Historical Perspectives

John von Neumann (1903-1957)

- Contributed to set theory, functional analysis, quantum mechanics, ergodic theory, economics, geometry, hydrodynamics, statistics, analysis, measure theory, ballistics, meteorology, ...
- Invented game theory (used in Cold War)
- Re-axiomatized set theory
- Principal member of Manhattan Project
- Helped design the hydrogen / fusion bomb
- Pioneered modern computer science
- Originated the "stored program"
- "von Neumann architecture" and "bottleneck"
- Helped design & build the EDVAC computer
- Created the field of cellular automata
- Investigated self-replication
- Invented merge sort



von Neumann and Morgenstern

Theory of Games and sixtieth-anniversary e













"Most mathematicians prove what they can; von Neumann proves what he wants."













JOHN VON NEUMANN and THE ORIGINS OF **MODERN COMPUTING** WILLIAM ASPRAY

von Neumann's Legacy

- Re-axiomatized set theory to address Russell's paradox
- Independently proved Godel's second incompleteness theorem: aximomatic systems are unable to prove their own consistency.
- Addressed Hilbert's 6th problem: axiomatized quantum mechanics using Hilbert spaces.
- Developed the game-theory based Mutually-Assured Destruction (MAD) strategic equilibrium policy still in effect today!
- von Neumann regular rings, von Neumann bicommutant theorem, von Neumann entropy, von Neumann programming languages



Von Neumann Architecture

"Surely there must be a less primitive way of making big changes in the store than by pushing vast numbers of words back and forth through the **von Neumann bottleneck**. Not only is this tube a literal bottleneck for the data traffic of a problem, but, more importantly, it is an intellectual bottleneck that has kept us tied to word-at-a-time thinking instead of encouraging us to think in terms of the larger conceptual units of the task at hand. Thus programming is basically planning and detailing the enormous traffic of words through the Von Neumann bottleneck, and much of that traffic concerns not significant data itself, but where to find it."

- John Backus, 1977 ACM Turing Award lecture



CPU

Functional programming

Programming Languages and Computer Architecture



The Craft of Functional Programming

ECOND EDITI

Simon Thompson



First Draft of a Report on the EDVAC

by

John von Neumann



Contract No. W-670-ORD-4926

Between the

United States Army Ordnance Department

and the

University of Pennsylvania

Moore School of Electrical Engineering University of Pennsylvania

June 30, 1945

This is an exact copy of the original typescript draft as obtained from the University of Pennsylvania Moore School Library except that a large number of typographical errors have been corrected and the forward references that von Neumann had not filled in are provided where possible. Missing references, mainly to unwritten Sections after 15.0, are indicated by empty {}. All added material, mainly forward references, is enclosed in {}. The text and figures have been reset using TEX in order to improve readability. However, the original manuscript layout has been adhered to very closely. For a more "modern" interpretation of the von Neumann design see M. D. Godfrey and D. F. Hendry, "The Computer as von Neumann Planned It," *IEEE Annals of the History of Computing*, vol. 15 no. 1, 1993.

Michael D. Godfrey, Information Systems Laboratory, Electrical Engineering Department Stanford University, Stanford, California, November 1992

EDVAC (1945):

- 1024 words (44-bits) 5.5KB
- 864 microsec / add (1157 / sec)
- 2900 microsec / multiply (345/sec)
- Magnetic tape (no disk), oscilloscope
- 6,000 vacuum tubes
- 56,000 Watts of power
- 17,300 lbs (7.9 tons), 490 sqft
- 30 people to operate
- Cost: **\$500,000**

THEORY OF SELF-REPRODUCING AUTOMATA



Self-Replication

- Biology / DNA
- Nanotechnology
- Computer viruses
- Space exploration
- Memetics / memes
- "Gray goo"





Problem (extra credit): write a program that prints out its own source code (no inputs of any kind are allowed).



nited States Patent	[19]
SELF REPRODUCING FUNDAMEN FABRICATING MACHINE SYSTEM	
Inventor: Charles M. Collins, 10800 Ct., Burke, Va. 22015	Oak Wilds
Appl. No.: 757,005	
Filed: Nov. 25, 1996	
Related U.S. Application Data	
Continuation-in-part of Ser. No. 364,926, I Pat. No. 5,659,477.	Dec. 28, 1994,
Int. Cl. ⁶ U.S. Cl. 364/468.01 Field of Search 364/46 364/468.19, 468.01, 468.2, 46 474.21, 478.01, 478.03, 47 478.13–478.18, 424.028, 424 180/168, 8.1–8.7; 104/88.03, 3 901/6–8, 1; 318/568.12, 587	l; 364/468.24 8.23, 468.22, 8.21, 468.24, 8.05, 478.06, .027, 424.07; 88.04, 88.02;
References Cited	
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,621,410 11/1986 Williamson ,669,168 6/1987 Tamura et al	

[11]	Pa	tent N	lumber:	5,764,518
[45]	Da	ate of]	Patent:	Jun. 9, 1998
4,870	.592	9/1989	Lampi et al	
4,964				1
5,084	.829			901/7 X
5,145	130			901/1 X
5.150	288			
5,214	,588			
-			seph Ruggiero m—Henry G.	
57]			ABSTRACT	

0.10.10p) including means of diverse materials consisting a plurality of pieces (20.22.23, 156-165) having at least e indicia (18) thereon for detection thereof, at least one joining means functioning according to instructions of a mputer program of a processor means for adjoining in any edetermined relation with other of the plurality of the eces (20, 22, 23, 156-165), and the processor means (30, 0, 166, 167) having the computer program instructions ing responsive to detection of the at least one indicia to ovide for arranging the other of the plurality of the pieces the predetermined relation for controlling the fabrication cans in assembling a given number of the plurality of the eces in the predetermined relation to comprise a produced rication means (10,10,10p) are selected from a group nsisting of a puzzle piece system, a construction system, not knife system, a holed piece system.

75 Claims, 30 Drawing Sheets



"In mathematics you don't understand things. You just get used to them." – John von Neumann





















Go Forth Replicate Birds do it, bees do it, but could machines do it?

New computer simulations suggest that the answer is yes

Apples beget apples, but can machines beget machines? Today it takes an elaborate manufacturing apparatus to build even a simple machine. Could we endow an artificial device with the ability to multiply on its own? Self-replication has long been considered one of the fundamental properties separating the living from the nonliving. Historically our limited understanding of how biological reproduction works has given it an aura of mystery and made it seem unlikely that it would ever be done by a man-made object. It is reported that when René Descartes averred to Queen Christina of Sweden that animals were just another form of mechanical automata, Her Majesty pointed to a clock and said, "See to it that it produces offspring."

The problem of machine self-replication moved from philosophy into the realm of science and engineering in the late 1940s with the work of eminent mathematician and physicist John von Neumann. Some researchers have actually constructed physical replicators. Forty years ago, for example, geneticist Lionel Penrose and his son, Roger (the famous physicist), built small assemblies of plywood that exhibited a simple form of self-replication [see "Self-Reproducing Machines," by Lionel

Penrose; SCIENTIFIC AMERICAN, June 1959]. But self-replication has proved to be so difficult that most researchers study it with the conceptual tool that von Neumann developed: twodimensional cellular automata.

Implemented on a computer, cellular automata can simulate a huge variety of self-replicators in what amount to austere universes with different laws of physics from our own. Such models free researchers from having to worry about logistical issues such as energy and physical construction so that they can focus on the fundamental questions of information flow. How is a living being able to replicate unaided, whereas mechanical objects must be constructed by humans? How does replication at the level of an organism emerge from the numerous interactions in tissues, cells and molecules? How did Darwinian evolution give rise to self-replicating organisms?

The emerging answers have inspired the development of selfrepairing silicon chips [see box on page 40] and autocatalyzing molecules [see "Synthetic Self-Replicating Molecules," by Julius Rebek, Jr.; SCIENTIFIC AMERICAN, July 1994]. And this may be just the beginning. Researchers in the field of nanotechnology have long proposed that self-replication will be crucial to manu-

By Moshe Sipper and James A. Reggia Photoillustrations by David Emmite

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facturing molecular-scale machines, and proponents of space exploration see a macroscopic version of the process as a way to colonize planets using in situ materials. Recent advances have given credence to these futuristic-sounding ideas. As with other scientific disciplines, including genetics, nuclear energy and chemistry, those of us who study self-replication face the twofold challenge of creating replicating machines and avoiding dystopian pre-

scription could be used in two distinct ways: first, as the instructions whose interpretation leads to the construction of an identical copy of the device; next, as data to be copied, uninterpreted, and attached to the newly created child so that it too possesses the ability to self-replicate. With this two-step process, the self-description need not contain a description of itself. In the architectural analogy, the blueprint would include a plan for building a phothe cellular-automata world. All decisions and actions take place locally; cells do not know directly what is happening outside their immediate neighborhood.

The apparent simplicity of cellular automata is deceptive; it does not imply ease of design or poverty of behavior. The most famous automata, John Horton Conway's Game of Life, produces amazingly intricate patterns. Many questions about the dynamic behavior of cellular cells contains a +, then the cell becomes a +; otherwise it becomes vacant. With this rule, a single + grows into four more +'s, each of which grows likewise, and so forth.

Such weedlike proliferation does not shed much light on the principles of replication, because there is no significant machine. Of course, that invites the question of how you would tell a "significant" machine from a trivially prolific automata. No one has yet devised a satisfactory answer. What is clear, however, is that the replicating structure must in some sense be complex. For example, it must consist of multiple, diverse components whose interactions collectively bring about replication-the proverbial "whole must be greater than the sum of the parts." The existence of multiple distinct components permits a self-description to be stored within the replicating structure.

In the years since von Neumann's seminal work, many researchers have probed the domain between the complex and the trivial, developing replicators that require fewer components, less space or simpler rules. A major step forward was taken in 1984 when Christopher G. Langton, then at the University of Michigan, observed that looplike storage devices-which had formed modules of earlier self-replicating machines-could be programmed to replicate on their own. These devices typically consist of two pieces: the loop itself, which is a string of components that circulate around a rectangle, and a construction arm, which protrudes from a corner of the rectangle into the surrounding space. The circulating components constitute a recipe for the loop-for example, "go three squares ahead, then turn left." When this recipe reaches the construction arm, the automata rules make a copy of it. One copy continues around the loop; the other goes down the arm, where it is interpreted as instructions.

By giving up the requirement of universal construction, which was central to von Neumann's approach, Langton showed that a replicator could be constructed from just seven unique components occupying only 86 cells. Even smaller and simpler self-replicating loops have been devised by one of us (Reggia) and our colleagues [see box on next page]. Be-



Her Majesty pointed to a clock and said, "See to it that it produces offspring."

dictions of devices running amok. The knowledge we gain will help us separate good technologies from destructive ones.

Playing Life

SCIENCE-FICTION STORIES often depict cybernetic self-replication as a natural development of current technology, but they gloss over the profound problem it poses: how to avoid an infinite regress. A system might try to build a clone using a blueprint-that is, a self-description. Yet the self-description is part of the machine, is it not? If so, what describes the description? And what describes the description of the description? Self-replication in this case would be like asking an architect to make a perfect blueprint of his or her own studio. The blueprint would have to contain a miniature version of the blueprint, which would contain a miniature version of the blueprint and so on. Without this information, a construction crew would be unable to re-create the studio fully: there would be a blank space where the blueprint had been.

Von Neumann's great insight was an explanation of how to break out of the infinite regress. He realized that the self-de-

> MOSHE SIPPER and JAMES A. REGGIA share a long-standing interest in how complex systems can self-organize. Sipper is a senior lecturer in the department of computer science at Ben-Gurion University in Israel and a visiting researcher at the Logic Systems Laboratory of the Swiss Federal Institute of Technology in Lausanne. He is interested mainly in bio-inspired computational paradigms such as evolutionary computation, self-replicating systems and cellular computing. Reggia is a professor of computer science and neurology, working in the Institute for Advanced Computer Studies at the University of Maryland. In addition to studying self-replication, he conducts research on computational models of the brain and its disorders, such as stroke.

tocopy machine. Once the new studio and the photocopier were built, the construction crew would simply run off a copy of the blueprint and put it into the new studio.

Living cells use their self-description, which biologists call the genotype, in exactly these two ways: transcription (DNA is copied mostly uninterpreted to form mRNA) and translation (mRNA is interpreted to build proteins). Von Neumann made this transcription-translation distinction several years before molecular bio logists did, and his work has been crucial in understanding self-replication in nature.

To prove these ideas, von Neumann and mathematician Stanislaw M. Ulam came up with the idea of cellular automata. A cellular-automata simulation involves a chessboardlike grid of squares, or cells, each of which is either empty or occupied by one of several possible components. At discrete intervals of time, each cell looks at itself and its neighbors and decides whether to metamorphose into a different component. In making this decision, the cell follows relatively simple rules, which are the same for all cells. These rules constitute the basic physics of how a pattern will unfold, you need to simulate it fully [see Mathematical Games, by Martin Gardner; SCIENTIFIC AMERICAN, October 1970 and February 1971; and "The Ultimate in Anty-Particles," by Ian Stewart, July 1994]. In its own way, a cellular-automata model can be just as complex as the real world.

automata are formally unsolvable. To see

Copy Machines

WITHIN CELLULAR AUTOMATA, selfreplication occurs when a group of components-a "machine"-goes through a sequence of steps to construct a nearby duplicate of itself. Von Neumann's machine was based on a universal constructor, a machine that, given the appropriate instructions, could create any pattern. The constructor consisted of numerous types of components spread over tens of thousands of cells and required a booklength manuscript to be specified. It has still not been simulated in its entirety, let alone actually built, on account of its complexity. A constructor would be even more complicated in the Game of Life because the functions performed by single cells in von Neumann's model-such as transmission of signals and generation of new components-have to be performed by composite structures in Life.

Going to the other extreme, it is easy to find trivial examples of self-replication. For example, suppose a cellular automata has only one type of component, labeled +, and that each cell follows only a single rule: if exactly one of the four neighboring cause they have multiple interacting components and include a self-description, they are not trivial. Intriguingly, asymmetry plays an unexpected role: the rules governing replication are often simpler when the components are not rotationally symmetric than when they are.

Emergent Replication

ALL THESE SELF-REPLICATING structures have been designed through ingenuity and much trial and error. This process is arduous and often frustrating; a small change to one of the rules results in an entirely different global behavior, most likely the disintegration of the structure in question. But recent work has gone beyond the direct-design approach. Instead of tailoring the rules to suit a particular type of structure, researchers have experimented with various sets of rules, filled the cellular-automata grid with a "primordial soup" of randomly selected components and checked whether selfreplicators emerged spontaneously.

In 1997 Hui-Hsien Chou, now at Iowa State University, and Reggia noticed that as long as the initial density of the free-floating components was above a certain threshold, small self-replicating loops reliably appeared. Loops that collided underwent annihilation, so there was an ongoing process of death as well as birth. Over time, loops proliferated, grew in size and evolved through mutations triggered by debris from past collisions. Although the automata rules were deterministic, these mutations were effectively random,

THE AUTHORS

because the system was complex and the components started in random locations.

Such loops are intended as abstract machines and not as simulacra of anything biological, but it is interesting to compare them with biomolecular structures. A loop loosely resembles circular DNA in bacteria, and the construction arm acts as the enzyme that catalyzes DNA replication. More important, replicating loops illustrate how complex global behaviors can arise from simple local interactions. For example, components move around a loop even though the rules say nothing about movement; what is actually happening is that individual cells are coming alive, dving or metamorphosing in such a way that a pattern is eliminated from one position and reconstructed elsewhere-a process that we perceive as motion. In short, cellular automata act locally but appear to think globally. Much the same is true of molecular biology.

In a recent computational experiment,

Jason Lohn, now at the NASA Ames Research Center, and Reggia experimented not with different structures but with different sets of rules. Starting with an arbitrary block of four components, they found they could determine a set of rules that made the block self-replicate. They discovered these rules via a genetic algorithm, an automated process that simulates Darwinian evolution.

The most challenging aspect of this work was the definition of the so-called

HOW TO PLAY: You will need two chessboards: one to

current configuration, consult the rules and place the

represent the current configuration, the other to show the

next configuration. For each round, look at each square of the

appropriate piece in the corresponding square on the other

and that of the four squares immediately to the left, to the

board. Each piece metamorphoses depending on its identity

right, above and below. When you have reviewed each square

and set up the next configuration, the round is over. Clear the

first board and repeat. Because the rules are complicated, it

The direction in which a knight faces is significant. In the

drawings here, we use standard chess conventions to indicate

the orientation of the knight: the horse's muzzle points forward.

have adjacent empty squares off the board. —M.S. and J.A.R.

If no rule explicitly applies, the contents of the square stay

the same. Squares on the edge should be treated as if they

takes a bit of patience at first. You can also view the

simulation at Islwww.epfl.ch/chess

1

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fitness function-the criteria by which sets of rules were judged, thus separating good solutions from bad ones and driving the evolutionary process toward rule sets that facilitated replication. You cannot simply assign high fitness to those sets of rules that cause a structure to replicate, because none of the initial rule sets is likely to allow for replication. The solution was to devise a fitness function composed of a weighted sum of three measures: a growth measure (the extent to which

each component type generates an increasing supply of that component), a relative position measure (the extent to which neighboring components stay together) and a replicant measure (a function of the number of actual replicators present). With the right fitness function, evolution can turn rule sets that are sterile into ones that are fecund; the process usually takes 150 or so generations.

Self-replicating structures discovered in this fashion work in a fundamentally different way than self-replicating loops do. For example, they move and deposit copies along the way-unlike replicating loops, which are essentially static. And although these newly discovered replicators consist of multiple, locally interacting components, they do not have an identifiable self-description-there is no obvious genome. The ability to replicate without a self-description may be relevant to questions about how the earliest biological Continued on page 43

REPLACE IT with a pawn.

BUILD YOUR OWN REPLICATOR

SIMULATING A SMALL self-replicating loop using an ordinary chess set is a good way to get an intuitive sense of how these systems work. This particular cellular-automata model has four different types of components: pawns, knights, bishops and rooks. The machine initially comprises four pawns, a knight and a bishop. It has two parts: the loop itself, which consists of a two-by-two square, and a construction arm, which sticks out to the right.

The knight and bishop represent the self-description: the knight, whose orientation is significant, determines which direction to grow, while the bishop tags along and determines how long the side of the loop should be. The pawns are fillers that define the rest of the shape of the loop, and the rook is a transient signal to guide the growth of a new construction arm.

As time progresses, the knight and bishop circulate counterclockwise around the loop. Whenever they encounter the arm, one copy goes out the arm while the original continues around the loop

11

1 1

1 The knight and

clockwise around

knight heads out

the arm.

bishop move counter-

the loop. A clone of the

STAGES OF REPLICATION



INITIALLY, the selfdescription, or "genome"-a knight followed by a bishop-is poised at the start of the construction arm.



2 The original knight-3 The knight triggers bishop pair continues the formation of two to circulate. The bishop corners of the child is cloned and follows loop. The bishop tags along, completing the new knight out the gene transfer. the arm.



4 The knight forges the remaining corner of the child loop. The loops are connected by the construction arm and a knight-errant.



5 The knight-errant moves up to endow the together with the parent with a new arm. pair, conjures up a A similar process, one step delayed, begins for the child loop. old arm is erased.

6 The knight-errant,

7 The rook kills the knight and generates original knight-bishop the new, upward arm. Another rook prepares rook. Meanwhile the to do the same for the child.



1



8 At last the two loops are separate and whole. The selfdescriptions continue to circulate, but otherwise all is calm.

9 The parent prepares to give birth again.

In the following step, the child too will begin to replicate.



AUGUST 2001

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SCIENTIFIC AMERICAN 39

KNIGHT IF THERE is a bishop just behind or to the left of the knight, replace the knight with another bishop.

OTHERWISE, if at least one of the 2 neighboring squares is occupied, 2 2 → remove the knight and leave the ? square empty.

PAWN

A A → **A**

🖆 🚼 🏒 → 🔩

📹 🧘 ? → 📹

Å

2

IF THERE is a neighboring knight, replace the pawn with a knight with a certain orientation, as follows:

> IF A NEIGHBORING knight is facing away from the pawn, the new knight faces the opposite way.

OTHERWISE, if there is exactly one neighboring pawn, the new knight faces that pawn.

OTHERWISE the new knight faces in the same direction as the neighboring knight.



BISHOP OR ROOK

€11 → 2

→ 1

→ 😩

IF THERE are two neighboring knights and either faces the empty square, fill the square with a rook.

IF THERE is only one neighboring knight and it faces the square, fill the square with a knight rotated 90 degrees counterclockwise.

IF THERE is a neighboring knight and its left side faces the square, and the other neighbors are empty, fill the square with a pawn.

IF THERE is a neighboring rook, and the other neighbors are empty, fill the square with a pawn.

IF THERE are three neighboring pawns, fill the square with a knight facing the fourth, empty neighbor.

ROBOT, HEAL THYSELF

Computers that fix themselves are the first application of artificial self-replication

LAUSANNE, SWITZERLAND—Not many researchers encourage the wanton destruction of equipment in their labs. Daniel Mange, however, likes it when visitors walk up to one of his inventions and press the button marked KILL. The lights on the panel go out; a small box full of circuitry is toast. Early in May his team unveiled its latest contraption at a science festival here—a wall-size digital clock whose components you can zap at will—and told the public: Give it your best shot. See if you can crash the system.

The goal of Mange and his team is to instill electronic circuits with the ability to take a lickin' and keep on tickin'—just like living things. Flesh-and-blood creatures might not be so good at calculating π to the millionth digit, but they can get through the day without someone pressing Ctrl-Alt-Del. Combining the precision of digital hardware with the resilience of biological wetware is a leading challenge for modern electronics.

Electronics engineers have been working on fault-tolerant circuits ever since there were electronics engineers [see "Redundancy in Computers," by William H. Pierce; SCIENTIFIC AMERICAN, February 1964]. Computer moders would still be dribbling data at 1200 baud if it weren't for error detection and correction. In many applications, simple quality-control checks, such as extra data bits, suffice. More complex systems provide entire backup computers. The space shuttle, for example, has five processors. Four of them perform the same calculations; the fifth checks whether they agree and pulls the plug on any dissenter. The problem with these systems, though, is that they rely on centralized control. What if that control unit goes bad?

Nature has solved that problem through radical decentralization. Cells in the body are all basically identical; each takes on a specialized task, performs it autonomously and, in the event of infection or failure, commits hara-kiri so that its tasks can be taken up by new cells. These are the attributes that Mange, a professor at the Swiss Federal Institute of Technology here, and others have sought since 1993 to emulate in circuitry, as part of the "Embryonics" (embryonic electronics) project.

One of their earlier inventions, the MICTREE (microinstruction tree) artificial cell, consisted of a simple processor and four bits of data storage. The cell is contained in a plastic box roughly the size of a pack of Post-its. Electrical contacts run along the sides so that cells can be snapped together like Legos. As in cellular automata, the models used to study the theory of self-replication, the MICTREE cells are connected only to their immediate neighbors. The communication burden on each cell is thus independent of the total number of cells. The system, in other words, is easily scalable— unlike many parallel-computing architectures.

Cells follow the instructions in their "genome," a program written in a subset of the Pascal computer language. Like their biological antecedents, the cells all contain the exact same genome and execute part of it based on their position within the array, which each cell calculates relative to its neighbors. Wasteful though it may seem, this redundancy allows the array to withstand the loss of any cell. Whenever someone presses the KILL button on a cell, that cell shuts down, and its left and right neighbors become directly connected. The right neighbor recalculates its position and starts executing the deceased's program. Its tasks, in turn, are taken up by the next cell to the right, and so on, until a cell designated as a spare is pressed into service.

Writing programs for any parallel processor is tricky, but the MICTREE array requires an especially unconventional approach. Instead of giving explicit instructions, the programmer must devise simple rules out of which the desired function will emerge. Being Swiss, Mange demonstrates by building a superreliable stopwatch. Displaying minutes and seconds requires four cells in a row, one for each digit. The genome allows for two cell types: a counter from zero to nine and a counter from zero to five. An oscillator feeds one pulse per second into the rightmost cell. After 10 pulses, this cell cycles back to zero and sends a pulse to the cell on its left, and so on down the line. The watch takes up part of an array of 12 cells; when you kill one, the clock transplants itself one cell over and carries on. Obviously, though, there is a limit to its resilience: the whole thing will fail after, at most, eight kills.

The prototype MICTREE cells are hardwired, so their processing power cannot be tailored to a specific application. In a finished product, cells would instead be implemented on a fieldprogrammable gate array, a grid of electronic components that can be reconfigured on the fly [see "Configurable Computing," by John Villasenor and William H. Mangione-Smith; SCIENTIFIC AMERICAN, June 1997]. Mange's team is now custom-designing a gate array,



known as MUXTREE (multiplexer tree), that is optimized for artificial cells. In the biological metaphor, the components of this array are the "molecules" that constitute a cell. Each consists of a logic gate, a data bit and a string of configuration bits that determines the function of this gate.

Building a cell out of such molecules offers not only flexibility but also extra endurance. Each molecule contains two copies of the gate and three of the storage bit. If the two gates ever give different results, the molecule kills itself for the greater good of the cell. As a last gasp, the molecule sends its data bit (preserved by the triplicate storage) and configuration to its right neighbor, which does the same, and the process continues until the rightmost molecule transfers its data to a spare. This second level of fault tolerance prevents a single error from wiping out an entire cell

A total of 2,000 molecules, divided into four 20-by-25 cells, make up the BioWall—the giant digital clock that Mange's team has just put on display. Each molecule is enclosed in a small box and includes a KILL button and an LED display. Some molecules are configured to perform computations; others serve as pixels in the clock display. Making liberal use of the KILL buttons, I did my utmost to crash the system, something I'm usually quite good at. But the plucky clock just wouldn't submit. The clock display did start to look funny—numerals bent over as their pixels shifted to the right—but at least it was still legible, unlike most faulty electronic signs.

That said, the system did suffer from display glitches, which Mange attributed mainly to timing problems. Although the processing power is decentralized, the cells still rely on a central oscillator to coordinate their communications; sometimes they fall out of sync. Another Embryonics team, led by Andy Tyrrell of the University of York in England, has been studying making the cells asynchronous, like their biological counterparts. Cells would generate handshaking signals to orchestrate data transfers. The present system is also unable to catch certain types of error, including damaged configuration strings. Tyrrell's team has proposed adding watchdog molecules—an immune system—that would monitor the configurations (and one another) for defects.

Although these systems demand an awful lot of overhead, so do other fault-tolerance technologies. "While Embryonics appears to be heavy on redundancy, it actually is not that bad when compared to other systems," Tyrrell argues. Moreover, MUXTREE should be easier to scale down to the nano level; the "molecules" are simple enough to really be molecules. Says Mange, "We are preparing for the situation where electronics will be at the same scale as biology."

On a philosophical level, Embryonics comes very close to the dream of building a self-replicating machine. It may not be quite as dramatic as a robot that can go down to Radio Shack, pull parts off the racks, and take them home to resolder a connection or build a loving mate. But the effect is much the same. Letting machines determine their own destiny—whether reconfiguring themselves on a silicon chip or reprogramming themselves using a neural network or genetic algorithm—sounds scary, but perhaps we should be gratified that machines are becoming more like us: imperfect, fallible but stubbornly resourceful.

-George Musser, imperfect but resourceful staff editor and writer





Continued from page 39

replicators originated. In a sense, researchers are seeing a continuum between nonliving and living structures.

Many researchers have tried other computational models besides the traditional cellular automata. In asynchronous cellular automata, cells are not updated in concert; in nonuniform cellular automata, the rules can vary from cell to cell. Another approach altogether is Core War [see Computer Recreations, by A. K. Dewdney; SCIENTIFIC AMERICAN, May 1984] and its successors, such as ecologist Thomas S. Ray's Tierra system. In these nents, one for the program and the other for data. The loops can execute an arbitrary program in addition to self-replicating. In a sense, they are as complex as the computer that simulates them. Their main limitation is that the program is copied unchanged from parent to child, so that all loops carry out the same set of instructions.

In 1998 Chou and Reggia swept away this limitation. They showed how selfreplicating loops carrying distinct information, rather than a cloned program, can be used to solve a problem known as satisfiability. The loops can be used to determine whether the variables in a logical exsigning a parallel computer from either transistors or chemicals [see "Computing with DNA," by Leonard M. Adleman; SCIENTIFIC AMERICAN, August 1998].

In 1980 a NASA team led by Robert Freitas, Jr., proposed planting a factory on the moon that would replicate itself, using local lunar materials, to populate a large area exponentially. Indeed, a similar probe could colonize the entire galaxy, as physicist Frank J. Tipler of Tulane University has argued. In the nearer term, computer scientists and engineers have experimented with the automated design of robots [see "Dawn of a New Species?" by George

In a sense, researchers are seeing a continuum between nonliving and living structures.

simulations the "organisms" are computer programs that vie for processor time and memory. Ray has observed the emergence of "parasites" that co-opt the selfreplication code of other organisms.

Getting Real

SO WHAT GOOD are these machines? Von Neumann's universal constructor can compute in addition to replicating, but it is an impractical beast. A major advance has been the development of simple vet useful replicators. In 1995 Gianluca Tempesti of the Swiss Federal Institute of Technology in Lausanne simplified the loop self-description so it could be interlaced with a small program-in this case, one that would spell the acronym of his lab, "LSL." His insight was to create automata rules that allow loops to replicate in two stages. First the loop, like Langton's loop, makes a copy of itself. Once finished, the daughter loop sends a signal back to its parent, at which point the parent sends the instructions for writing out the letters.

Drawing letters was just a demonstration. The following year Jean-Yves Perrier, Jacques Zahnd and one of us (Sipper) designed a self-replicating loop with universal computational capabilities—that is, with the computational power of a universal Turing machine, a highly simplified but fully capable computer. This loop has two "tapes," or long strings of compopression can be assigned values such that the entire expression evaluates to "true." This problem is NP-complete—in other words, it belongs to the family of nasty puzzles, including the famous travelingsalesman problem, for which there is no known efficient solution. In Chou and Reggia's cellular-automata universe, each replicator received a different partial solution. During replication, the solutions mutated, and replicators with promising solutions were allowed to proliferate while those with failed solutions died out.

Although various teams have created cellular automata in electronic hardware, such systems are probably too wasteful for practical applications; automata were never really intended to be implemented directly. Their purpose is to illuminate the underlying principles of replication and, by doing so, inspire more concrete efforts. The loops provide a new paradigm for deMusser; SCIENTIFIC AMERICAN, November 2000]. Although these systems are not truly self-replicating—the offspring are much simpler than the parent—they are a first step toward fulfilling the queen of Sweden's request.

Should physical self-replicating machines become practical, they and related technologies will raise difficult issues, including the Terminator film scenario in which artificial creatures outcompete natural ones. We prefer the more optimistic. and more probable, scenario that replicators will be harnessed to the benefit of humanity [see "Will Robots Inherit the Earth?" by Marvin Minsky; SCIENTIFIC AMERICAN, October 1994]. The key will be taking the advice of 14th-century English philosopher William of Ockham: entia non sunt multiplicanda praeter necessitatem-entities are not to be multiplied beyond necessity.

MORE TO EXPLORE

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Animations of self-replicating loops can be found at necsi.org/postdocs/sayama/sdsr/java/ For John von Neumann's universal constructor, see alife.santafe.edu/alife/topics/jvn/jvn.html

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Historical Perspectives

- Claude Shannon (1916-2001)
- Invented electrical digital circuits (1937)
- Founded information theory (1948)
- Introduced sampling theory, coined term "bit"
- Contributed to genetics, cryptography
- Joined Institute for Advanced Study (1940) Influenced by Turing, von Neumann, Einstein
- Originated information entropy, Nyquist–Shannon, sampling theorem, Shannon-Hartley theorem, Shannon switching game, Shannon-Fano coding, Shannon's source coding theorem, Shannon limit, Shannon decomposition / expansion, Shannon #
- Other hobbies & inventions: juggling, unicycling, computer chess, rockets, motorized pogo stick, flame-throwers, Rubik's cube solver, wearable computer, mathematical gambling, stock markets
 "AT&T Shannon Labs"











BY M. MITCHECK, WALDHOP

Reluctant Father Claude Shannon

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CLAUDE ELWOOD SHANNON **Collected Papers** Edited by N. J. A. Sloane Aaron D. Wyner



ASS. INST. TECO	58
(DEC 201940)	TA HLE OF CONTENTS
A SYMBOLIC ANALYSIS	Page
OF	I Introduction; Types of Problems 1
RELAY AND SWITCHING CIRCUITS	II Series-Parallel Two-Terminal Circuits 4
	Fundamental Definitions and Postulates 4
òy	Theorems 6
Claude Elwood Shannon	Analogue with the Calculus of Propositions 8
B.S., University of Michigan	
1956	III Multi-Terminal and Non-Series-Parallel Networks18
	Equivalence of n-Terminal Networks 13
Submitted in Partial Fulfillment of the	Star-Mesh and Delta-Wye Transformations 19
Requirements for the Degree of	Hinderance Function of a Non-Series-Parallel Network 21
NASTER OF SCIENCE	Simultaneous Equations 24
from the	Matrix Methods 25
Massachusetts Institute of Technology	Special Types of Relays and Switches 28
1940	IV Synthesis of Networks 31
	General Theorems on Networks and Functions 31
	Dual Networks 36
Signature of Author	Synthesis of the General Symmetric Function 39
Department of Electrical Engineering, August 10, 1937	Equations from Given Operating Characteristics 47
Signature of Professor	
in Charge of Research	V <u>Illustrative Examples</u>
Signature of Chairman of Department,	
Committee on Graduate Students	An Electric Combination Lock 55
,	A Vote Counting Circuit 58 An Adder to the Base Two 59
	A Factor Table Machine 62
	References -240579 69

•

Theseus: Shannon's electro-mechanical mouse (1950): first "learning machine" and AI experiment

AUTOMATA STUDIES

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Chess champion Ed Lasker looking at Shannon's chess-playing machine



Shannon's home study room





Shannon's On/Off machine









THE MATHEMATICAL THEORY OF COMMUNICATION

by Claude E. Shannon and Warren Weaver



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Introduction

The recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist¹ and Hartley² on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.

The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have meaning; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one selected from a set of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design. If the number of messages in the set is finite then this number or any monotonic function of this number can be regarded as a measure of the information produced when one message is chosen from the set, all choices being equally likely. As was pointed out by Hartley the most natural choice is the logarithmic function. Although this definition must be generalized considerably when we consider the influence of the statistics of the message and when we have a continuous range of messages, we will in all cases use an essentially logarithmic measure.

The logarithmic measure is more convenient for various reasons:

1. It is practically more useful. Parameters of engineering importance such as time, bandwidth, number of relays, etc., tend to vary linearly with the logarithm of the number of possibilities. For example, adding one relay to a group doubles the number of possible states of the relays. It adds 1 to the base 2 logarithm of this number. Doubling the time roughly squares the number of possible messages, or doubles the logarithm, etc.

2. It is nearer to our intuitive feeling as to the proper measure. This is closely related to (1) since we intuitively measure entities by linear comparison with common standards. One feels, for example, that two punched cards should have twice the capacity of one for information storage, and two identical channels twice the capacity of one for transmitting information.

3. It is mathematically more suitable. Many of the limiting operations are simple in terms of the logarithm but would require clumsy restatement in terms of the number of possibilities.

The choice of a logarithmic base corresponds to the choice of a unit for measuring information. If the base 2 is used the resulting units may be called binary digits, or more briefly *bits*, a word suggested by J. W. Tukey. A device with two stable positions, such as a relay or a flip-flop circuit, can store one bit of information. N such devices can store N bits, since the total number of possible states is 2^N and $\log_2 2^N = N$. If the base 10 is used the units may be called decimal digits. Since

$$\log_2 M = \log_{10} M / \log_{10} 2$$

= 3.32 log_{10} M,

¹ Nyquist, H., "Certain Factors Affecting Telegraph Speed," Bell System Technical Journal, April 1924, p. 324; "Certain Topics in Telegraph Transmission Theory," A.I.E.E. Trans., v. 47, April 1928, p. 617.

² Hartley, R. V. L., "Transmission of Information," Bell System Technical Journal, July 1928, p. 535.

Discrete Noiseless Systems

1. The Discrete Noiseless Channel

Teletype and telegraphy are two simple examples of a discrete channel for transmitting information. Generally, a discrete channel will mean a system whereby a sequence of choices from a finite set of elementary symbols $S_1 \cdot \cdot \cdot \cdot S_n$ can be transmitted from one point to another. Each of the symbols S_i is assumed to have a certain duration in time t_i seconds (not necessarily the same for different S_i , for example the dots and dashes in telegraphy). It is not required that all possible sequences of the S_i be capable of transmission on the system; certain sequences only may be allowed. These will be possible signals for the channel. Thus in telegraphy suppose the symbols are: (1) A dot, consisting of line closure for a unit of time and then line open for a unit of time; (2) A dash, consisting of three time units of closure and one unit open; (3) A letter space consisting of, say, three units of line open; (4) A word space of six units of line open. We might place the restriction on allowable sequences that no spaces follow each other (for if two letter spaces are adjacent, they are identical with a word space). The question we now consider is how one can measure the capacity of such a channel to transmit information.

In the teletype case where all symbols are of the same duration, and any sequence of the 32 symbols is allowed, the answer is easy. Each symbol represents five bits of information. If the system transmits n symbols per second it is natural to say that the channel has a capacity of 5n bits per second. This does not mean that the teletype channel will always be transmitting information at this rate — this is the maximum possible rate and whether or not the actual rate reaches this maximum depends on the source

In the more general case with different lengths of symbols and constraints on the allowed sequences, we make the following definition: The capacity C of a discrete channel is given by

of information which feeds the channel, as will appear later.

$$C = \lim_{T \to \infty} \frac{\log N(T)}{T}$$

where N(T) is the number of allowed signals of duration T.

It is easily seen that in the teletype case this reduces to the previous result. It can be shown that the limit in question will exist as a finite number in most cases of interest. Suppose all sequences of the symbols S_1, \cdots, S_n are allowed and these symbols have durations t_1, \cdots, t_n . What is the channel capacity? If N(t) represents the number of sequences of duration t we have

$$N(t) = N(t - t_1) + N(t - t_2) + \cdots + N(t - t_n).$$

The total number is equal to the sum of the numbers of sequences ending in S_1, S_2, \cdots, S_n and these are $N(t - t_1), N(t - t_2), \cdots, N(t - t_n)$, respectively. According to a well-known result in finite differences, N(t) is the asymptotic for large t to AX_{δ}^{t} where A is constant and X_0 is the largest real solution of the characteristic equation:

$$X^{-t_1} + X^{-t_2} + \cdots + X^{-t_n} = 1$$

and therefore

$$C = \lim_{T \to \infty} \frac{\log A X_0^T}{T} = \log X_0.$$

In case there are restrictions on allowed sequences we may still often obtain a difference equation of this type and find C from the characteristic equation. In the telegraphy case mentioned above

$$N(t) = N(t-2) + N(t-4) + N(t-5) + N(t-7) + N(t-8) + N(t-10)$$

a decimal digit is about $3\frac{1}{3}$ bits. A digit wheel on a desk computing machine has ten stable positions and therefore has a storage capacity of one decimal digit. In analytical work where integration and differentiation are involved the base *e* is sometimes useful. The resulting units of information will be called natural units. Change from the base *a* to base *b* merely requires multiplication by $\log_b a$.

By a communication system we will mean a system of the type indicated schematically in Fig. 1. It consists of essentially five parts:

1. An information source which produces a message or sequence of messages to be communicated to the receiving terminal. The message may be of various types: (a) A sequence of letters as in a telegraph or teletype system; (b) A single function of time f(t) as in radio or telephony; (c) A function of time and other variables as in black and white television - here the message may be thought of as a function f(x, y, t) of two space coordinates and time, the light intensity at point (x, y) and time t on a pickup tube plate; (d) Two or more functions of time, say f(t), q(t), h(t) — this is the case in "three-dimensional" sound transmission or if the system is intended to service several individual channels in multiplex; (e) Several functions of several variables - in color television the message consists of three functions f(x, y, t), g(x, y, t), h(x, y, t) defined in a threedimensional continuum --- we may also think of these three functions as components of a vector field defined in the region similarly, several black and white television sources would produce "messages" consisting of a number of functions of three variables; (f) Various combinations also occur, for example in television with an associated audio channel.

2. A transmitter which operates on the message in some way to produce a signal suitable for transmission over the channel. In telephony this operation consists merely of changing sound pressure into a proportional electrical current. In telegraphy we have an encoding operation which produces a sequence of dots, dashes and spaces on the channel corresponding to the message. In a multiplex PCM system the different speech functions must be sampled, compressed, quantized and encoded, and finally interleaved properly to construct the signal. Vocoder systems, television and frequency modulation are other examples of complex operations applied to the message to obtain the signal.

Fig. 1. — Schematic diagram of a general communication system.

3. The *channel* is merely the medium used to transmit the signal from transmitter to receiver. It may be a pair of wires, a coaxial cable, a band of radio frequencies, a beam of light, etc. During transmission, or at one of the terminals, the signal may be perturbed by noise. This is indicated schematically in Fig. 1 by the noise source acting on the transmitted signal to produce the received signal.

4. The *receiver* ordinarily performs the inverse operation of that done by the transmitter, reconstructing the message from the signal.

5. The *destination* is the person (or thing) for whom the message is intended.

We wish to consider certain general problems involving communication systems. To do this it is first necessary to represent the various elements involved as mathematical entities, suitably idealized from their physical counterparts. We may roughly classify communication systems into three main categories: discrete, continuous and mixed. By a discrete system we will mean one in which both the message and the signal are a sequence of discrete symbols. A typical case is telegraphy where the message is a sequence of letters and the signal a sequence of dots, dashes and spaces. A continuous system is one in which the



34

Suppose we have a set of possible events whose probabilities of occurrence are p_1, p_2, \cdots, p_n . These probabilities are known but that is all we know concerning which event will occur. Can we find a measure of how much "choice" is involved in the selection of the event or of how uncertain we are of the outcome?

If there is such a measure, say $H(p_1, p_2, \cdots, p_n)$, it is reasonable to require of it the following properties:

- 1. H should be continuous in the p_i .
- 2. If all the p_i are equal, $p_i = \frac{1}{n}$, then H should be a monotonic increasing function of n. With equally likely events there is more choice, or uncertainty, when there are more possible events.
- 3. If a choice be broken down into two successive choices, the original H should be the weighted sum of the individual values of H. The meaning of this is illustrated in Fig. 6. At the left we have three possibilities $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{6}$. On the right we first choose between two possibilities each with probability $\frac{1}{2}$, and if the second occurs make another choice with probabilities $\frac{2}{3}$, $\frac{1}{3}$. The final results have the same probabilities as before. We require, in this special case, that

 $H(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}) = H(\frac{1}{2}, \frac{1}{2}) + \frac{1}{2} H(\frac{2}{3}, \frac{1}{3}).$

The coefficient $\frac{1}{2}$ is the weighting factor introduced because this second choice only occurs half the time.



Fig. 6. - Decomposition of a choice from three possibilities.

In Appendix 2, the following result is established:

Theorem 2: The only H satisfying the three above assumptions is of the form:

$$H = -K \sum_{i=1}^{n} p_i \log p_i$$

where K is a positive constant.

This theorem, and the assumptions required for its proof, are in no way necessary for the present theory. It is given chiefly to lend a certain plausibility to some of our later definitions. The real justification of these definitions, however, will reside in their implications.

Quantities of the form $H = -\Sigma p_i \log p_i$ (the constant K merely amounts to a choice of a unit of measure) play a central role in information theory as measures of information, choice and uncertainty. The form of H will be recognized as that of entropy



as defined in certain formulations of statistical mechanics⁸ where p_i is the probability of a system being in cell *i* of its phase space.

⁸See, for example, R. C. Tolman, *Principles of Statistical Mechanics*, Oxford, Clarendon, 1938.

50

49

quence of symbols x_i ; and let β be the state of the transducer, which produces, in its output, blocks of symbols y_i . The combined system can be represented by the "product state space" of pairs (α, β) . Two points in the space (α_1, β_1) and (α_2, β_2) , are connected by a line if α_1 can produce an x which changes β_1 to β_2 , and this line is given the probability of that x in this case. The line is labeled with the block of y_1 symbols produced by the transducer. The entropy of the output can be calculated as the weighted sum over the states. If we sum first on β each resulting term is less than or equal to the corresponding term for α , hence the entropy is not increased. If the transducer is non-singular let its output be connected to the inverse transducer. If H'_1, H'_2 and H'_3 are the output entropies of the source, the first and second transducers respectively, then $H'_1 \geq H'_2 \geq H'_3 = H'_1$ and therefore $H'_1 = H'_2$.

Suppose we have a system of constraints on possible sequences of the type which can be represented by a linear graph as in Fig. 2. If probabilities $p_{ij}^{(s)}$ were assigned to the various lines connecting state *i* to state *j* this would become a source. There is one particular assignment which maximizes the resulting entropy (see Appendix 4).

Theorem 8: Let the system of constraints considered as a channel have a capacity $C = \log W$. If we assign

$$p_{ij}^{(s)} = -\frac{B_j}{B_i} W^{-l_{ij}^{(s)}}$$

where $l_{ij}^{(s)}$ is the duration of the sth symbol leading from state i to state j and the B_i satisfy

$$B_i = \sum_{s,j} B_j W^{-l_{ij}^{(s)}}$$

then H is maximized and equal to C.

By proper assignment of the transition probabilities the entropy of symbols on a channel can be maximized at the channel capacity.

9. The Fundamental Theorem for a Noiseless Channel

We will now justify our interpretation of H as the rate of gen-

erating information by proving that H determines the channel capacity required with most efficient coding.

Theorem 9: Let a source have entropy H (bits per symbol) and a channel have a capacity C (bits per second). Then it is possible to encode the output of the source in such a way as to transmit at the average rate $\frac{C}{H} - \epsilon$ symbols per second over the channel where ϵ is arbitrarily small. It is not possible to transmit at an average rate greater than $\frac{C}{H}$.

The converse part of the theorem, that $\frac{C}{H}$ cannot be exceeded,

may be proved by noting that the entropy of the channel input per second is equal to that of the source, since the transmitter must be non-singular, and also this entropy cannot exceed the channel capacity. Hence $H' \leq C$ and the number of symbols per second $= H'/H \leq C/H$.

The first part of the theorem will be proved in two different ways. The first method is to consider the set of all sequences of N symbols produced by the source. For N large we can divide these into two groups, one containing less than $2^{(H+\eta)N}$ members and the second containing less than 2^{RN} members (where R is the logarithm of the number of different symbols) and having a total probability less than μ . As N increases η and μ approach zero. The number of signals of duration T in the channel is greater than $2^{(C-\theta)T}$ with θ small when T is large. If we choose

$$T = \left(\frac{H}{C} + \lambda\right) N$$

then there will be a sufficient number of sequences of channel symbols for the high probability group when N and T are sufficiently large (however small λ) and also some additional ones. The high probability group is coded in an arbitrary one-to-one way into this set. The remaining sequences are represented by larger sequences, starting and ending with one of the sequences not used for the high probability group. This special sequence acts as a start and stop signal for a different code. In between a sufficient time is allowed to give enough different sequences for all the low probability messages. This will require

Entropy and Randomness

- Entropy measures the expected "uncertainly" (or "surprise") associated with a random variable.
- Entropy quantifies the "information content" and represents a lower bound on the best possible lossless compression.
- Ex: a random fair coin has entropy of 1 bit.
 A biased coin has lower entropy than fair coin. A two-headed coin has zero entropy.



- The string 0000000000000... has zero entropy.
- English text has entropy rate of 0.6 to 1.5 bits per letter.
- Q: How do you simulate a fair coin with a biased coin of unknown but fixed bias?

A [von Neumann]: Look at pairs of flips. HT and TH both occur with equal probability of p(1-p), and ignore HH and TT pairs.

Entropy and Randomness

- Information entropy is an analogue of thermodynamic entropy in physics / statistical mechanics, and von Neumann entropy in quantum mechanics.
- Second law of thermodynamics: entropy of an isolated system can not decrease over time.
- Entropy as "disorder" or "chaos".
- Entropy as the "arrow of time".
- "Heat death of the universe" / black holes
- Quantum computing uses a quantum information theory to generalize classical information theory.
- Theorem: String compressibility decreases as entropy increases. Theorem: Most strings are not (losslessly) compressible. Corollary: Most strings are random!





"My greatest concern was what to call it. I thought of calling it 'information', but the word was overly used, so I decided to call it 'uncertainty'. When I discussed it with John von Neumann, he had a better idea. Von Neumann told me, 'You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, nobody knows what entropy really is, so in a debate you will always have the advantage.'"

DEPT. OF ENTROPY

- Claude Shannon on his conversation with John von Neumann regarding what name to give to the "measure of uncertainty" or attenuation in phone-line signals (1949)







Historical Perspectives

- Stephen Kleene (1909-1994)
- Founded recursive function theory
- Pioneered theoretical computer science
- Student of Alonzo Church; was at the Institute for Advanced Study (1940)
- Invented regular expressions
- Kleene star / closure, Kleene algebra, Kleene recursion theorem, Kleene fixed point theorem, Kleene-Rosser paradox



"Kleeneliness is next to Gödeliness"





Regular Expression

a celebration of powerful string manipulation JUNE 1ST // 2008

Historical Perspectives

Noam Chomsky (1928-)

- Linguist, philosopher, cognitive scientist, political activist, dissident, author
- Father of modern linguistics
- Pioneered formal languages
- Developed generative grammars Invented context-free grammars
- Defined the Chomsky hierarchy
- Influenced cognitive psychology, philosophy of language and mind
- Chomskyan linguistics, Chomskyan syntax, Chomskyan models
- Critic of U.S. foreign policy
- Most widely cited living scholar Eighth most-cited source overall!



















ANARCHISM Ur doin it wrong







"...I must admit to taking a copy of Noam Chomsky's 'Syntactic Structures' along with me on my honeymoon in 1961 ... Here was a marvelous thing: a mathematical theory of language in which I could use as a computer programmer's intuition!"

- Don Knuth on Chomsky's influence



"One of the great voices of reason of our time." - NEW YORK DAILY NEWS





The most important intellectual alive -THE NEW YORK TIMES

America's most useful citizen" -THE BOSTON GLOBE











Noam Chomsky

www.PostmodernHaircut.com

If we don't believe in freedom of expression for people we despise, we don't believe in it at all.

"Propaganda is to a democracy what the bludgeon is to a totalitarian state" - Noam Chomsky









TURING CENTENARY CONFERENCE CiE 2012 - How the World Computes

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Biotchies Park

University of Cambridge 18 June - 23 June, 2012

CIE 2012 is one of a series of special events, running throughout the Alan Turing Year, celebrating Turing's unique impact on mathematics, computing, computer science, informatics, morphogenesis, philosophy and the wider scientific world. Its central theme is the computability-theoretic concerns underlying the broad spectrum of Turing's interests, and the contemporary research areas founded upon and animated by them. In this sense, CiE 2012, held in Cambridge in the week running up to the centenary of Turing's birthday, deals with the essential core of what made Turing's contribution so influential and long-lasting. CiE 2012 promises to be an event worthy of the remarkable scientific career it commemorates.

Programme Committee: S Barry Cooper (Leeds, **Co-chair**), Anuj Dawar (Cambridge, **Co-chair**)

Organising Committee: Luca Cardelli, S Barry Cooper (Leeds), Ann Copestake, Anuj Dawar (Chair), Martin Hyland, Andrew Pitts



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17.8.09 Andrew Hodges to speak at CiE 2012

22.7.09 Cambridge confirmed for CiE12

News

31.12.07 Turing Advisory Group founded



Picture of King's College Chapel in Cambridge



Picture of Bletchley Park Bombe rebuild



The Alan Turing Memorial in Sackville Park Manchester