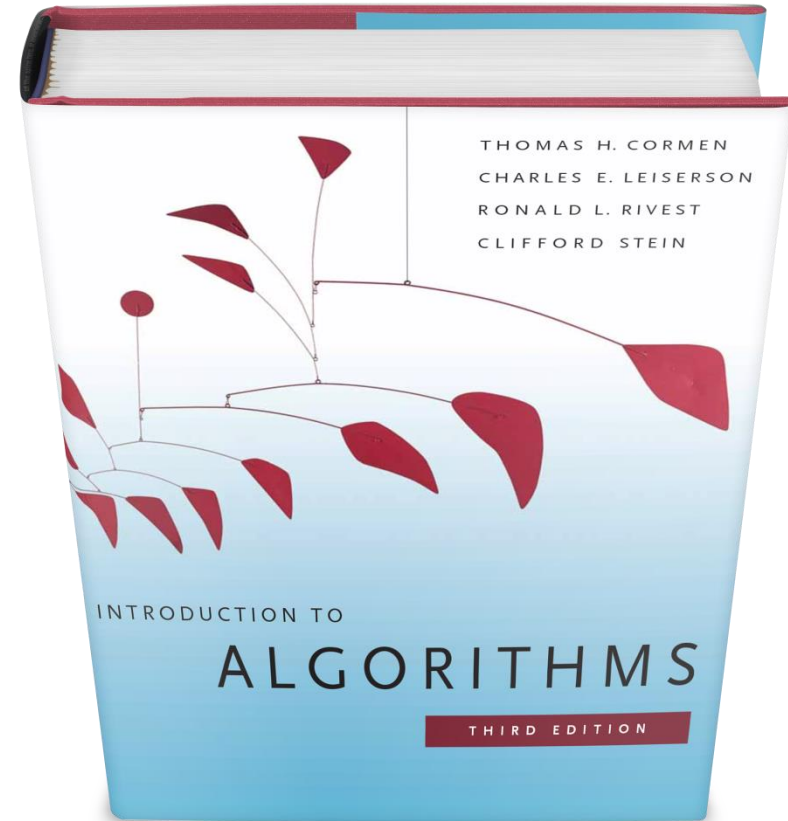
- A classic on **problem solving**



George Polya (1887-1985)

Theory of Computation (CS3102)

Good algorithms textbook:
Introduction to Algorithms
by Cormen et al (MIT)
Third Edition, 2009



Thomas Cormen



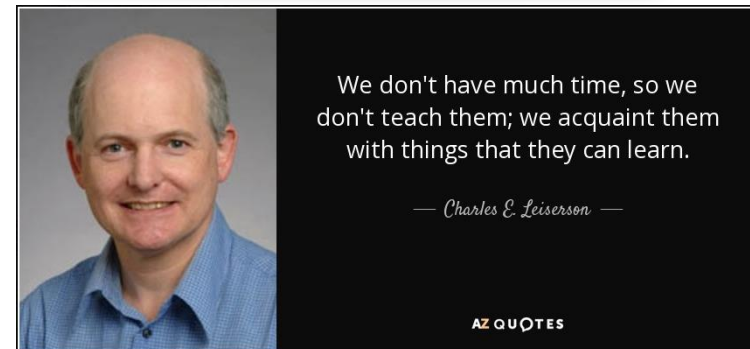
Charles Leiserson



Ronald Rivest



Clifford Stein



Introduction to Algorithms

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein
Third Edition

Some books on algorithms are rigorous but incomplete; others cover masses of material but lack rigor. *Introduction to Algorithms* uniquely combines rigor and comprehensiveness. The book covers a broad range of algorithms in depth, yet makes their design and analysis accessible to all levels of readers. Each chapter is relatively self-contained and can be used as a unit of study. The algorithms are described in English and in a pseudocode designed to be readable by anyone who has done a little programming. The explanations have been kept elementary without sacrificing depth of coverage or mathematical rigor.

The first edition became a widely used text in universities worldwide as well as the standard reference for professionals. The second edition featured new chapters on the role of algorithms, probabilistic analysis and randomized algorithms, and linear programming. The third edition has been revised and updated throughout. It includes two completely new chapters, on van Emde Boas trees and multithreaded algorithms, and substantial additions to the chapter on recurrences (now called "Divide-and-Conquer"). It features improved treatment of dynamic programming and greedy algorithms and a new notion of edge-based flow in the material on flow networks. Many new exercises and problems have been added for this edition.

As of the third edition, this textbook is published exclusively by the MIT Press.

Thomas H. Cormen is Professor of Computer Science and former Director of the Institute for Writing and Rhetoric at Dartmouth College. Charles E. Leiserson is Professor of Computer Science and Engineering at MIT. Ronald L. Rivest is Andrew and Erna Viterbi Professor of Electrical Engineering and Computer Science at MIT. Clifford Stein is Professor of Industrial Engineering and Operations Research at Columbia University.

"In light of the explosive growth in the amount of data and the diversity of computing applications, efficient algorithms are needed now more than ever. This beautifully written, thoughtfully organized book is the definitive introductory book on the design and analysis of algorithms. The first half offers an effective method to teach and study algorithms; the second half then engages more advanced readers and curious students with compelling material on both the possibilities and the challenges in this fascinating field."

—Shang-Hua Teng, University of Southern California

"*Introduction to Algorithms*, the 'bible' of the field, is a comprehensive textbook covering the full spectrum of modern algorithms: from the fastest algorithms and data structures to polynomial-time algorithms for seemingly intractable problems, from classical algorithms in graph theory to special algorithms for string matching, computational geometry, and number theory. The revised third edition notably adds a chapter on van Emde Boas trees, one of the most useful data structures, and on multithreaded algorithms, a topic of increasing importance."

—Daniel Spielman, Department of Computer Science, Yale University

"As an educator and researcher in the field of algorithms for over two decades, I can unequivocally say that the Cormen book is the best textbook that I have ever seen on this subject. It offers an incisive, encyclopedic, and modern treatment of algorithms, and our department will continue to use it for teaching at both the graduate and undergraduate levels, as well as a reliable research reference."

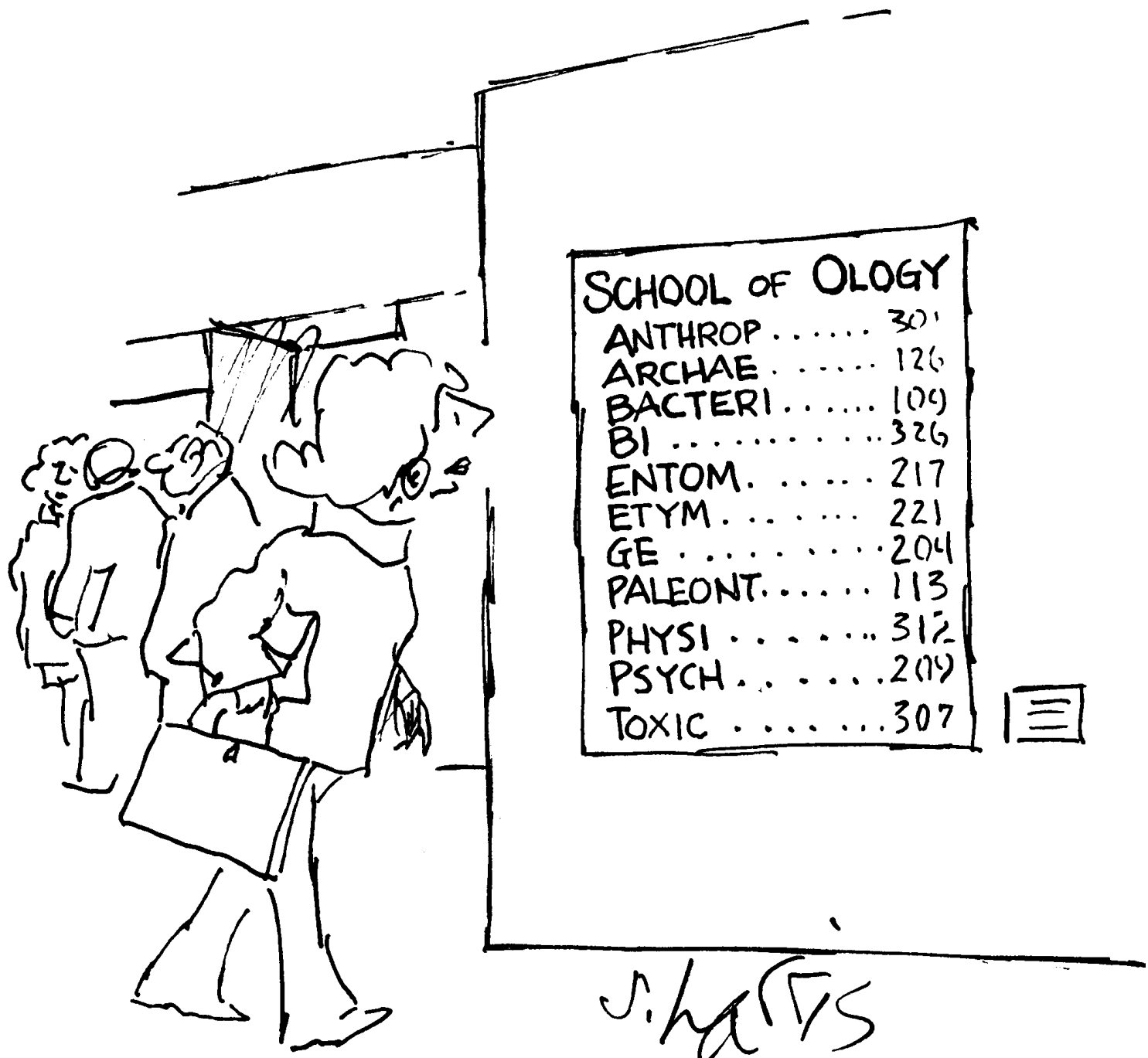
—Gabriel Robins, Department of Computer Science, University of Virginia

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The MIT Press
Massachusetts Institute of Technology
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<http://mitpress.mit.edu>

978-0-262-03384-8





SCHOOL OF OLOGY	
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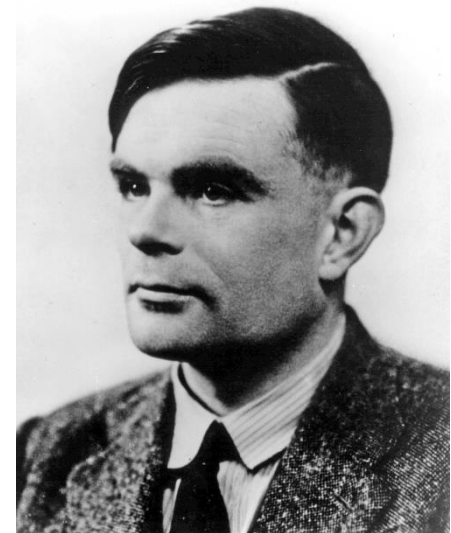
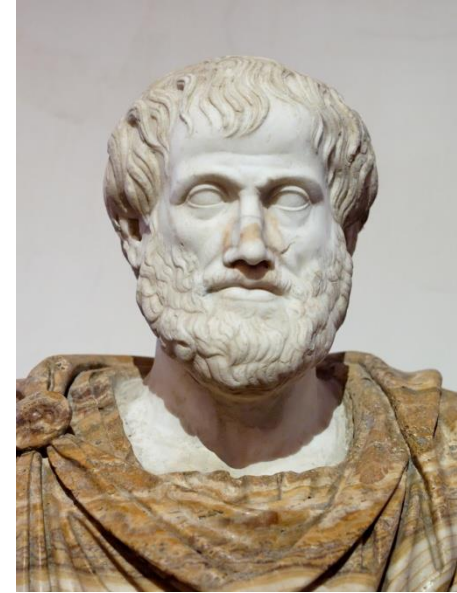


J. Hart's

Theory of Computation (CS3102) - Syllabus

A brief history of computing:

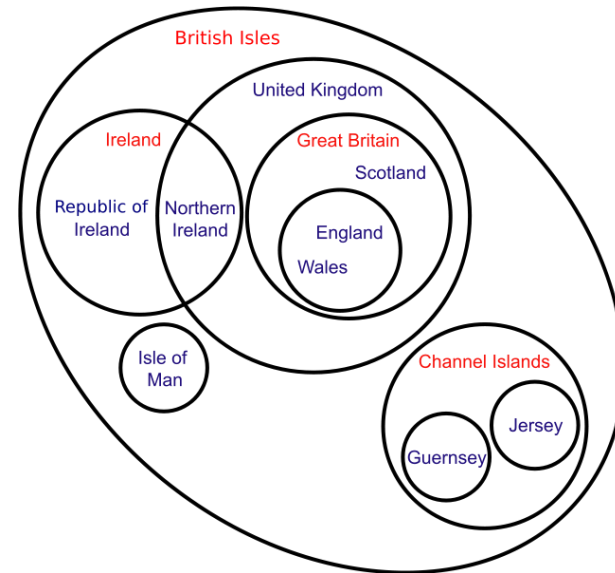
- Aristotle, **Euclid**, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- **Boole**, **De Morgan**, **Babbage**, Ada Augusta
- Venn, Carroll, **Cantor**, **Hilbert**, **Russell**
- Hardy, Ramanujan, Ramsey
- Godel, **Church**, **Turing**, **von Neumann**
- Shannon, **Kleene**, **Chomsky**



Theory of Computation Syllabus (continued)

Fundamentals:

- Set theory
- Predicate logic
- Formalisms and notation
- Infinities and countability
- Dovetailing / diagonalization
- Proof techniques
- Problem solving
- Asymptotic growth
- Review of graph theory

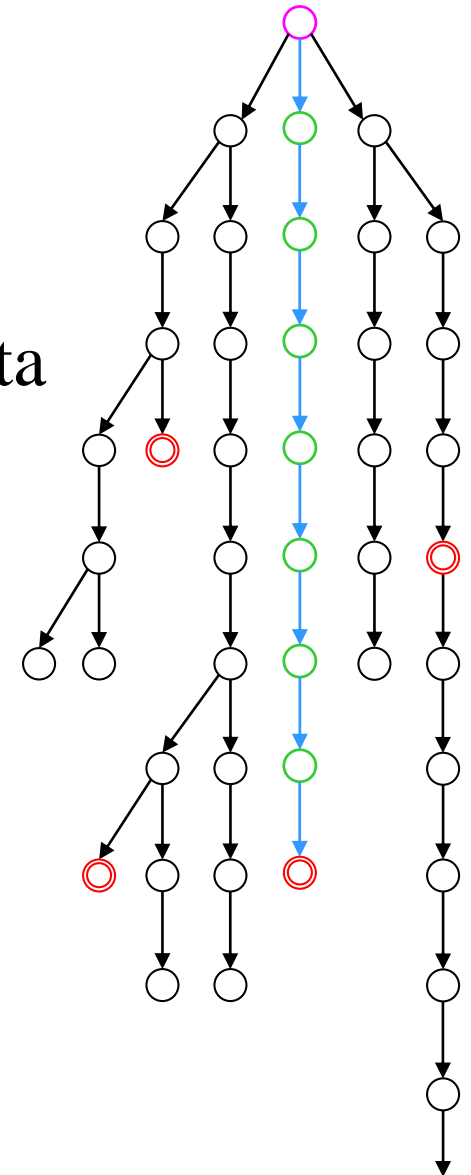


7	$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$	$\frac{7}{7}$	$\frac{7}{8}$...
6	$\frac{6}{1}$	$\frac{6}{2}$	$\frac{6}{3}$	$\frac{6}{4}$	$\frac{6}{5}$	$\frac{6}{6}$	$\frac{6}{7}$	$\frac{6}{8}$...
5	$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$...
4	$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$...
3	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$...
2	$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$	$\frac{2}{8}$...
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$...
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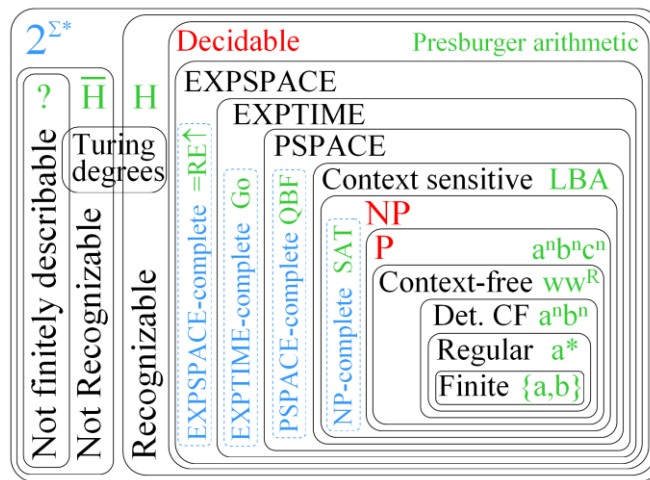
Theory of Computation Syllabus (continued)

Formal languages and machine models:

- The Chomsky hierarchy
- Regular languages / finite automata
- Context-free grammars / pushdown automata
- Unrestricted grammars / Turing machines
- Non-determinism
- Closure operators
- Pumping lemmas
- Non-closures
- Decidable properties



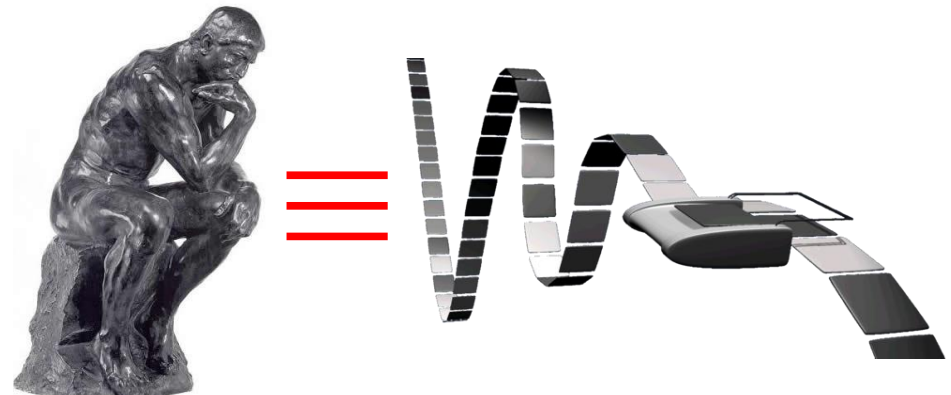
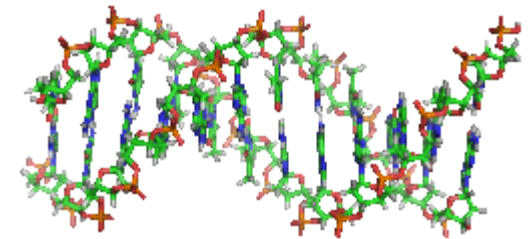
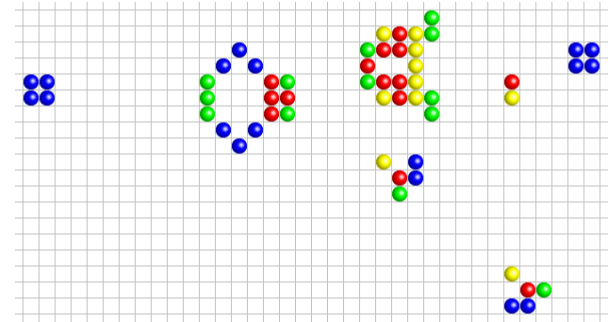
The Extended Chomsky Hierarchy



Theory of Computation Syllabus (continued)

Computability and undecidability:

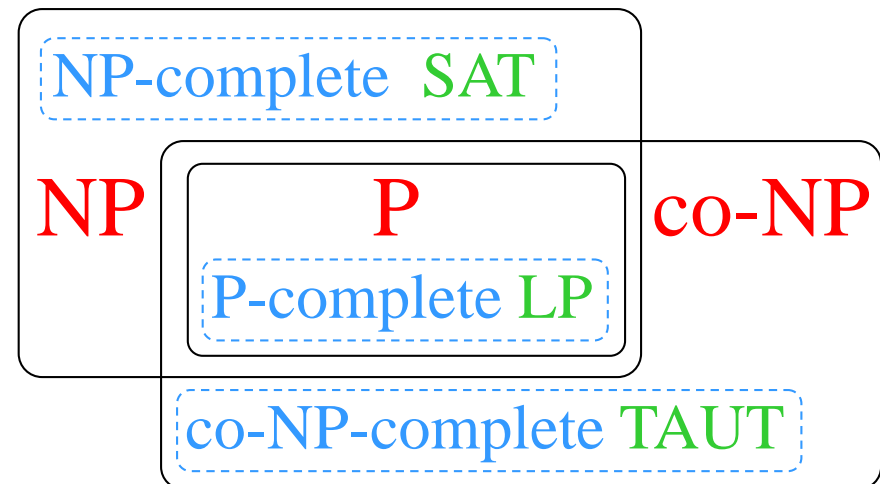
- Basic models
- Modifications and extensions
- Computational universality
- Decidability
- Recognizability
- Undecidability
- Church-Turing thesis
- Rice's theorem



Theory of Computation Syllabus (continued)

NP-completeness:

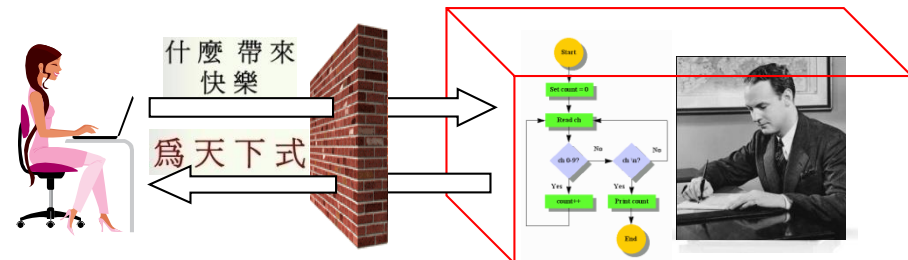
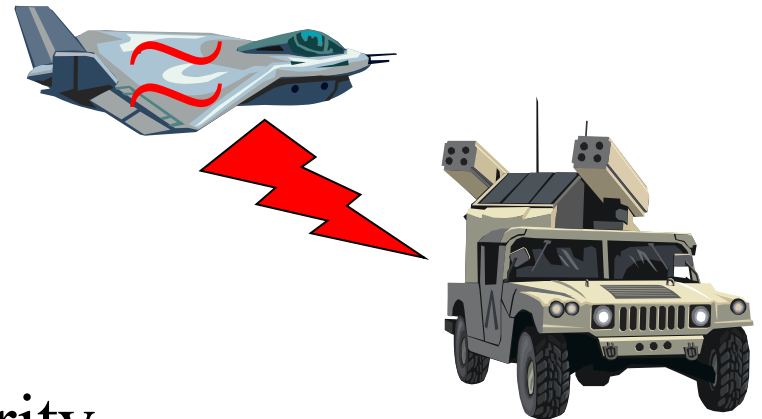
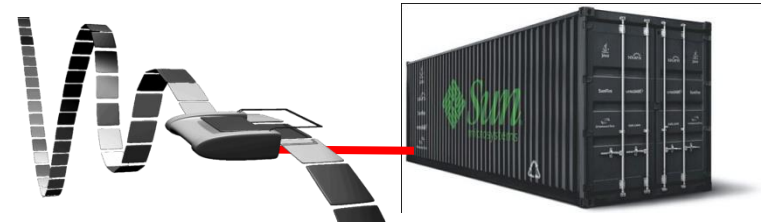
- Resource-constrained computation
- Complexity classes
- Intractability
- Boolean satisfiability
- Cook-Levin theorem
- Transformations
- Graph clique problem
- Independent sets
- Hamiltonian cycles
- Colorability problems
- Heuristics



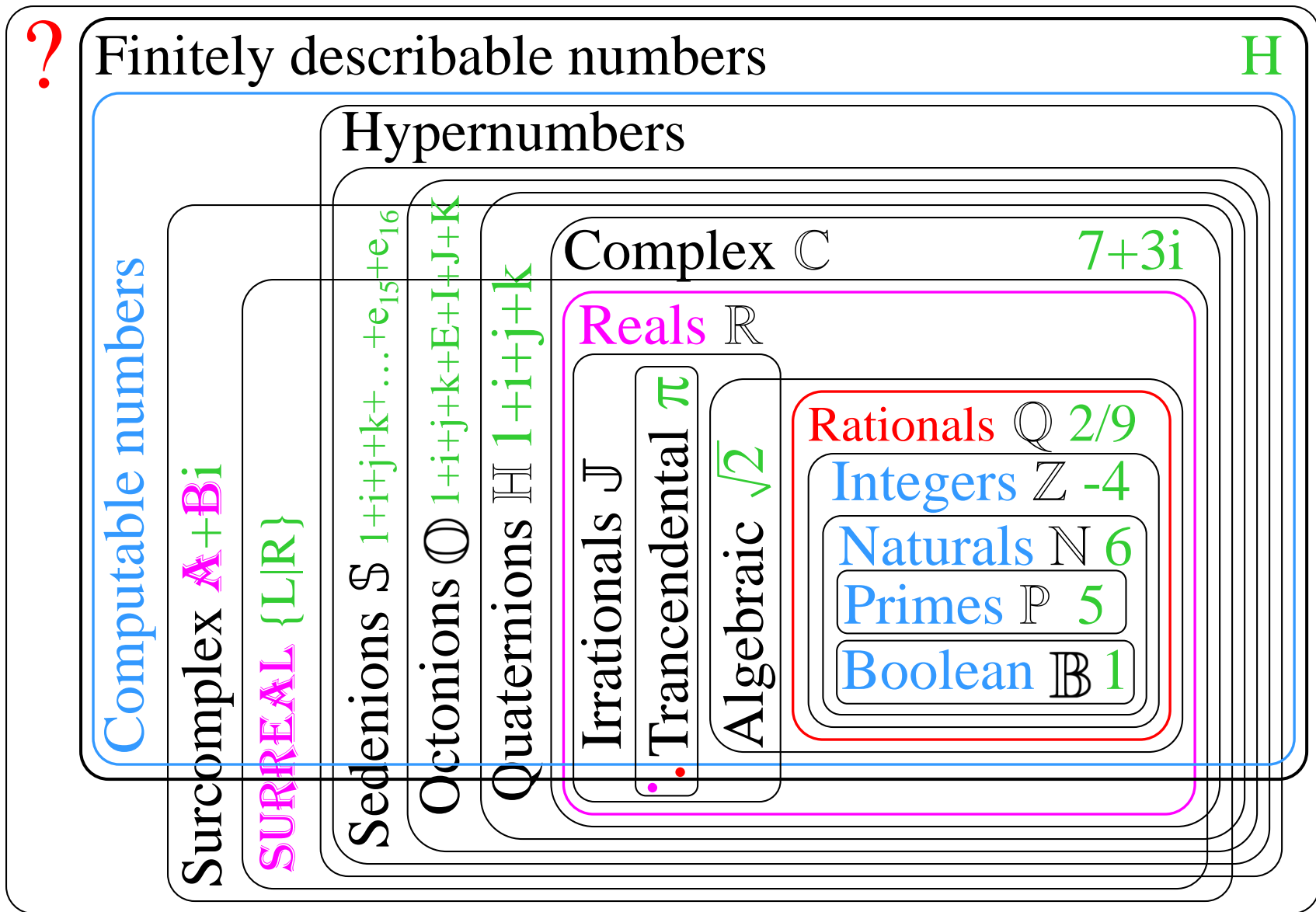
Theory of Computation Syllabus (continued)

Other topics (as time permits):

- Generalized number systems
- Oracles and relativization
- Zero-knowledge proofs
- Cryptography & mental poker
- The Busy Beaver problem
- Randomness and compressibility
- The Turing test
- AI and the Technological Singularity



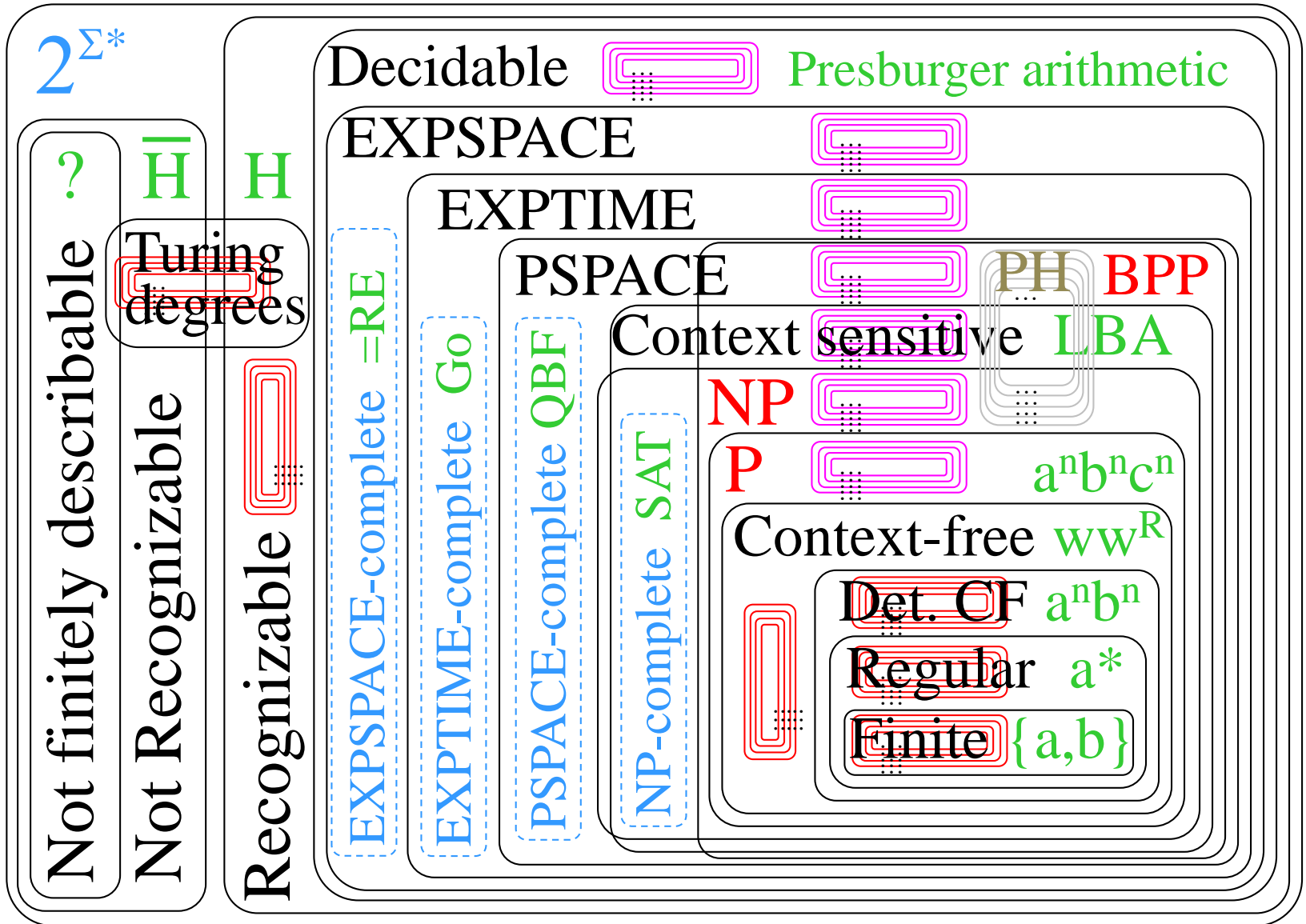
Generalized Numbers



Theorem: some real numbers are not finitely describable!

Theorem: some finitely describable real numbers are not computable!

The Extended Chomsky Hierarchy

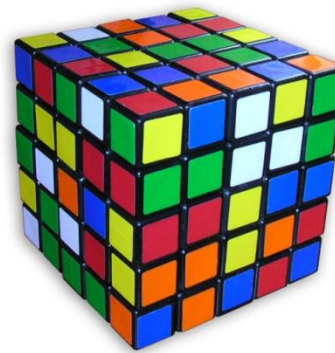


Dense infinite time & space complexity hierarchies

Other infinite complexity & descriptive hierarchies

Overarching Philosophy

- Focus on the “big picture” & “scientific method”
- Emphasis on **problem solving** & creativity
- Discuss applications & practice
- A primary objective: have **fun!**



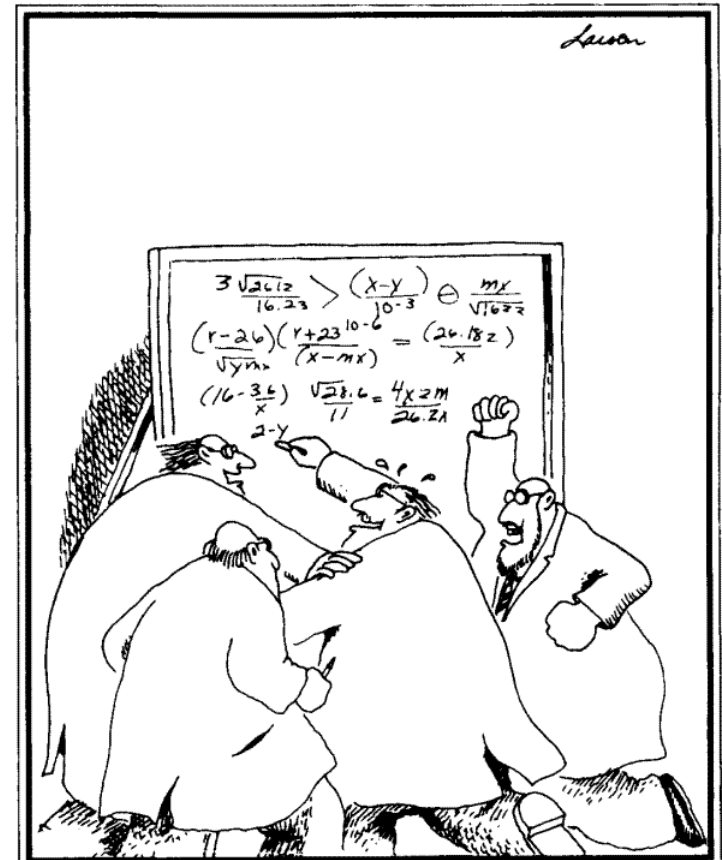
Prerequisites

- Some **discrete math & algorithms** knowledge
- Ideally, should have taken CS2102
- Course will “**bootstrap**” (albeit quickly) from **first principles**
- Critical: **Tenacity**, **patience**



Course Organization

- **Exams:** probably take home
 - Decide by vote
 - Flexible exam schedule
- **Problem sets:**
 - Lots of problem solving
 - **Work in groups!** (max size 6 people)
 - Not formally graded
 - **Most exam questions will come from these sets!**
- **Homeworks:**
 - Will come from problem sets
 - Formally graded
- **Readings:** papers / videos / books
- **Extra credit** problems
 - In class & take-home
 - Find mistakes in slides, handouts, etc.
- Course materials posted on Web site
www.cs.virginia.edu/robins/theory



"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"

Grading Scheme

• Attendance	10%
• Homeworks	20%
• Readings	20%
• Midterm	25%
• Final	25%
• Extra credit	10%
<hr/>	
Total:	110% +

Best strategy:

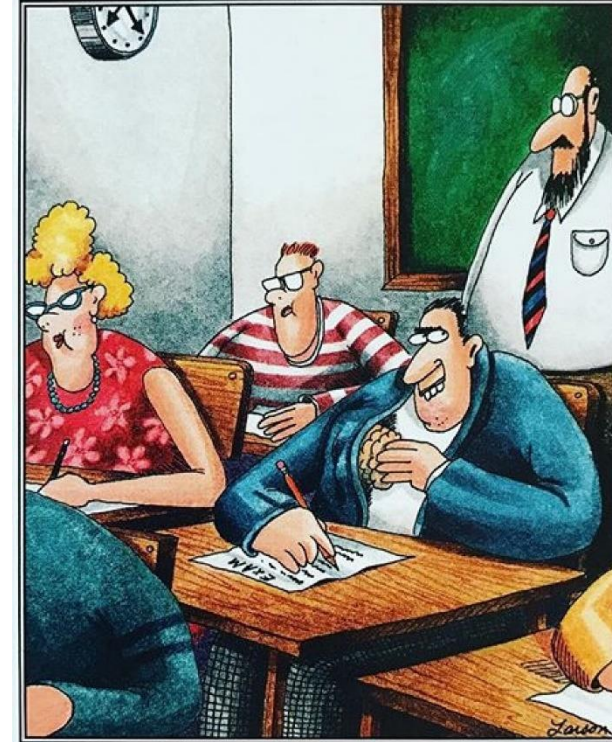
- **Solve lots of problems!**
- **Do lots of readings / EC!**
- “Ninety percent of success is just **showing up.**” – Woody Allen



“Mr. Osborne, may I be excused? My brain is full.”

Cheating Policy

- Cheating / plagiarism is strictly prohibited
- Serious penalties for violators
- Please review the UVa Honor Code
- Examples of Cheating / plagiarism:
 - Mass-copying of solutions from others / Web
 - Mass-sharing of solutions with others / Web
 - Cutting-and-pasting from other people / Web
 - Copying article/book/movie reviews from people / Web
 - Other people / Web solving entire problems for you
 - Providing other people / Web with verbatim solutions
 - This list is not exhaustive!
- We have automated cheating / plagiarism detection tools!
- We encourage collaborations / brainstorming
- Lets keep it positive (and not play “gotcha”)



Midway through the exam, Allen pulls out a bigger brain.

Contact Information

Professor Gabriel Robins

Office: 406 Rice Hall

Phone: (434) 982-2207

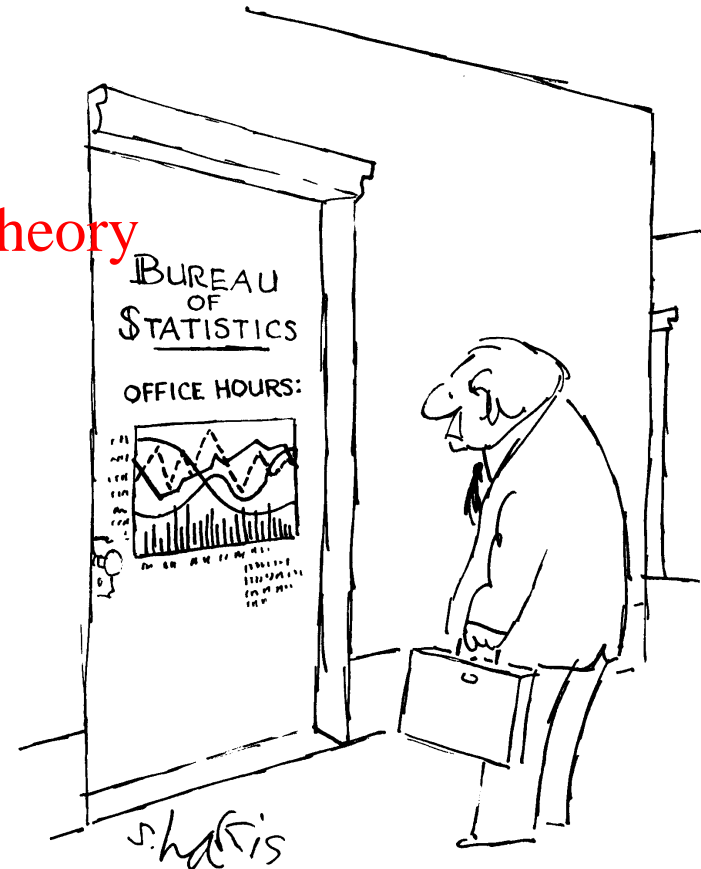
Email: robins@cs.virginia.edu

Web: www.cs.virginia.edu/robins

www.cs.virginia.edu/robins/theory

Office hours: right after class

- Any other time
- [By email](#) (preferred)
- By appointment
- Q&A blog posted on class Web site



Course Readings

www.cs.virginia.edu/robins/CS_readings.html

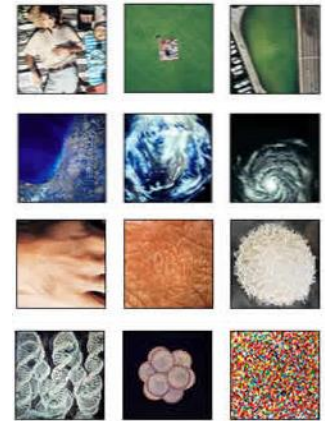
Goal: broad exposure to lots of cool **ideas & technologies!**

- **Required:** total of at least 36 items over the semester
- **Diversity:** minimums in each of 3 categories:
 1. Minimum of 15 videos
 2. Minimum of 15 papers / Web sites
 3. Minimum of 6 books
- More than 36 total is even better! (extra credit)
- Some required items in each category
 - Remaining “elective” items should be a diverse mix
- Email all submissions to: homework.cs3102@gmail.com

Required Readings

www.cs.virginia.edu/robins/CS_readings.html

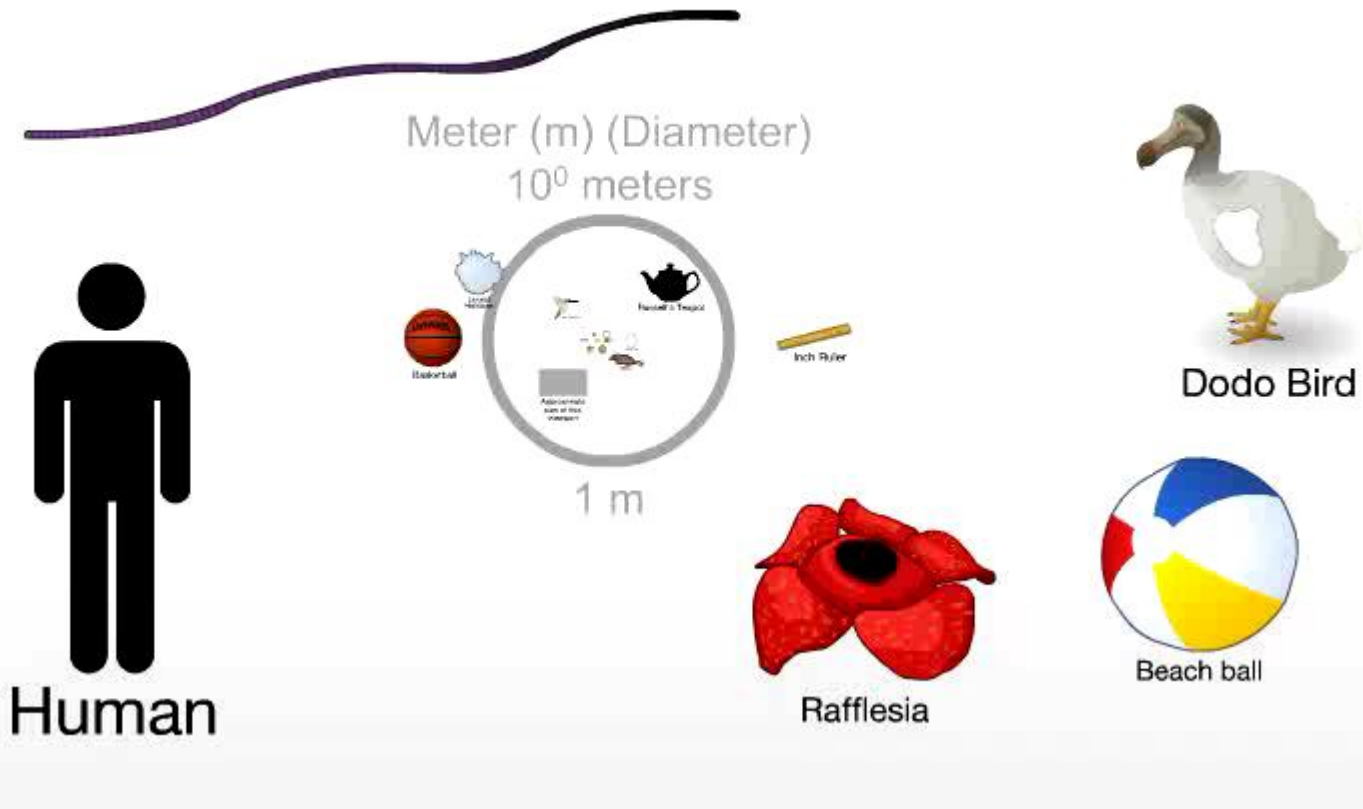
- **Required** videos:
 - Last Lecture, Randy Pausch, 2007
 - Time Management, Randy Pausch, 2007
 - Powers of Ten, Charles and Ray Eames, 1977



Required Reading

- “[Scale of the Universe](#)”, Cary and Michael Huang, 2012

Giant Earthworm



- 10^{-24} to 10^{26} meters \Rightarrow 50 orders of magnitude!

Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- More **required** videos:
 - Claude Shannon - Father of the Information Age, UCTV
 - [The Pattern Behind Self-Deception](#), Michael Shermer, 2010

Claude Shannon
(1916–2001)



Michael Shermer

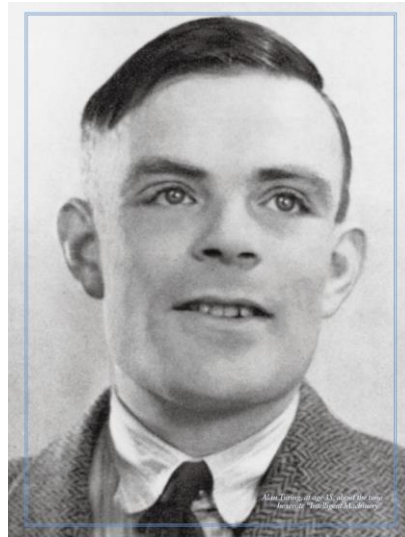
Required Readings

www.cs.virginia.edu/robins/CS_readings.html

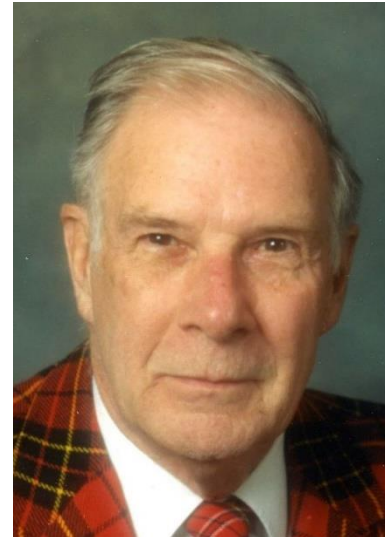
- **Required** articles:
 - Decoding an Ancient Computer, Freeth, 2009
 - Alan Turing's Forgotten Ideas, Copeland and Proudfoot, 1999
 - You and Your Research, Richard Hamming, 1986
 - **Who Can Name the Bigger Number**, Scott Aaronson, 1999



Antikythera computer, 200BC



Alan Turing



Richard Hamming



Scott Aaronson

"BENEDICT CUMBERBATCH IS OUTSTANDING"

RADIO TIMES

"THE BEST BRITISH FILM OF THE YEAR"



THE INDEPENDENT

"AN INSTANT CLASSIC"



GLAMOUR

"A SUPERB THRILLER"



EMPIRE



TIME OUT

THE TIMES

THE IMITATION GAME

BENEDICT CUMBERBATCH

KEIRA KNIGHTLEY

12A MODERATE SEX REFERENCES

BASED ON THE INCREDIBLE TRUE STORY

BLACK BEAR PICTURES presents in association with FILMATION ENTERTAINMENT. BLACK BEAR PICTURES production. BRISTOL AUTOMOTIVE production. "THE IMITATION GAME" BENEDICT CUMBERBATCH KEIRA KNIGHTLEY MATTHEW GOODE RUBY KINNEAR CHARLES DANCE AND MARK STRONG. COSTUME DESIGNER: ANNA SOLO. EXECUTIVE PRODUCERS: ANNA PRINCE. PRODUCED BY SAMMY SHELDON OFFER. EXECUTIVE PRODUCERS: ANNA TUJIKOVIC. EXECUTIVE PRODUCERS: ALEXANDRE DESPLAT AND WILLIAM GOLDENBERG. EXECUTIVE PRODUCERS: OSCAR FANDR. EXECUTIVE PRODUCERS: PETER HESLOP. PRODUCED BY GRAHAM MOORE. DIRECTOR OF PHOTOGRAPHY: DO OSTRONOWSKI. EDITOR: TEOBY SCHWARZMAN. EXECUTIVE PRODUCERS: GRAHAM MOORE. EXECUTIVE PRODUCERS: MORTEN TYLUND. EXECUTIVE PRODUCERS: STUDIOCANAL. www.imitationgame.com

[f /ImitationGameUK](#)

IN CINEMAS NOVEMBER 14

Extra credit!

Basic Concepts and Notation

Gabriel Robins

"When I use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean -- neither more nor less."

Required reading

A **set** is formally an undefined term, but intuitively it is a (possibly empty) collection of arbitrary objects. A set is usually denoted by curly braces and some (optional) restrictions. Examples of sets are $\{1,2,3\}$, $\{\text{hi, there}\}$, and $\{k \mid k \text{ is a perfect square}\}$. The symbol \in denotes set **membership**, while the symbol \notin denotes set **non-membership**; for example, $7 \in \{p \mid p \text{ prime}\}$ states that 7 is a prime number, while $q \notin \{0,2,4,6,\dots\}$ states that q is not an even number.

Some **common sets** are denoted by special notation:

The **natural numbers**:

$$\mathbb{N} = \{1,2,3,\dots\}$$

The **integers**:

$$\mathbb{Z} = \{\dots,-3,-2,-1,0,1,2,3,\dots\}$$

The **rational numbers**:

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a,b \in \mathbb{Z}, b \neq 0 \right\}$$

The **real numbers**:

$$\mathbb{R} = \{x \mid x \text{ is a real number}\}$$

The **empty set**:

$$\emptyset = \{\}$$

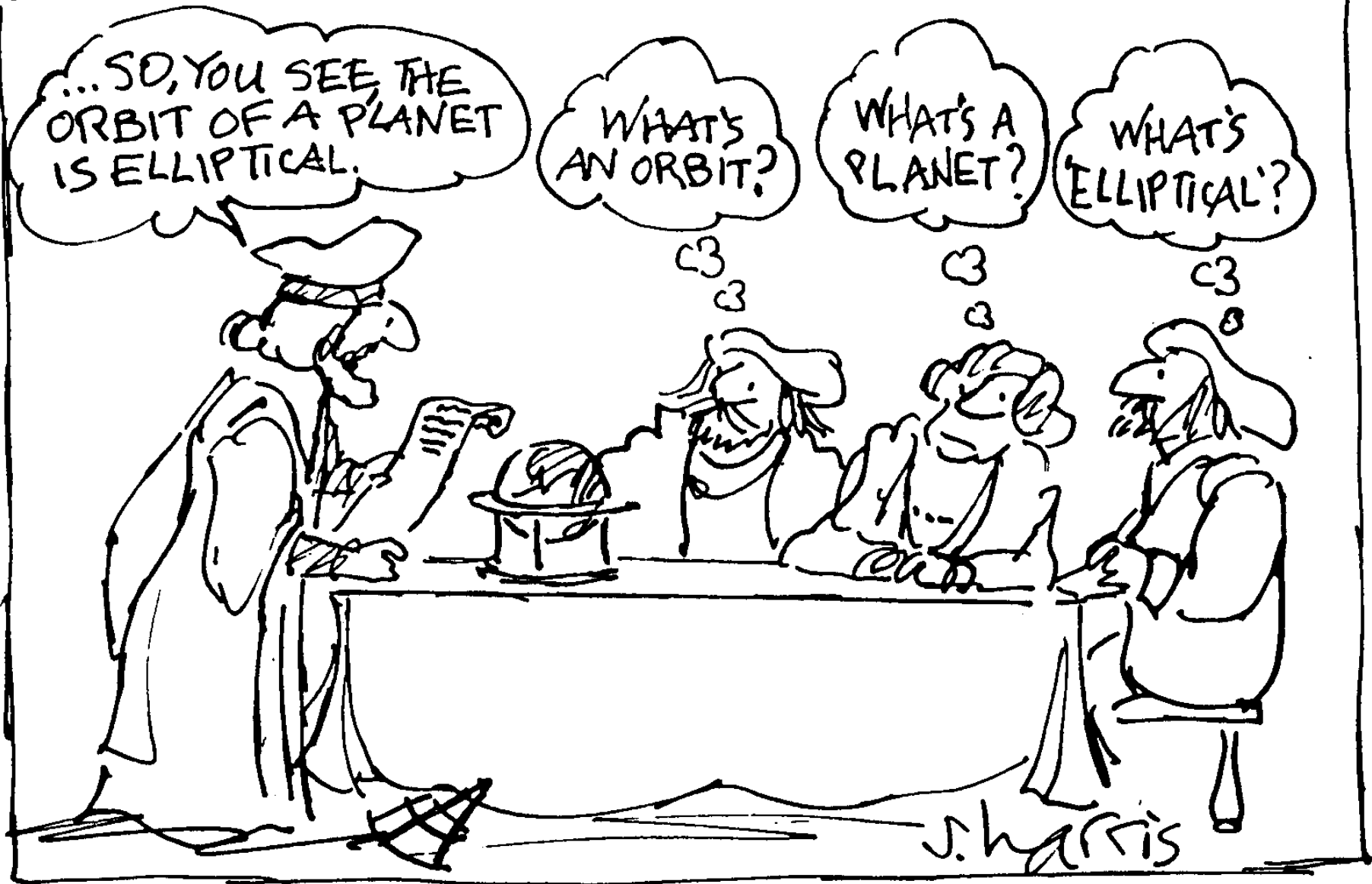
JOHANNES KEPLER'S UPHILL BATTLE

...SO, YOU SEE, THE ORBIT OF A PLANET IS ELLIPTICAL.

WHAT'S AN ORBIT?

WHAT'S A PLANET?

WHAT'S 'ELLIPTICAL'?



J. HARRIS

Discrete Math Review Slides

Symbolic Logic

Def: *proposition* - statement either true (T) or false (F)

- Ex: $1 + 1 = 2$
 $2 + 2 = 3$
 $3 < 7$
 $x + 4 = 5$
 "today is Monday"

Boolean Functions

- "and" \wedge
- "or" \vee
- "not" \neg
- "xor" \oplus
- "nand" $\bar{\wedge}$
- "nor" $\bar{\vee}$
- "implication" \Rightarrow
- "equivalence" \Leftrightarrow

Logical Implication

Truth table:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Ex: $(x < 0) \wedge (x > 0) \Rightarrow (x = 0)$
 $1 < x < y \Rightarrow x^3 < y^3$
 "today is Sunday" $\Rightarrow 1 + 1 = 3$

Logical Equivalence

"biconditional" \Leftrightarrow
 or "if and only if" ("iff")
 or "necessary and sufficient"
 or "logically equivalent" =

Truth table:

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Ex: $p \Leftrightarrow p$
 $[(x=0) \vee (y=0)] \Leftrightarrow (xy=0)$
 $\min(x,y) = \max(x,y) \Leftrightarrow x=y$

Predicates

Def: *predicate* - a function or formula involving some variables

Ex: let $P(x) = "x > 3"$
 x is the variable
 $"x > 3"$ is the predicate

$P(5)$
 $P(1)$

Ex: $Q(x,y,z) = "x^2 + y^2 = z^2"$
 $Q(2,3,4)$
 $Q(3,4,5)$

Quantifiers

- Universal: "for all" \forall
 $\forall x P(x)$
 $\Leftrightarrow P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \dots$
 Ex: $\forall x \ x < x + 1$
 $\forall x \ x < x^2$
- Existential: "there exists" \exists
 $\exists x P(x)$
 $\Leftrightarrow P(x_1) \vee P(x_2) \vee P(x_3) \vee \dots$
 Ex: $\exists x \ x = x^2$
 $\exists x \ x < x - 1$

Combinations:
 $\forall x \exists y \ y > x$

Sets

Def: *set* - an **unordered** collection of elements

Ex: $\{1, 2, 3\}$ or $\{3, 1, 2\}$ (here)

Venn Diagram:

Def: two sets are **equal** iff they contain the **same** elements

Ex: $\{1, 2, 3\} = \{2, 3, 1\}$
 $\{0\} \neq \{1\}$
 $\{3, 5\} = \{3, 5, 3, 3, 5\}$

Common Sets

Naturals: $\mathbb{N} = \{1, 2, 3, 4, \dots\}$
Integers: $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
Rationals: $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\}$
Reals: $\mathbb{R} = \{x \mid x \text{ a real } \# \}$
Empty set: $\emptyset = \{ \}$
 \mathbb{Z}^+ = non-negative integers
 \mathbb{R}^- = non-positive reals, etc.

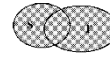
Subsets

- Subset notation:** \subseteq
 $S \subseteq T \Leftrightarrow (x \in S \Rightarrow x \in T)$

- Proper subset:** \subset
 $S \subset T \Leftrightarrow ((S \subseteq T) \wedge (S \neq T))$
 $S = T \Leftrightarrow ((T \subseteq S) \wedge (S \subseteq T))$
 $\forall S \ \emptyset \subseteq S$
 $\forall S \ S \subseteq S$

Union:

$S \cup T = \{x \mid x \in S \vee x \in T\}$



Intersection:

$S \cap T = \{x \mid x \in S \wedge x \in T\}$



- Universal set: U (everything)
- Set **complement:** S' or \bar{S}
 $S' = \{x \mid x \notin S\} = U - S$

- Disjoint sets:** $S \cap T = \emptyset$

$S - T = S \cap T'$
 $S - S = \emptyset$

DeMorgan's Laws

$(S \cup T)' = S' \cap T'$

$(S \cap T)' = S' \cup T'$

Boolean logic version:
 $(X \wedge Y)' = X' \vee Y'$
 $(X \vee Y)' = X' \wedge Y'$

Function Types

- One-to-one** function: "1-1"
 $a, b \in S \wedge a \neq b \Rightarrow f(a) \neq f(b)$

Ex: $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x$ is 1-1
 $g(x) = x^2$ is not 1-1

- Onto** function:
 $\forall t \in T \ \exists s \in S \ \ni f(s) = t$

Ex: $f: \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = 13 - x$ is onto
 $g(x) = x^2$ is not onto

1-to-1 Correspondence

- 1-to-1 correspondence:** $f: S \leftrightarrow T$
 f is **both** 1-1 and onto

Ex: $f: \mathbb{R} \leftrightarrow \mathbb{R} \ \ni f(x) = x$ (identity)

$h: \mathbb{N} \leftrightarrow \mathbb{Z} \ \ni h(x) = \frac{x-1}{2}, x \text{ odd},$
 $\frac{-x}{2}, x \text{ even}.$

Generalized Cardinality

- S is **at least as large** as T :
 $|S| \geq |T| \Rightarrow \exists f: S \rightarrow T, f$ onto
 i.e., " S covers T "

Ex: $f: \mathbb{R} \rightarrow \mathbb{Z}, f(x) = \text{round}(x)$
 $\Rightarrow |\mathbb{R}| \geq |\mathbb{Z}|$

- S and T have **same cardinality**:
 $|S| = |T| \Rightarrow |S| \geq |T| \wedge |T| \geq |S|$
 or
 \exists 1-1 correspondence $S \leftrightarrow T'$

- Generalizes finite cardinality:
 $\{1, 2, 3, 4, 5\} \geq \{a, b, c\}$

Infinite Sets

- Infinite set:** $|S| > k \ \forall k \in \mathbb{Z}$
 or
 \exists 1-1 corres. $f: S \leftrightarrow T, S \subset T$

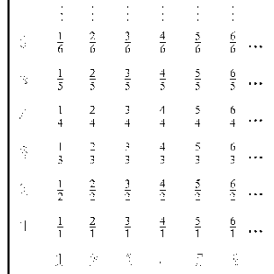
Ex: $\{p \mid p \text{ prime}\}, \mathbb{R}$

- Countable set:** $|S| \leq \mathbb{N}$
 Ex: $\emptyset, \{p \mid p \text{ prime}\}, \mathbb{N}, \mathbb{Z}$

- S is **strictly smaller** than T :
 $|S| < |T| \Rightarrow |S| \leq |T| \wedge |S| \neq |T|$

- Uncountable set:** $|\mathbb{N}| < |S|$
 Ex: $|\mathbb{N}| < \mathbb{R}$
 $|\mathbb{N}| < [0, 1] = \{x \mid x \in \mathbb{R}, 0 \leq x \leq 1\}$

Thm: \exists 1-1 correspondence $\mathbb{Q} \leftrightarrow \mathbb{N}$



Thm: $|\mathbb{R}| > \mathbb{N}$

Pf (diagonalization):
 Assume \exists 1-1 corres. $f: \mathbb{R} \leftrightarrow \mathbb{N}$
 Construct $X \in \mathbb{R}$:

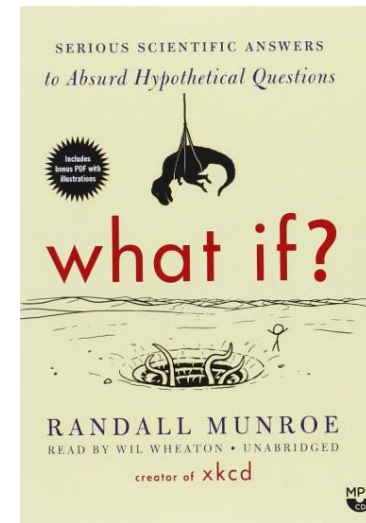
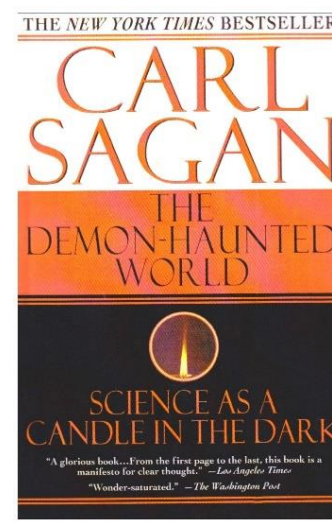
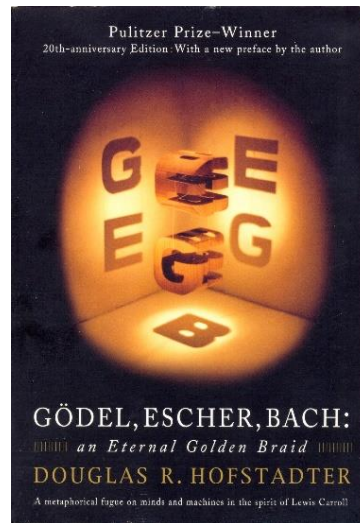
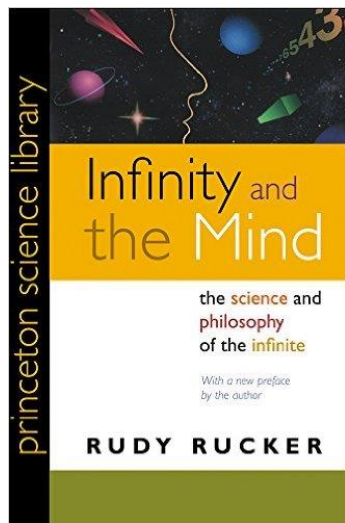
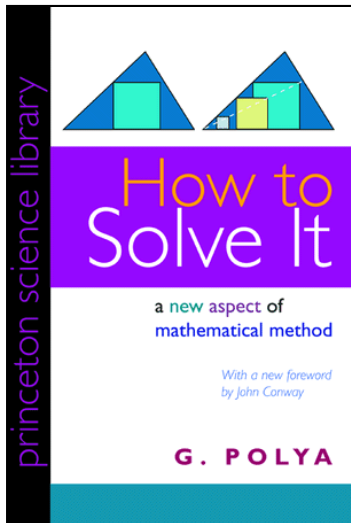
$f(1) = 2.18281828\dots$	$\rightarrow ?$
$f(2) = 2.14213562\dots$	$\rightarrow ?$
$f(3) = 2.167033969\dots$	$\rightarrow ?$

$X = 0.\underline{1}71\dots \neq f(k) \ \forall k \in \mathbb{N}$
 $\Rightarrow f$ not a 1-1 correspondence
 \rightarrow contradiction
 $\Rightarrow \mathbb{R}$ is uncountable

Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- **Required** books:
 - “How to Solve It”, Polya, 1957
 - “Infinity and the Mind”, Rucker, 1995
 - “Godel, Escher, Bach”, Hofstadter, 1979
 - “The Demon-Haunted World”, Sagan, 2009
 - “What If”, Munroe, 2014



Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Remaining videos / articles / books are “electives”
- Pacing: at least 2 submissions per week (due 5pm Monday)
 - Policy intended to help you avoid “cramming”
- Length: 1-2 paragraphs per article / video
1-2 pages per book
- Books are worth more credit than articles / videos
- Email all submissions to: homework.cs3102@gmail.com
- Additional readings beyond 36 are welcome! (extra credit)

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- Theory and Algorithms:

- **Who Can Name the Bigger Number**, Scott Aaronson, 1999
- The Limits of Reason, Gregory Chaitin, Scientific American, March 2006, pp. 74-81.
- Breaking Intractability, Joseph Traub and Henryk Wozniakowski, Scientific American, January 1994, pp. 102-107.
- Confronting Science's Logical Limits, John Casti, Scientific American, October 1996, pp. 102-105.
- **Go Forth and Replicate**, Moshe Sipper and James Reggia, Scientific American, August 2001, pp. 34-43.
- The Science Behind Sudoku, Jean-Paul Delahaye, Scientific American, June 2006, pp. 80-87.
- The Traveler's Dilemma, Kaushik Basu, Scientific American, June 2007, pp. 90-95.

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- **Biological Computing:**

- Computing with DNA, Leonard Adleman, Scientific American, August 1998, pp. 54-61.
- Bringing DNA Computing to Life, Ehud Shapiro and Yaakov Benenson, Scientific American, May 2006, pp. 44-51.
- Engineering Life: Building a FAB for Biology, David Baker et al., Scientific American, June 2006, pp. 44-51.
- Big Lab on a Tiny Chip, Charles Choi, Scientific American, October 2007, pp. 100-103.
- DNA Computers for Work and Play, Macdonald et al, Scientific American, November 2007, pp. 84-91.

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Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- **Quantum Computing:**

- Quantum Mechanical Computers, Seth Lloyd, *Scientific American*, 1997, pp. 98-104.
- Quantum Computing with Molecules, Gershenfeld and Chuang, *Scientific American*, June 1998, pp. 66-71.
- Black Hole Computers, Seth Lloyd and Jack Ng, *Scientific American*, November 2004, pp. 52-61.
- Computing with Quantum Knots, Graham Collins, *Scientific American*, April 2006, pp. 56-63.
- The Limits of Quantum Computers, Scott Aaronson, *Scientific American*, March 2008, pp. 62-69.
- Quantum Computing with Ions, Monroe and Wineland, *Scientific American*, August 2008, pp. 64-71.

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- **History of Computing:**

- The Origins of Computing, Campbell-Kelly, Scientific American, September 2009, pp. 62-69.
- Ada and the First Computer, Eugene Kim and Betty Toole, Scientific American, April 1999, pp. 76-81.

- **Security and Privacy:**

- Malware Goes Mobile, Mikko Hypponen, Scientific American, November 2006, pp. 70-77.
- RFID Powder, Tim Hornyak, Scientific American, February 2008, pp. 68-71.
- Can Phishing be Foiled, Lorrie Cranor, Scientific American, December 2008, pp. 104-110.

Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- **Future of Computing:**

- Microprocessors in 2020, David Patterson, Scientific American, September 1995, pp. 62-67.
- Computing Without Clocks, Ivan Sutherland and Jo Ebergen, Scientific American, August 2002, pp. 62-69.
- Making Silicon Lase, Bahram Jalali, Scientific American, February 2007, pp. 58-65.
- A Robot in Every Home, Bill Gates, Scientific Am, January 2007, pp. 58-65.
- Ballbots, Ralph Hollis, Scientific American, October 2006, pp. 72-77.
- Dependable Software by Design, Daniel Jackson, Scientific American, June 2006, pp. 68-75.
- Not Tonight Dear - I Have to Reboot, Charles Choi, Scientific American, March 2008, pp. 94-97.
- Self-Powered Nanotech, Zhong Lin Wang, Scientific American, January 2008, pp. 82-87.

Other “Elective” Readings

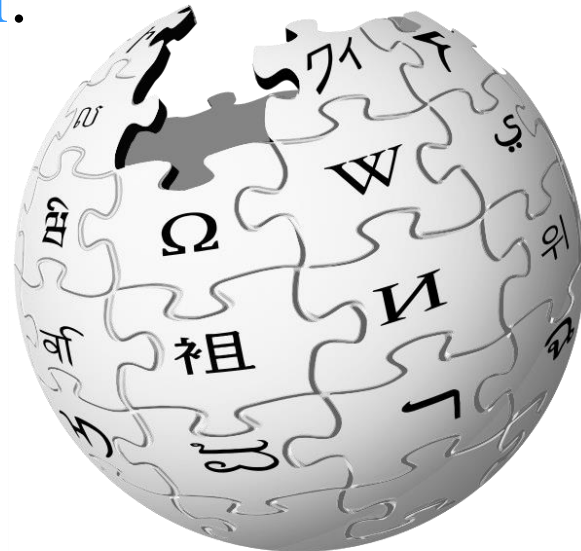
www.cs.virginia.edu/robins/CS_readings.html

- **The Web:**

- The Semantic Web in Action, Lee Feigenbaum et al., Scientific American, December 2007, pp. 90-97.
- Web Science Emerges, Nigel Shadbolt and Tim Berners-Lee, Scientific American, October 2008, pp. 76-81.

- **The Wikipedia Computer Science Portal:**

- Theory of computation and Automata theory
- Formal languages and grammars
- Chomsky hierarchy and the Complexity Zoo
- Regular, context-free & Turing-decidable languages
- Finite & pushdown automata; Turing machines
- Computational complexity
- List of data structures and algorithms



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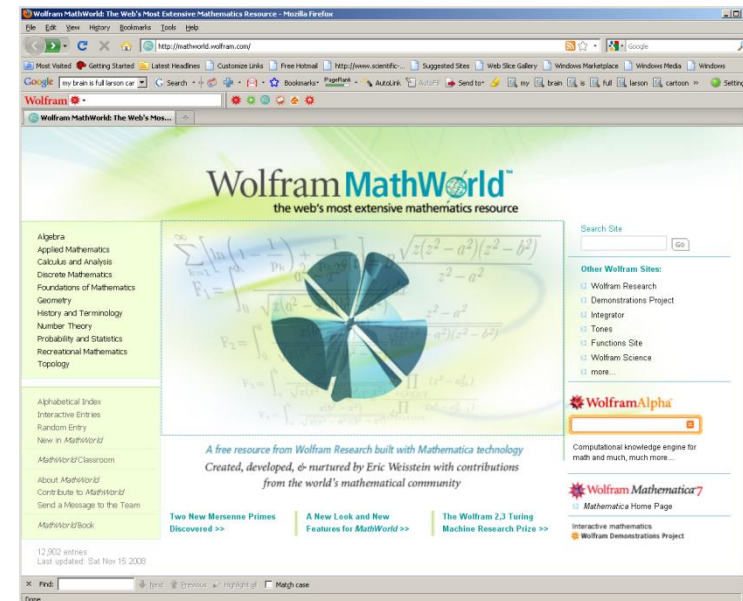
Other “Elective” Readings

www.cs.virginia.edu/robins/CS_readings.html

- The Wikipedia Math Portal:
 - Problem solving
 - List of Mathematical lists
 - Sets and Infinity
 - Discrete mathematics
 - Proof techniques and list of proofs
 - Information theory & randomness
 - Game theory
- Mathematica's “Math World”

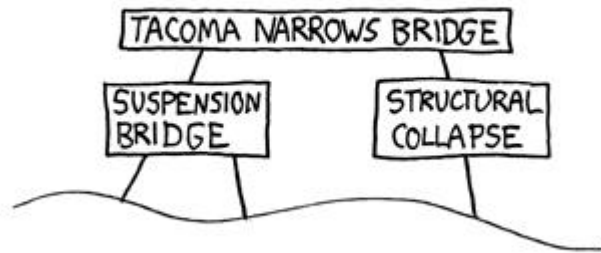


WIKIPEDIA
The Free Encyclopedia

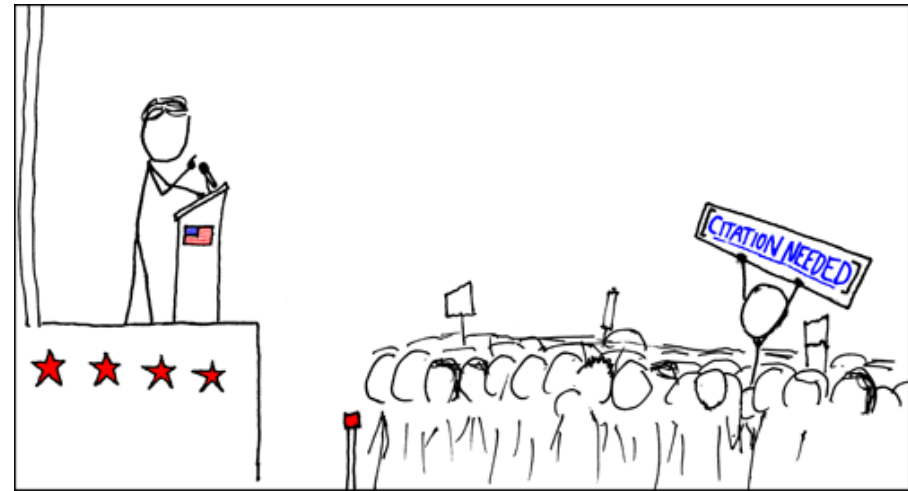
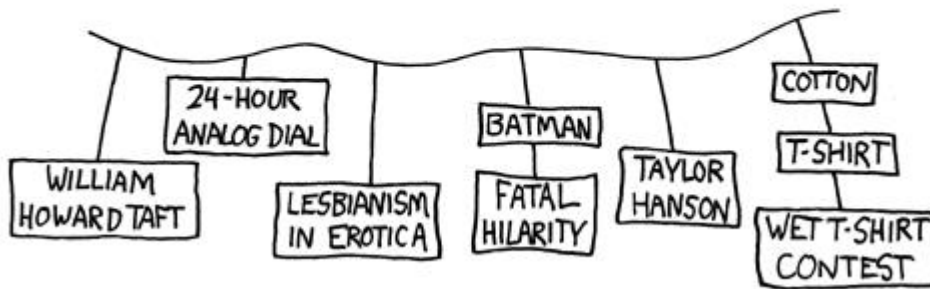


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THE PROBLEM WITH WIKIPEDIA:



[THREE HOURS OF
FASCINATED CLICKING]



WIKIFRIENDS:

I REALLY LIKED
THAT MOVIE.

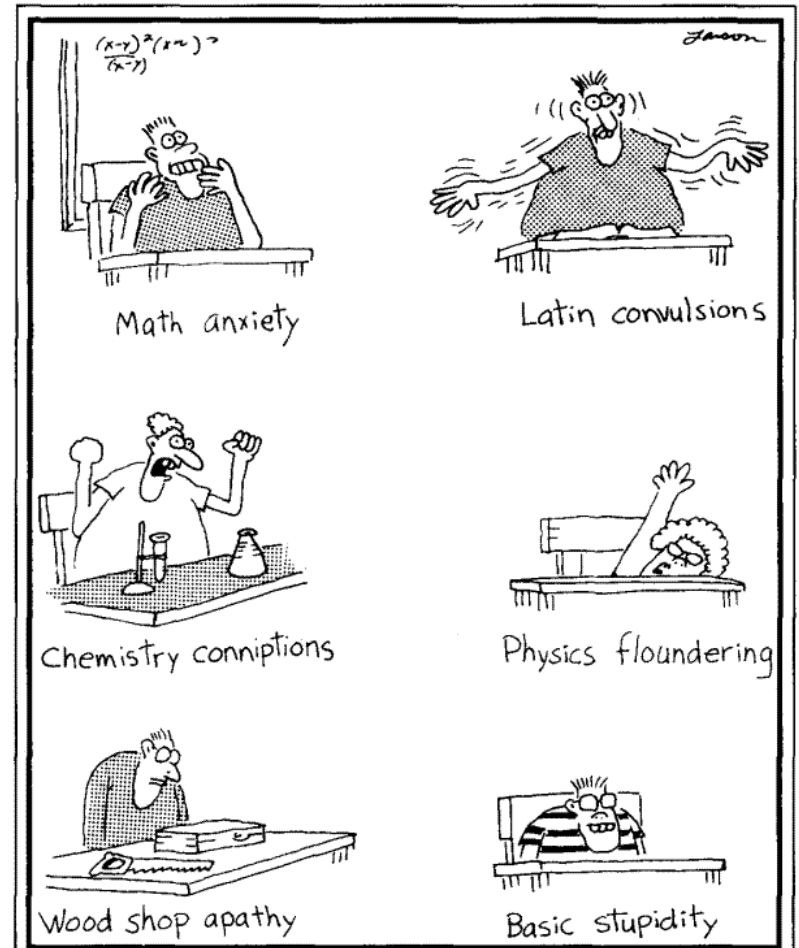
I HATED
THAT MOVIE.

ME TOO.



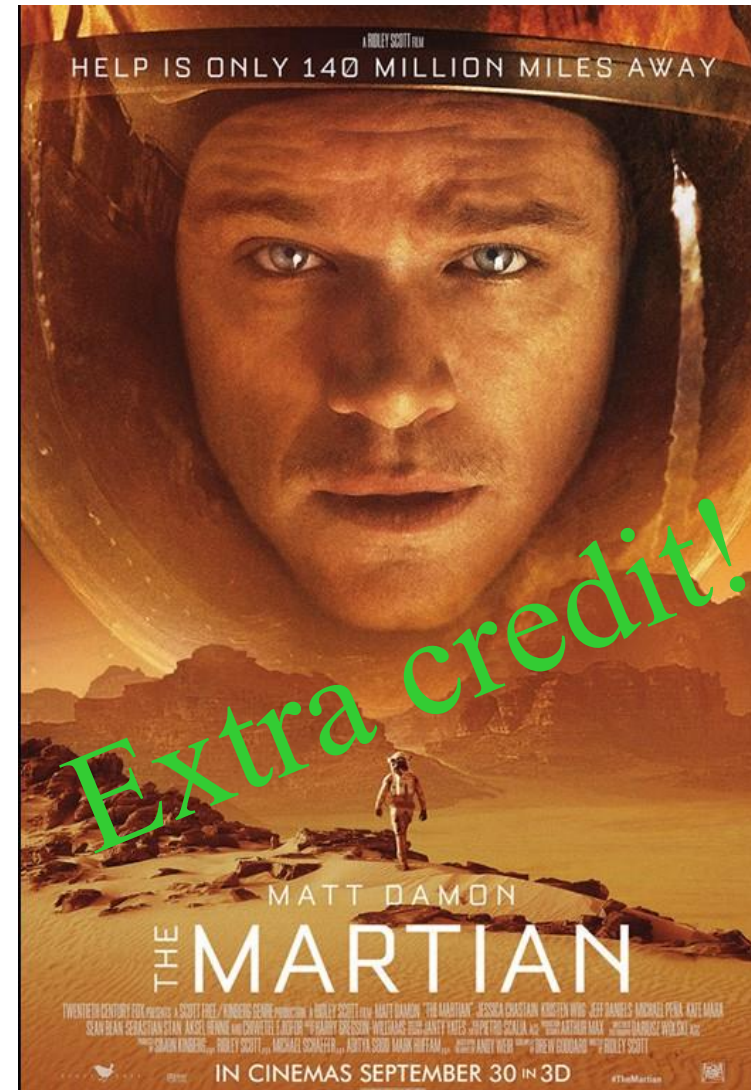
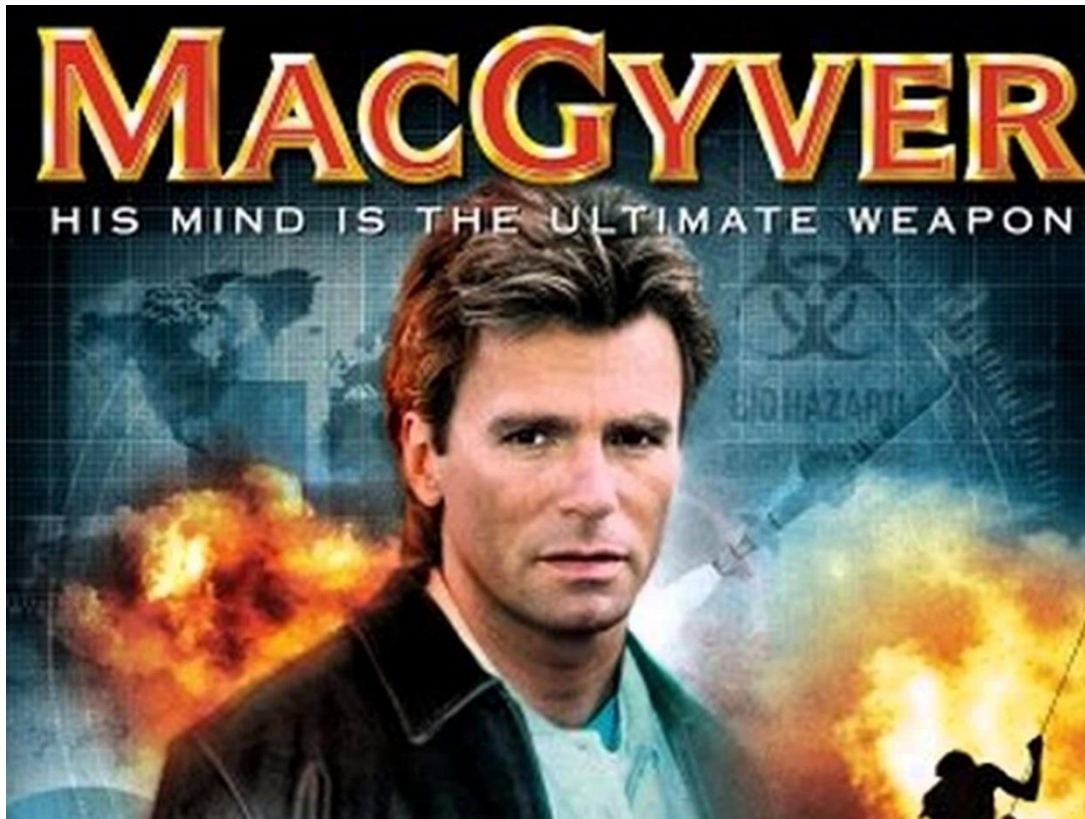
Good Advice

- Ask questions ASAP
- Solve problems ASAP
- **Work in study groups**
- Do not fall behind
- “Cramming” won’t work
- Do lots of extra credit
- Attend every lecture
- Visit class Website often
- **Solve lots of problems**



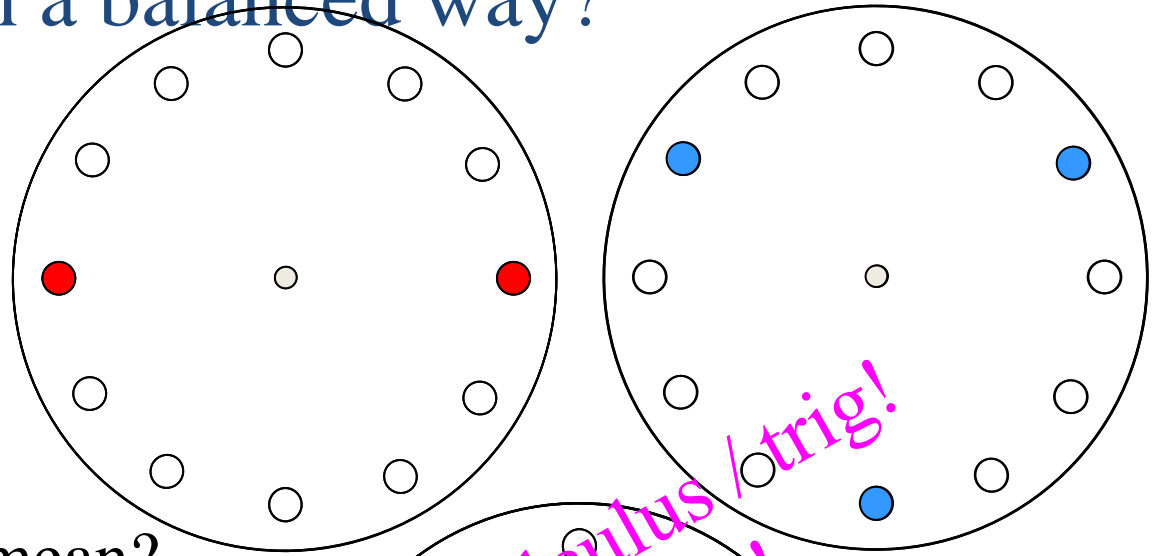
Classroom afflictions

Goal: Become a more effective problem solver!



Email all submissions to: homework.cs3102@gmail.com

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?



- What does “balanced” mean?
- Why are 3 test tubes balanced?
- **Symmetry!**
- Can you merge solutions?
- **Superposition!**
- **Linearity!** $f(x + y) = f(x) + f(y)$
- Can you spin 7 test tubes?
- **Complementarity!**
- Empirical testing...

No vector calculus / trig!
No equations!
Truth is guaranteed!
Fundamental principles exposed!
Easy to generalize!
High elegance / beauty!

Problem: $1 + 2 + 3 + 4 + \dots + 100 = ?$

Proof: Induction...



$$= (100 * 101) / 2$$

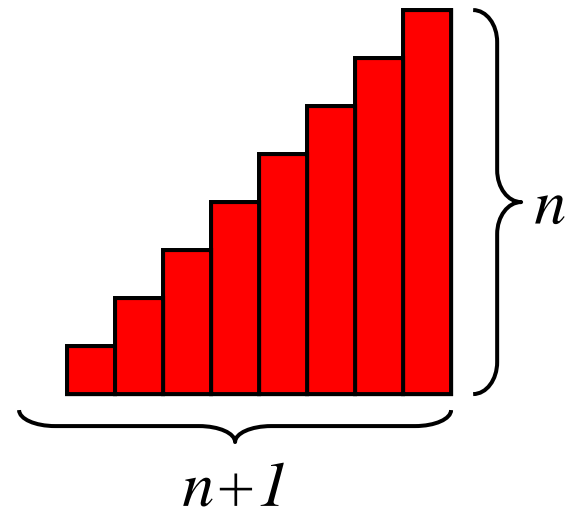
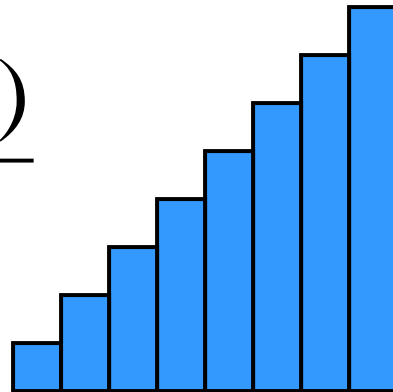
$$= 5050$$

$$1 + 2 + 3 + \dots + 99 + 100$$

$$100 + 99 + 98 + \dots + 2 + 1$$

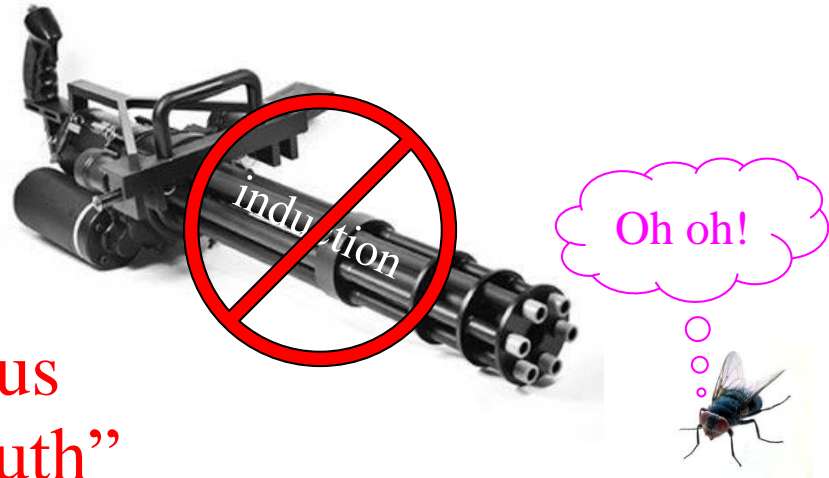
$$101 + 101 + 101 + \dots + 101 + 101 = 100 * 101$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$



Drawbacks of Induction

- You must **a priori** know the formula / result
- Easy to make **mistakes** in inductive proof
- Mostly “mechanical” – **ignores intuitions**
- **Tedious** to construct
- **Difficult** to check
- **Hard** to understand
- **Not very convincing**
- Generalizations **not obvious**
- Does not “**shed light on truth**”
- **Obfuscates** connections



Conclusion: only use induction as a **last resort!** (i.e., **rarely**)

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \dots = ?$

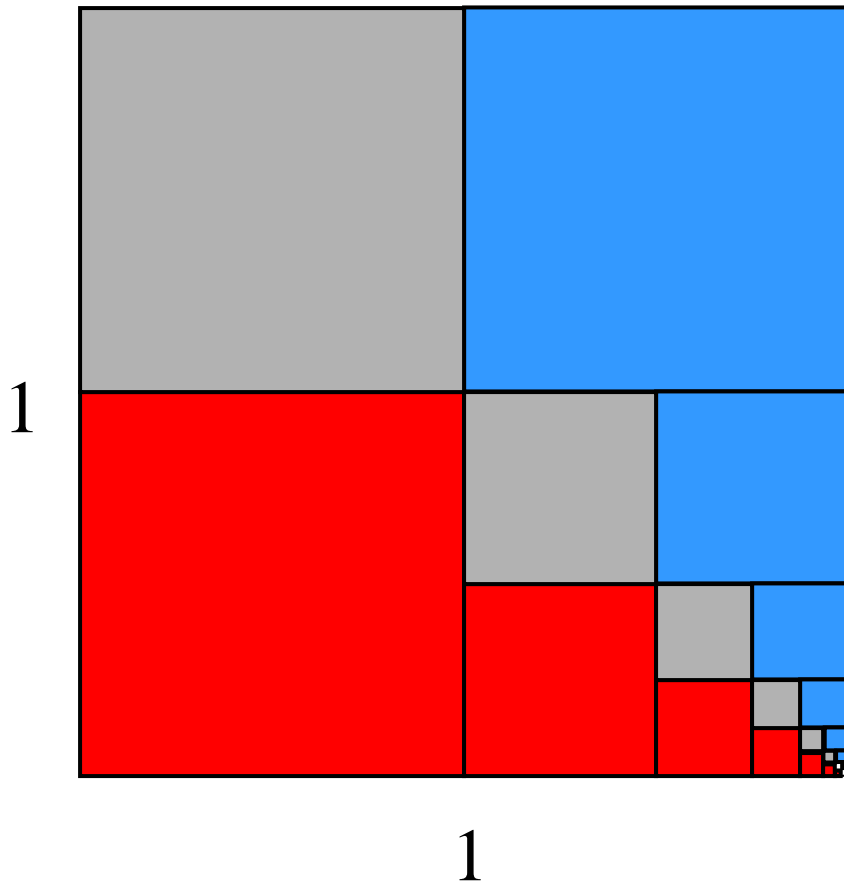
$$\sum_{i=1}^{\infty} \frac{1}{4^i} = ?$$

Extra Credit:

Find a short, **geometric**, induction-free proof.

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \dots = ?$

Find a short, **geometric**, induction-free proof.



$$\sum_{i=1}^{\infty} \frac{1}{4^i} = \frac{1}{3}$$

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \dots = ?$

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = ?$$

Extra Credit:

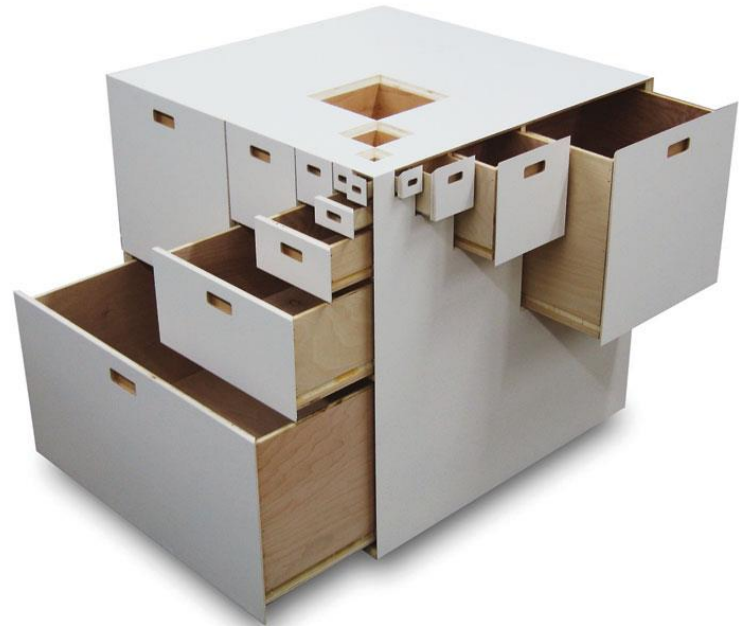
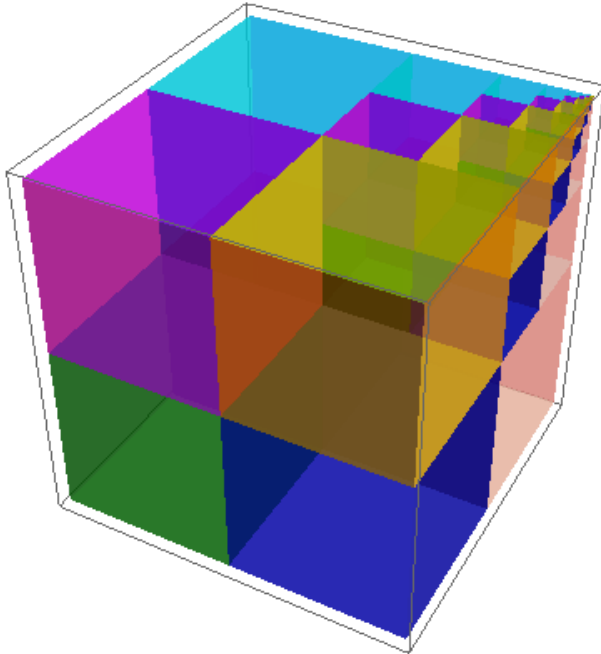
Find a short, **geometric**, induction-free proof.

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \dots = ?$

Find a short, **geometric**, induction-free proof.



$$\sum_{i=1}^{\infty} \frac{1}{8^i} = \frac{1}{7}$$

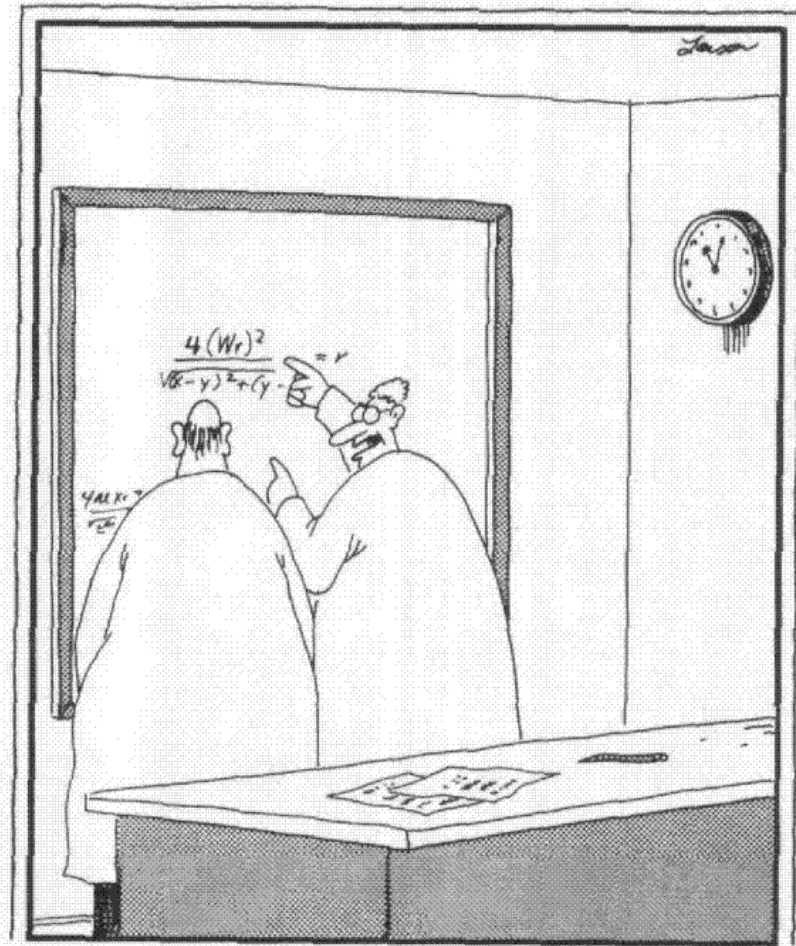


Problem: $1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = ?$

$$\sum_{i=1}^n i^3 = ?$$

Extra Credit:

find a short, **geometric**,
induction-free proof.

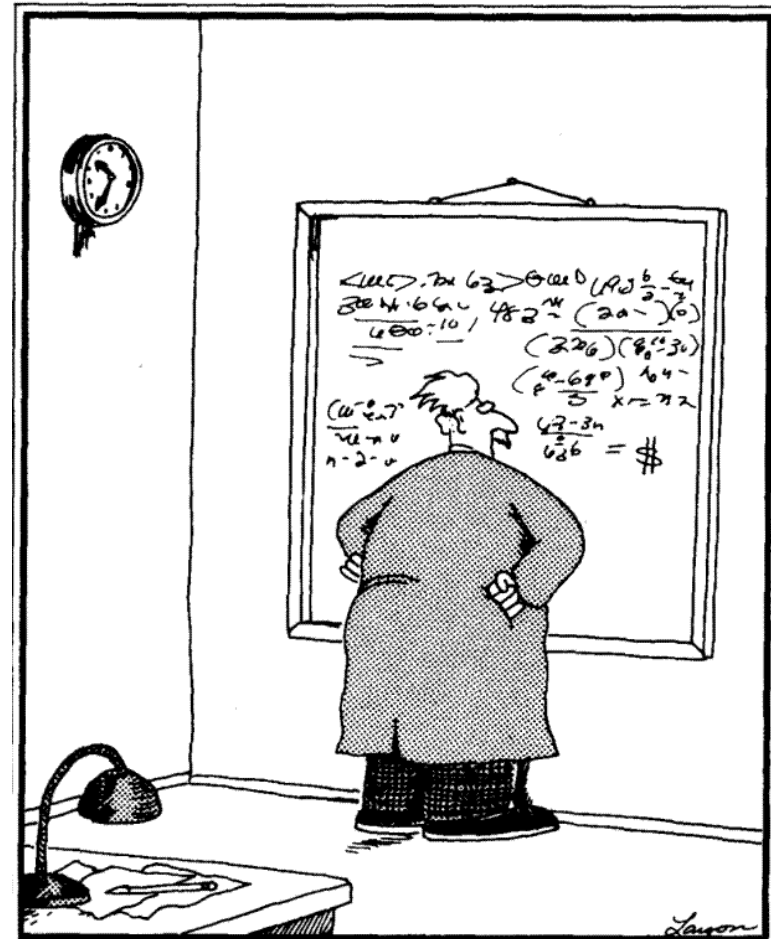


"Yes, yes, I know that, Sidney ... everybody knows that! ... But look: Four wrongs squared, minus two wrongs to the fourth power, divided by this formula, do make a right."

Problem: Prove that $\sqrt{2}$ is irrational.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Einstein discovers that time is actually money.

Problem: Prove that there are an infinity of primes.

Extra Credit: Find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

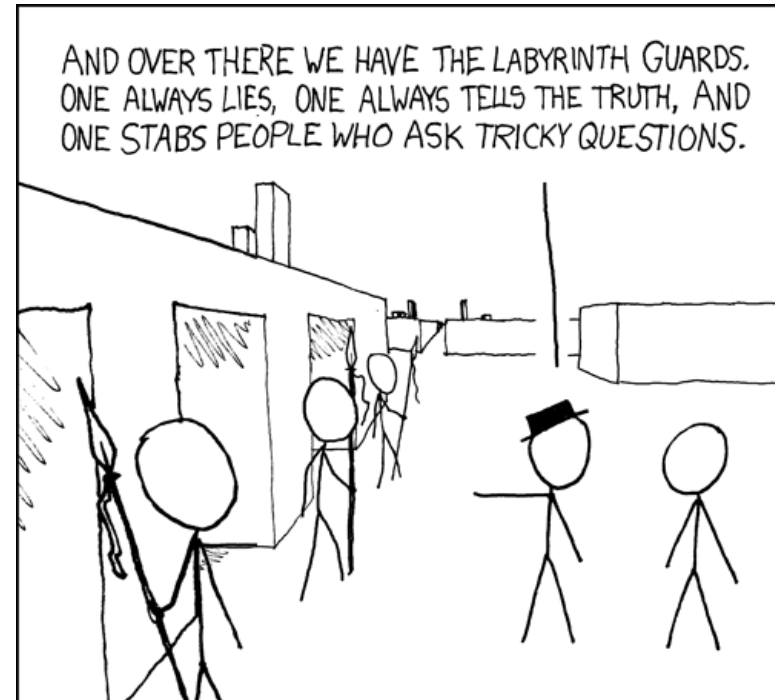


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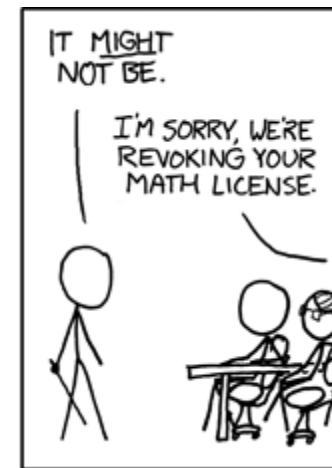
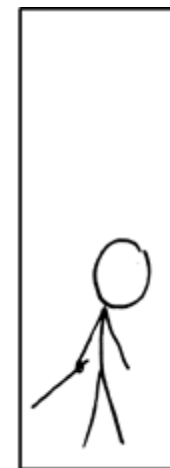
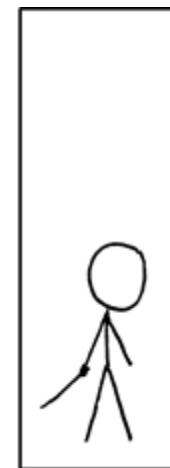
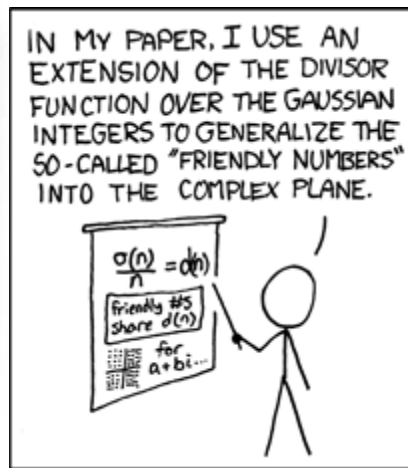
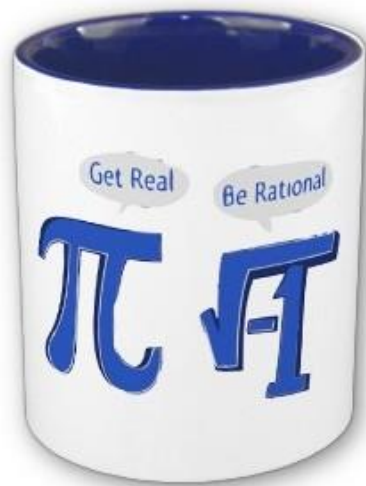
Problem: True or false: there arbitrary long blocks of consecutive composite integers.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i ?



Problem: Does exponentiation preserve irrationality?
i.e., are there two irrational numbers x and y such
that x^y is rational?

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Problem: Solve the following equation for X:

$$X^{X^{X^{X^{\dots}}}} = 2$$

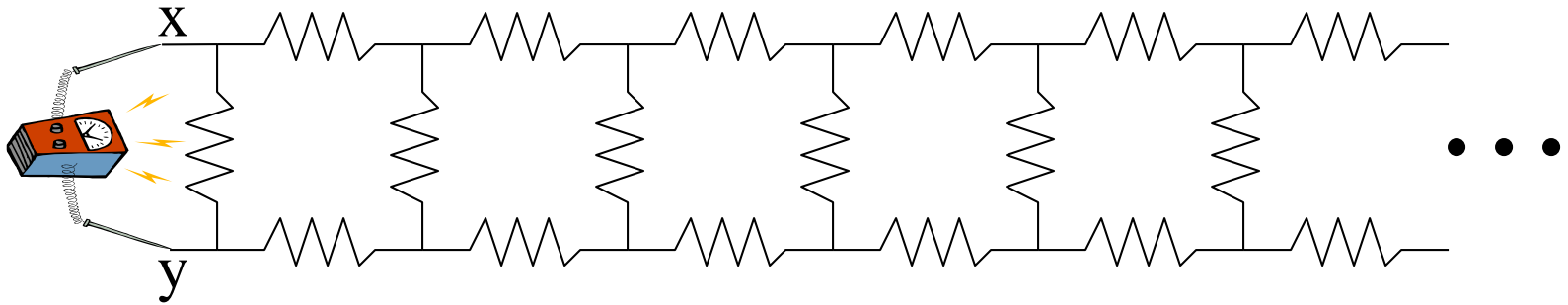
where the stack of exponentiated x's extends forever.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

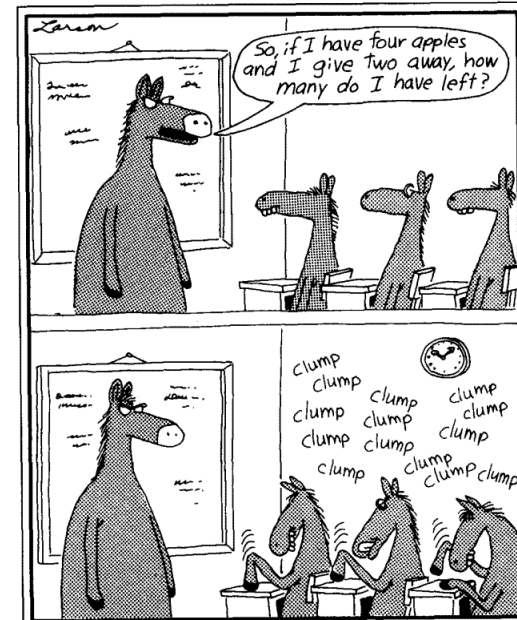


"Mr. Osborne, may I be excused? My brain is full."

Problem: For the given infinite ladder of resistors of resistance R each, what is the resistance measured between points x and y ?

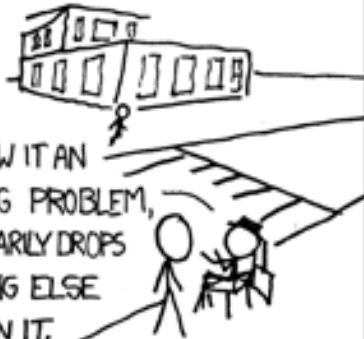


- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



THERE'S A CERTAIN TYPE OF BRAIN THAT'S EASILY DISABLED.

IF YOU SHOW IT AN INTERESTING PROBLEM, IT INVOLUNTARILY DROPS EVERYTHING ELSE TO WORK ON IT.



THIS HAS LED ME TO INVENT A NEW SPORT: NERD SNIPING.

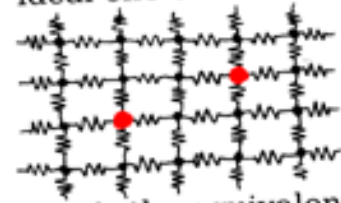
SEE THAT PHYSICIST CROSSING THE ROAD?



HEY!



On this infinite grid of ideal one-ohm resistors,



what's the equivalent resistance between the two marked nodes?

IT'S... HMM. INTERESTING. MAYBE IF YOU START WITH ... NO, WAIT. HMM... YOU COULD—



I WILL HAVE NO PART IN THIS.

C'MON, MAKE A SIGN. IT'S FUN! PHYSICISTS ARE TWO POINTS, MATHEMATICIANS THREE.



Historical Perspectives



Historical Perspectives

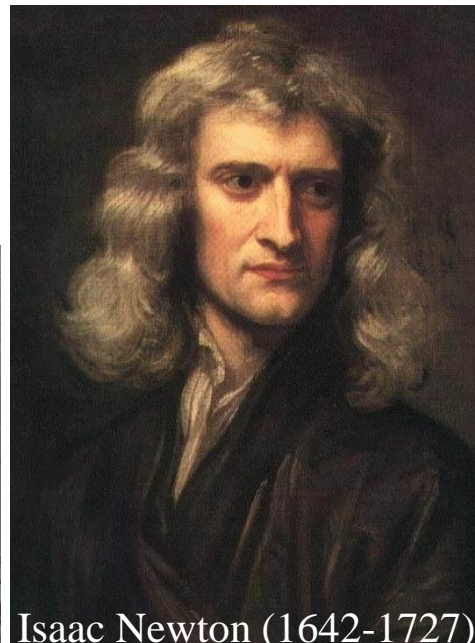
- Knowing the “big picture” is empowering
- Science and mathematics builds heavily on past
- Often the simplest ideas are the most subtle
- Most fundamental progress was done by a few
- We learn much by observing the best minds
- Research benefits from seeing connections
- The field of computer science has many “parents”
- We get inspired and motivated by excellence
- The giants can show us what is possible to achieve
- It is fun to know these things!

“Standing on the Shoulders of Giants”

- Aristotle, **Euclid**, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, **Euler**, Gauss, Hamilton
- **Boole**, **De Morgan**
- **Babbage**, **Ada Lovelace**
- Venn, Carroll



Ada Lovelace
(1815-1852)



Isaac Newton (1642-1727)

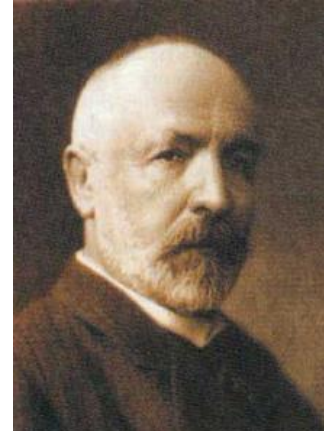


Euclid (300 BC)

“Standing on the Shoulders of Giants”

- **Cantor**, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- **Gödel**, Church, **Turing**
- **von Neumann**, **Shannon**
- Kleene, **Chomsky**
- Hoare, McCarthy, Erdos
- Knuth, Backus, Dijkstra

Many others...



Georg Cantor (1845-1918)



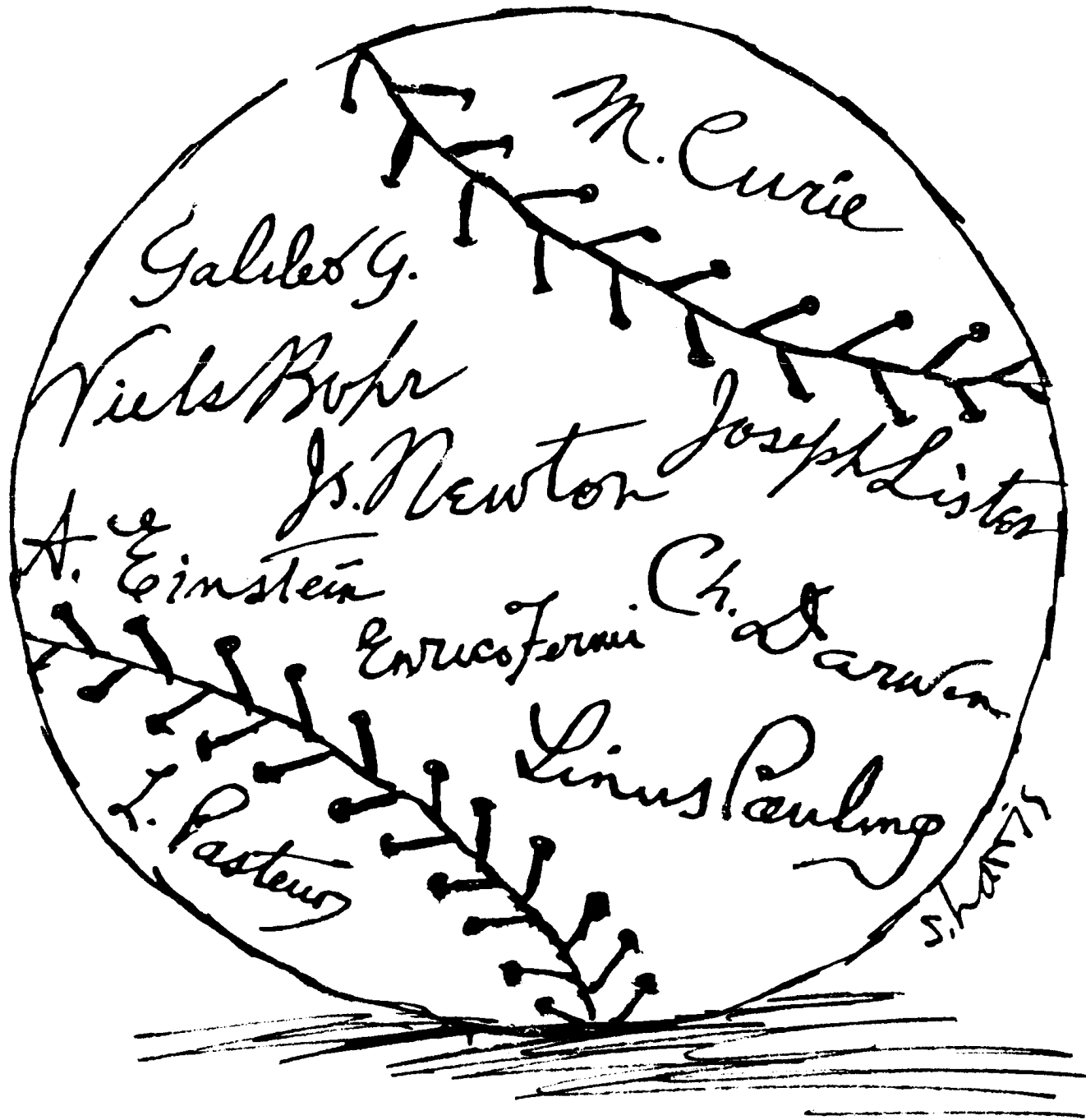
Bertrand Russell (1872-1970)



David Hilbert (1862-1943)



Kurt Gödel (1906-1978)



M. Curie

Galileo G.

Niels Bohr

J. Newton Joseph Lister

A. Einstein

Enrico Fermi Ch. Darwin

I. Pasteur

Linus Pauling

S. HART

Gauss
 Newton
 Archimedes
 Euler
 Cauchy
 Poincare
 Riemann
 Cantor
 Cayley
 Hamilton
 Eisenstein
 Pascal
 Abel
 Hilbert
 Klein
 Leibniz
 Descartes
 Galois
 Mobius
 Jacob
 Johann Bernoulli
 Daniel Bernoulli
 Dirichlet
 Fermat
 Pythagoras
 Laplace
 Lagrange
 Kronecker
 Jacobi
 Bolyai
 Lobatchewsky
 Noether
 Germain
 Euclid
 Legendre

$$(p/q)(q/p) = -1^{(p-1)(q-1)/4}$$

$$\text{num} = \Delta + \Delta + \Delta$$

$$\pi(n) = \frac{n}{\ln n}$$

$$(a/p) = -1^{\eta(p,a)}$$



$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \dots$$

$$\int_b^a f(x)dx = F(b) - F(a); \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\frac{dF(x)}{dx} = f(x)$$

$$F(s) = s^{-2}$$

$$(abcdef) = (ab)(ac)(ad)(ae)(af)$$

$$\int_{\gamma} f(z) dz = 0$$

$$|a \cdot b| \leq |a||b|$$

$$\text{Gal}(E/F);$$

$$E_H = \{x \in E \mid \phi(x) = x \forall \phi \in H\}$$

$$f'(c)(b-a) = f(b) - f(a)$$

$$u_{tt} = c^2 u_{xx}; \quad 0 < x < 1$$

$$u(0,t) = 0 = u(1,t)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

$$D = R[x]$$



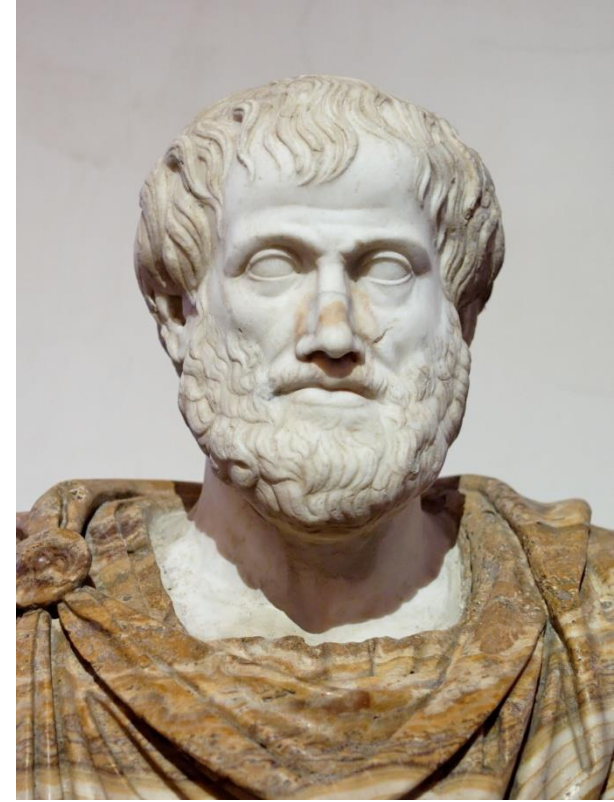
MAKING PHILOSOPHY ACCESSIBLE: POP-UP PLATO

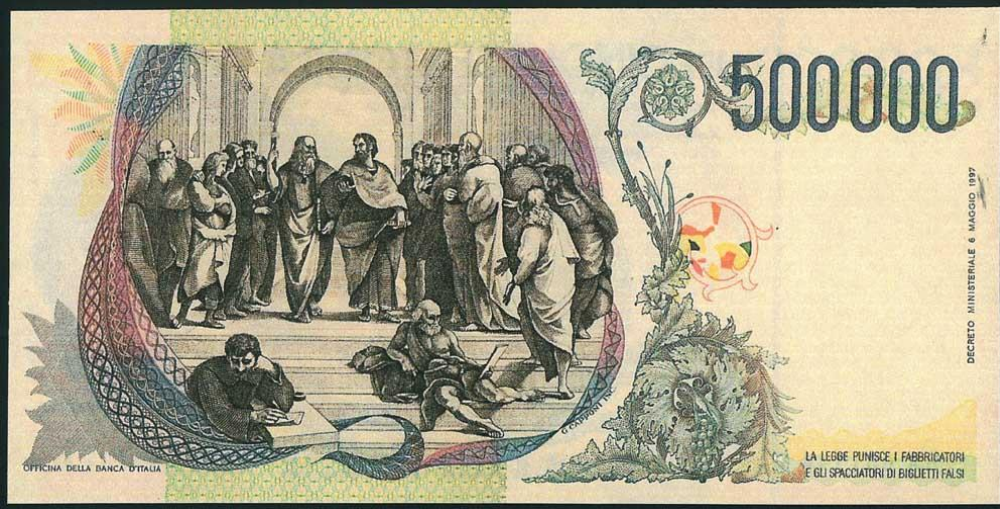


Historical Perspectives

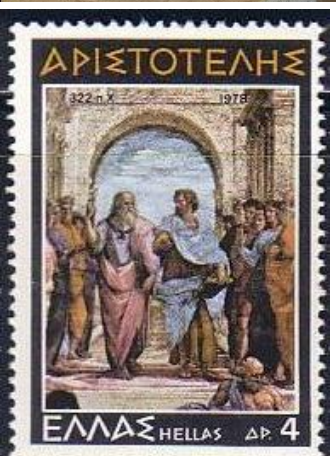
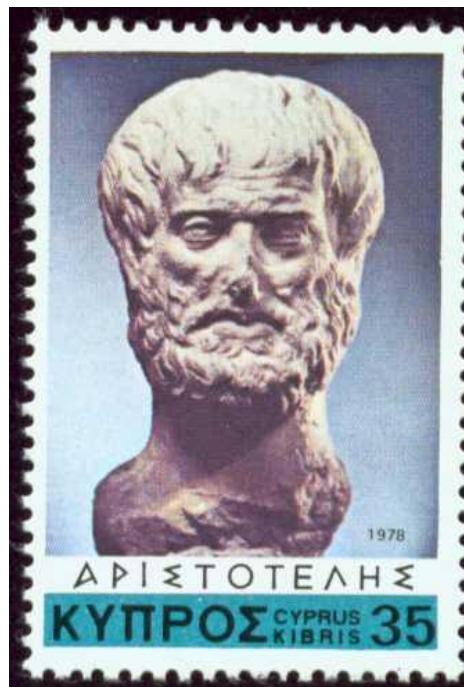
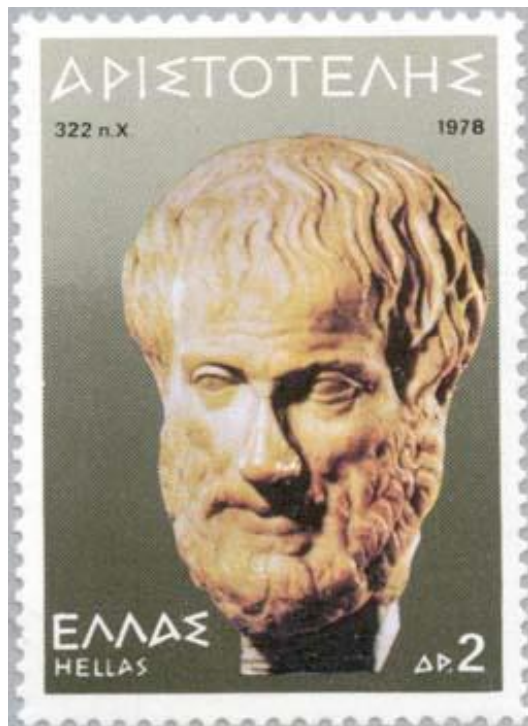
Aristotle (384BC-322BC)

- Founded Western philosophy
 - Student of Plato
 - Taught Alexander the Great
 - “Aristotelianism”
 - Developed the “scientific method”
 - One of the most influential people ever
 - Wrote on physics, theatre, poetry, music, logic, rhetoric, politics, government, ethics, biology, zoology, morality, optics, science, aesthetics, psychology, metaphysics, ...
 - Last person to know everything known in his own time!
- “Almost every serious intellectual advance has had to begin with an attack on some Aristotelian doctrine.” – Bertrand Russell





“Wit is educated insolence.”
- Aristotle (384-322 B.C.)





“The School of Athens” (by Raphael, 1483-1520)



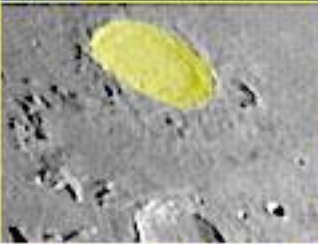
ARISTOTELES

87 km

97 / 10 / 09 D=254mm FD=10



"LIGIA"
Moon



© Antonio J. Cidadão

8

B/W QuickCam

a.cidadao@mail.telepac.pt



Birds fly because they're
lighter than air.
Some trees have different
fruits each year.
At night, clouds rest on
the ground.

Are you sure
he's Aristotle?



J. Harris



“What I especially like about being a philosopher-scientist is that I don’t have to get my hands dirty.”

PEDIMENT

CORNICE

FRIEZE

TRIGLYPH

METOPE

ARCHITRAVE

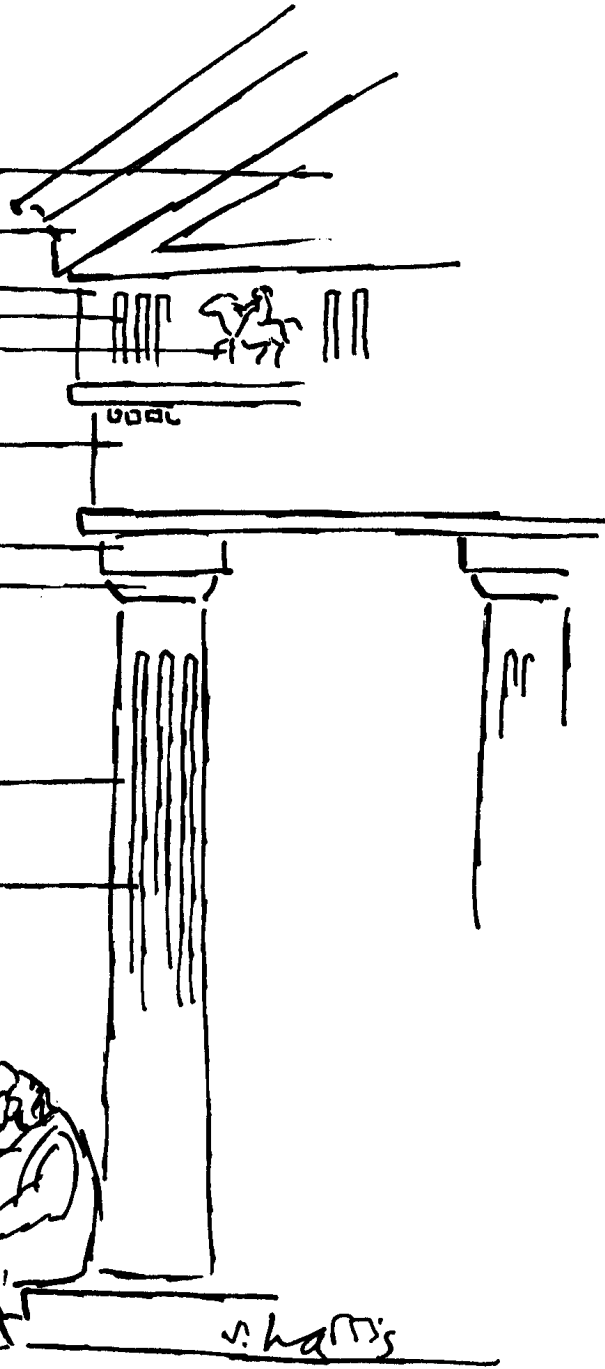
ABACUS

ECHINUS

SHAFT

FLUTE

SOCRATES

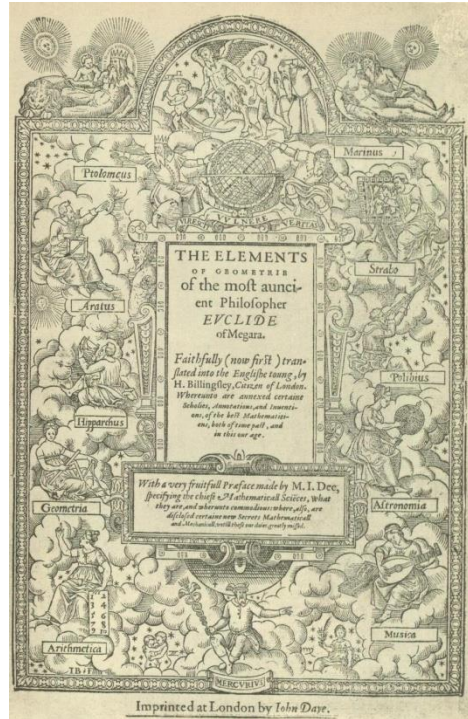


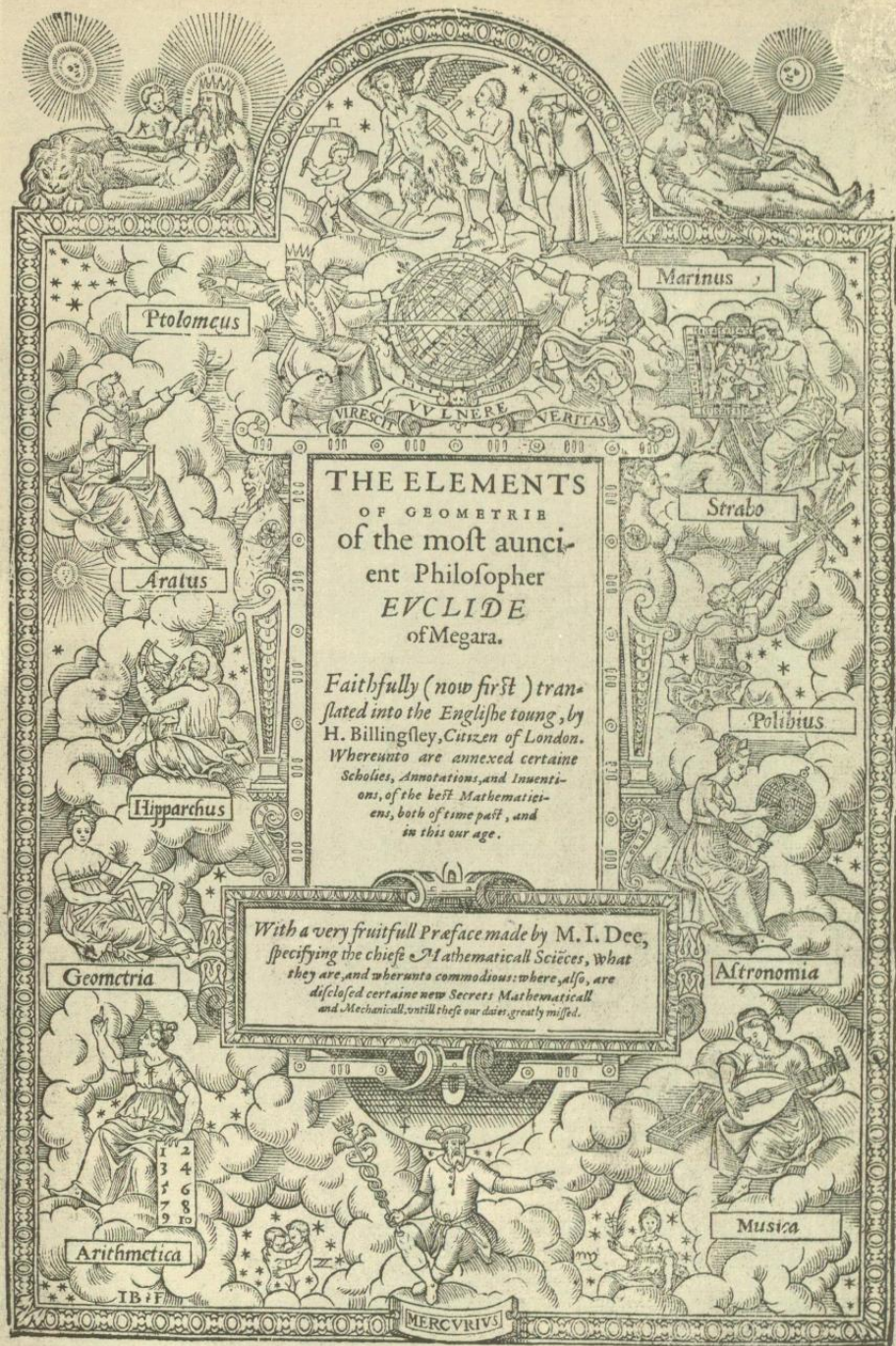
s. hartis

Historical Perspectives

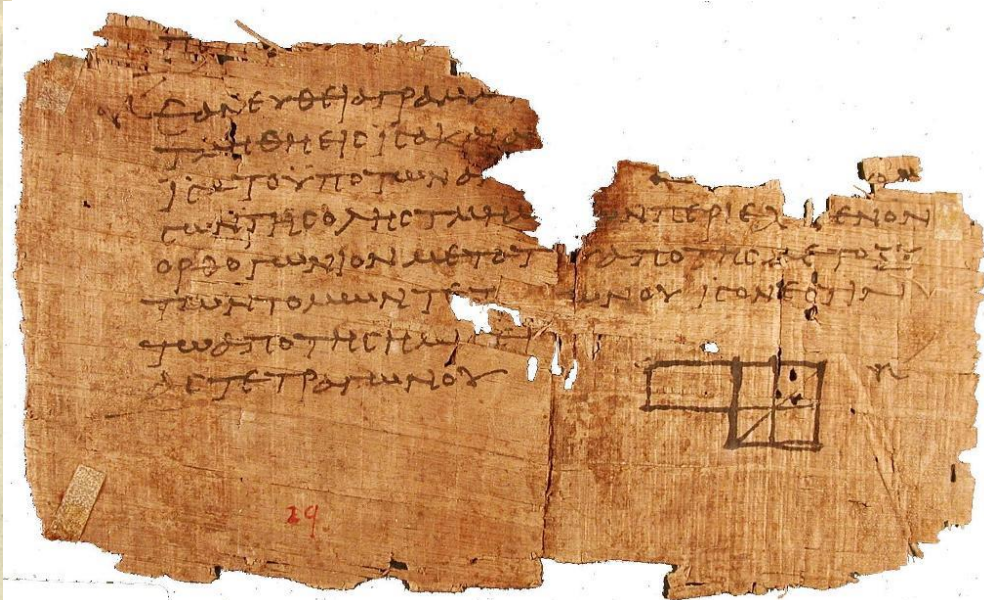
Euclid (325BC-265BC)

- Founder of geometry & the **axiomatic method**
- “**Elements**” – oldest and most impactful textbook
- Unified logic & math
- Introduced rigor and “**Euclidean**” geometry
- Influenced all other fields of science: Copernicus, Kepler, Galileo, Newton, Russell, Lincoln, Einstein & many others

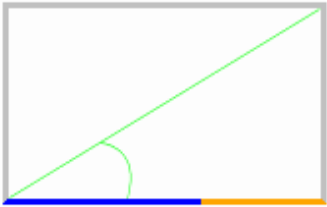
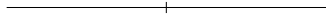




Imprinted at London by Iohn Daye.



Euclid's Straight-Edge and Compass Geometric Constructions



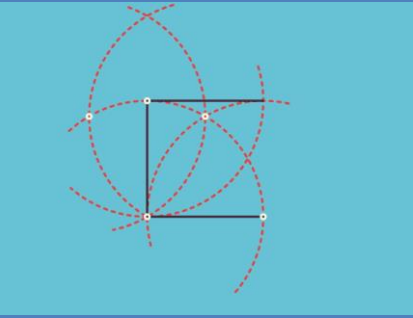
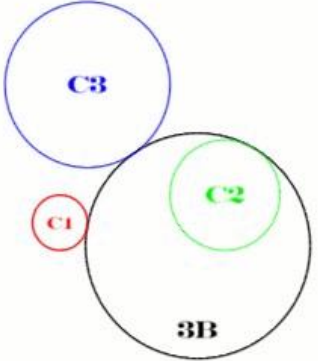
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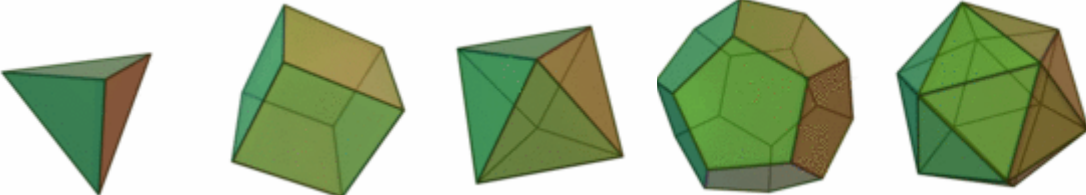
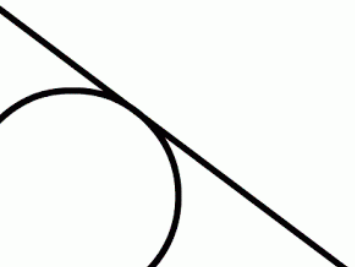
A •

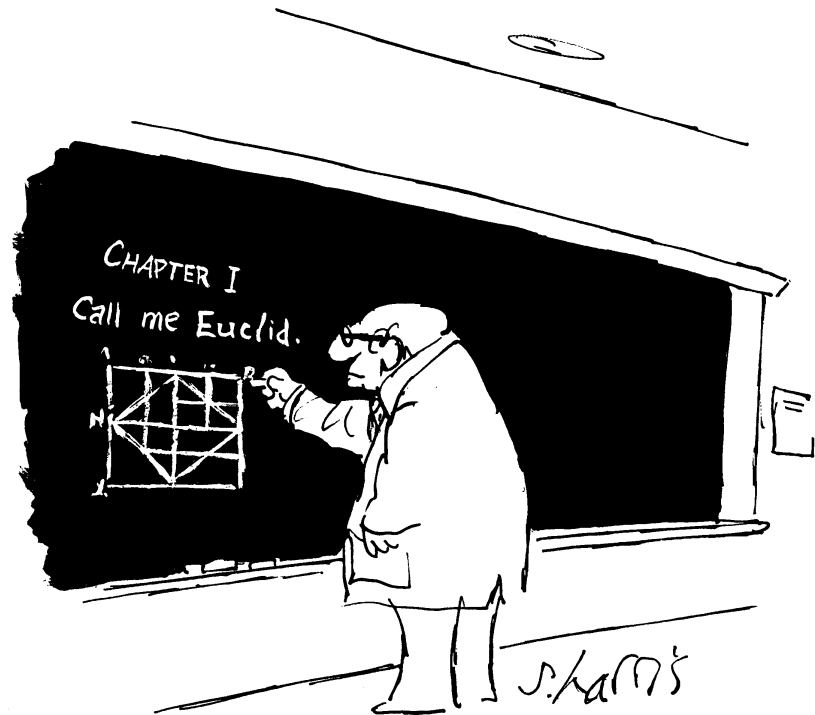
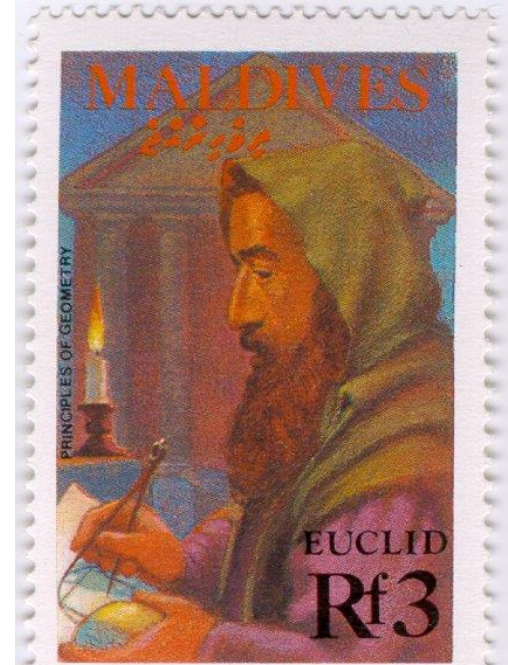
B •

x



Extra credit!





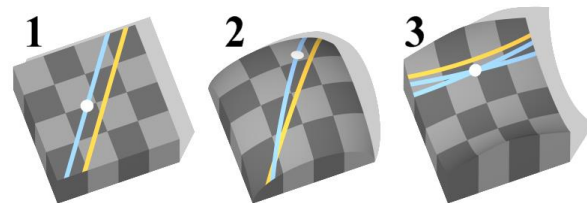
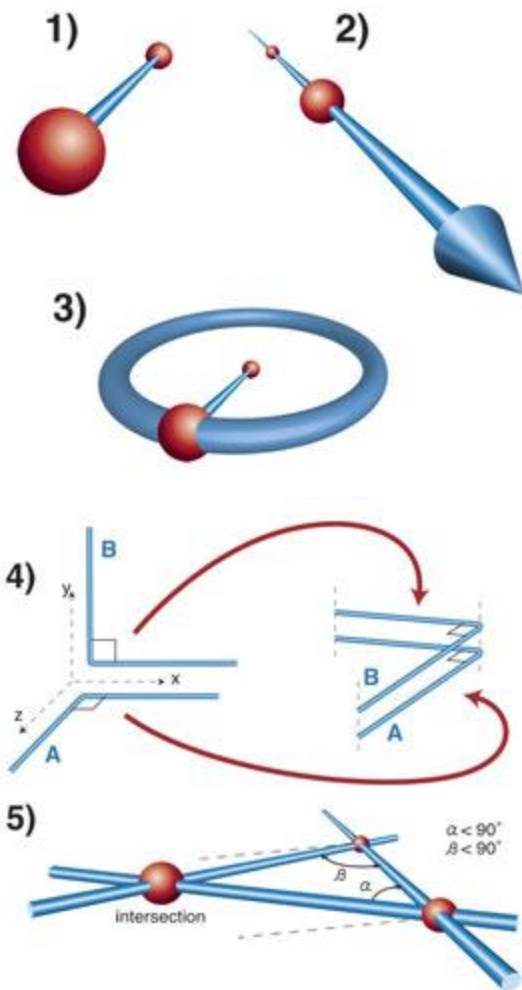
Euclid's Axioms

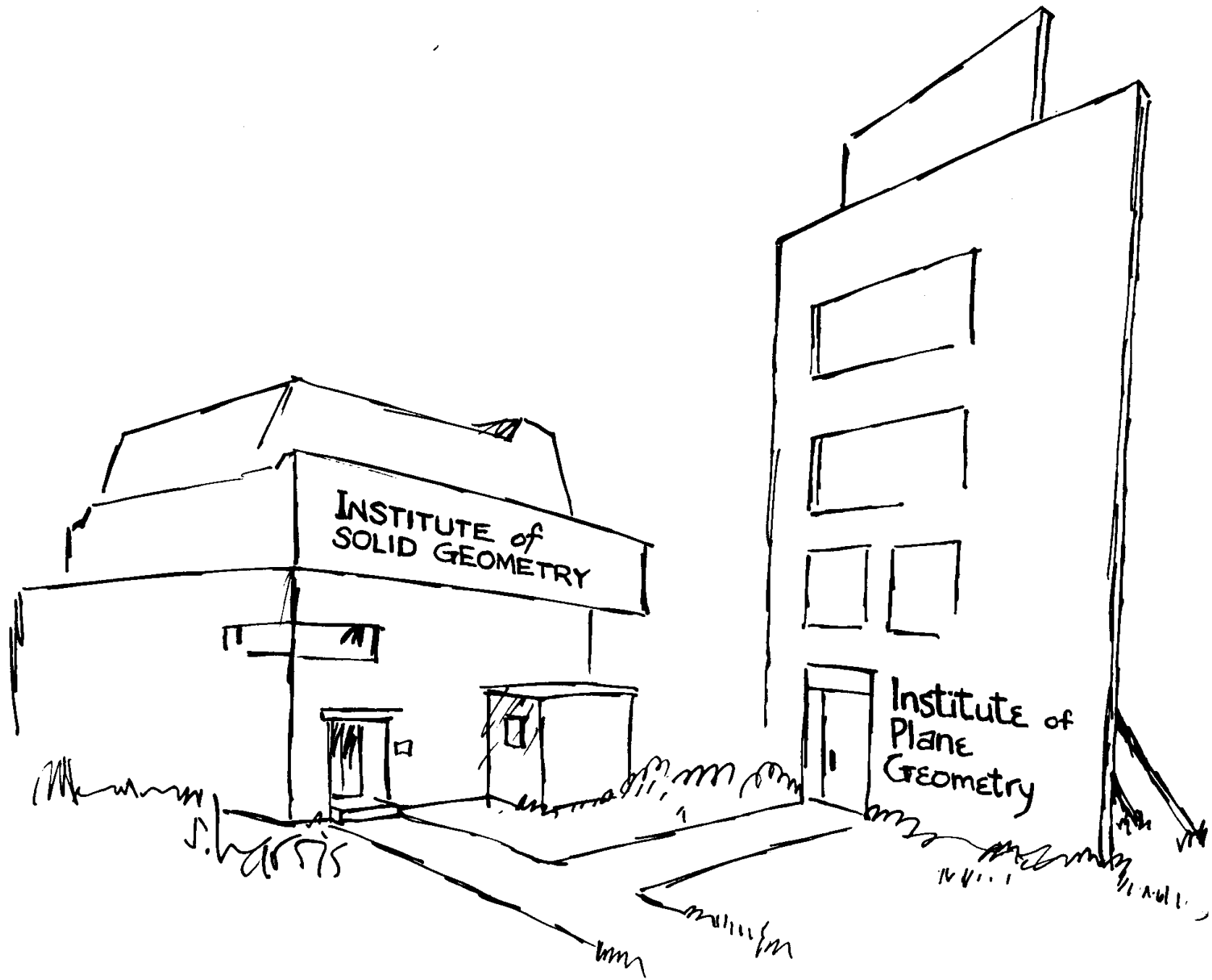
- 1: Any two points can be connected by exactly one straight line.
- 2: Any segment can be extended indefinitely into a straight line.
- 3: A circle exists for any given center and radius.
- 4: All right angles are equal to each other.
- 5: The **parallel postulate**: Given a line and a point off that line, there is exactly one line passing through the point, which does not intersect the first line.

The first 28 propositions of Euclid's Elements were proven without using the parallel postulate!

Theorem [Beltrami, 1868]: The parallel postulate is **independent** of the other axioms of Euclidean geometry.

The parallel postulate can be **modified** to yield **non-Euclidean geometries**!





INSTITUTE of
SOLID GEOMETRY

Institute of
Plane
Geometry

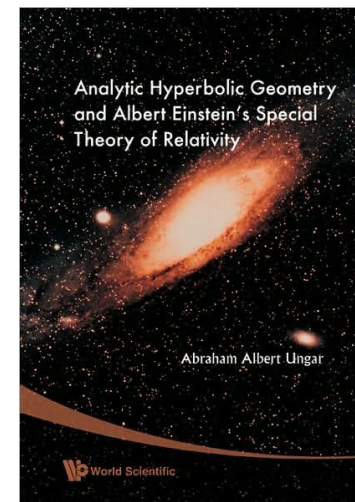
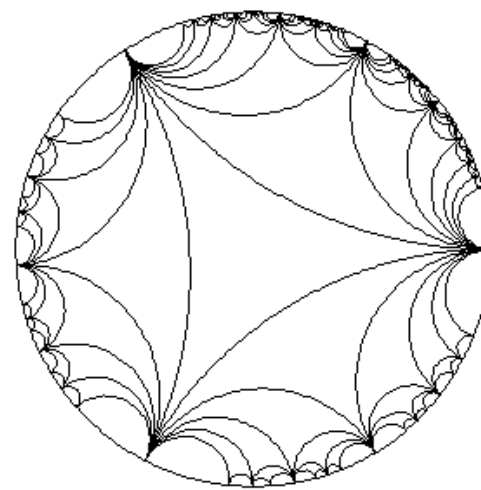
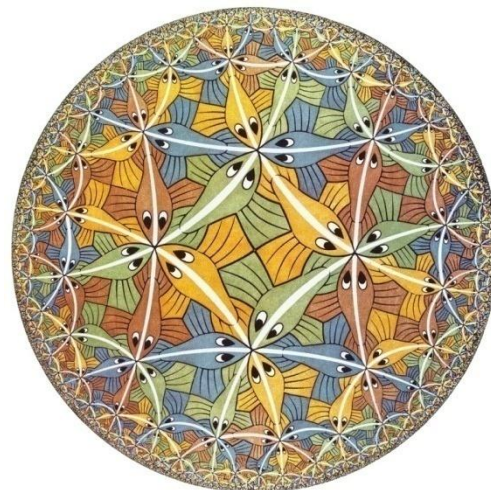
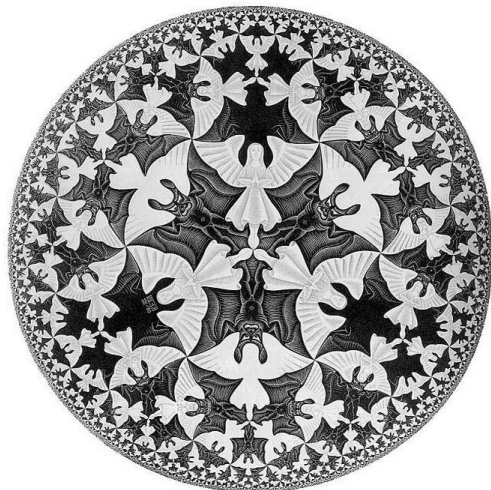
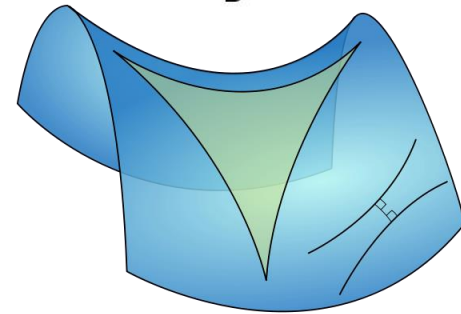
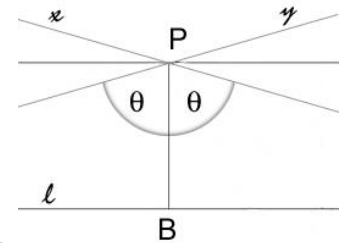
S. Shrivastava

14.11.11

Non-Euclidean Geometries

Hyperbolic geometry: Given a line and a point off that line, there are an **infinity of lines** passing through that point that do not intersect the first line.

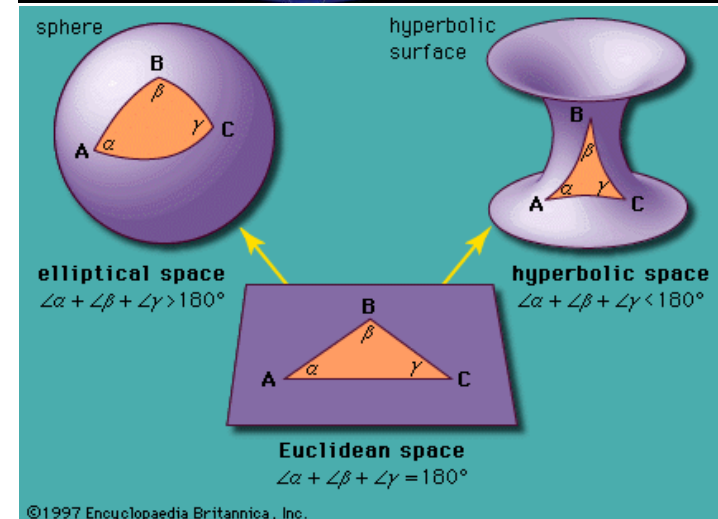
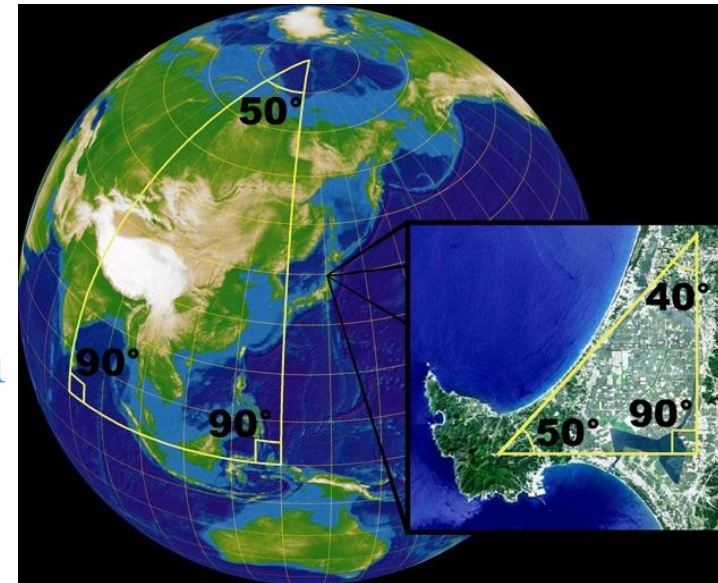
- Sum of triangle angles is **less than 180°**
- Different triangles have **different angle sum**
- Triangles with **same angles** have **same area**
- There are **no similar triangles**
- Used in **relativity theory**

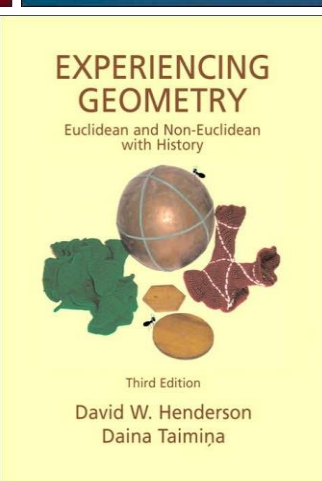
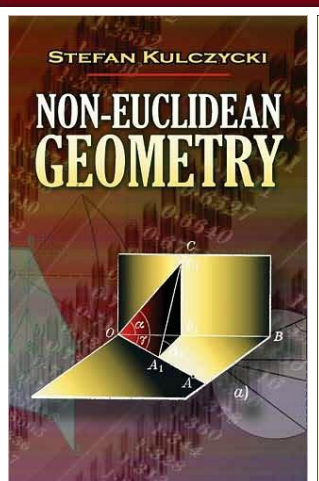
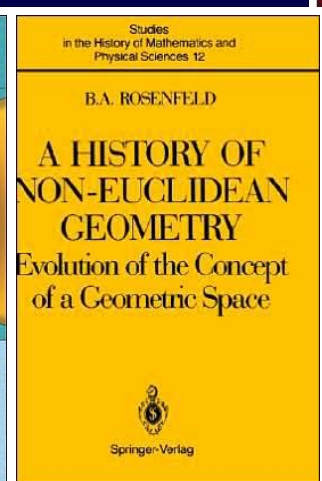
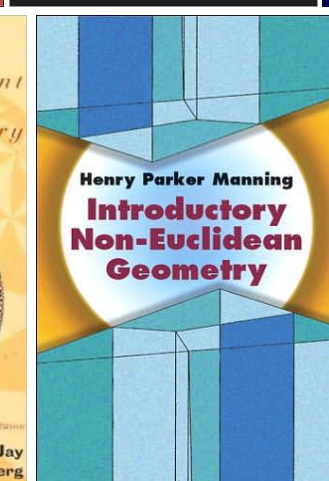
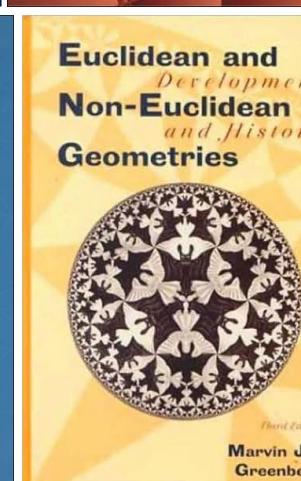
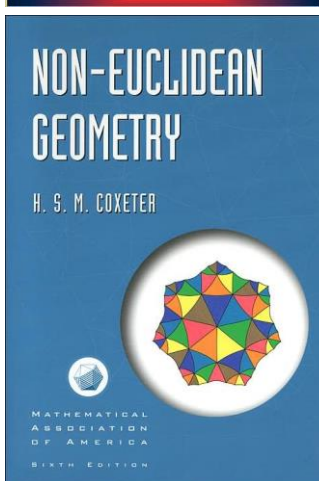
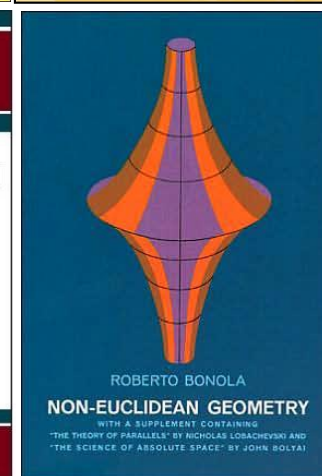
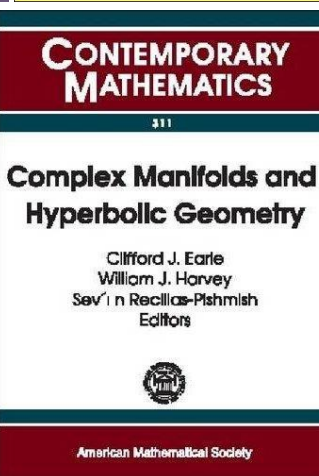
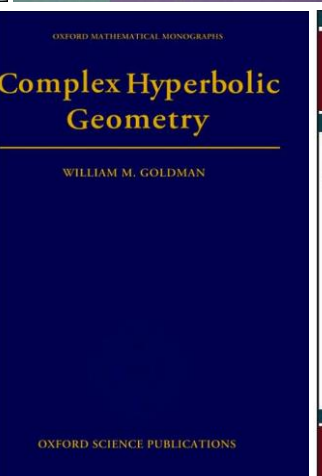
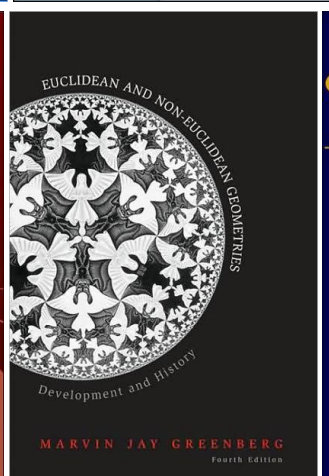
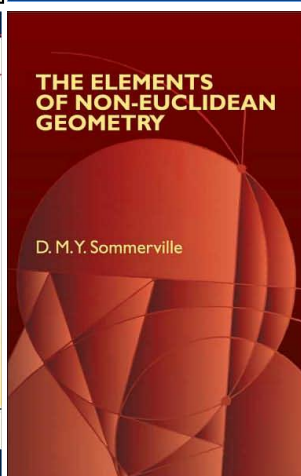
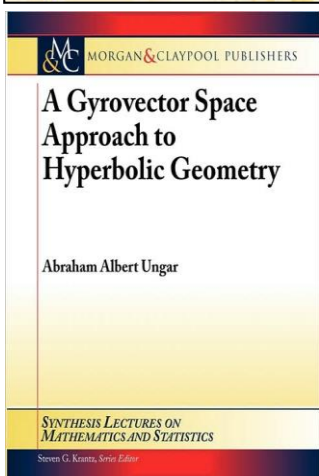
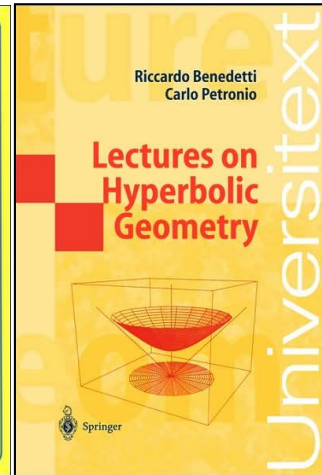
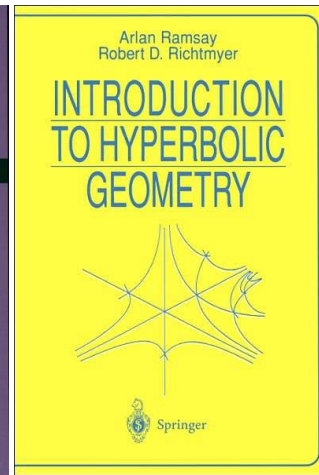
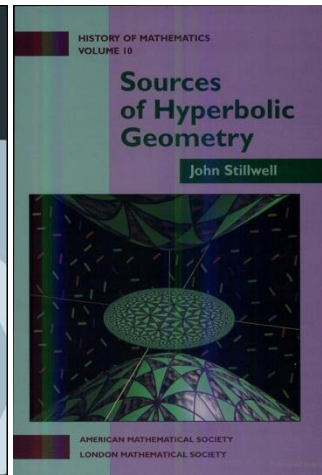
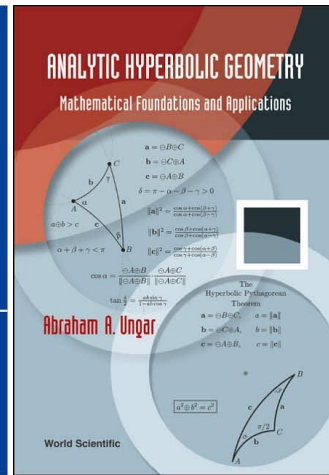
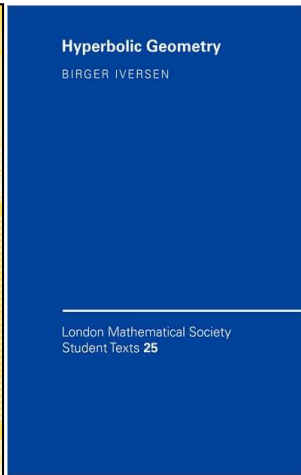
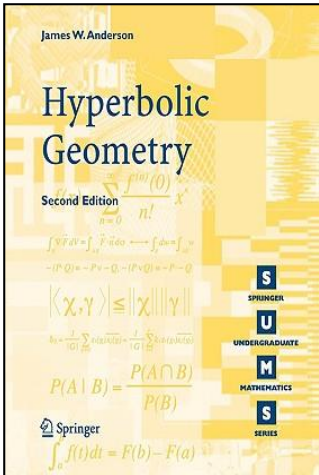


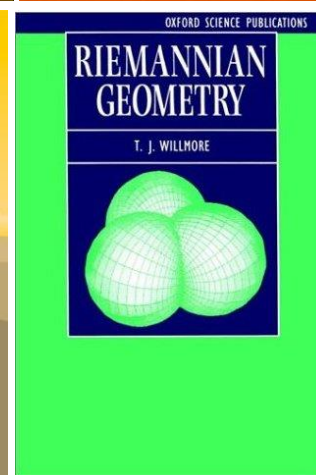
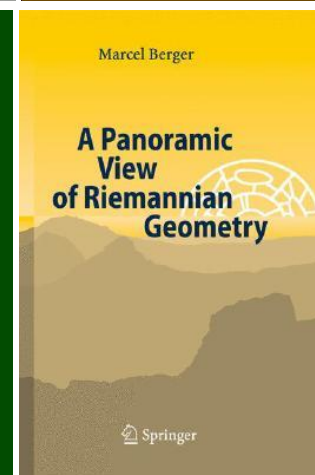
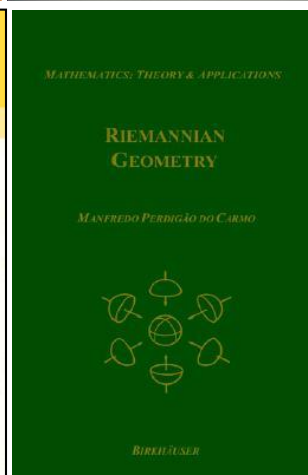
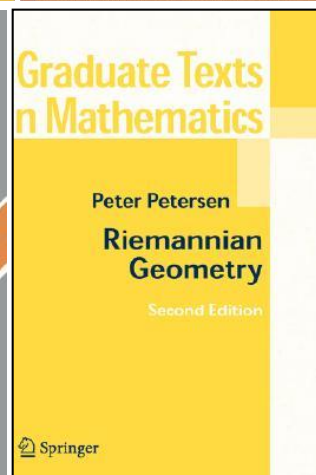
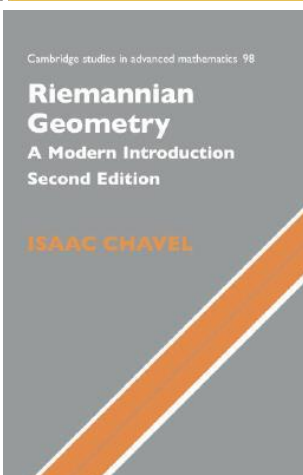
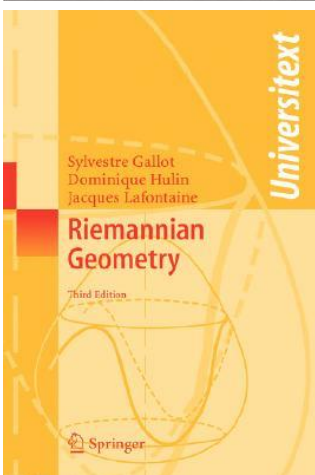
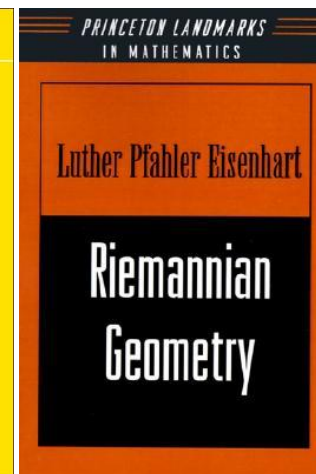
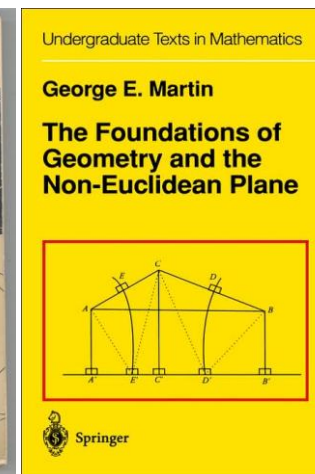
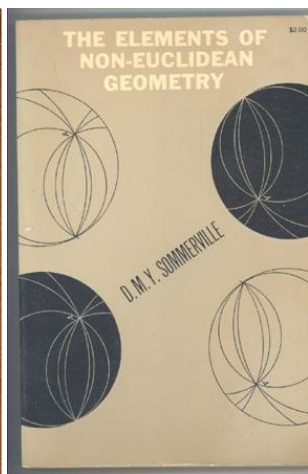
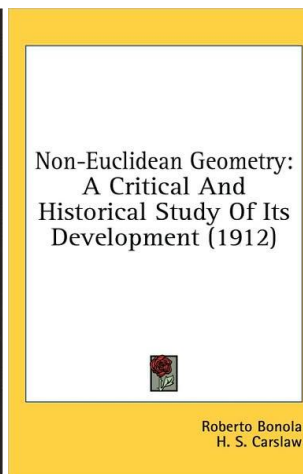
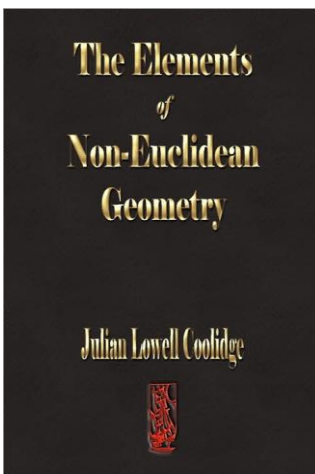
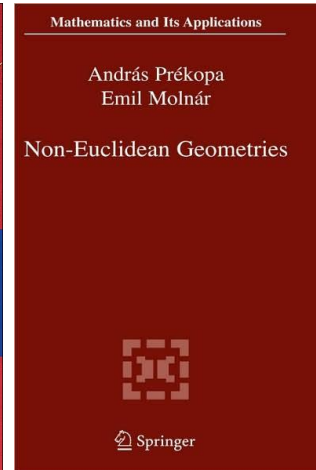
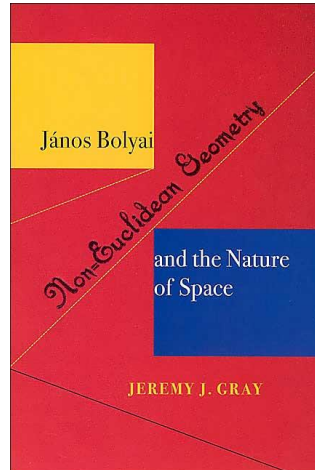
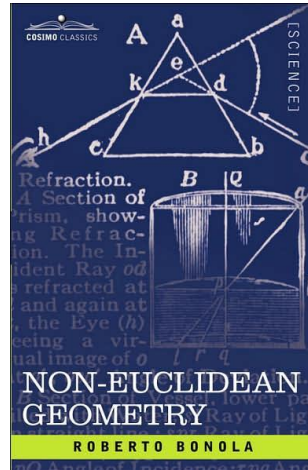
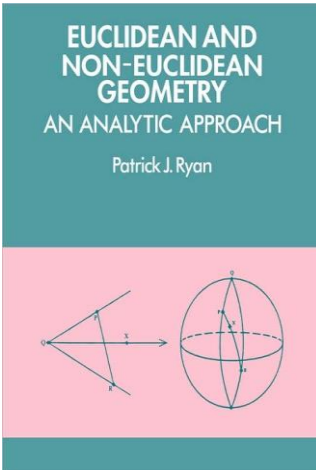
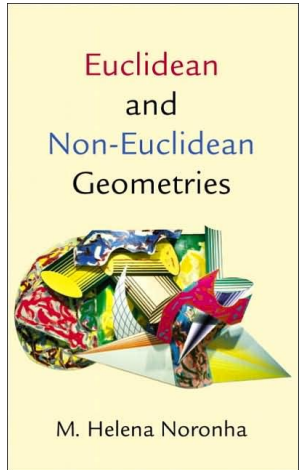
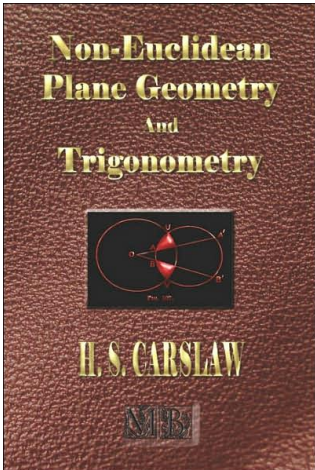
Non-Euclidean Geometries

Spherical / Elliptic geometry: Given a line and a point off that line, there are **no lines** passing through that point that do not intersect the first line.

- Lines are **geodesics** - “great circles”
- Sum of triangle angles is $> 180^\circ$
- Not all triangles have same **angle sum**
- Figures can not scale up indefinitely
- **Area** does not scale as the **square**
- **Volume** does not scale as the **cube**
- The **Pythagorean theorem** fails
- **Self-consistent**, and **complete**

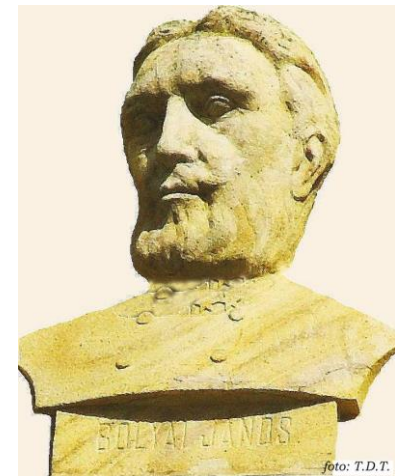
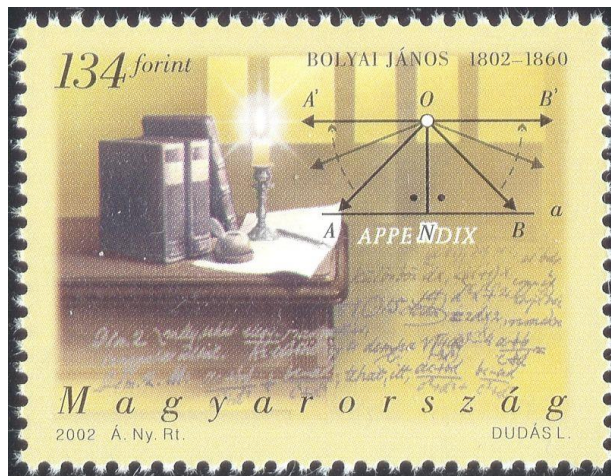






Founders of Non-Euclidean Geometry

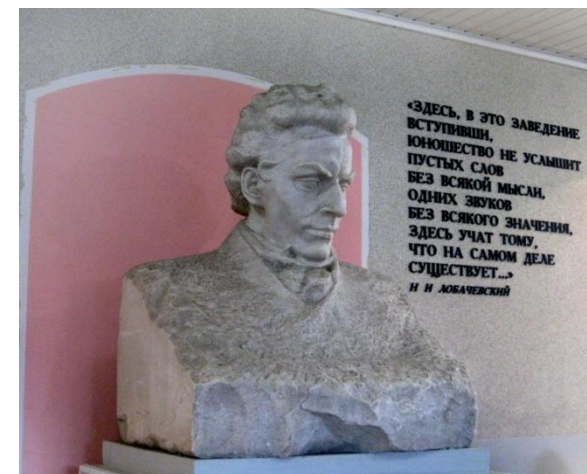
János **Bolyai** (1802-1860)



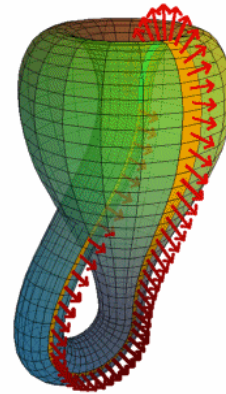
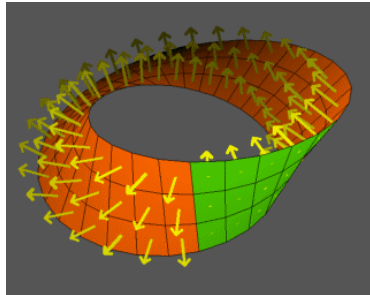
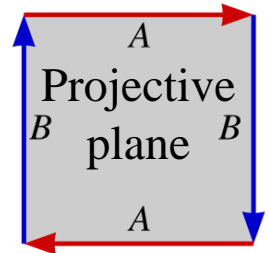
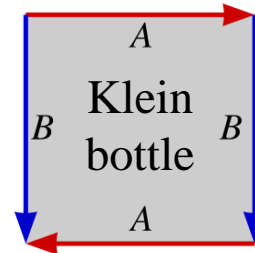
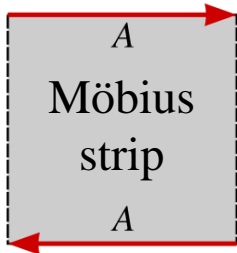
Nikolai Ivanovich **Lobachevsky** (1792-1856)



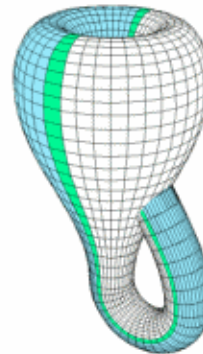
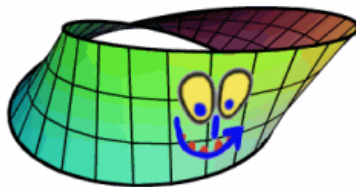
N. I. Lobachevsky



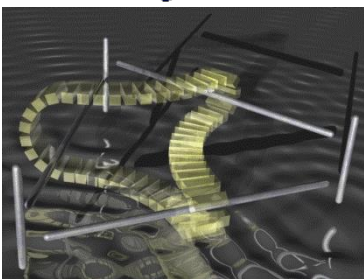
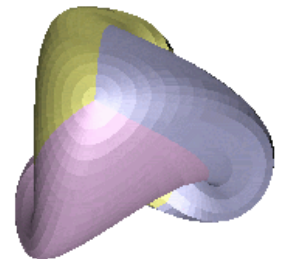
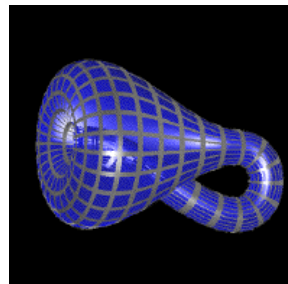
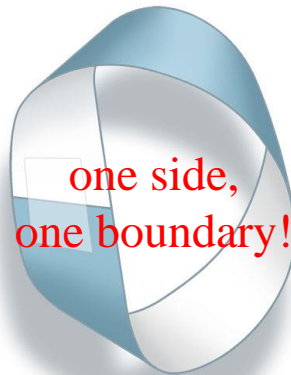
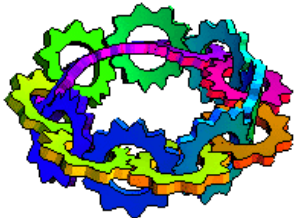
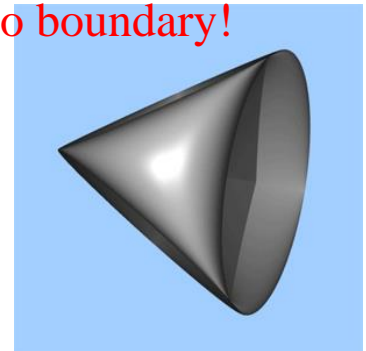
Non-Euclidean Non-Orientable Surfaces



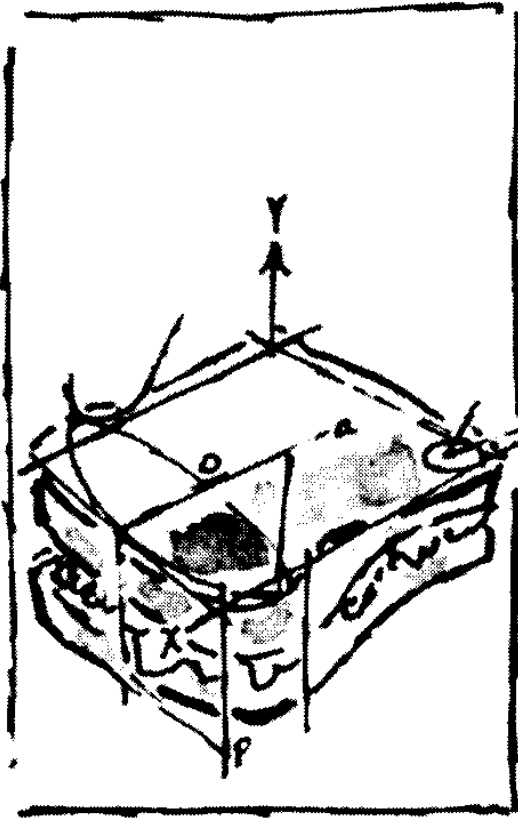
one side,
no boundary!



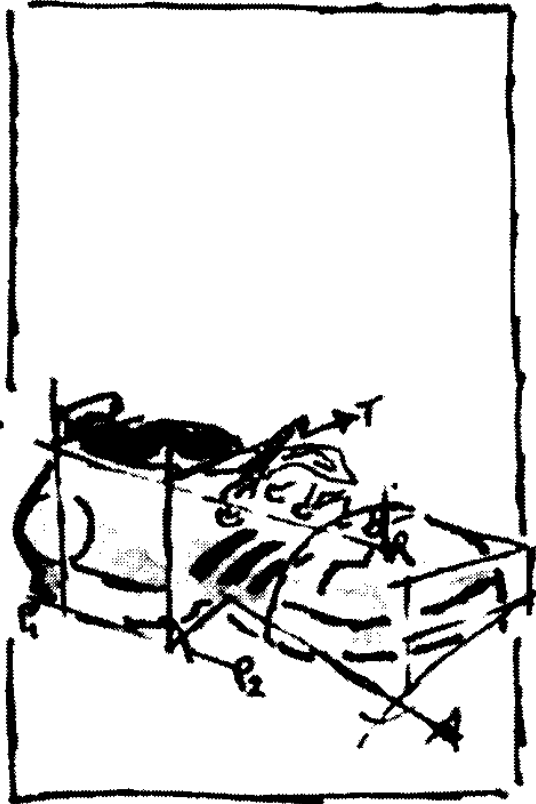
one side,
no boundary!



THE GEOMETRY OF EVERYDAY LIFE



TUNA SANDWICH



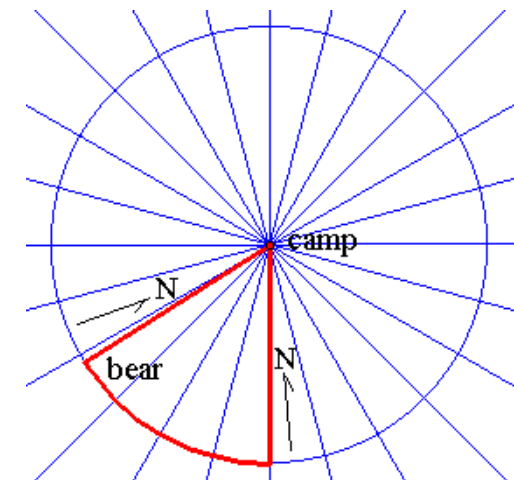
SNEAKER



GRANDMA

sharis

Problem: A man leaves his house and walks one mile south. He then walks one mile west and sees a Bear. Then he walks one mile north back to his house. What color was the bear?



Problem: Is the house location unique?