

CS6160 Theory of Computation

Problem Set 3

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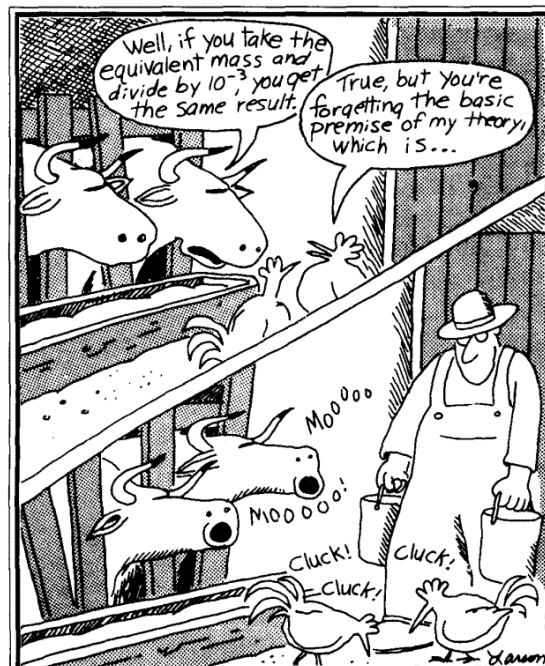
Please start solving these problems immediately, and work in study groups. Please prove all your answers; informal arguments are acceptable, but please make them precise / detailed / convincing enough so that they can be easily made rigorous.

Important note: this is not a “due homework”, but rather a “pool of problems” meant to calibrate the scope and depth of the knowledge & skills in CS theory that you (eventually) need to have for exams, PhD quals, becoming a better problem-solver, thinking more abstractly and generally, performing more effective research, etc. Please solve as many of these problems as you can, and use this problem list as a resource to improve your problem-solving skills, abstract thinking, and to find out what topics you need to further focus and improve upon. Recall that most (and perhaps even all) exam questions in this course will come from these problem sets, so your best strategy of studying for the exams in this course is to solve (in study groups) as many of these problems as possible, and the sooner the better!

Advice: Please try to solve the easier problems first (where the meta-problem here is to figure out which are the easier ones. ☺) Please don't spend too long on any single problem without also attempting (in parallel) to solve other problems as well; this way, the easiest problems (at least easier for you) will reveal themselves much sooner (think about this as a “hedging strategy”).



Before paper and scissors



1. The following problems are from [Sipser, Second Edition]:

Page 294-300: 7.6, 7.7, 7.9, 7.10, 7.11, 7.13, 7.14, 7.17, 7.21, 7.26, 7.32, 7.33, 7.36, 7.41, 7.42, 7.44.

Pages 329-332: 8.4, 8.6, 8.8, 8.9, 8.11, 8.17, 8.18, 8.20, 8.21, 8.22, 8.23, 8.24, 8.25.

Pages 361-363: 9.8, 9.12, 9.13, 9.14.

2. Two cyborgs walk into your home, both claiming to be oracles for the graph 3-colorability decision problem. They both always give a yes/no answer in constant time for any instance, and are each self-consistent (i.e. each always gives the same answer for the same instance). However, one is a true oracle and the other is a shameless impostor, and you have a large instance of 3-colorability upon which they disagree. Prove whether it is possible to expose the impostor within time polynomial in the size of that instance.
3. Modify Turing machines so that they can insert new tape cells into their tape(s), and also remove old tape cells from their tape(s), instead of only (over)writing existing tape cells. (a) Define carefully the transition function and the computational behavior of such machines. (b) Show that such a machine can be simulated by an ordinary Turing machine with at most a quadratic loss of efficiency.
4. True or false: any two-tape Turing machine that uses constant space (aside from the read-only space occupied by the input string) recognizes a regular language.
5. True or false: if L is Turing recognizable, then there is a Turing machine M that enumerates L without ever repeating an element of L .
6. True or false: If a rooted binary tree has infinitely many nodes, then it has an infinitely long path from the root.
7. True or false: If S is an infinite set of Boolean expressions, and every finite subset of S is satisfiable, then S itself is satisfiable.
8. True or false: Most Boolean functions on N inputs have an exponentially long (as a function of N) minimal description (in any fixed reasonable encoding / formalism).
9. True or false: Most Boolean functions on N inputs require an exponential (as a function of N) number of 2-input Boolean gates to implement.
10. Prove that NP is not equal to $DSPACE(N)$. (Note: it is still an open question whether either one contains the other, although we know that they are not equal.)
11. Is NP closed under the Kleene-star operator?
12. Is P closed under the Kleene-star operator?
13. Explain whether or not matrix inversion is NP -complete.
14. Is NP countable?

15. Is PSPACE countable?