CS6160 – Theory of Computation

Final Examination – Fall 2013

University of Virginia

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- This is a take-home open-book open notes pledged exam.
- Note: while for your convenience the "possession time" of this exam is extended up to 24 hours, the actual "work time" for this exam is up to **six** contiguous hours.
- No collaborations, Web searches, nor communications with others are allowed during the exam.
- Do as many of the problems as you can; please explain/prove all answers.
- Shorter proofs / explanations are much preferable to longer ones (Occam's razor!).
- Clearly state the short answer / proof idea first, and then your complete proof / explanation.
- Submit only the pages provided (use more sheets only if absolutely necessary).
- Derive answers on scratch paper first, then copy them neatly onto these pages.

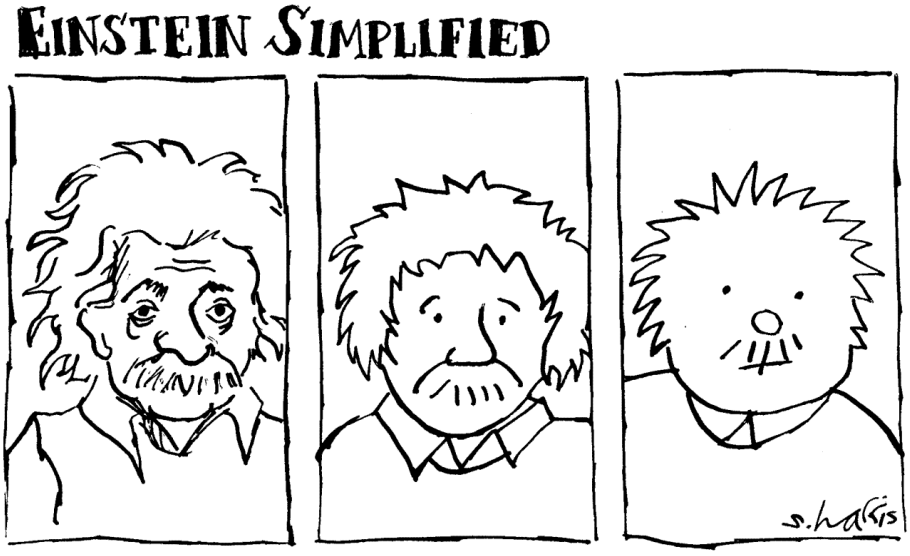
During the exam, please feel free to ask clarifying questions using Email; responses will be posted to the class Web page (so please look at the class Web page often during this exam).

When you are done with this exam, please return it to me (you may slip your completed exam under my office door at 406 Rice Hall).

Good Luck!

Name: _____

"Make everything as simple as possible, but not simpler."
- Albert Einstein (1879-1955)



Problem 1:	**20**	_____
Problem 2:	**20**	_____
Problem 3:	**20**	_____
Problem 4:	**20**	_____
Problem 5:	**20**	_____
Problem 6:	**20**	_____
Problem 7:	**20**	_____
Problem 8:	**20**	_____
Total:	**160**	_____

1) True, false, or open: Most Boolean functions on N inputs require an exponential (as a function of N) number of 2-input Boolean hardware gates to implement.

Short answer (circle one): **True** **False** **Open**

Proof idea:

Proof:

2) True, false, or open: is PSPACE countable?

Short answer (circle one): **True** **False** **Open**

Proof idea:

Proof:

3) If we had free access to an oracle that computes the Busy Beaver function in constant time, prove either that all functions (mapping naturals to naturals) are computable relative to such an oracle, or else give a counter-example.

Proof idea:

Proof:

4) Solve problem 7.21 on page 296 in the [Sipser] textbook.

Proof idea:

Proof:

5) Solve problem 7.42 on page 299 in the [Sipser] textbook.

Proof idea:

Proof:

6) Solve problem 8.17 on page 331 in the [Sipser] textbook.

Proof idea:

Proof:

7) Are the non-finitely-describable languages closed under (a) Union? (b) Complementation? (c) Kleene closure?

Short answer (circle one): (a) True False Open

Proof:

Short answer (circle one): (b) True False Open

Proof:

Short answer (circle one): (c) True False Open

Proof:

8) Two cyborgs walk into your home, both claiming to be oracles for the graph 3-colorability decision problem. They both always give a yes/no answer in constant time for any instance, and are each self-consistent (i.e. each always gives the same answer for the same instance). However, one is a true oracle and the other is a shameless impostor, and you have a large instance of 3-colorability upon which they disagree. Prove whether it is possible to expose the impostor within time polynomial in the size of that instance.

Short answer (circle one): **Possible** **Not possible** **Open question**

Proof idea:

Proof:

"Once you eliminate the impossible, whatever remains,
no matter how improbable, must be the truth."
- Sherlock Holmes (by Sir Arthur Conan Doyle, 1859-1930)