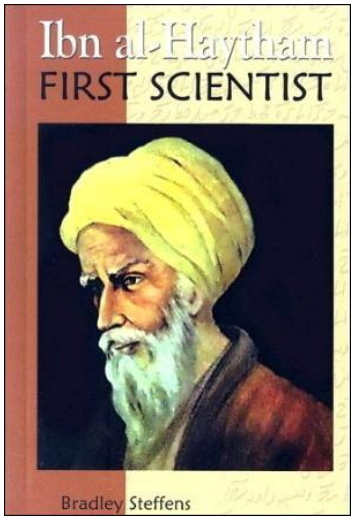
Historical Perspectives

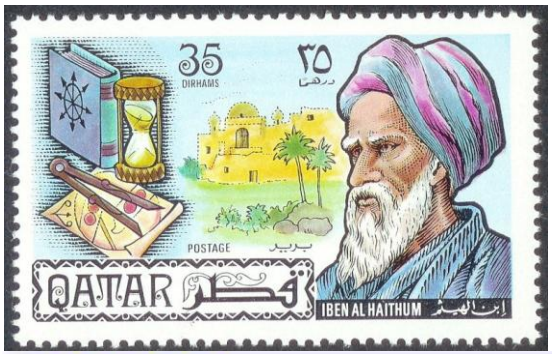
Abu Ali al-Hasan ibn al-Haytham (965-1039)

- AKA Alhazen or "The Physicist"
- Greatest scientist of the middle ages
- Contributed to mathematics, physics, optics, astronomy, anatomy, medicine, engineering, philosophy, psychology
- Pioneered the scientific method, modern optics and experimental physics
- Polymath: authored over 200 treatises, including influential "Book of Optics"
- Influenced Leonardo da Vinci, Bacon, Descartes, Kepler, Galileio and Newton

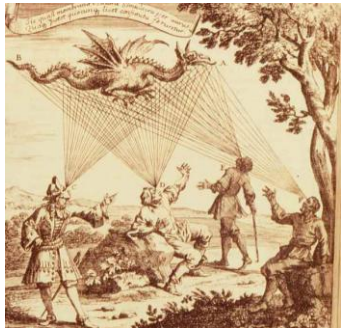
البنك المركزي العراقي
١٠٠٠٠
عشرة آلاف دينار
ورقة نقدية صادرة بموجب القانون
عن البنك المركزي العراقي
والمحافظ

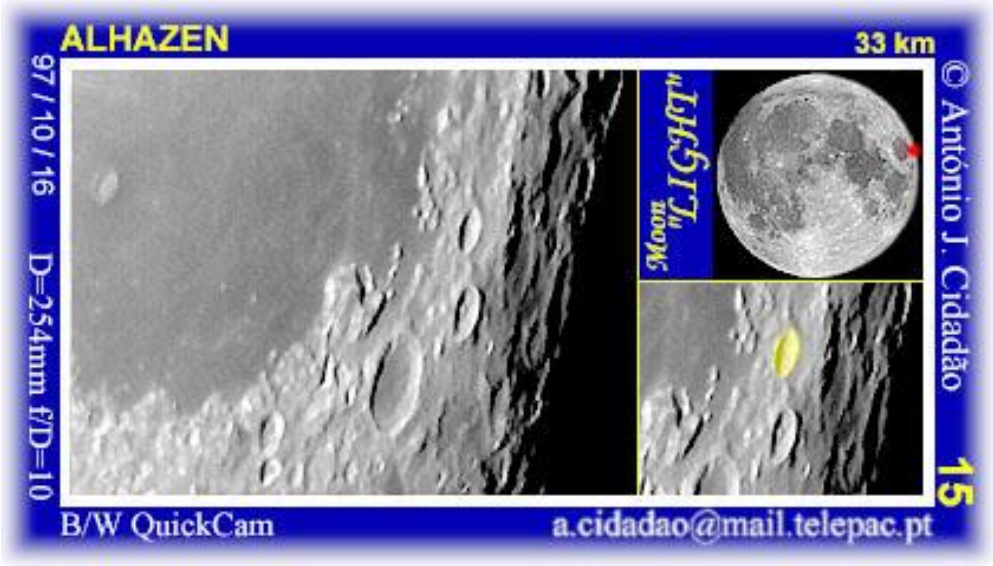

Ibn al-Haytham
FIRST SCIENTIST
Bradley Steffens

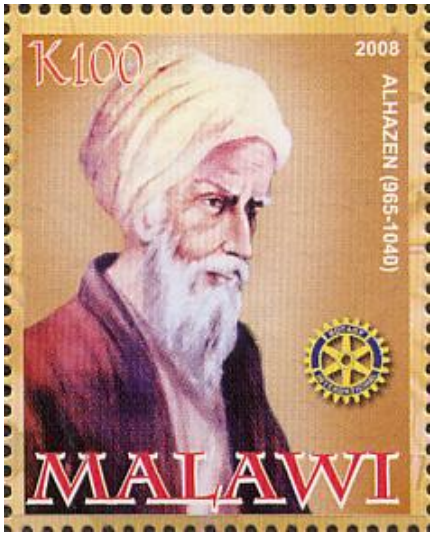
ALHAZEN FILII
ALHAYZEN OPTICAE
LIBER PRIMVS


35
DIRHAMS
٣٥
درهما
POSTAGE
بريد
QATAR
قطر
IBEN AL HAITHUM

IBN-AL-HAITHAM
FATHER OF OPTICS
(965-1039)
20
PAISA
POSTAGE
PAKISTAN
پاکستان
पाकिस्तान




ALHAZEN
33 km
97/10/16
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
15
B/W QuickCam
a.cidadao@mail.telepac.pt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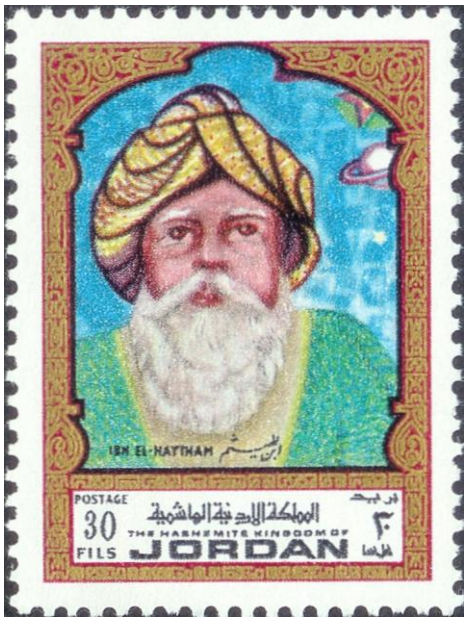
ALHAZEN (965-1040)
MALAWI


IBN EL-HAITHAM
POSTAGE
30
FILS
THE HASHEMITE KINGDOM OF
JORDAN

THE OLD SCIENTIFIC METHOD

Formulate a hypothesis.
Accumulate data.
Do extensive experimentation.

THE NEW SCIENTIFIC METHOD

Formulate a hypothesis.
Patent it.
Raise $17 million.

Historical Perspectives

Leonardo of Pisa (1170–1250)

- Better known as "Fibonacci"
- Considered the most talented mathematician of the middle ages
- Published (1202) "Liber Abaci" – "The Book of Calculation"
- Introduced Hindu-Arabic positional number system in Europe
- Popularized Fibonacci sequence

1 1 2 3 5 8 13 21 34 55 89

European	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	٠	١	٢	٣	۴	۵	۶	٧	٨	٩
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९
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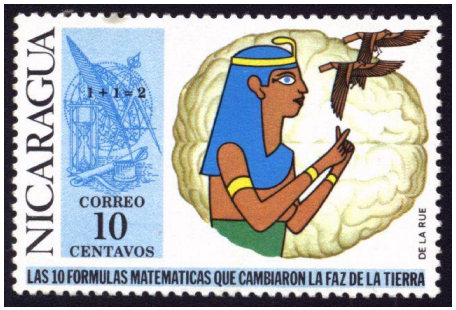
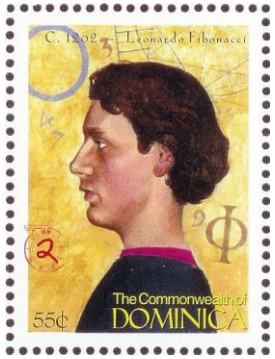
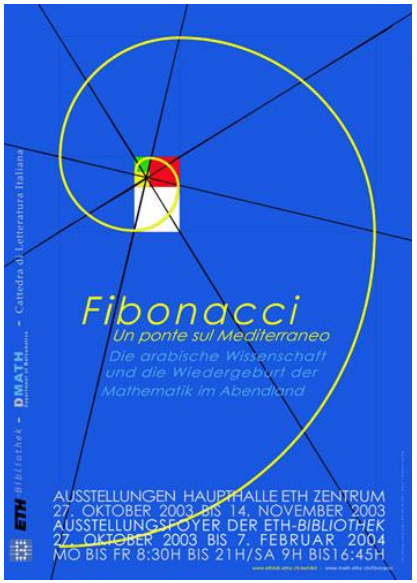
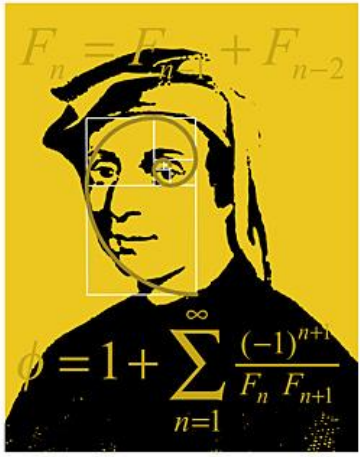
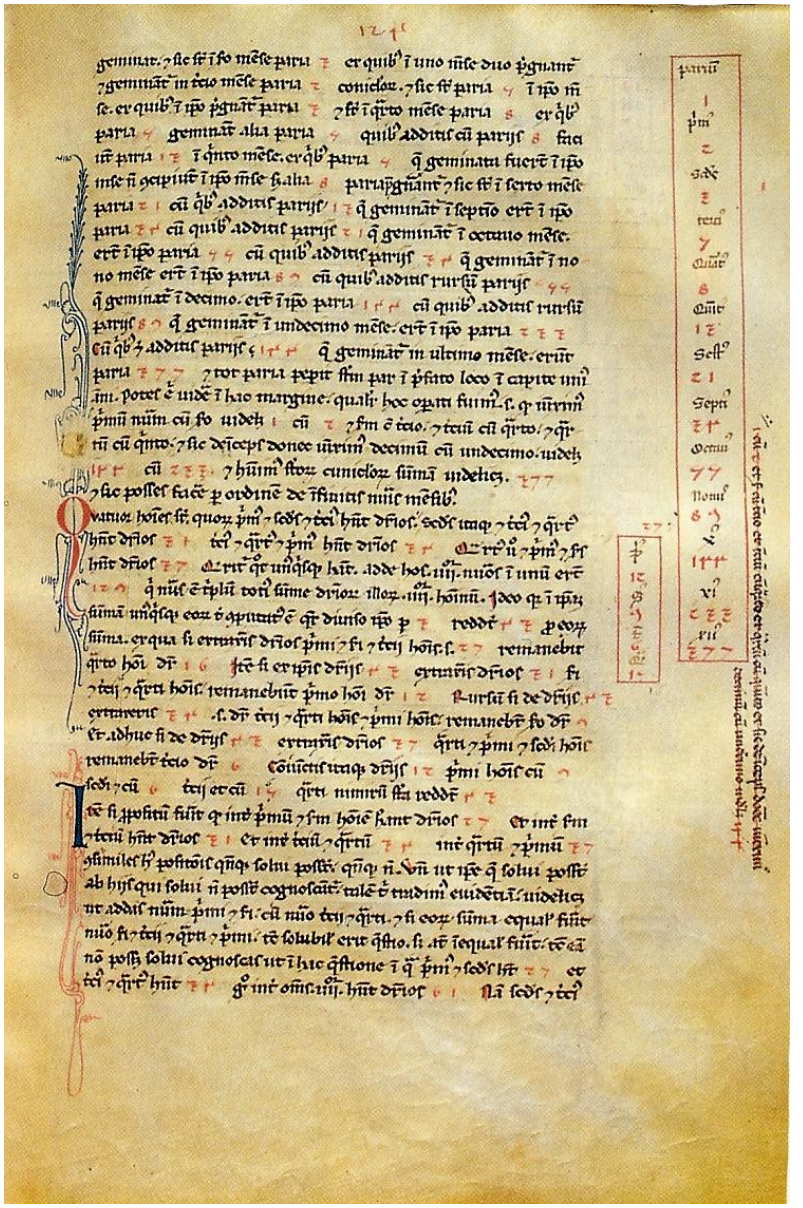






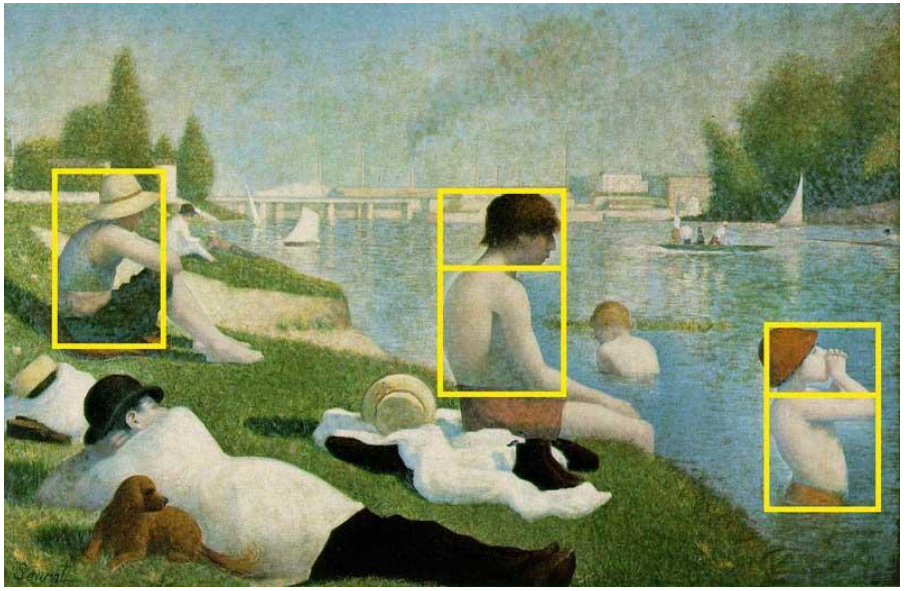


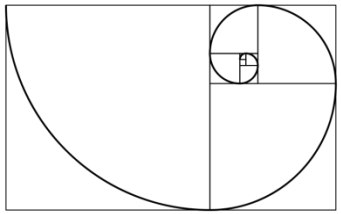
Leonardo Pisano Fibonacci
1170 - 1250

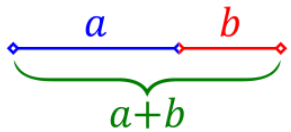


The Fibonacci Quarterly

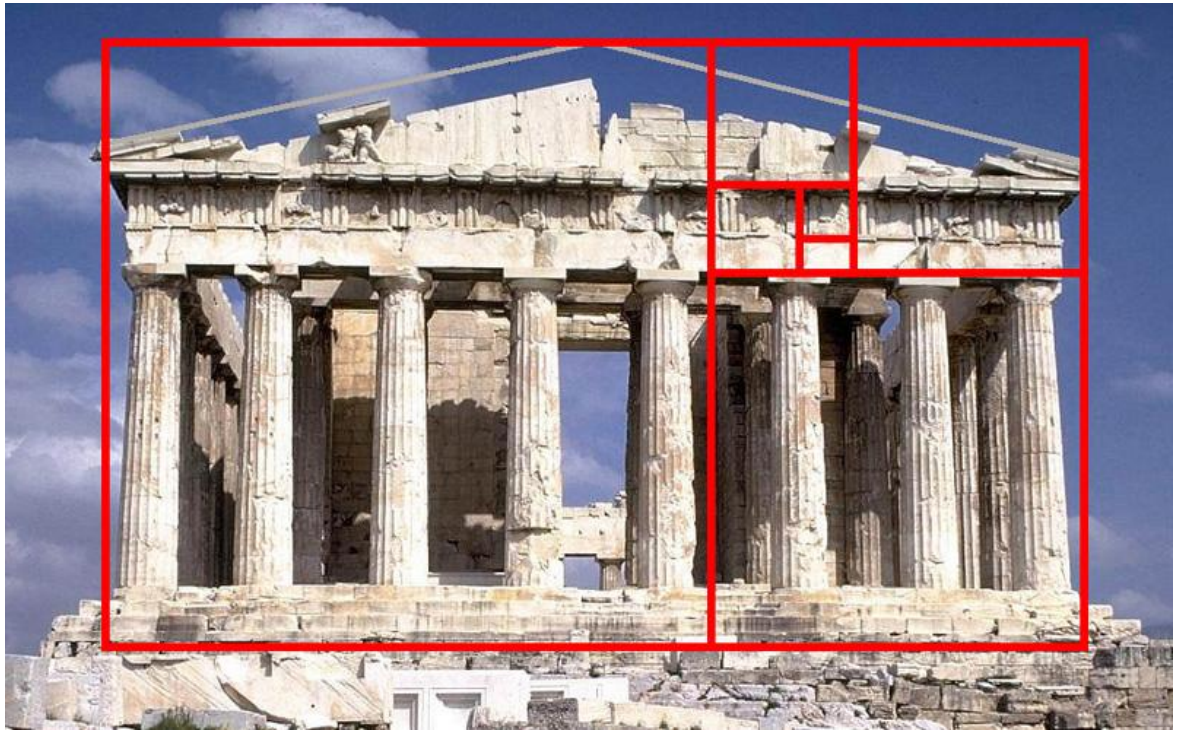
Official Publication of The Fibonacci Association

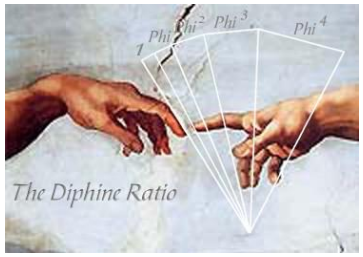






$a+b$ is to a as a is to b




Phi 1
Phi 2
Phi 3
Phi 4
The Diphinc Ratio

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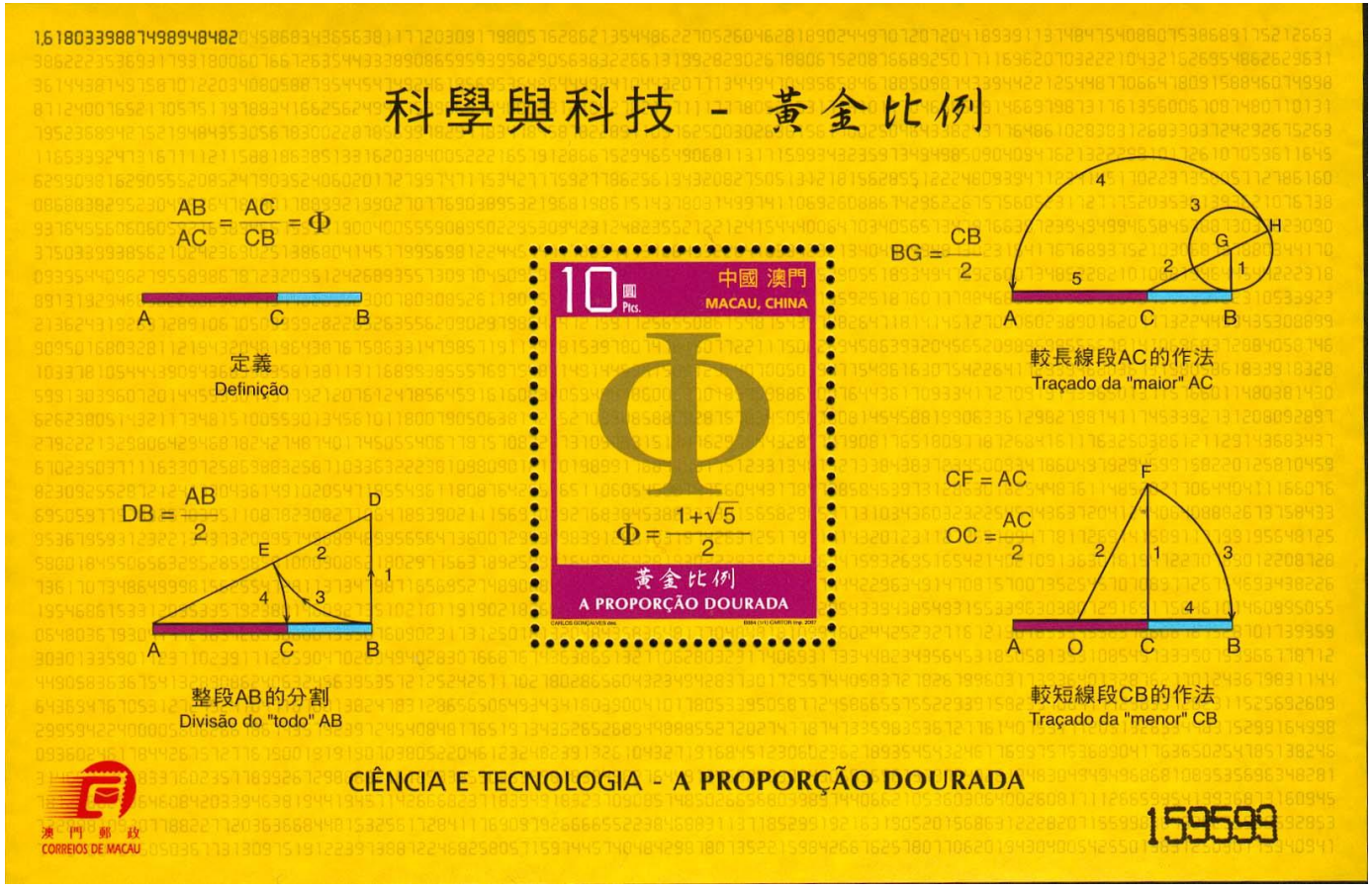
LANCE STIRLING
RBA

日本郵便
NIPPON
60

1.5
中國 澳門
MACAU, CHINA
黃金比例 • 費波納奇數列
A PROPORÇÃO DOURADA • SEQUÊNCIA DE FIBONACCI
2.0
中國 澳門
MACAU, CHINA
黃金比例 • 向日葵螺線
A PROPORÇÃO DOURADA • ESPIRAL DO GIRASSOL
3.5
中國 澳門
MACAU, CHINA
黃金比例 • 鸚鵡螺
A PROPORÇÃO DOURADA • NAUTILUS
2.5
中國 澳門
MACAU, CHINA
黃金比例 • 彭羅斯鑲嵌
A PROPORÇÃO DOURADA • LADRILHOS DE PENROSE

HELVETIA
Sika 1837-1987
80
STEPHAN BUNDI
1987
COURVOISIER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AB/AC = AC/CB = Φ
A C B
定義
Definição
10 圓 Pcs.
中國 澳門
MACAU, CHINA
Φ
Φ = (1+√5)/2
黃金比例
A PROPORÇÃO DOURADA
BG = CB/2
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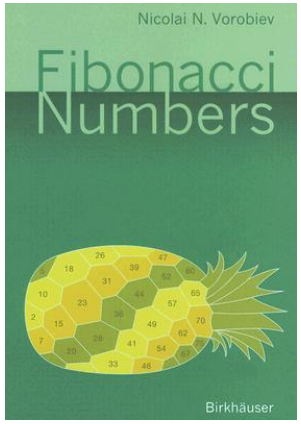
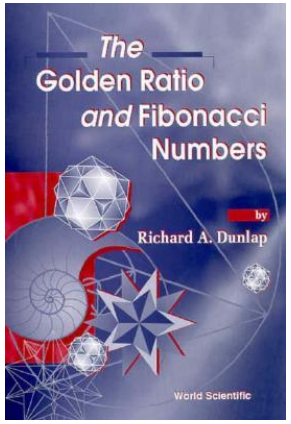
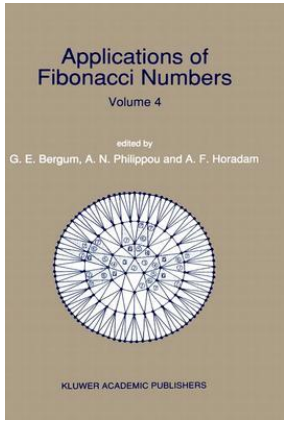
3
H
G
2
1
5
A C B
較長線段AC的作法
Traçado da "maior" AC
DB = AB/2
D
E
2
1
4
3
A C B
整段AB的分割
Divisão do "todo" AB
CF = AC
OC = AC/2
F
2
1
3
4
A O C B
較短線段CB的作法
Traçado da "menor" CB
CIÊNCIA E TECNOLOGIA - A PROPORÇÃO DOURADA
澳門郵政
CORREIOS DE MACAU
159599


Nicolai N. Vorobiev
Fibonacci Numbers
Birkhäuser


The Golden Ratio and Fibonacci Numbers
by
Richard A. Dunlap
World Scientific


Applications of Fibonacci Numbers
Volume 4
edited by
G. E. Bergum, A. N. Philippou and A. F. Horadam
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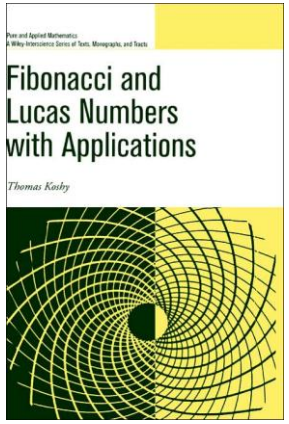
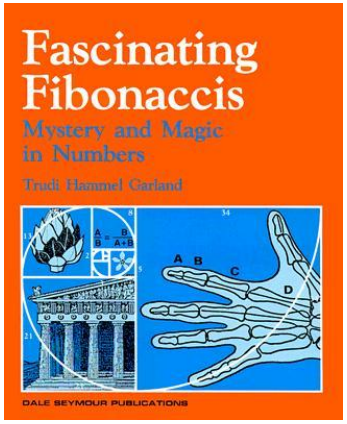

Fibonacci and Lucas Numbers with Applications
Thomas Koshy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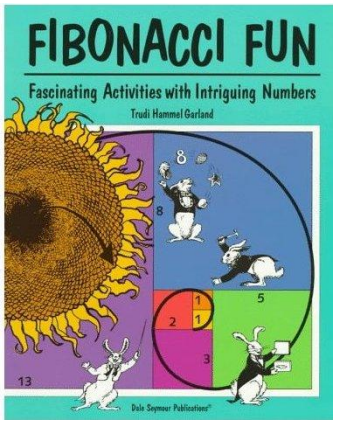
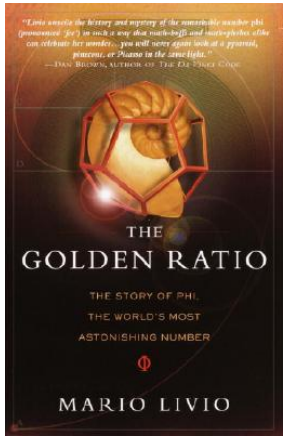
Mystery and Magic in Numbers
Trudi Hammel Garland
DALE SEYMOUR PUBLICATIONS


FIBONACCI FUN
Fascinating Activities with Intriguing Numbers
Trudi Hammel Garland
Dale Seymour Publications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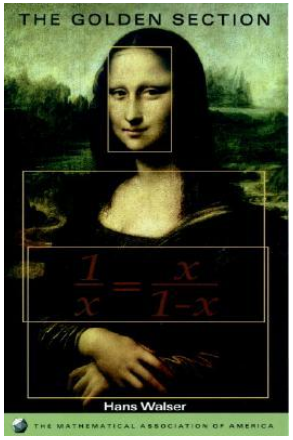
THE STORY OF PHI
THE WORLD'S MOST ASTONISHING NUMBER
MARIO LIVIO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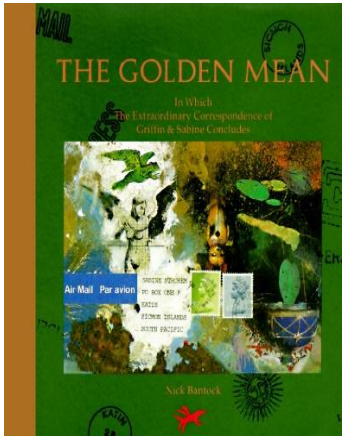
Hans Walser
THE MATHEMATICAL ASSOCIATION OF AMERICA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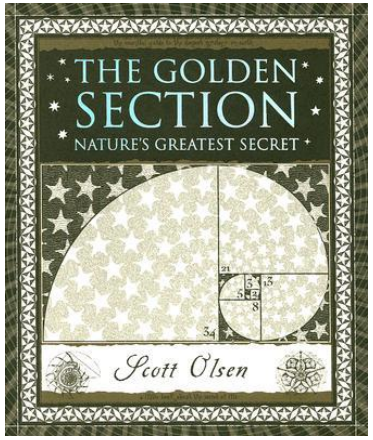
In Which
The Extraordinary Correspondence of Griffin & Sabine Concludes
Nick Bantock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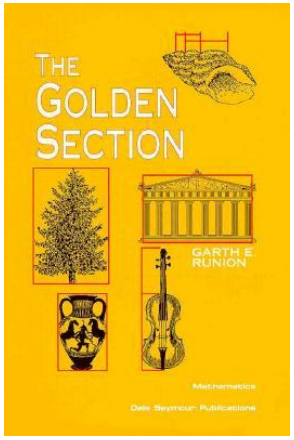
NATURE'S GREATEST SECRET
Scott Olsen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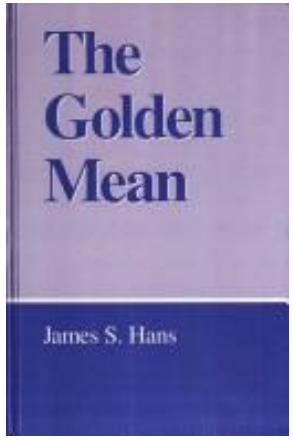
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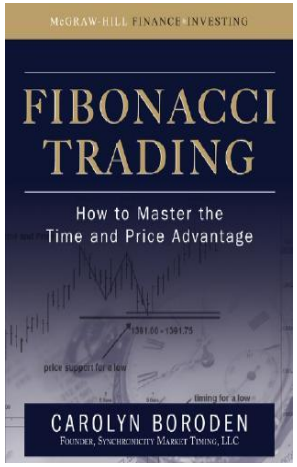
James S. Hans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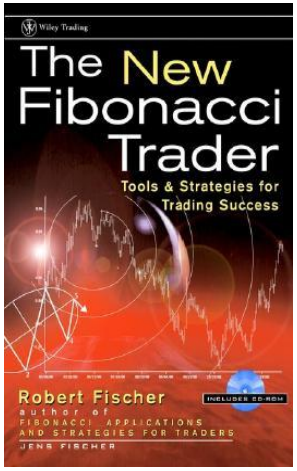
FIBONACCI TRADING
How to Master the Time and Price Advantage
CAROLYN BORODEN
Founder, Synchronicity Market Timing, LLC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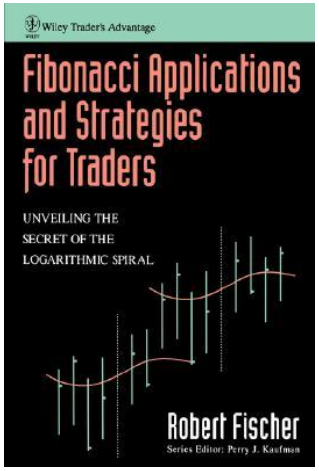
Tools & Strategies for Trading Success
Robert Fischer
author of
FIBONACCI APPLICATIONS AND STRATEGIES FOR TRADERS
JENS FISCHER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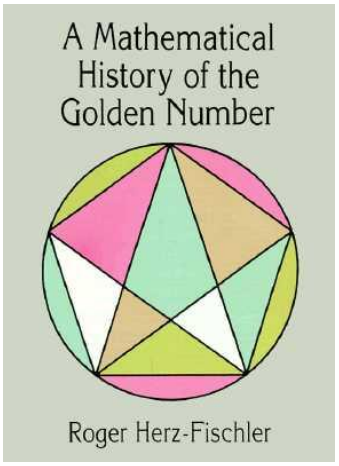
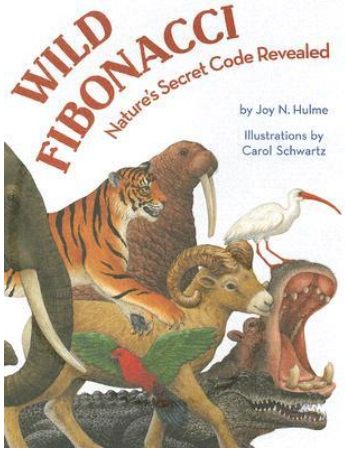
Fibonacci Applications and Strategies for Traders
UNVEILING THE SECRET OF THE LOGARITHMIC SPIRAL
Robert Fischer
Series Editor: Perry J. Kaufman

Bloomberg MARKET ESSENTIALS
TECHNICAL ANALYSIS
FIBONACCI ANALYSIS
Constance BROWN


A Mathematical History of the Golden Number
Roger Herz-Fischler


WILD FIBONACCI
Nature's Secret Code Revealed
by Joy N. Hulme
Illustrations by Carol Schwartz

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020547185549611808764 2 6576511060545881 47560443178479858 4
539731286301625448761 14 8520217064404111 66076695059775783 25
703951108782308271064 78 9390211156910392 76838453863333215658
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251797141432012311279551 89 4778172691415891 177991956481255800184 5
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466747593269516542140210913630 1819472270789012 208728736170734864999815625547
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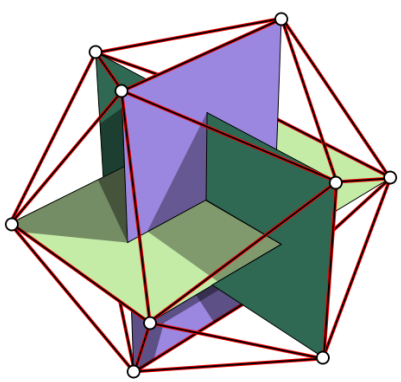
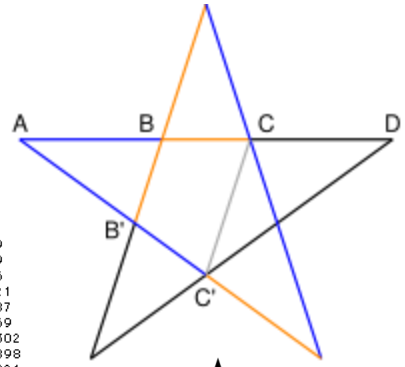
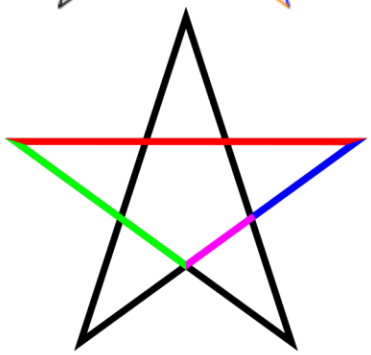
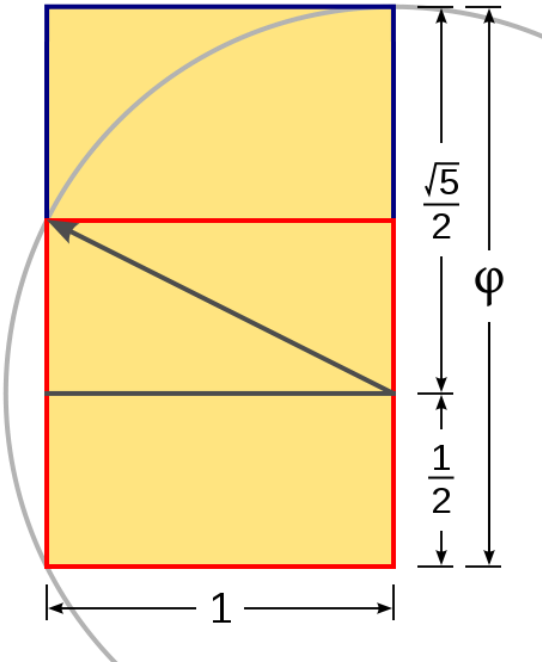
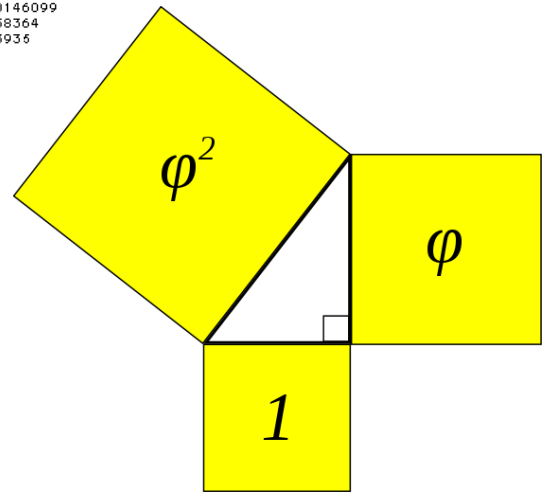
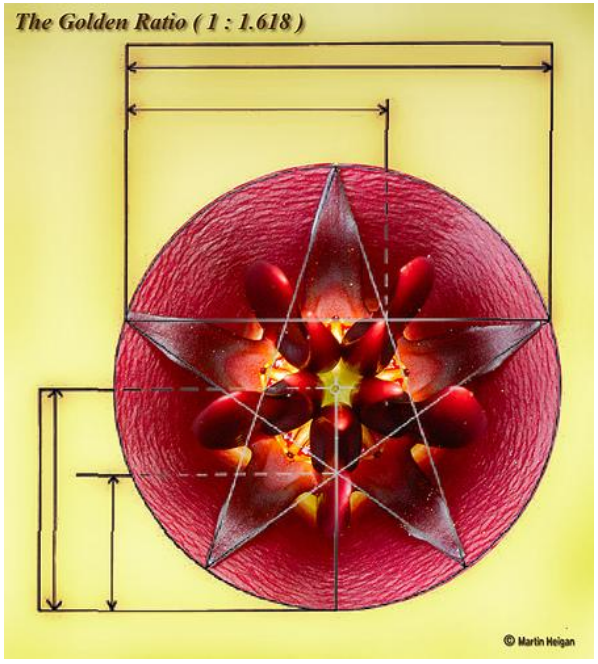










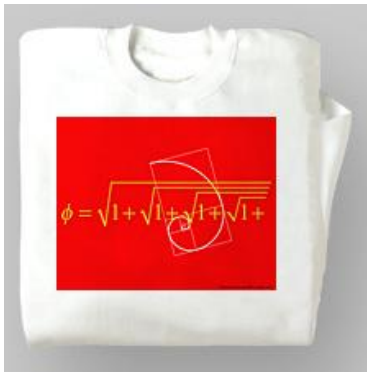


Fibonacci

235813
FIBONACCI
.618033
GOLDEN

1.6180339887498948482
04586834365638117720
30917980576286213544
86227052604628189024
4970720[illegible]89391137
4847540[illegible]38689175
2126633[illegible]35369317
9318006076[illegible]726354433
38908659593958290563
83226613199282902678
80675208766892501711
696207032221043216.....

$\phi = \frac{1}{\phi - 1}$
$\phi(\phi - 1) = 1$
$\phi^2 - \phi - 1 = 0$
$\phi = \frac{1+\sqrt{5}}{2}$
$\phi = 1.618033989...$


$\phi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+}}}}$

1
1
2
3
5
8
13
21

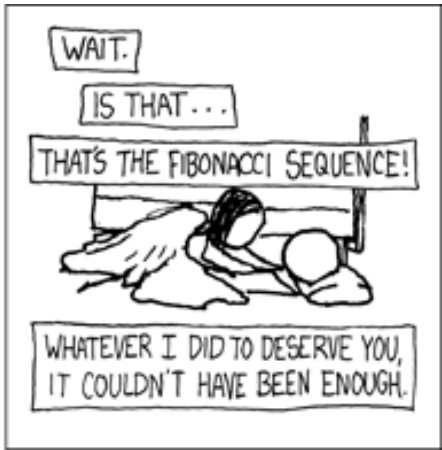
WANT MY NUMBER?
1.6180339887498948482...

(defun fib (n)
(if (< n 2)
n
(+ (fib (- n 2)) (fib (- n 1)))))

...1,2,3,5,8...
Fibonacci!

MADE IN THE USA

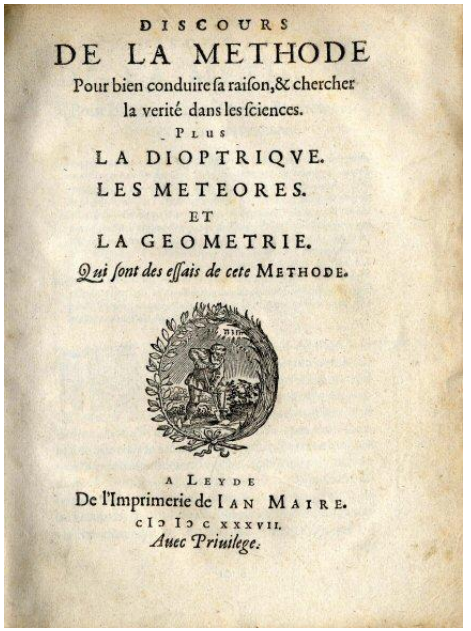
"This must be Fibonacci's."



Historical Perspectives

René Descartes (1596-1650)

- Father of modern philosophy
- Invented Cartesian coordinates, analytic geometry, heuristics
- Characterized paradoxes & falacies
- Discovered momentum conservation
- Authored "Principia philosophiae"
- Pioneered methodological skepticism "Cogito ergo sum" - "Je pense, donc je suis "
- "Discours de la méthode" (1637) - one of the most influential works in modern science
- Pioneered the scientific method & revolution


DISCOURS
DE LA METHODE
Pour bien conduire ſa raiſon,& chercher
la verité dans les ſciences.
PLUS
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui ſont des eſſais de cete METHODE.
A LEYDE
De l'Imprimerie de IAN MAIRE.
cIↃ IↃ c xxxvii.
Auec Priuilege.

"For it is not enough to have a good mind: one must use it well." - Descartes

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BANQUE DE FRANCE
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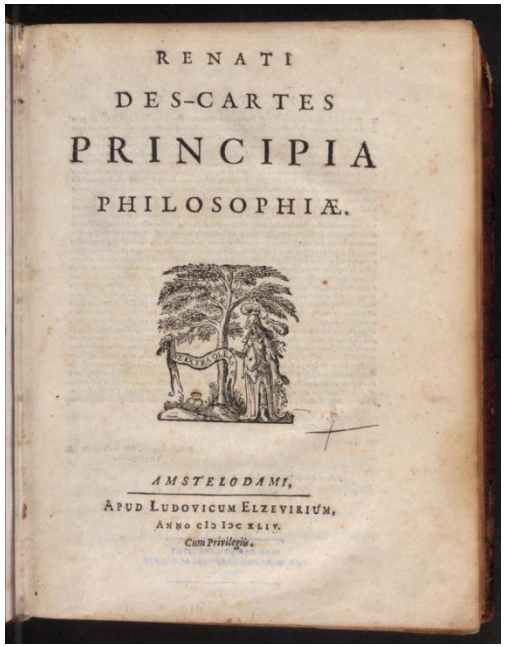
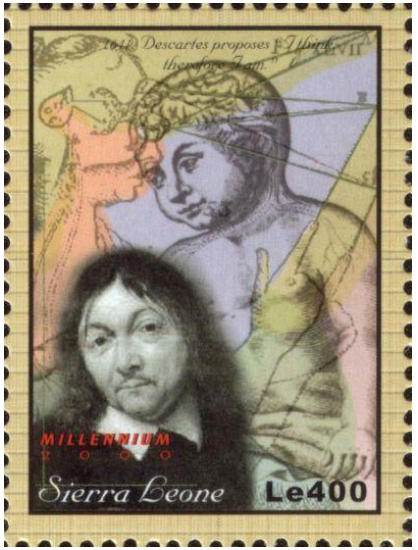
E.7
CENT
FRANCS
11736
LE CAISSIER GÉNÉRAL,
LE SECRÉTAIRE GÉNÉRAL,
11736
G.15-5-1942.G.
E.7

COGITO, ERGO SUM!


RENATI
DES-CARTES
PRINCIPIA
PHILOSOPHIÆ.
AMSTELODAMI,
APUD LUDOVICUM ELZEVIRIUM,
ANNO CIↃ IↃC XLIV.
Cum Privilegio.

DESCARTES
Rene
Cartesins
1596-1650
85
LEKE
FILOZOFE DHE MATEMATIKANE TE SHQUAR
SHQIPERIA
M. TEMO
1996
A. MATSOUKIS S.A.

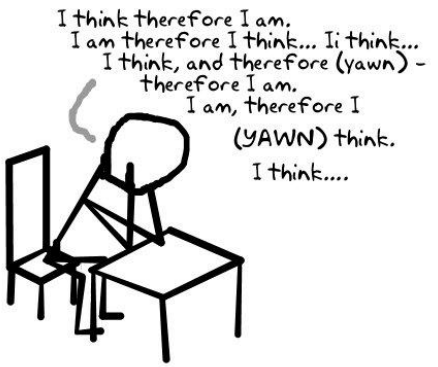
5,00
MONACO
DESCARTES René
1596-1996


MILLENNIUM
Sierra Leone
Le400

MILLENNIUM
Grenada
50¢

I am a think tank, therefore I am.

COGITO
ERGO
OBAMA


I think therefore I am.
I am therefore I think... Ii think...
I think, and therefore (yawn) -
therefore I am.
I am, therefore I
(YAWN) think.
I think....

ZZZZzzzz
ZZzzzzz.

So, If Rene Descartes never blogged... Did he really exist?
Pirillo & Fitz
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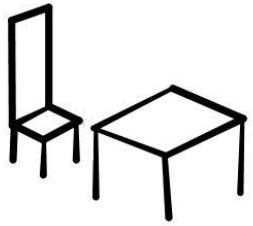
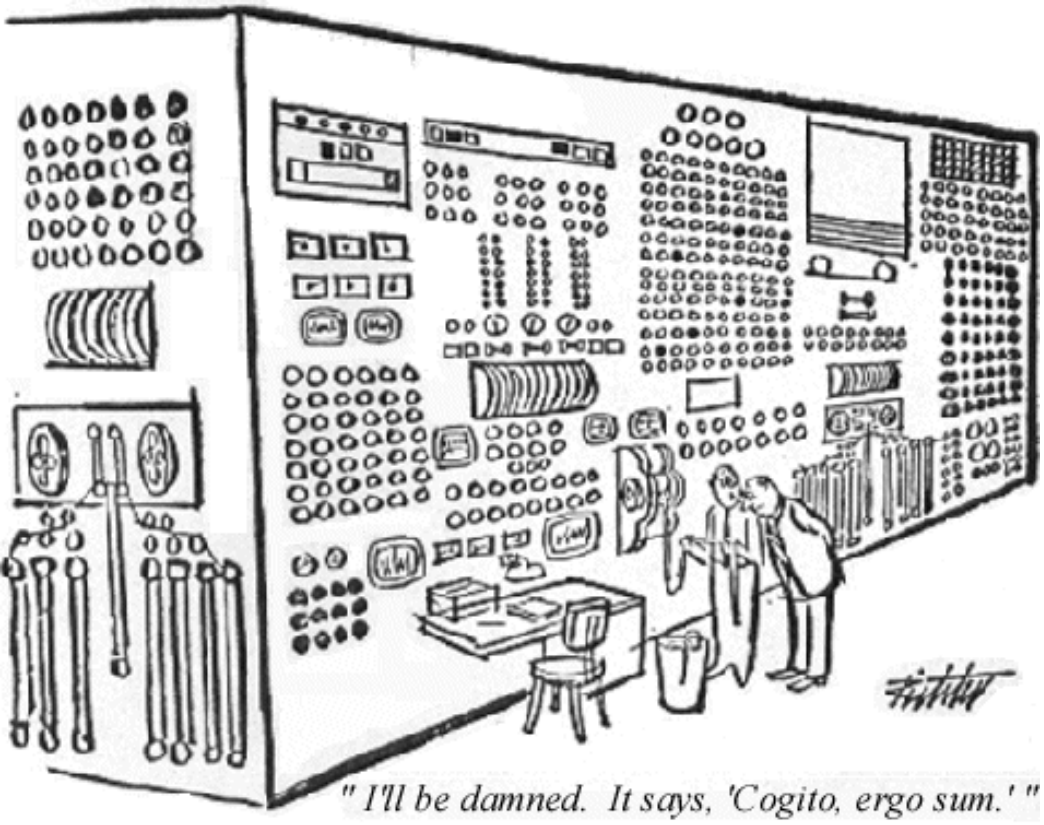
RUE
RENE DESCARTES

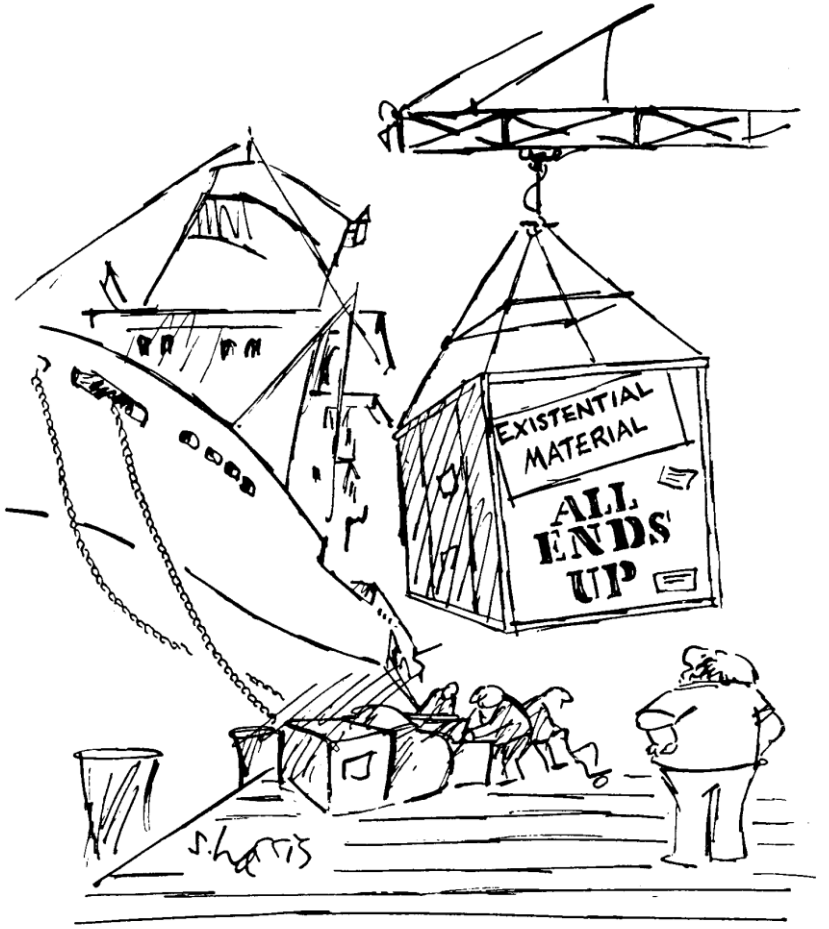
POP!

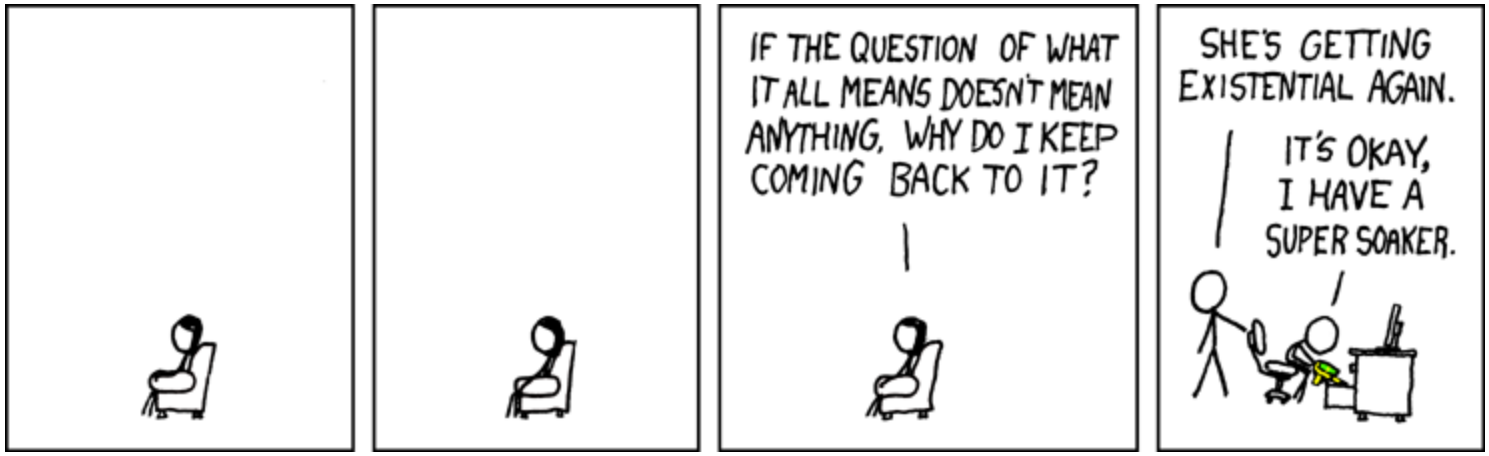


DJL



" I'll be damned. It says, 'Cogito, ergo sum.' "



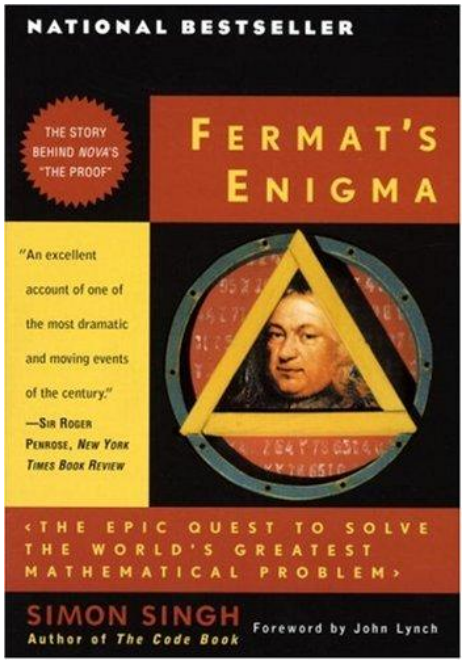


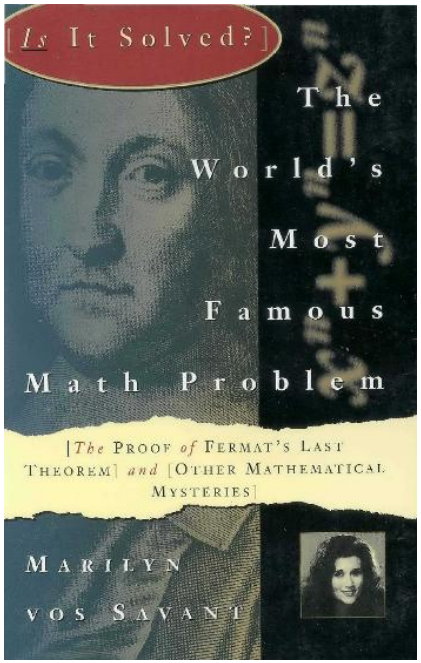
René Descartes 1596 - 1650

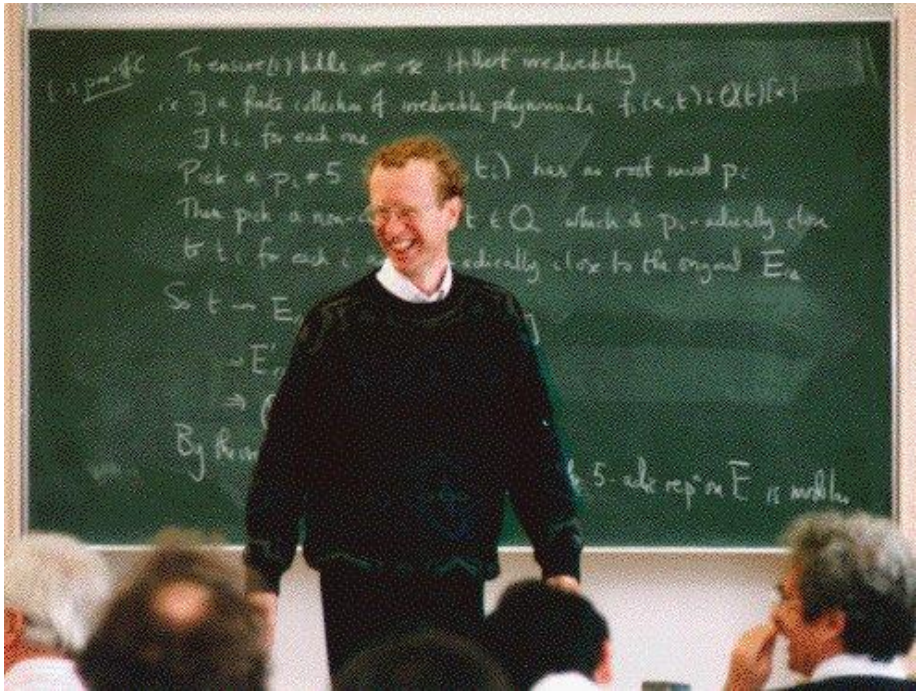
Historical Perspectives

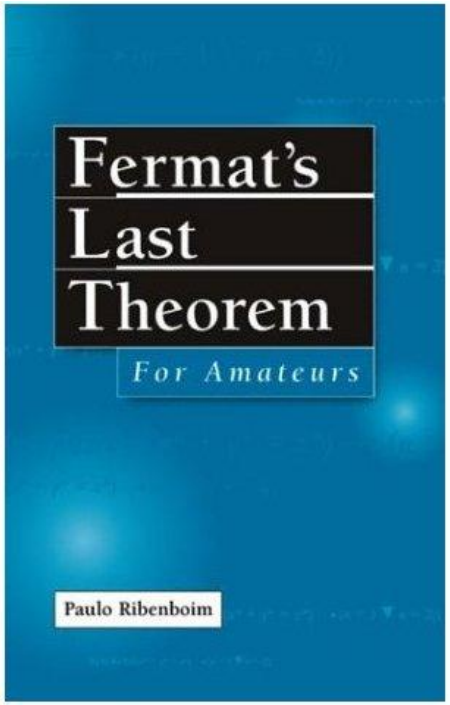
Pierre de Fermat (1601-1665)

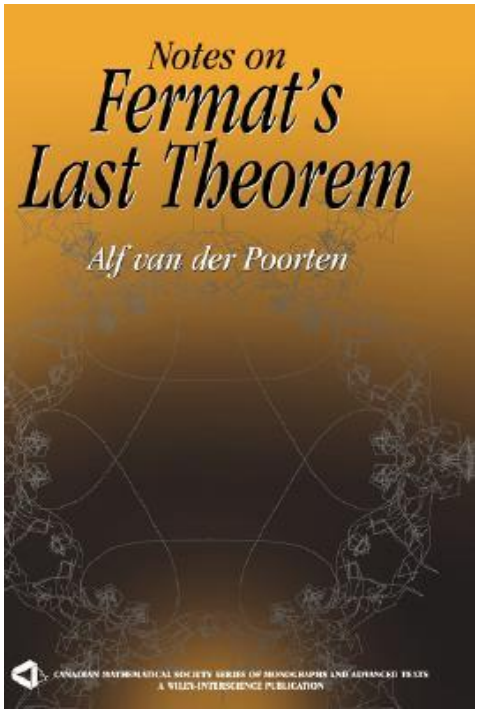
- Father of modern number theory
- Lawyer, Parlement of Toulouse
- Laid groundwork for calculus
- Contributions to optics, probability, and analytic geometry
- Fermat numbers, primes, perfect #'s
- Descartes' Law of refraction
- Reponsible for many open problems
- "Fermat's Last Theorem" (1637-1995)
- Recognized "principle of least action" and "principle of least time" in physics
- Influenced Newton and Leibniz

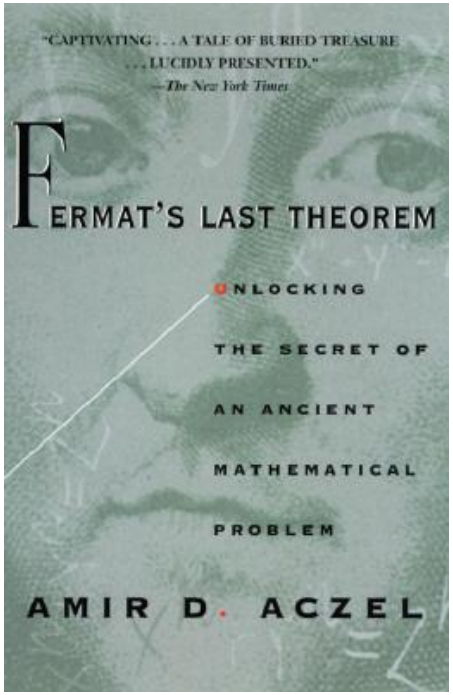

NATIONAL BESTSELLER
THE STORY BEHIND NOVA'S "THE PROOF"
FERMAT'S ENIGMA
"An excellent account of one of the most dramatic and moving events of the century."
—SIR ROGER PENROSE, NEW YORK TIMES BOOK REVIEW
‹THE EPIC QUEST TO SOLVE THE WORLD'S GREATEST MATHEMATICAL PROBLEM›
SIMON SINGH
Author of The Code Book
Foreword by John Lynch


[Is It Solved?]
The World's Most Famous Math Problem
[The Proof of Fermat's Last Theorem] and [Other Mathematical Mysteries]
MARILYN VOS SAVANT




Fermat's Last Theorem
For Amateurs
Paulo Ribenboim


Notes on
Fermat's Last Theorem
Alf van der Poorten


"CAPTIVATING . . . A TALE OF BURIED TREASURE . . . LUCIDLY PRESENTED."
—The New York Times
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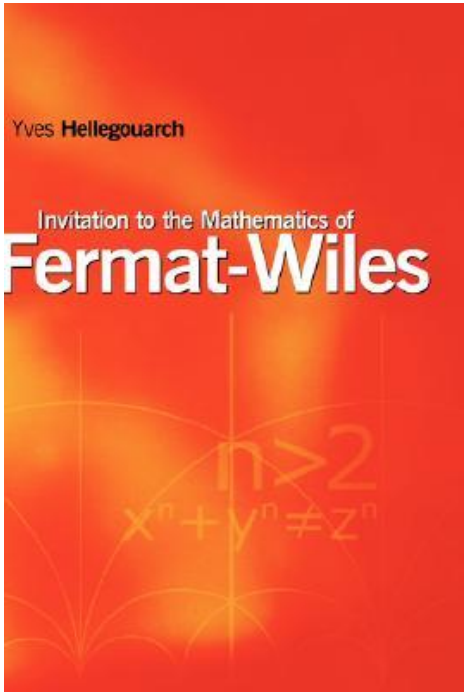
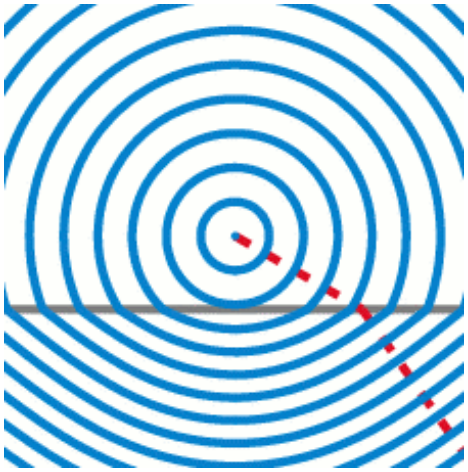
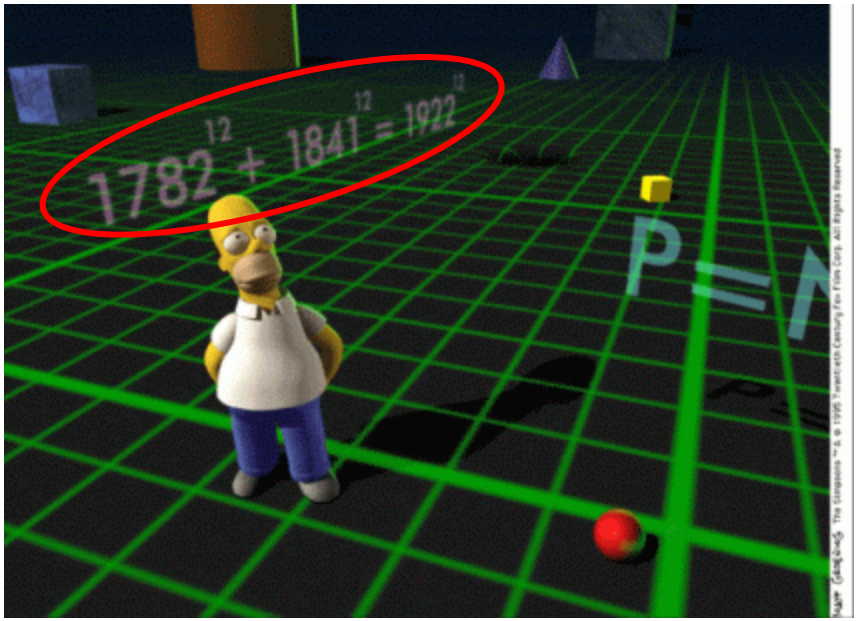
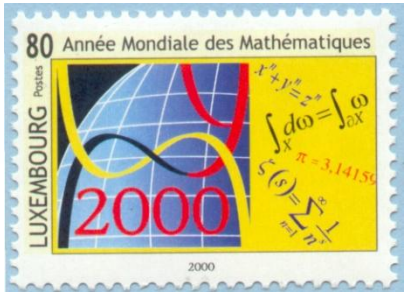
UNLOCKING THE SECRET OF AN ANCIENT MATHEMATICAL PROBLEM
AMIR D. ACZEL


Yves Hellegouarch
Invitation to the Mathematics of
Fermat-Wiles

Pierre de Fermat
1601 - 1665







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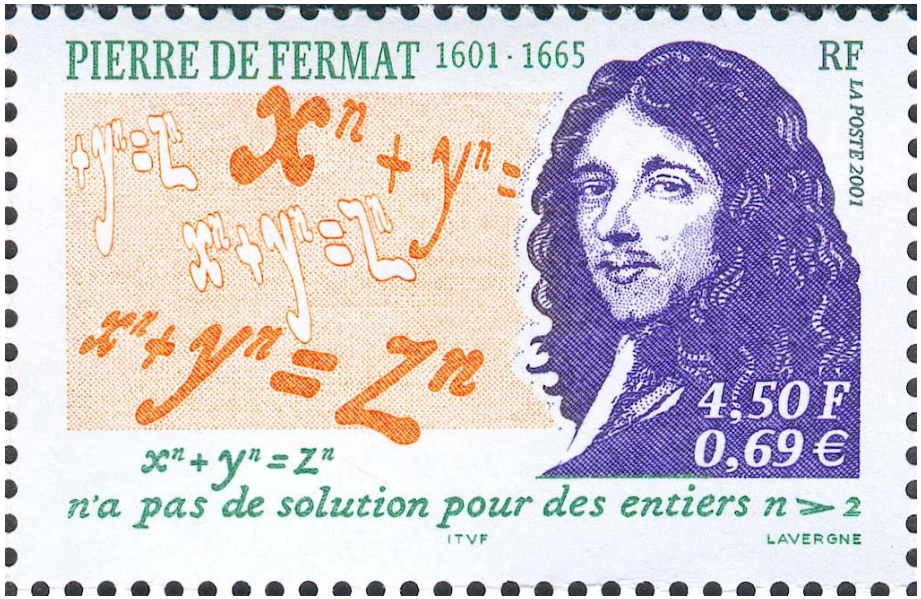
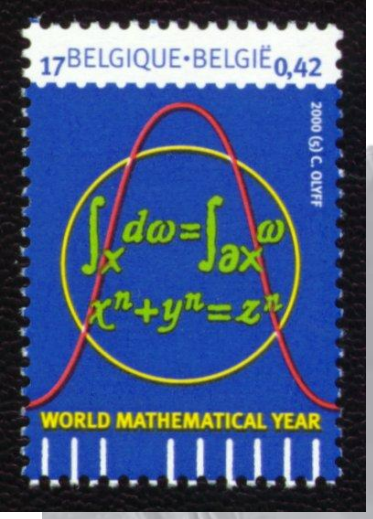
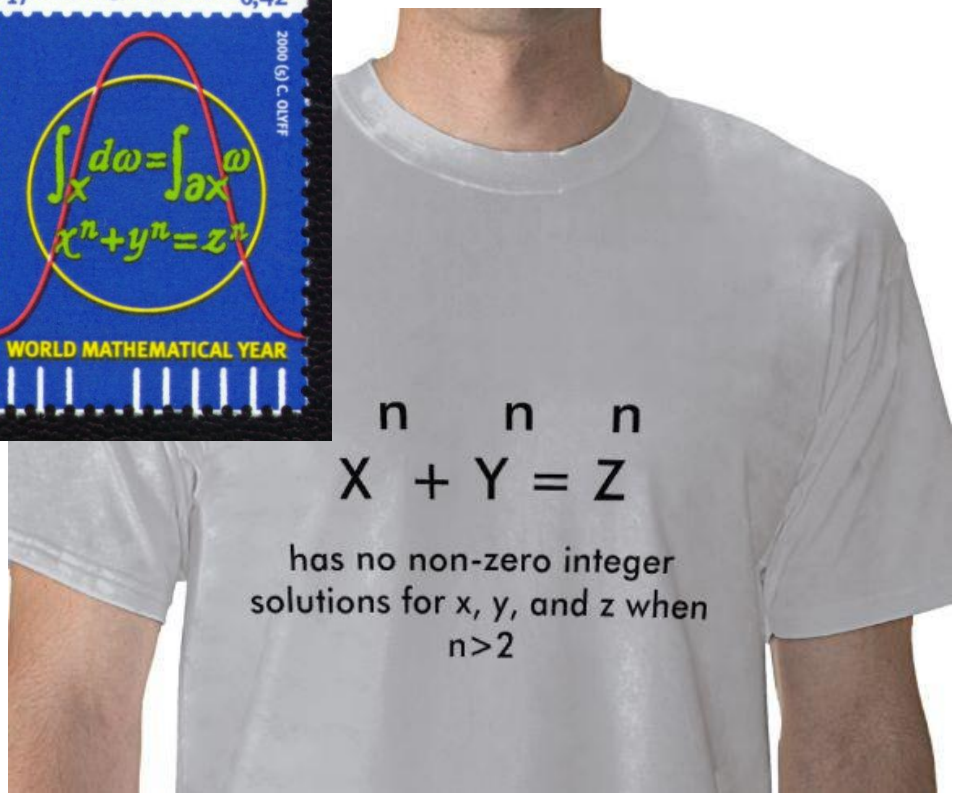
INSTITUT de MATHEMATIQUES de TOULOUSE

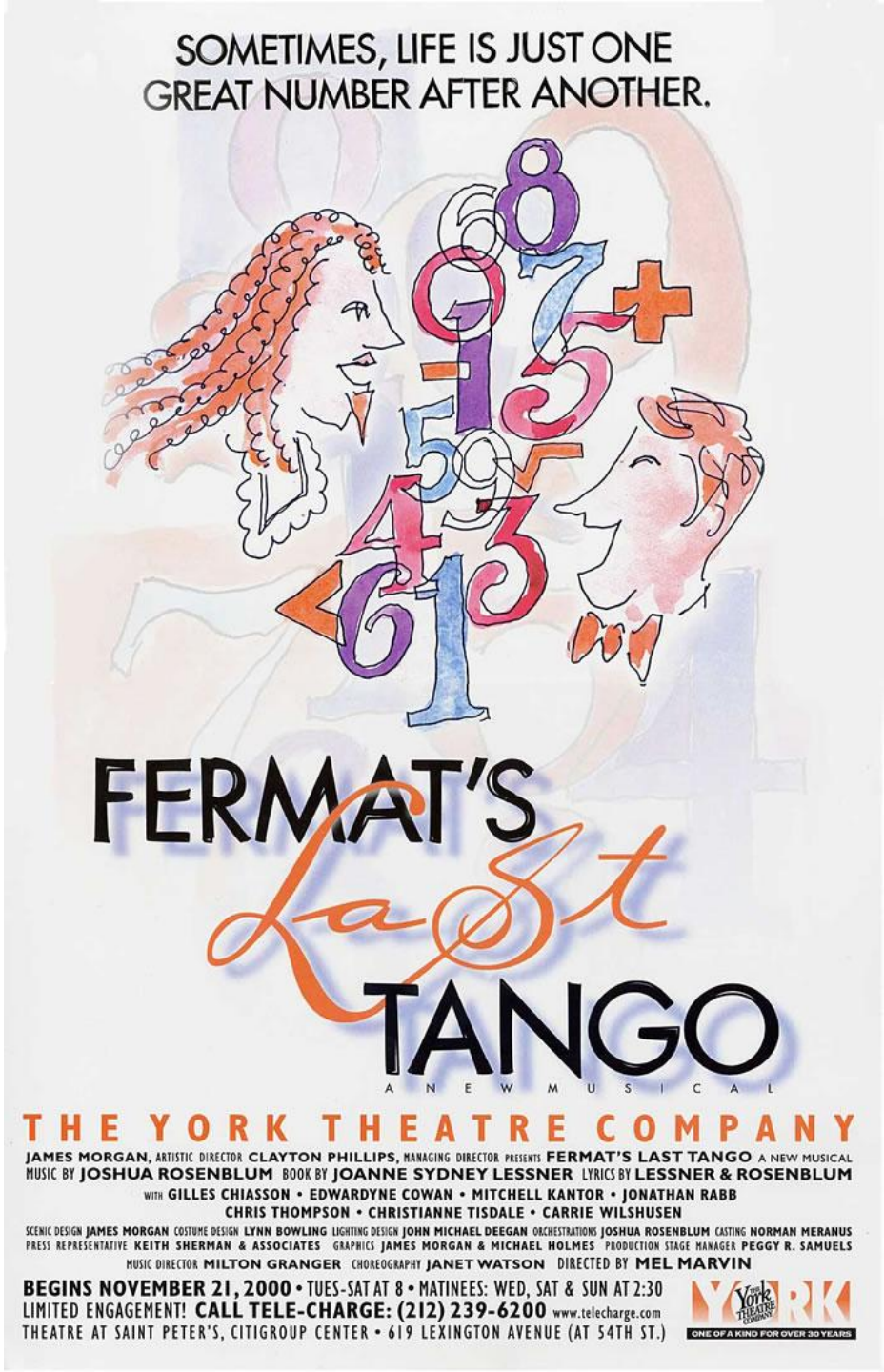
Fermat Prize for Mathematics Research




17 BELGIQUE·BELGIË 0,42
2000 (c) C. OLYFF
$\int_X d\omega = \int_{\partial X} \omega$
$x^n + y^n = z^n$
WORLD MATHEMATICAL YEAR


n n n
X + Y = Z
has no non-zero integer solutions for x, y, and z when n>2

FERMAT
39 km
97/10/08
D=254mm f/D=10
Moon LIGHT
© António J. Cidadão
B/W QuickCam
a.cidadao@mail.telepac.pt


SOMETIMES, LIFE IS JUST ONE GREAT NUMBER AFTER ANOTHER.
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JAMES MORGAN, ARTISTIC DIRECTOR CLAYTON PHILLIPS, MANAGING DIRECTOR PRESENTS FERMAT'S LAST TANGO A NEW MUSICAL
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FERMAT'S Last TANGO

A NEW MUSICAL

THE YORK THEATRE COMPANY

A Musical Fantasy inspired by Andrew Wiles and his encounters with Fermat's Last Theorem

"Rollicking! Whimsical! Catchy & Clever!" - The New York Times

FERMAT'S LAST TANGO

Followed by an Interview with Andrew Wiles

A CMI production

VHS

In 1993 Andrew Wiles stunned the world when he announced a solution to "Fermat's Last Theorem," the famous unsolved mathematics problem set forth by Pierre de Fermat in 1637. In the musical *Fermat's Last Tango*, the fictional character Daniel Keane earns overnight acclaim when he presents his findings. However, fanfare soon gives way to doubt when the reincarnated Fermat discovers a hole in Keane's proof. The singular pursuit by Keane to correct this flaw results in a love triangle involving himself, his wife, and mathematics—the story of which is brought to life by Fermat and his immortal friends from the "AfterMath," namely: Pythagoras, Euclid, Newton, and Gauss. The musical is both a cheerful romp through history and a personal confrontation with destiny. It provides a testament to the extraordinary excitement of mathematics and its unparalleled beauty.

The Composer Joshua Rosenblum enjoyed mathematics while studying music at Yale along with the author, his wife Joanne Sydney Lessner. They both take an active role in the New York music community. This recording was captured by David Stern and his Emmy Award-winning crew during a performance at the York Theatre Company in New York City.

STARRING

Carl Friedrich Gauss / Reporter	GILLES CHIASSON
Anna Keane	EDWARDYNE COWAN
Pythagoras / Reporter	MITCHELL KANTOR
Pierre de Fermat	JONATHAN RABB
Daniel Keane	CHRIS THOMPSON
Euclid / Reporter	CHRISTIANNE TISDALE
Sir Isaac Newton / Reporter	CARRIE WILSHUSEN

Approximate Running Time:
100 minutes
Color/Not Rated/VHS/NTSC
Produced by The Clay Mathematics Institute, Cambridge, MA
Arthur Jaffe, *Producer*
David Stern, *Director*
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Illustrated Guide Enclosed

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Cambridge, MA 02140
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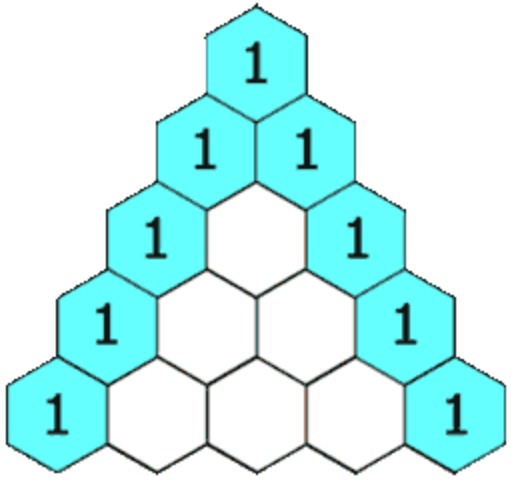
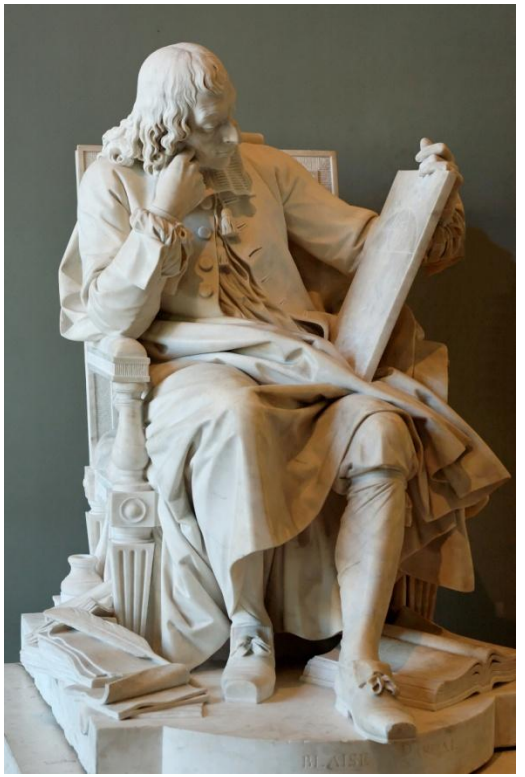
Historical Perspectives

Blaise Pascal (1623-1662)

- Mathematician, physicist, philosopher
- Studied fluids, pressure, vacuum
- Helped pioneer projective geometry, probability, and the scientific method
- Influenced modern economics
- "Pascal's triangle", "Pascal's law"
- Invented hydraulic press and syringe
- Constructed a mechanical calculator
- Used humor, wit, and satire in writings
- Influenced Voltaire and Rousseau
- Inagurated the world's first bus line
- SI unit of pressure - "pascal"





THE

PROVINCIAL LETTERS:

MORAL TEACHINGS OF THE JESUIT FATHERS
OPPOSED TO THE CHURCH OF ROME
AND LATIN VULGATE.

BY

Blaise Pascal.

"I speak as to wise men, judge ye what I say."—1 Cor. x. 15.

Translated from the original French into various languages.

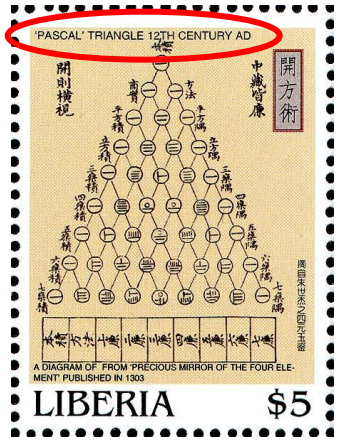
Paris.

Vienna. London.
Cologne. Edinburgh.
Basle. Glasgow.

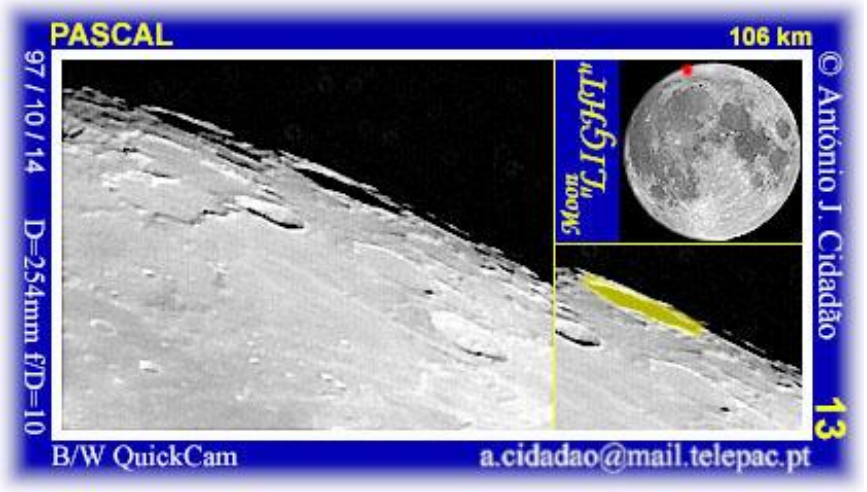
0,50
PASCAL
1623-1662
REPUBLIQUE FRANÇAISE
POSTES

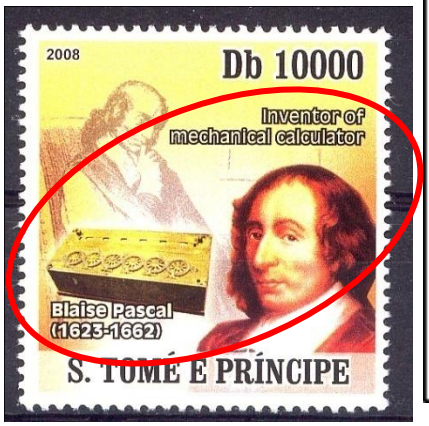
BLAISE PASCAL
1623-MONACO-1973 0,50

1623 PASCAL 1662
POSTES 1f20+2f80
FRANCE

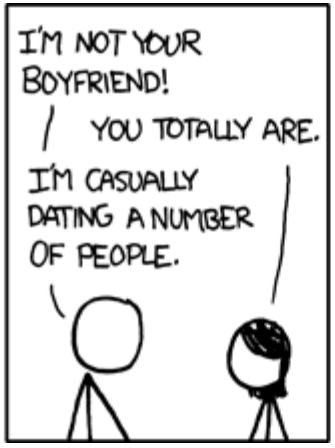

'PASCAL' TRIANGLE 12TH CENTURY AD
A DIAGRAM OF 'FROM 'PRECIOUS MIRROR OF THE FOUR ELEMENT' PUBLISHED IN 1303
LIBERIA
$5

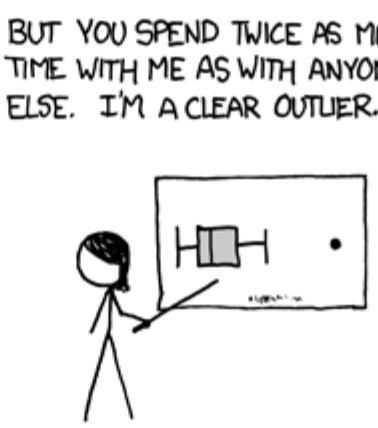
The heart has its reasons which reason cannot know
Blaise Pascal

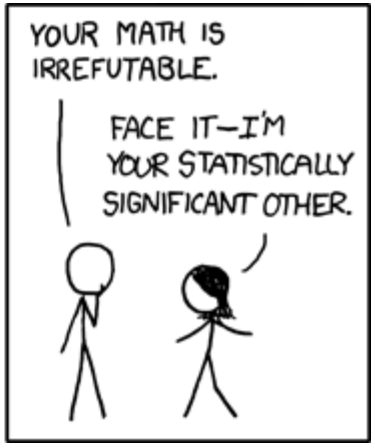

PASCAL
106 km
97/10/14 D=254mm f/D=10
"LIGHT" moon
© António J. Cidadão
13
B/W QuickCam
a.cidadao@mail.telepac.pt


2008
Db 10000
Inventor of mechanical calculator
Blaise Pascal (1623-1662)
S. TOMÉ E PRÍNCIPE

CAN MY BOYFRIEND COME ALONG?


I'M NOT YOUR BOYFRIEND!
YOU TOTALLY ARE.
I'M CASUALLY DATING A NUMBER OF PEOPLE.

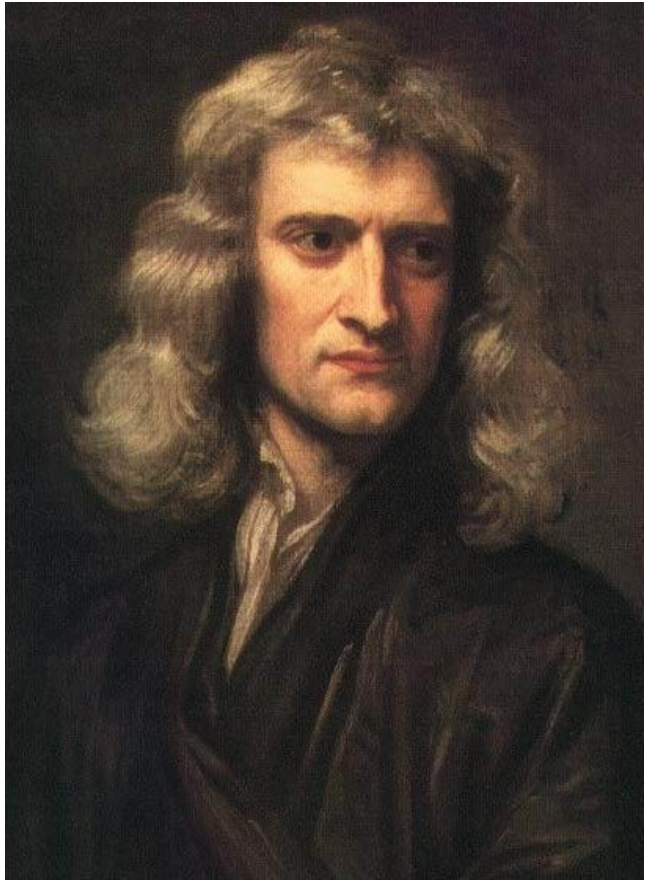

BUT YOU SPEND TWICE AS MUCH TIME WITH ME AS WITH ANYONE ELSE. I'M A CLEAR OUTLIER.

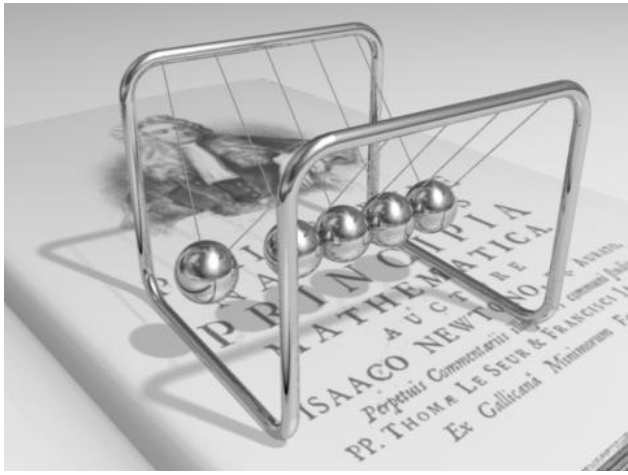

YOUR MATH IS IRREFUTABLE.
FACE IT—I'M YOUR STATISTICALLY SIGNIFICANT OTHER.

Historical Perspectives

Sir Isaac Newton (1643-1727)

- Mathematician, physicist, astronomer, philosopher, alchemist, theologian
- One of history's most influential people
- "Principia Mathematica" (1687)
- Invented calculus, theory of gravitation
- Founded "Newtonian mechanics"
- Discovered laws of motion, inertia
- "Newtonian fluid", "Newtonian Universe"
- Advanced the Scientific Revolution
- Developed practical reflecting telescope, theory of color, "Newton's method"
- SI unit of force: newton





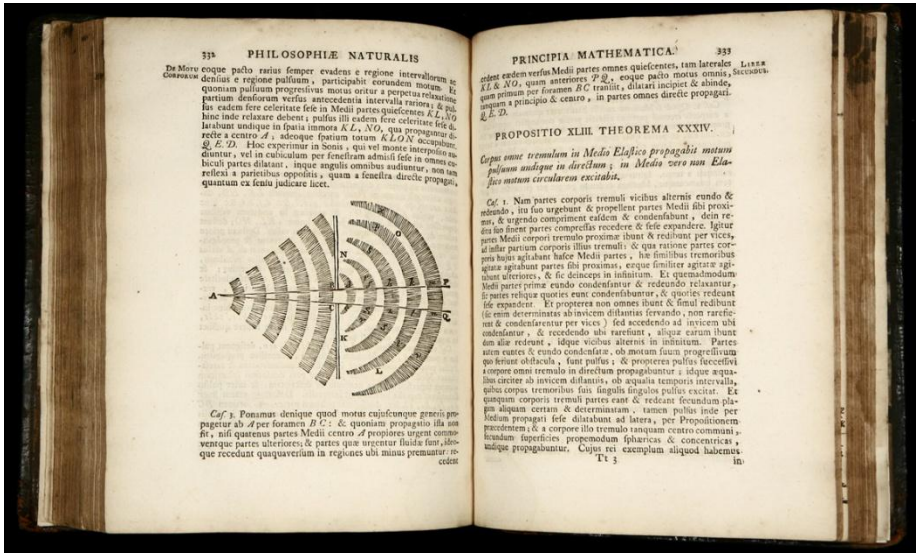
PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.



332 PHILOSOPHIÆ NATURALIS

De Motu Corporum

eoque pacto rarius semper evadens e regione intervallorum ac densius e regione pulsuum, participabit eorundem motum. Et quoniam pulsuum progressivus motus oritur a perpetua relaxatione partium densiorum versus antecedentia intervalla rariora; & pulsus eadem fere celeritate sese in Medii partes quiescentes *KL*, *NO* inde relaxare debent; pulsus illi eadem fere celeritate sese dirigent undique in spatia immota *KL*, *NO*, qua propagantur directe a centro *A*, adeoque spatium totum *KLON* occupabunt. Hoc experimur in sonis, qui vel monte interposito audiuntur, vel in cubiculum per fenestram admissi sese in omnes cubiculi partes dilatant, inque angulis omnibus audiuntur, non tam reflexi a parietibus oppositis, quam a fenestra directe propagati, quantum ex sensu judicare licet.

Cas 3. Ponamus denique quod motus cujuscunque generis propagetur ab *A* per foramen *BC*: & quoniam propagatio illa non fit, nisi quatenus partes Medii centro *A* propiores urgent commoventque partes ulteriores; & partes quæ urgentur fluidæ sunt, idoque recedunt quaquaversum in regiones ubi minus premuntur: recedent

PRINCIPIA MATHEMATICA. 333

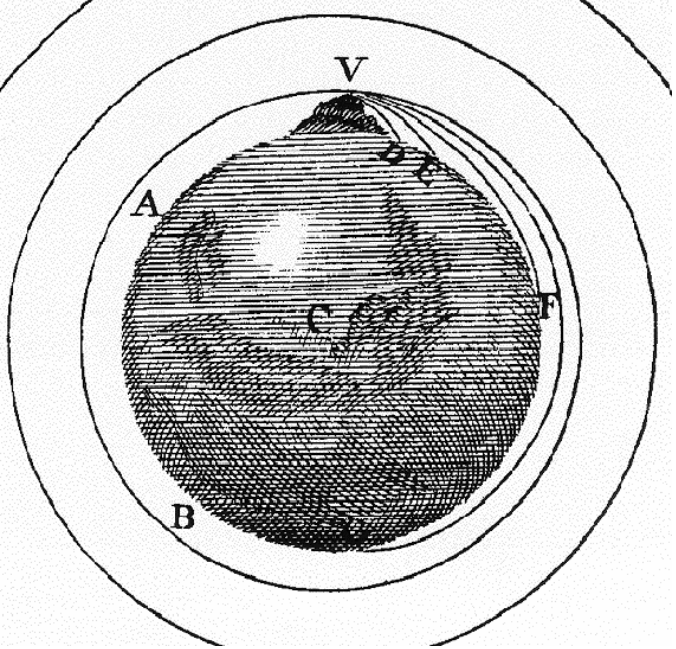
cedent eædem versus Medii partes omnes quiescentes, tam laterales *KL* & *NO*, quam anteriores *PQ*, eoque pacto motus omnis, quousque a principio & centro, in partes omnes directe propagari. LIBER SECUNDUS.

Q. E. D.

PROPOSITIO XLIII. THEOREMA XXXIV.

Corpus omne tremulum in Medio Elastico propagabit motum pulsuum undique in directum; in Medio vero non Elastico motum circularem excitabit.

Cas 1. Nam partes corporis tremuli vicibus alternis eundo & redeundo, ita suo urgebunt & propellent partes Medii sibi proximas, & urgendo compriment easdem & condensabunt, dein redeuntes sinent partes compressas recedere & sese expandere. Igitur partes Medii corpori tremulo proximæ ibunt & redibunt per vices, ad instar partium corporis illius tremuli: & qua ratione partes corporis hujus agitabant hasce Medii partes, hæ similibus tremoribus agitatæ agitabunt partes sibi proximas, eoque similiter agitatæ agitabunt ulteriores, & sic deinceps in infinitum. Et quemadmodum Medii partes primæ eundo condensantur & redeundo relaxantur, sic partes reliquæ quoties eunt condensabuntur, & quoties redeunt sese expandent. Et propterea non omnes ibunt & simul redibunt (sic enim determinatas ab invicem distantias servando, non rarefierent & condensarentur per vices) sed accedendo ad invicem ubi condensantur, & recedendo ubi rarefiunt, aliquæ earum ibunt dum aliæ redeunt, idque vicibus alternis in infinitum. Partes autem euntes & eundo condensatæ, ob motum suum progressivum quo feriunt obstacula, sunt pulsus; & propteres pulsus successivi a corpore omni tremulo in directum propagabuntur: idque æqualibus circiter ab invicem distantiis, ob æqualia temporis intervalla, quibus corpus tremoribus suis singulis singulos pulsus excitat. Et quanquam corporis tremuli partes eant & redeant secundum plagam aliquam certam & determinatam; tamen pulsus inde per Medium propagati sese dilatabunt ad latera, per Propositionem præcedentem, & a corpore illo tremulo tanquam centro communi, secundum superficies propemodum sphæricas & concentricas, undique propagabuntur. Cujus rei exemplum aliquod habemus in



Arithmetica Universalis;

SIVE

DE COMPOSITIONE

ET

RESOLUTIONE

ARITHMETICA

LIBER.

Cui accessit

HALLEIANA

Æquationum Radices Arithmetice inveniendi methodus.

In Usum Juventutis Academicæ.

CANTABRIGIÆ

TYPIS ACADEMICIS.

LONDINI, Impensis *Benj. Tooke* Bibliopolæ juxta Medii Templi Portam in vico vulgo vocato *Fleetstreet*. A.D. MDCCVII.

OPTICKS:

OR, A

TREATISE

OF THE

REFLEXIONS, REFRACTIONS, INFLEXIONS and COLOURS

OF

LIGHT.

ALSO

TWO TREATISES

OF THE

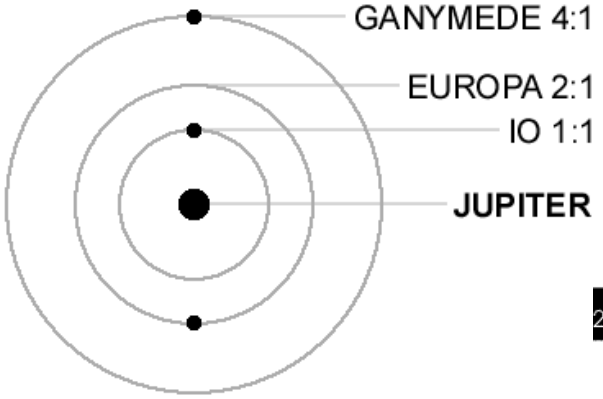
SPECIES and MAGNITUDE

OF

Curvilinear Figures.

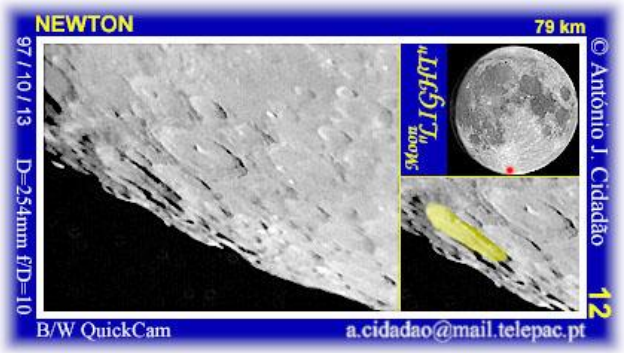
LONDON,

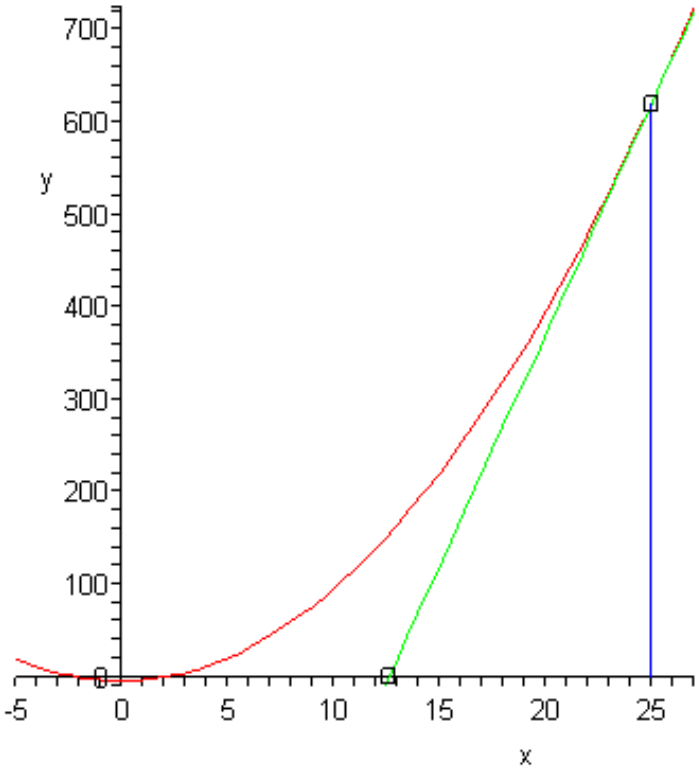
Printed for SAM. SMITH, and BENJ. WALFORD, Printers to the Royal Society, at the *Prince's Arms* in St. *Paul's* Church-yard. MDCCIV.



2m

m




700
600
y
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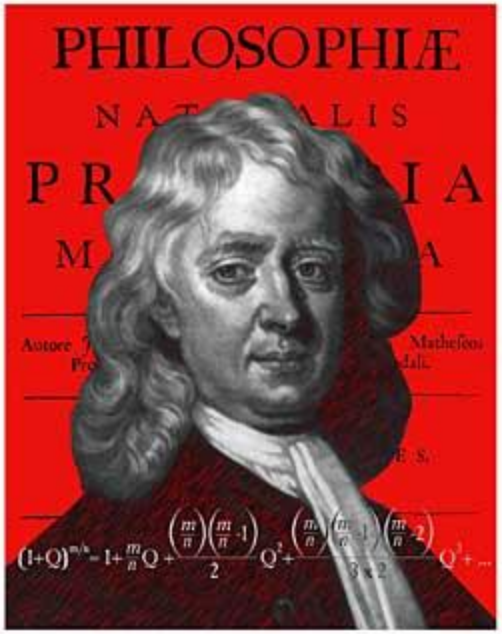
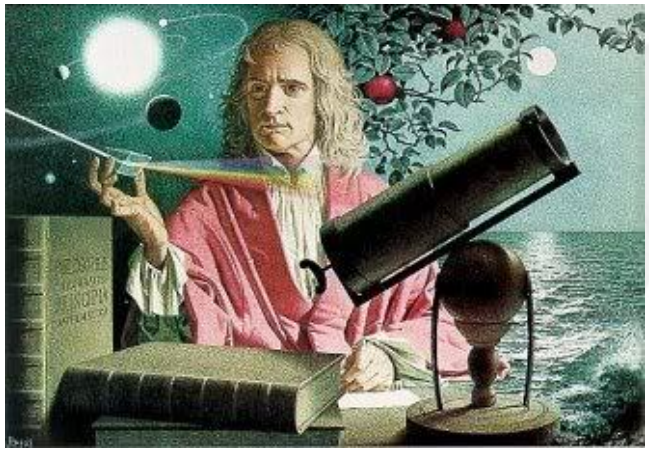
400
300
200
100
0
-5
0
5
10
15
20
25
x

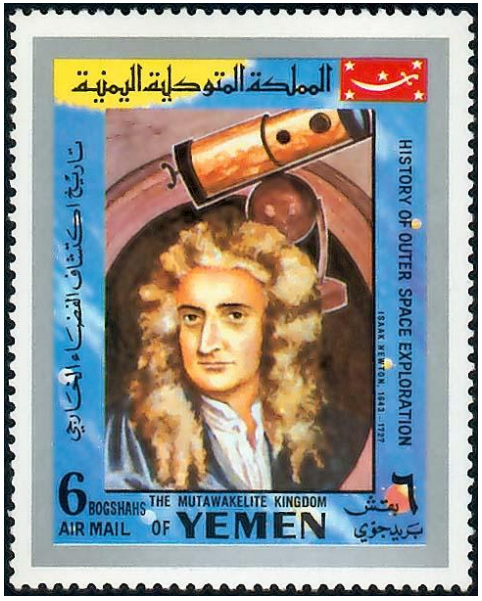
JOHANNES KEPLER'S UPHILL BATTLE
...SO, YOU SEE, THE ORBIT OF A PLANET IS ELLIPTICAL.
WHAT'S AN ORBIT?
WHAT'S A PLANET?
WHAT'S 'ELLIPTICAL'?
J. harris



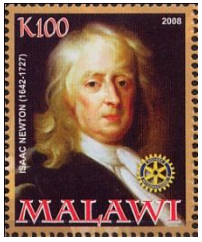
Sir Isaac Newton 1642 [Julian New Style Calendar] - 1727



ONE POUND
£1
Sir Isaac Newton
1642 - 1727


المملكة المتوكلية اليمنية
تاريخ اكتشاف الفضاء الخارجي
HISTORY OF OUTER SPACE EXPLORATION
ISAAK NEWTON, 1643 - 1727
6 BOGSHAHS
AIR MAIL
THE MUTAWAKELITE KINGDOM OF YEMEN
بريد جوي

REPUBLIQUE DU NIGER
100F
LUNA 16 — Sept. 1970
PREMIERS PRELEVEMENTS AUTOMATIQUES SUR LA LUNE
POSTE AERIENNE
150F
1970
PRECURSEUR DE L'ESPACE - ISAAC NEWTON - 1642 - 1727


K100
2008
ISAAC NEWTON (1642-1727)
MALAWI

REDONDA
20¢
SIR ISAAC NEWTON
CRANE COURT, HOME OF THE ROYAL SOCIETY

REPUBLIQUE POPULAIRE DU CONGO
140F
POSTE AERIENNE
1977
ISAAC NEWTON 1642.1727

1 RIVAL
AIRMAIL
SIR ISAAC NEWTON, F.R.S. 1642-1727
دبي
DUBAI

1 00
ПОЩА БЪЛГАРИЯ
BULGARIA
ИСАК НЮТОН
1643
1727
F=k mM/r²
ГРУЕВ 1993

IZAAK NEWTON
1642-1727
POLSKA
1 ZŁ 55
STCZ-CHŁUDZIŃSKI PWPW B.KOWALSKA SC

ISAAC NEWTON
1643-1727
Δ(mv)=FΔt
DEUTSCHE
BUNDESPOST
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1993

VIỆT NAM
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BƯU CHÍNH
Í-XÁC NÍU-TƠN
1642-1727
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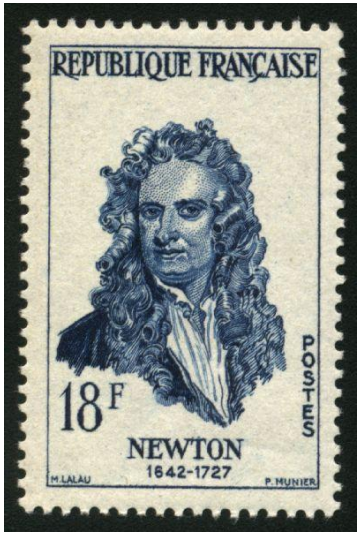
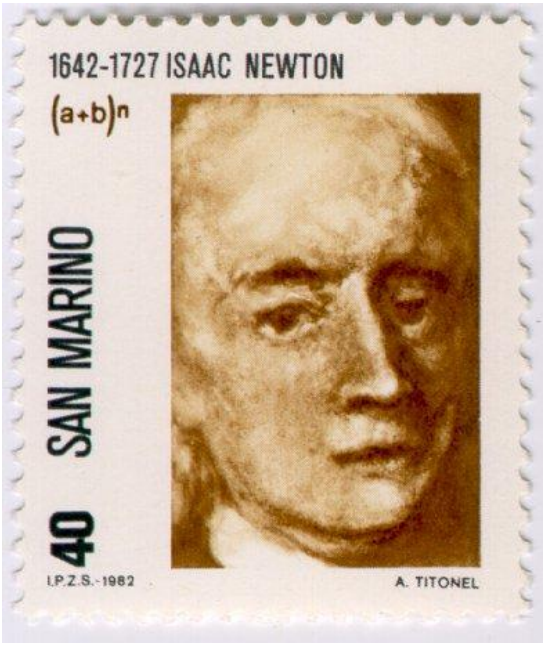
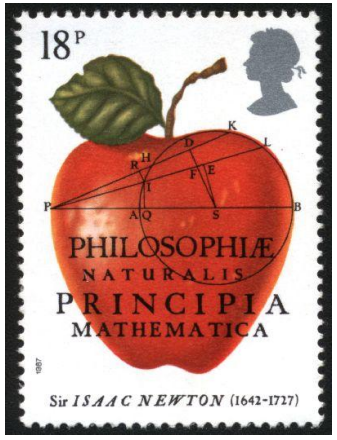

REPUBLIQUE FRANÇAISE
18F
NEWTON
1642-1727
M.LALAU
P.MUNIER
POSTES

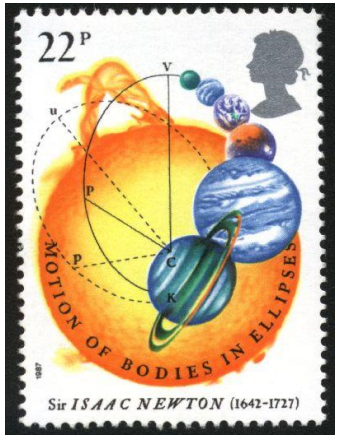

1642-1727 ISAAC NEWTON
(a+b)n
SAN MARINO
40
I.P.Z.S.-1982
A. TITONEL

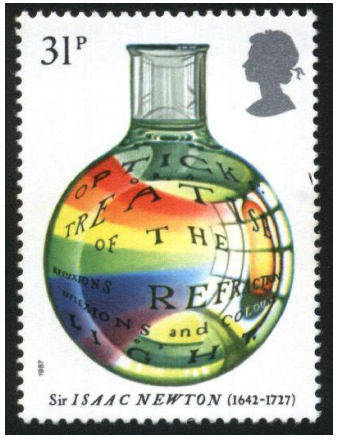
Почта СССР 1987 К
1643
1727
Исаак НЬЮТОН
1687
„Математические начала натуральной философии" – вершина творчества И.Ньютона. Впервые была создана единая система земной и небесной механики, которая легла в основу классической физики.

REPUBLIQUE POPULAIRE DU BENIN
150F
POSTE AERIENNE
1977
250e ANNIVERSAIRE DE LA MORT D'ISAAC NEWTON
1642-1727

Isaac Newton 1643-1727
3Ft
MAGYAR POSTA
PRINCIPIA MATHEMATICA

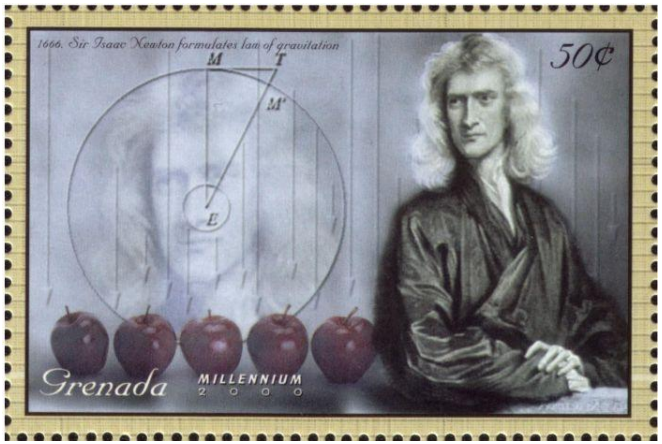

18P
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA
Sir ISAAC NEWTON (1642-1727)


22P
MOTION OF BODIES IN ELLIPSES
Sir ISAAC NEWTON (1642-1727)


31P
OPTICKS
TREATISE
of THE
REFLECTIONS
and
Sir ISAAC NEWTON (1642-1727)

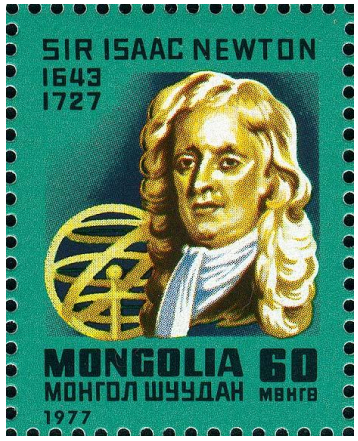
34P
THE SYSTEM OF THE WORLD
Sir ISAAC NEWTON (1642-1727)

L'Histoire de
L'Espace
SIR ISAAC NEWTON (1642-1727) inventa le téloscope à réflexion utilisant une lentille et un miroir parabolique
BURUNDI 1500 F

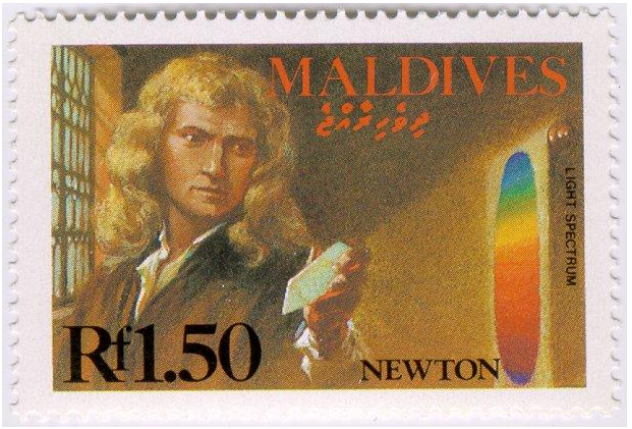

1666. Sir Isaac Newton formulates law of gravitation
50¢
Grenada
MILLENNIUM
2000

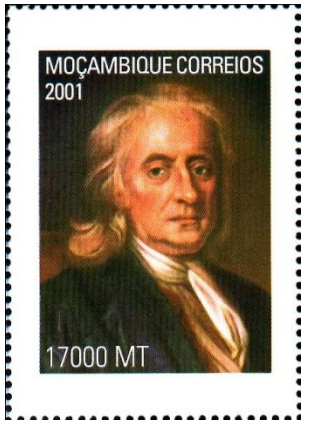
REPUBLIQUE TOGOLAISE
La comète de Halley
Edmond HALLEY 1656-1742
1000F
Isaac NEWTON 1642-1727
Poste Aérienne 1986

POSTES LAO 1984


SIR ISAAC NEWTON
1643
1727
MONGOLIA 60
МОНГОЛ ШУУДАН
1977

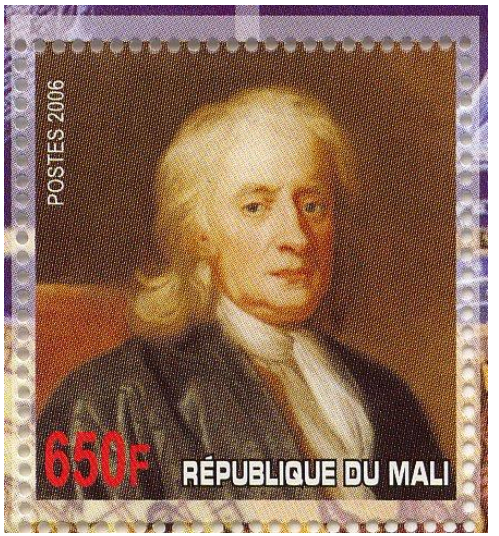
Isaac Newton
URUGUAY
$7.50
CORREOS
(1642 - 1727)
BARREIRO 96
M.A.BIA


MALDIVES
LIGHT SPECTRUM
Rf1.50
NEWTON


MOÇAMBIQUE CORREIOS
2001
17000 MT

HALLEY ET NEWTON
HALLEY'S COMET
1985 - OFFICIAL - 1986
1682
1759
POSTE AÉRIENNE
REPUBLIQUE DE GUINÉE 300F

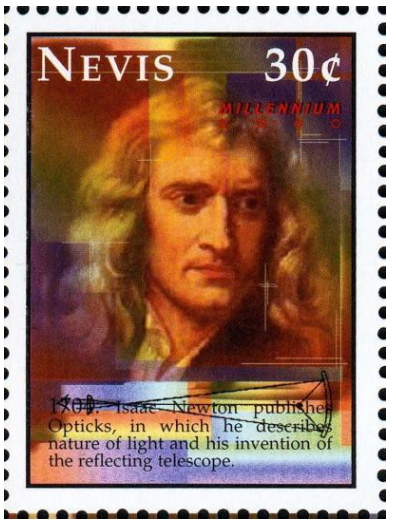
HALLEY'S COMET
1985 - OFFICIAL - 1986
HALLEY ET NEWTON OBSERVANT LA COMETE
300f
REPUBLIQUE CENTRAFRICAINE
IMPRESSOR S.A. (SWITZERLAND) • W.SPINNER • 1985


POSTES 2006
650F
RÉPUBLIQUE DU MALI

USA EXPLORER VII
13.10.1959
DISCOVERIES OF UNIVERSE
ISAAC NEWTON
1642 - 1727
ENGLAND
10B
YEMEN ARAB REPUBLIC
AIR MAIL

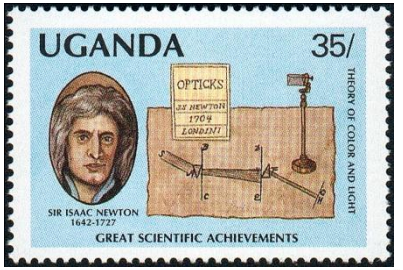
1668: Newton builds first reflecting telescope
MILLENNIUM
2000
Grenada
50¢

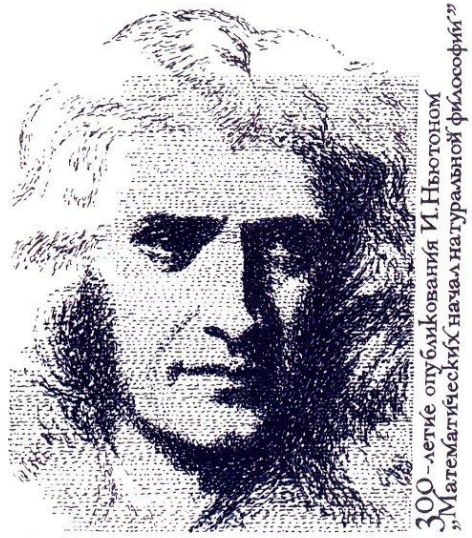
RÉPUBLIQUE DE GUINÉE
300 F
POSTES
Sir Isaac Newton
1642-1727
Sonde
Voyager
PUVILLAND

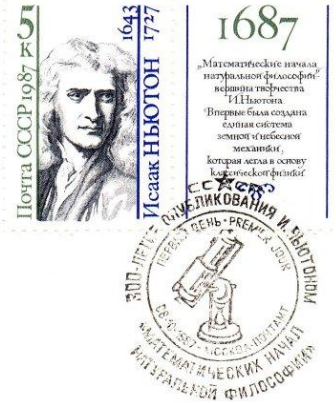

NEVIS
30¢
MILLENNIUM
1704: Isaac Newton published Opticks, in which he describes nature of light and his invention of the reflecting telescope.

LESOTHO
5s
REFLECTING TELESCOPE
Sir Isaac Newton
1642-1727

MONGOLIA
200₮

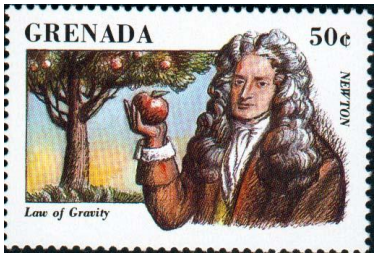

UGANDA
35/
OPTICKS
SIR NEWTON
1704
LONDINI
THEORY OF COLOR AND LIGHT
SIR ISAAC NEWTON
1642-1727
GREAT SCIENTIFIC ACHIEVEMENTS

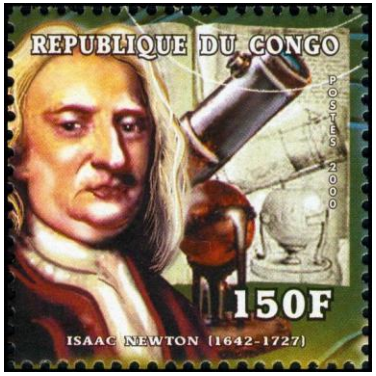

300-летие опубликования И.Ньютоном
„Математических начал натуральной философии"


5 к
1643
1727
Почта СССР 1987
Исаак НЬЮТОН
1687
Математические начала натуральной философии
великим творением
И.Ньютона
Впервые была создана
единая система
земной и небесной
механики,
которая легла в основу
классического
естествознания
300-ЛЕТИЕ ОПУБЛИКОВАНИЯ И.НЬЮТОНОМ
МАТЕМАТИЧЕСКИХ НАЧАЛ
НАТУРАЛЬНОЙ ФИЛОСОФИИ

GHANA
C200
HALLEY'S COMET 1985-6

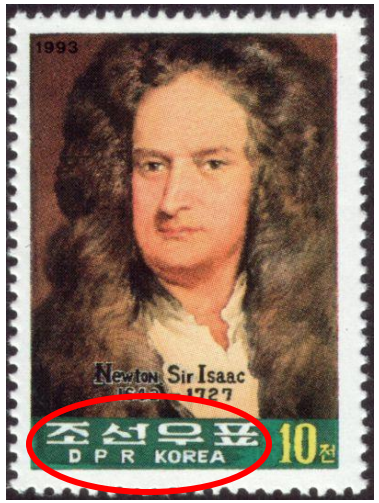
CORREO DEL PARAGUAY
ÓRBITAS ELÍPTICAS
LUNA
TIERRA
GRAVITACIÓN UNIVERSAL
ÓPTICA
ISAAC NEWTON
* 25 DICIEMBRE 1642
CÁLCULO INFINITESIMAL Y DIFERENCIAL
BINOMIO
TANGENTES
G 0.10


GRENADA
50¢
NEWTON
Law of Gravity


REPUBLIQUE DU CONGO
POSTES
2000
150F
ISAAC NEWTON (1642-1727)

CORREO DEL PARAGUAY
ÓRBITAS ELÍPTICAS
LUNA
TIERRA
GRAVITACIÓN UNIVERSAL
ÓPTICA
ISAAC NEWTON
* 25 DICIEMBRE 1642
CÁLCULO INFINITESIMAL Y DIFERENCIAL
BINOMIO
TANGENTES
G 0.30

1993
Newton, Sir Isaac
1642 – 1727
조선우표 50전
D P R KOREA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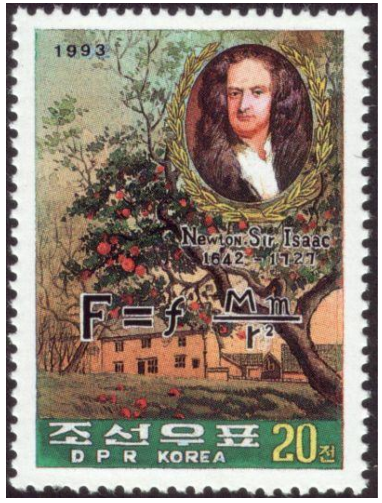
Newton, Sir Isaac
조선우표 10전
D P R KOREA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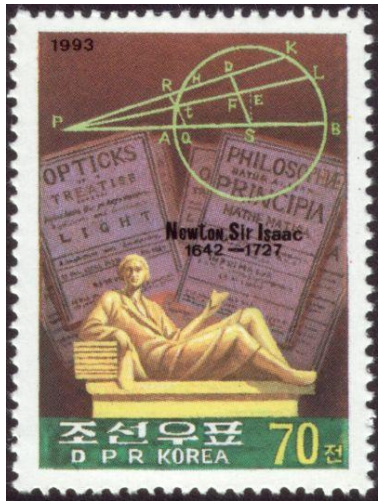
Newton, Sir Isaac
1642 – 1727
조선우표 20전
D P R KOREA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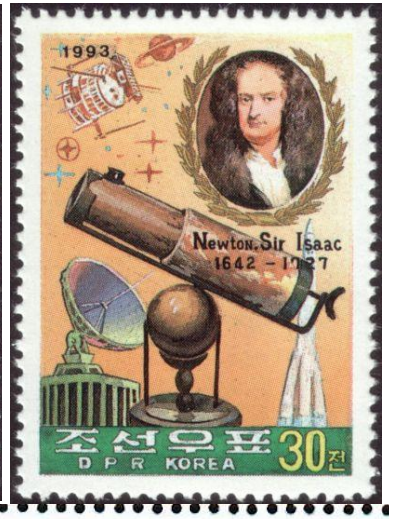
OPTICKS
PHILOSOPHIA
PRINCIPIA
Newton, Sir Isaac
1642 – 1727
조선우표 70전
D P R KOREA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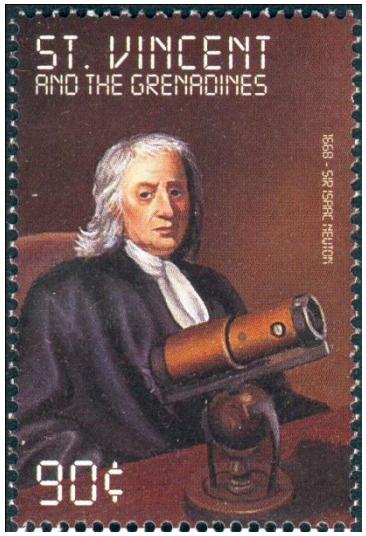
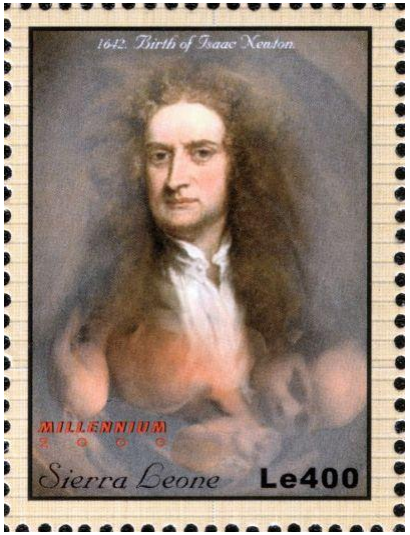
Newton, Sir Isaac
조선우표 30전
D P R KOREA

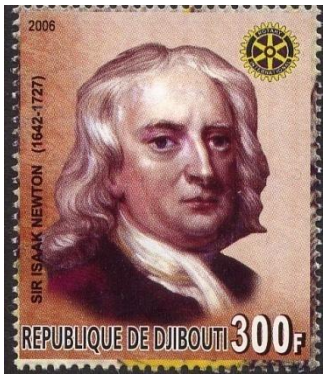

ST. VINCENT
AND THE GRENADINES
1668 - SIR ISAAC NEWTON
90¢

EXPLOITATION DES RESSOURCES DE L'ESPACE
SIR ISAAC NEWTON – SATELLITE TDF 1
REPUBLIQUE DU TCHAD 1500F


1642. Birth of Isaac Newton.
MILLENNIUM
Sierra Leone
Le400

Postes 1999
150F
1600 ~ Sir Isaac Newton, 1642-1727
République du Tchad

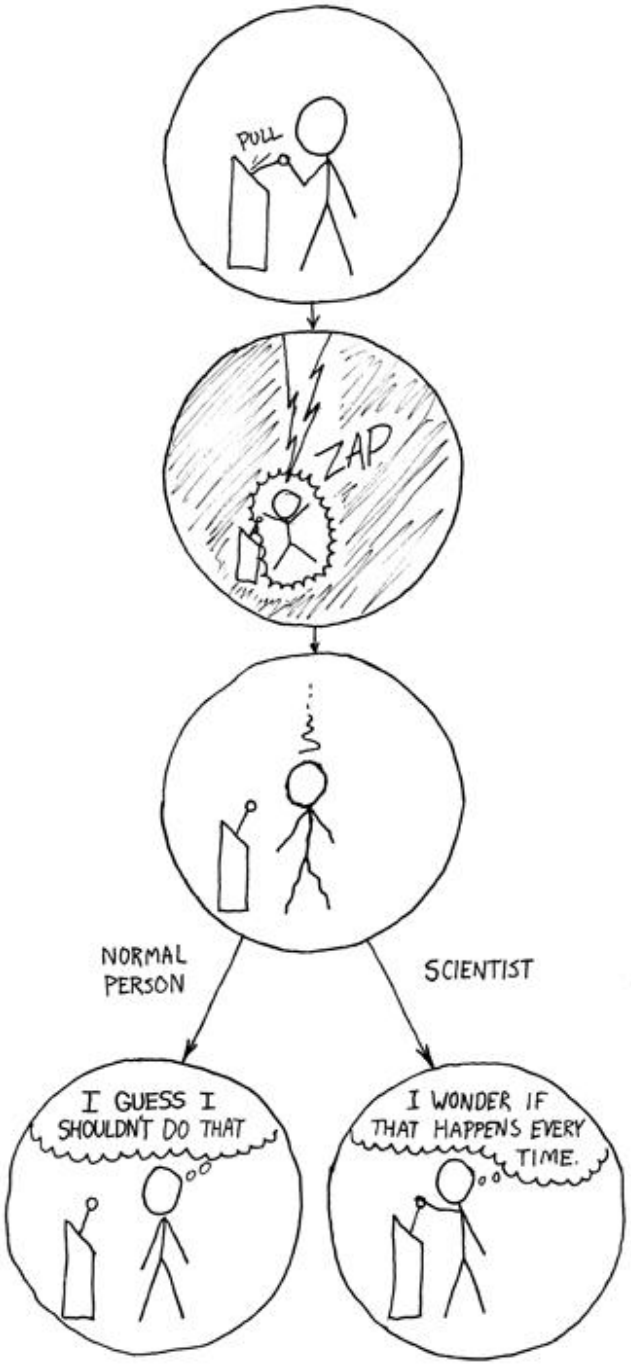
ISAAC NEWTON,
MATHEMATICIAN
& ASTRONOMER
2½p
ASCENSION


2006
SIR ISAAK NEWTON (1642-1727)
REPUBLIQUE DE DJIBOUTI 300F

Isaac Newton (1642-1726)
Telescopio de Newton 1688
CORREOS 1994
C1.50
NICARAGUA

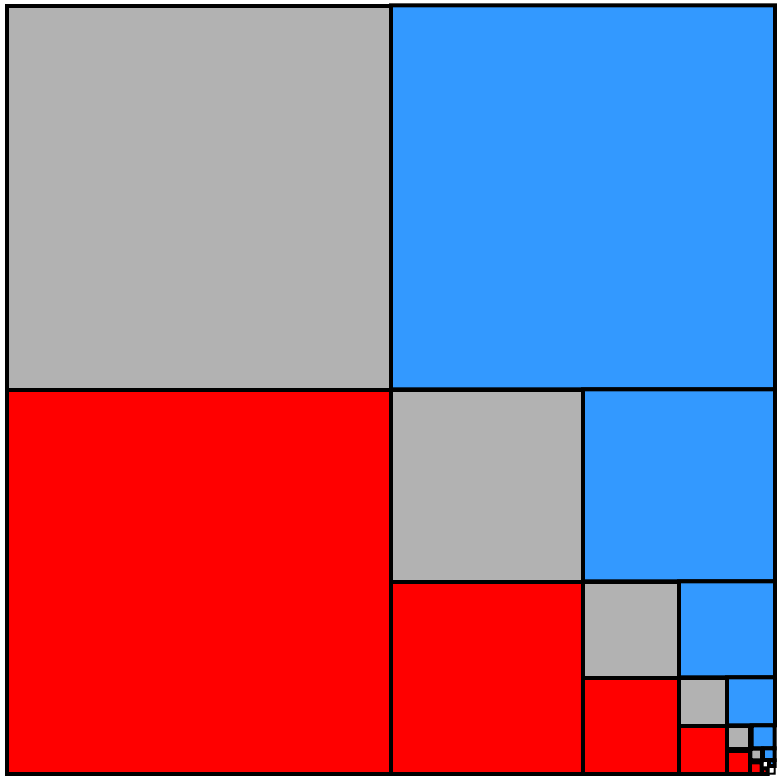
ISAAC NEWTON 1643-1727
400F
REPUBLIQUE DU MALI

THE UNKNOWN SCIENTIST
(WHO DID SOME VERY IMPORTANT GROUNDWORK)


PULL
ZAP
NORMAL PERSON
SCIENTIST
I GUESS I SHOULDN'T DO THAT
I WONDER IF THAT HAPPENS EVERY TIME.

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.

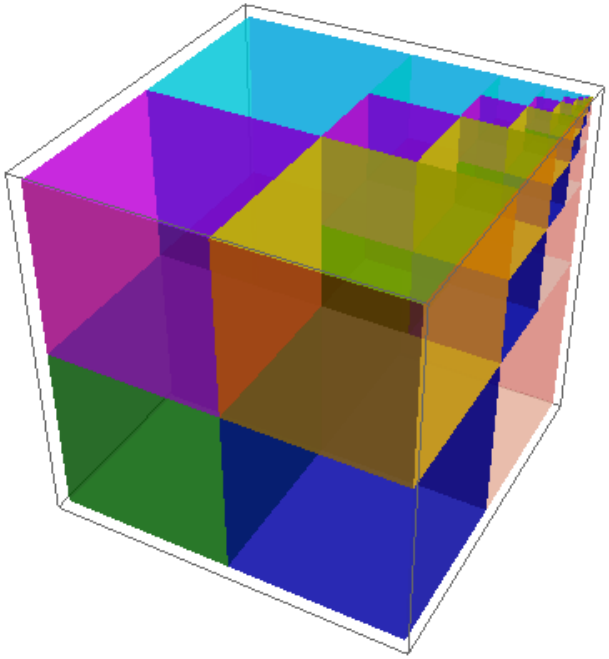


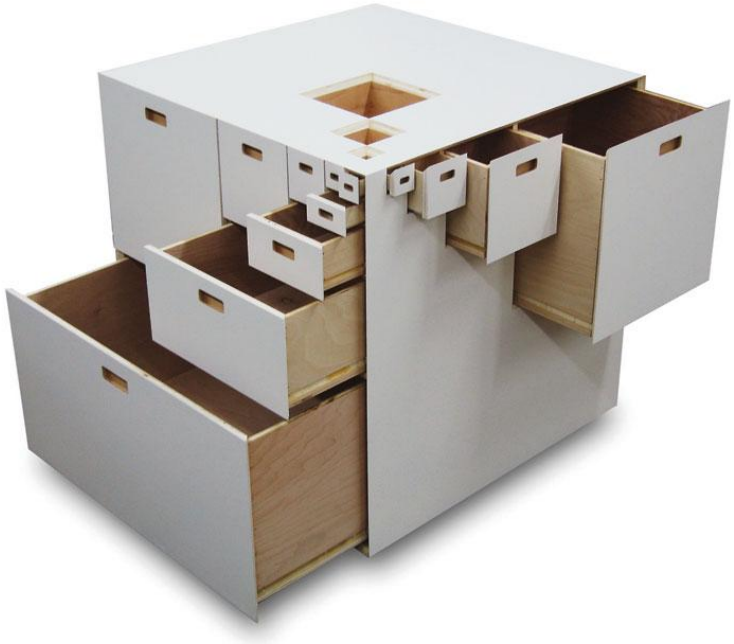
$$\sum_{i=1}^{\infty} \frac{1}{4^i} = \frac{1}{3}$$

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = \frac{1}{7}$$





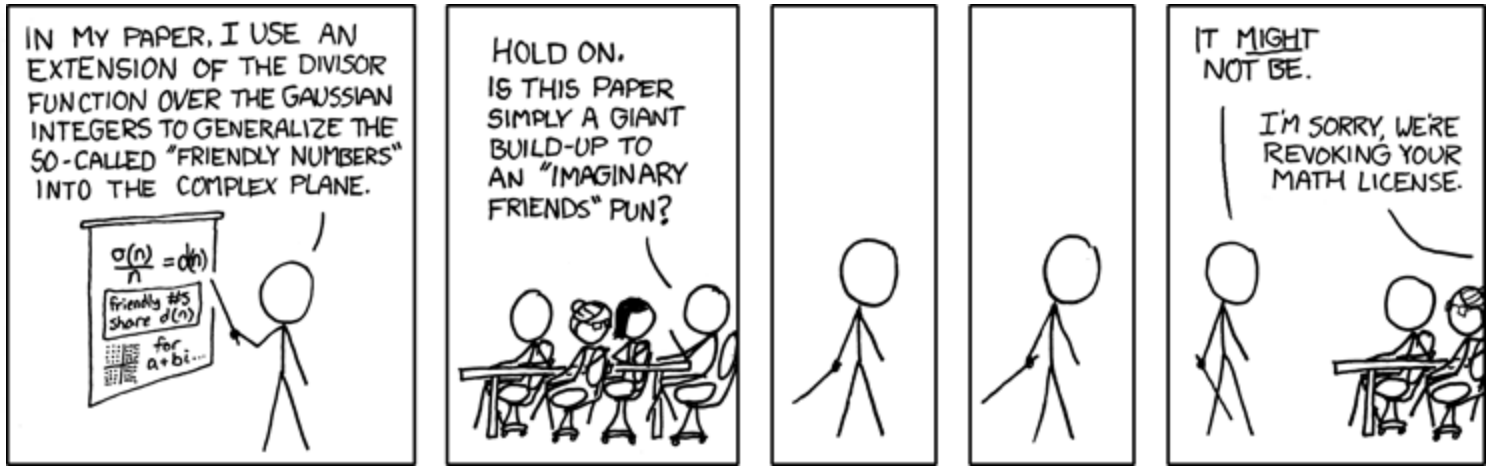
Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i?

$$i^i = \frac{1}{\sqrt{e^{\pi}}} = 0.207879..$$

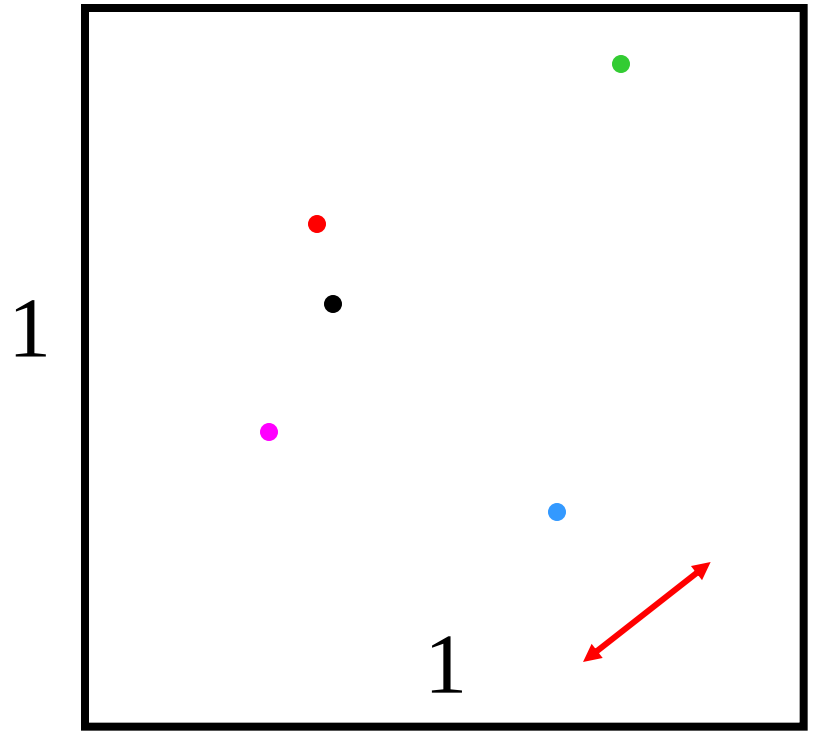
$$e^{ix} = \cos(x) + i \sin(x)$$

$$i^i = \frac{1}{\sqrt{e^{\pi+2k\pi}}}$$

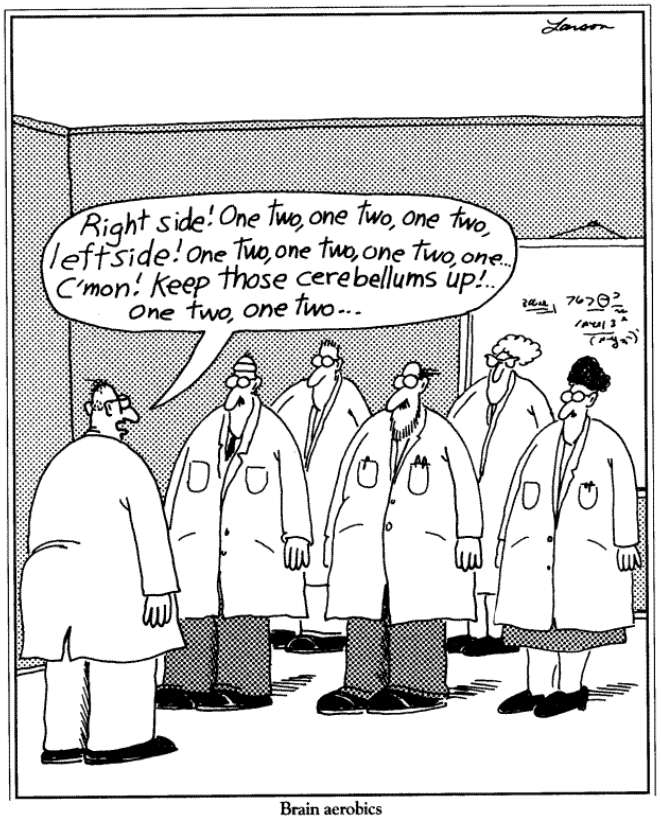
i^i is multi-valued!



Problem: Given any five points in/on the unit square, is there always a pair with distance $\leq \frac{1}{\sqrt{2}}$?

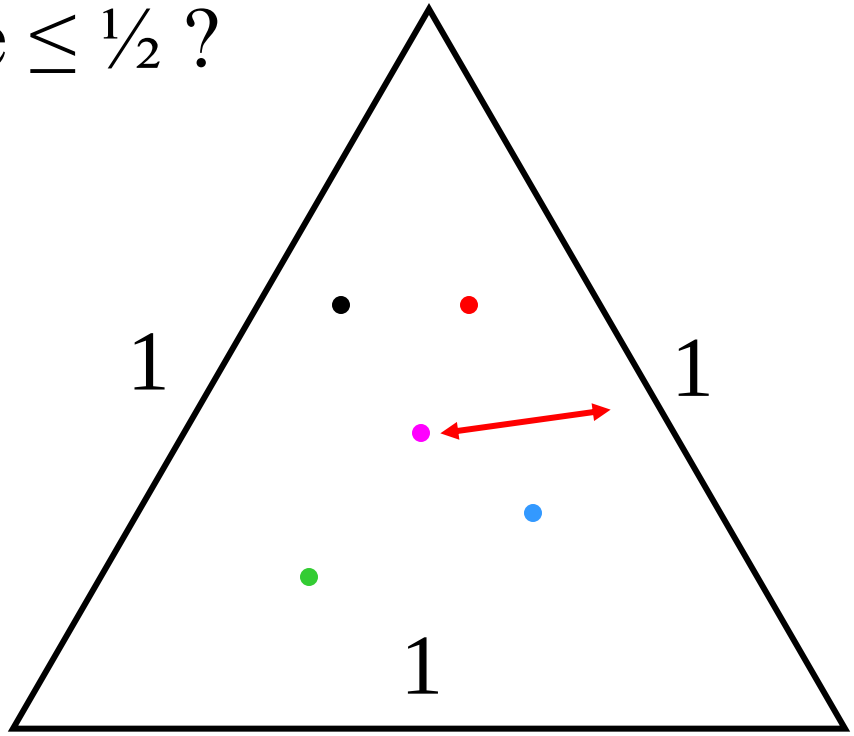


- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

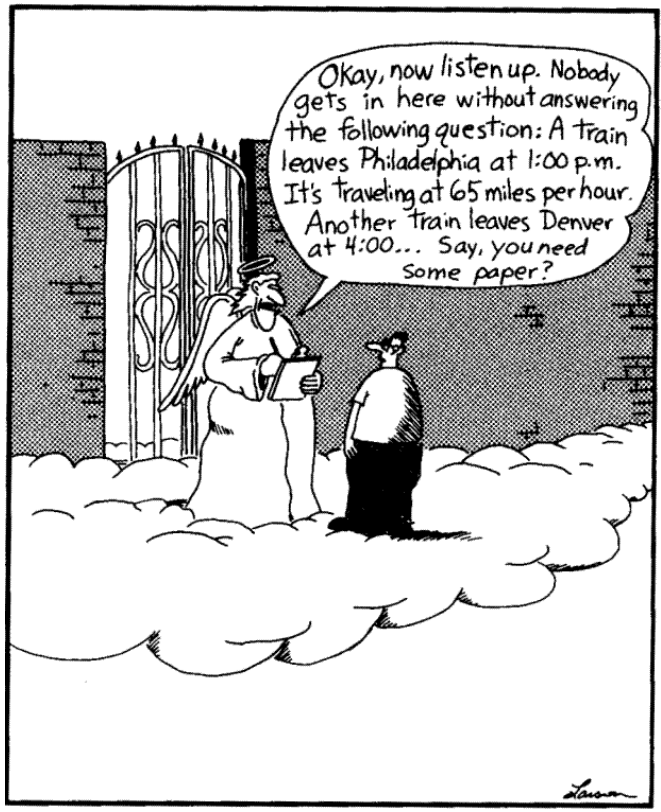


Brain aerobics

Problem: Given any five points in/on the unit equilateral triangle, is there always a pair with distance $\leq \frac{1}{2}$?



- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Math phobic's nightmare

Problem: Solve the following equation for X:

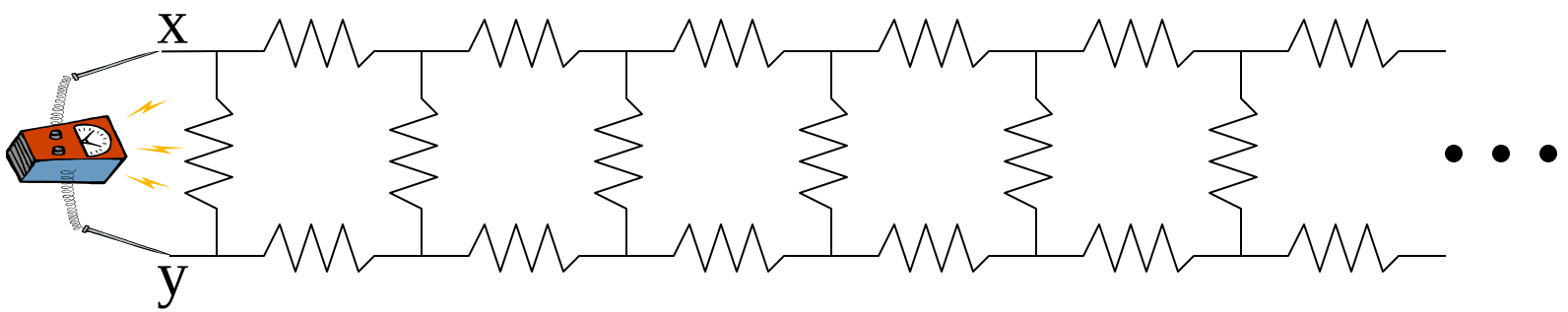
$$X^{X^{X^{X^{\cdots}}}} = 2$$

where the stack of exponentiated x's extends forever.

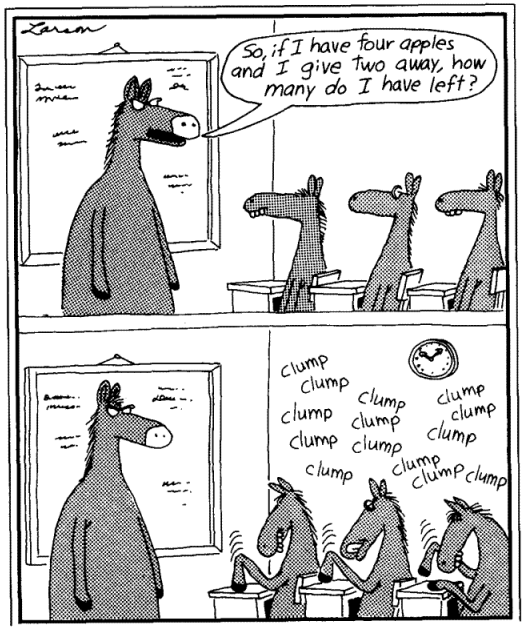
- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

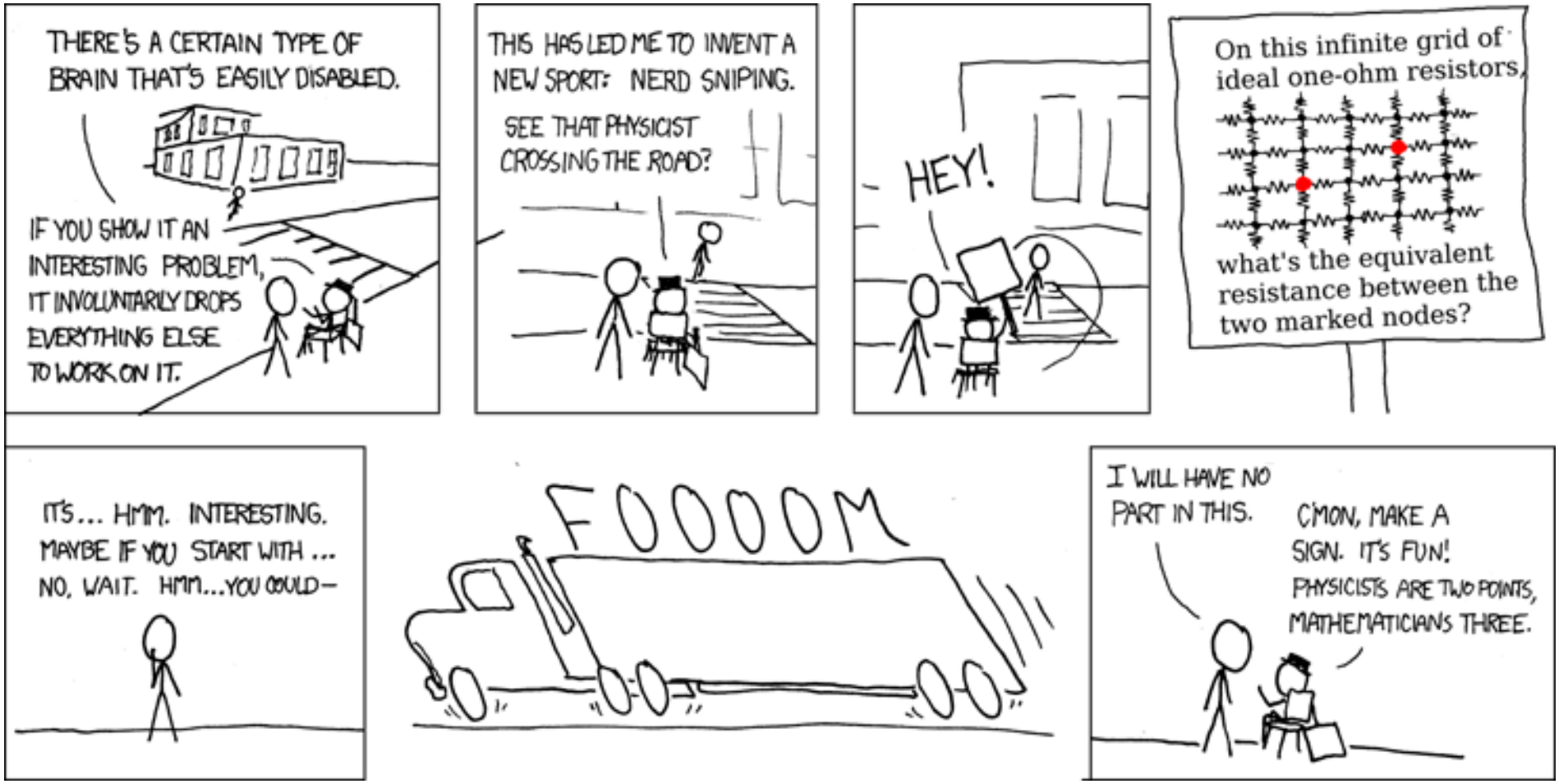
"Mr. Osborne, may I be excused? My brain is full."

Problem: For the given infinite ladder of resistors of resistance R each, what is the resistance measured between points x and y?



- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

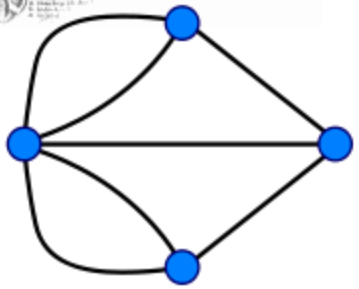


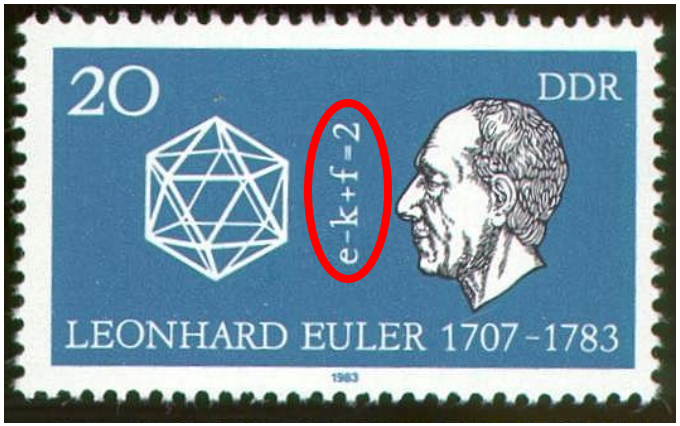

THERE'S A CERTAIN TYPE OF BRAIN THAT'S EASILY DISABLED.
IF YOU SHOW IT AN INTERESTING PROBLEM, IT INVOLUNTARILY DROPS EVERYTHING ELSE TO WORK ON IT.
THIS HAS LED ME TO INVENT A NEW SPORT: NERD SNIPING.
SEE THAT PHYSICIST CROSSING THE ROAD?
HEY!
On this infinite grid of ideal one-ohm resistors,
what's the equivalent resistance between the two marked nodes?
IT'S... HMM. INTERESTING. MAYBE IF YOU START WITH ... NO, WAIT. HMM...YOU COULD—
FOOOOM
I WILL HAVE NO PART IN THIS.
C'MON, MAKE A SIGN. IT'S FUN! PHYSICISTS ARE TWO POINTS, MATHEMATICIANS THREE.

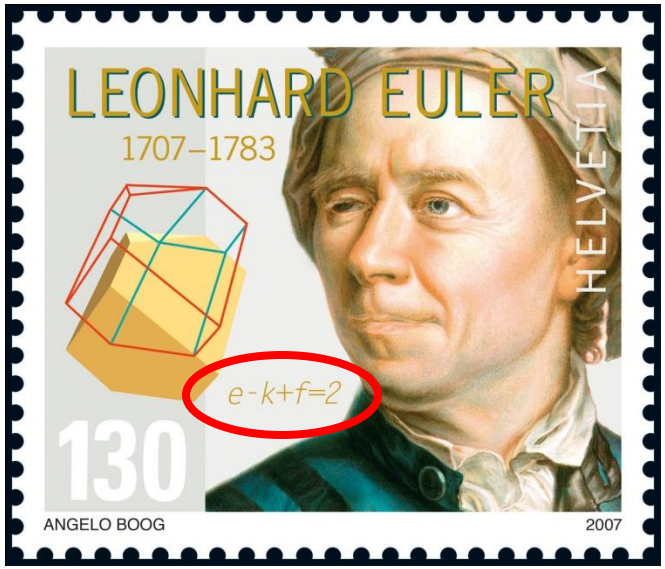
Historical Perspectives

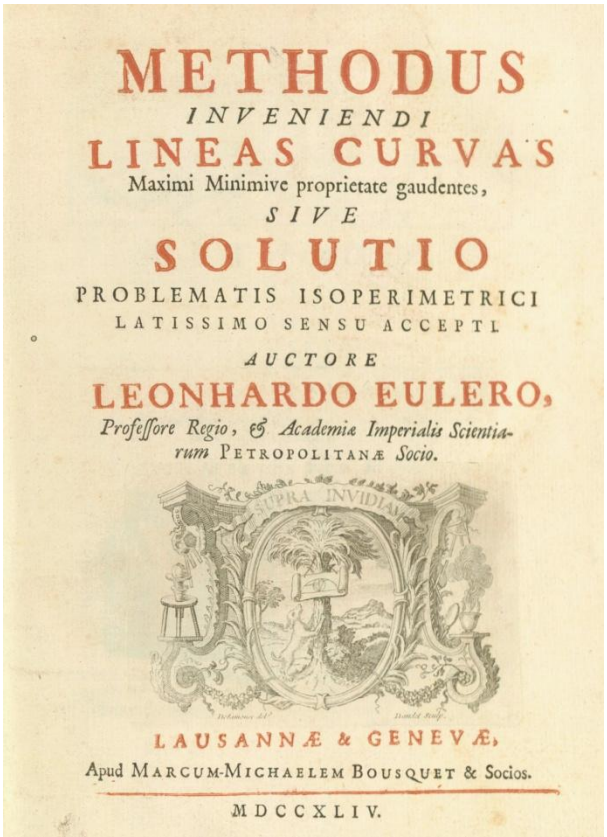
Leonhard Euler (1707–1783)

- Invented graph theory
- "Bridges of Königsberg", Prussia
- Eulerian tour
- Euler's formula: $V + F = E + 2$
- Euler's number: e
- Euler's identity: $e^{i\pi} + 1 = 0$
- Major contributions to analysis, algebra, calculus, number theory, topology, optics, fluid dynamics, mechanics, astronomy, education









METHODUS

INVENIENDI

LINEAS CURVAS

Maximi Minimive proprietate gaudentes,

SIVE

SOLUTIO

PROBLEMATIS ISOPERIMETRICI

LATISSIMO SENSU ACCEPTI

AUCTORE

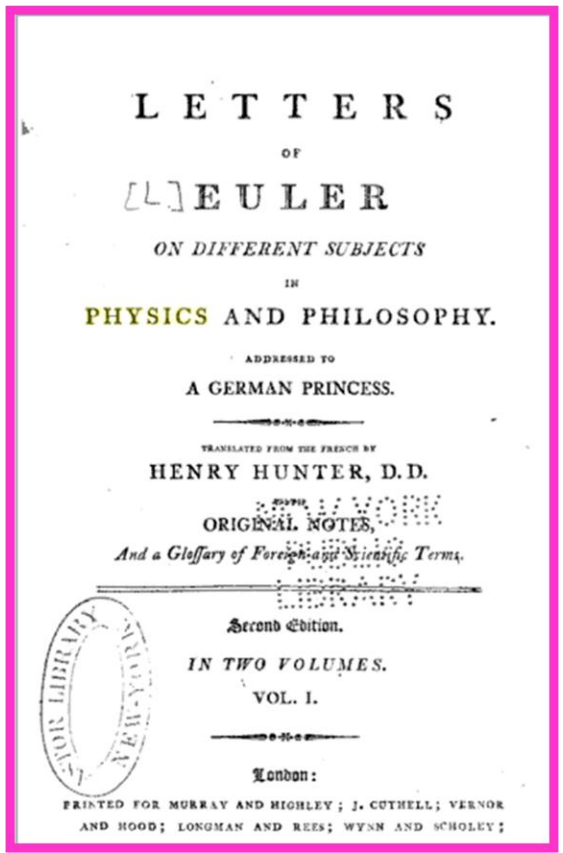
LEONHARDO EULERO,

Professore Regio, & Academiæ Imperialis Scientiarum PETROPOLITANÆ *Socio.*

LAUSANNÆ & GENEVÆ,

Apud MARCUM-MICHAELEM BOUSQUET & Socios.

MDCCXLIV.



LETTERS

OF

EULER

ON DIFFERENT SUBJECTS

IN

PHYSICS AND PHILOSOPHY.

ADDRESSED TO

A GERMAN PRINCESS.

TRANSLATED FROM THE FRENCH BY

HENRY HUNTER, D.D.

WITH

ORIGINAL NOTES,

And a Glossary of Foreign and Scientific Terms.

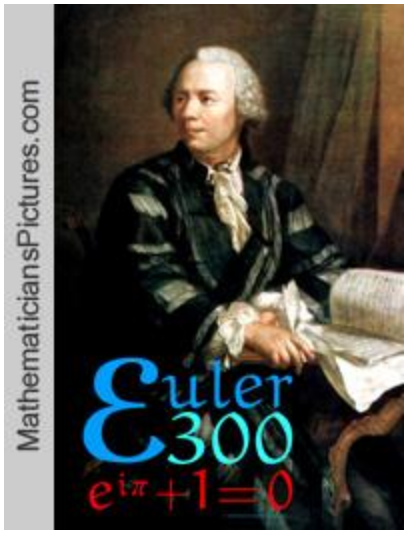
Second Edition.

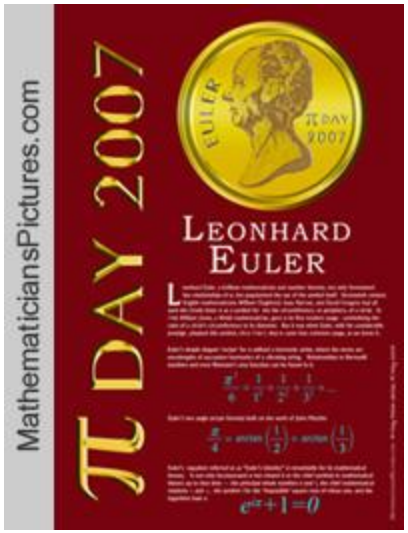
IN TWO VOLUMES.

VOL. I.

London:

PRINTED FOR MURRAY AND HIGHLEY; J. CUTHELL; VERNOR AND HOOD; LONGMAN AND REES; WYNN AND SCHOLEY;


MathematiciansPictures.com
Euler
300
$e^{i\pi}+1=0$


MathematiciansPictures.com
π DAY 2007
LEONHARD EULER
$e^{i\pi}+1=0$

MathematiciansPictures.com
Euler
300
$e^{i\pi}+1=0$
© MATHEMATICIANSPICTURES.COM

$$e^{i\pi}+1=0$$

$$e^{iu}=\cos(u)+i\sin(u)$$

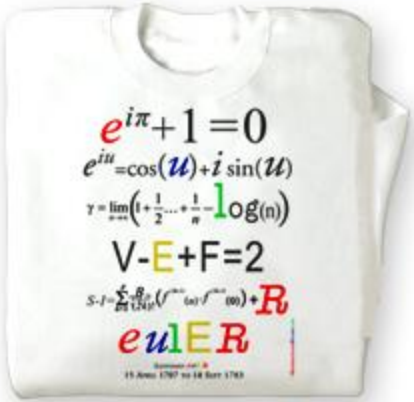
$$\gamma=\lim_{n\to\infty}\left(1+\frac{1}{2}\ldots+\frac{1}{n}-\log(n)\right)$$

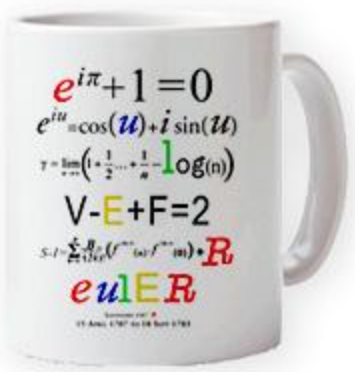
$$V-E+F=2$$

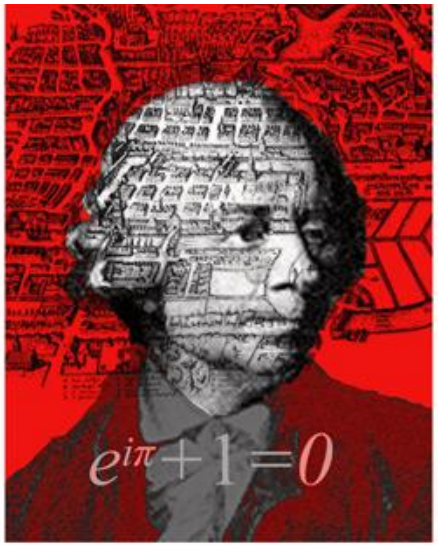
$$S-I=\sum_{k=1}^{p}\gamma\frac{B_{2k}}{(2k)!}\left(f^{(2k-1)}(n)-f^{(2k-1)}(0)\right)+R$$

$$euleR$$

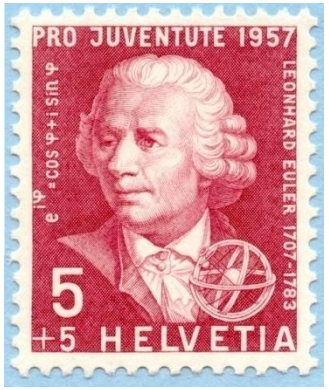
Leonhard euleR
15 April 1707 to 18 Sept 1783







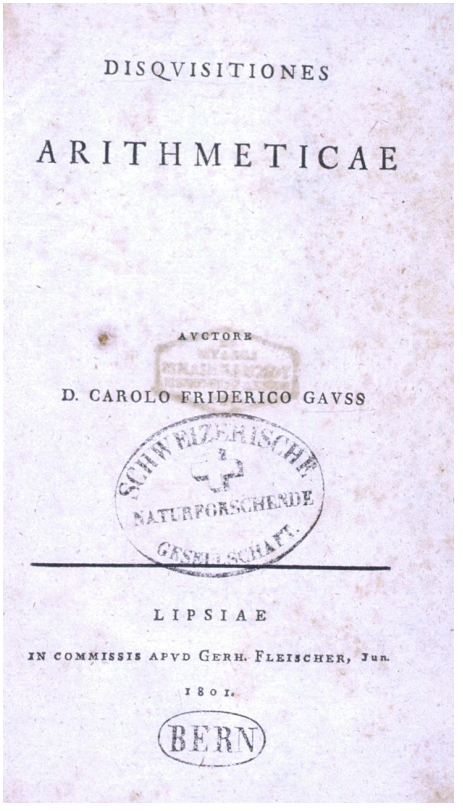
Leonhard Euler
1707 - 1783



Historical Perspectives

Carl Friedrich Gauss (1777–1855)

- "Prince of Mathematics"
- Founded modern number theory
- Authored "Disquisitiones Arithmeticae"
- Fundamental Theorem of Algebra
- Major contributions to astronomy, optics electromagnetism, statistics, geometry
- Gaussian distribution, Gaussian elimination Gaussian noise, Gaussian integers & primes Gauss' Law, Gauss' constant, "degaussing"
- SI unit of magnetic field strength: gauss
- Students: Dedekind, Riemann, Bessel


DISQVISITIONES

ARITHMETICAE

AVCTORE

D. CAROLO FRIDERICO GAVSS

SCHWEIZERISCHE NATURFORSCHENDE GESELLSCHAFT

LIPSIAE

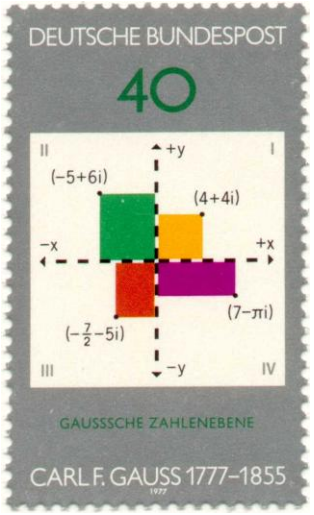
IN COMMISSIS APVD GERH. FLEISCHER, Jun.

1801.

BERN

GU567297252
German Marks
DEUTSCHE BUNDESBANK
Banknote
10
ZEHN DEUTSCHE MARK
1777–1855 Carl Friedr. Gauß
GU5672972S2
CARL FRIEDRICH GAUSS
1777
1855
DDR
LEIP
20

TERRES AUSTRALES ET ANTARCTIQUES FRANÇAISES
9,00
NAVIRE SCIENTIFIQUE
"GAUSS"
1901-1903
POSTE AERIENNE


DEUTSCHE BUNDESPOST
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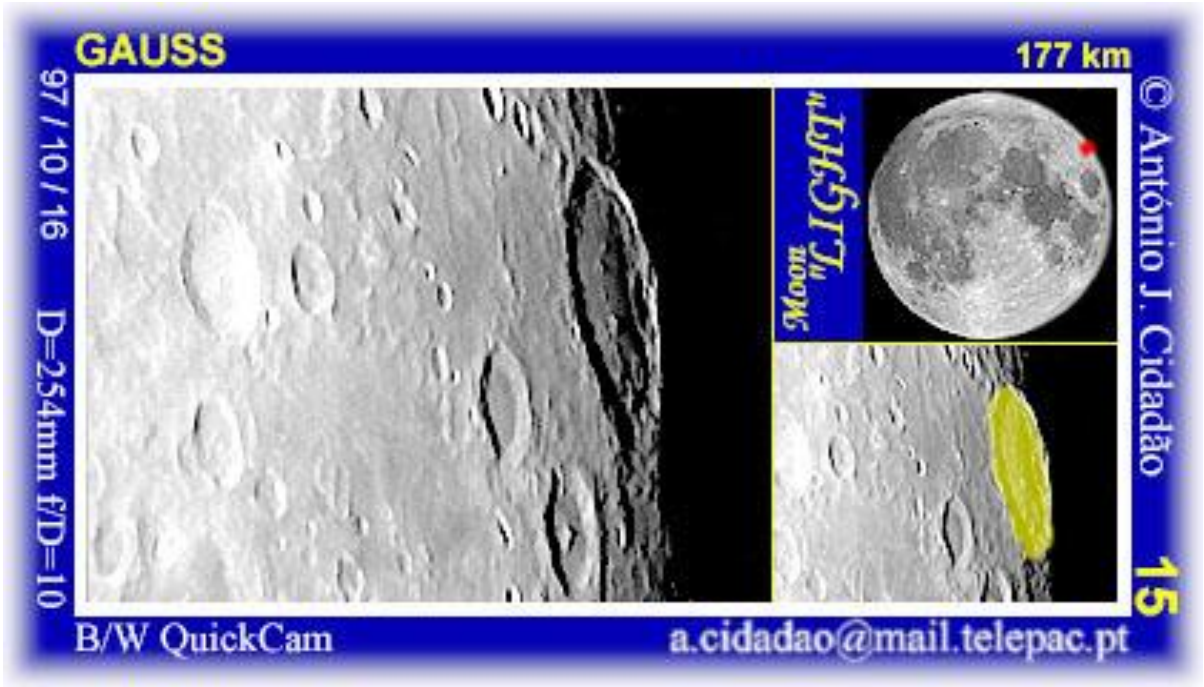
GAUSSSCHE ZAHLENEBENE
CARL F. GAUSS 1777–1855


GAUSS
177 km
97/10/16
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
15
B/W QuickCam
a.cidadao@mail.telepac.pt

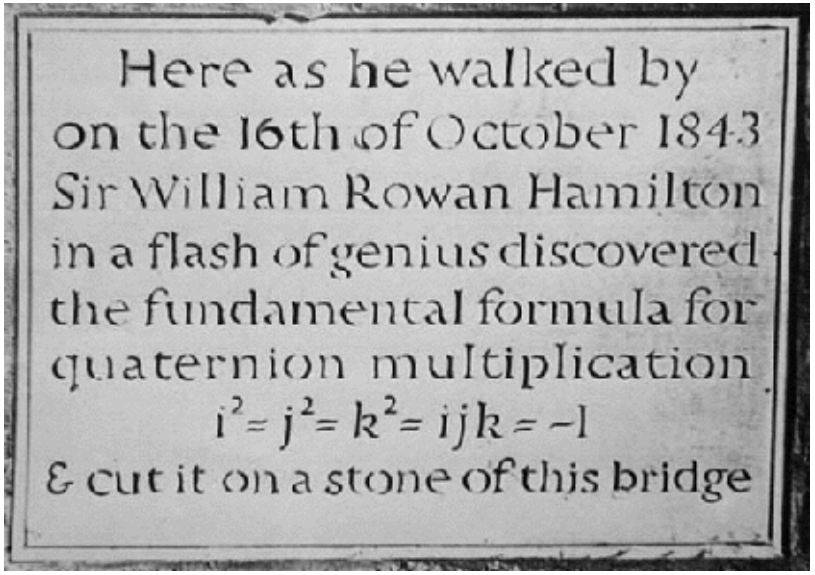
C.F.GAUSS * 1777 † 1855
DEUTSCHE
BUNDESPOST
10

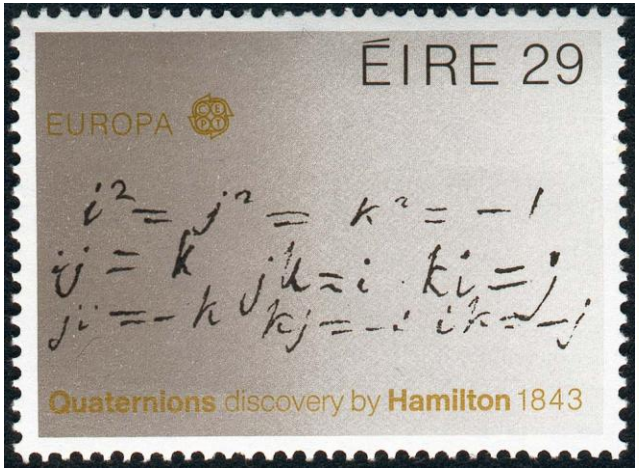
NICARAGUA
CORREOS 1994
C1.50
Gottinga
Karl Friedrich Gauss (1777-1855)

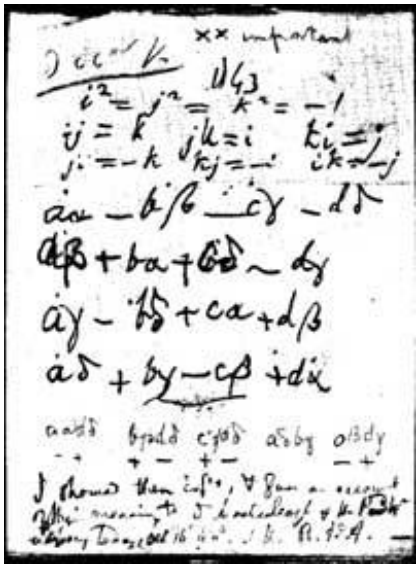
Historical Perspectives

William R. Hamilton (1805-1865)

- Mathematician, physicist, and astronomer
- Contributed to algebra, mechanics, optics
- Formulated Hamiltonian mechanics
- Discovered quaternions, conical refraction, Hamilton function, Hamilton principle, Hamiltonian group
- Invented "Icosian Calculus", dot & cross products, Hamiltonian paths
- Influenced computer graphics, mechanics, electromagnetism, relativity, quantum theory, vector algebra







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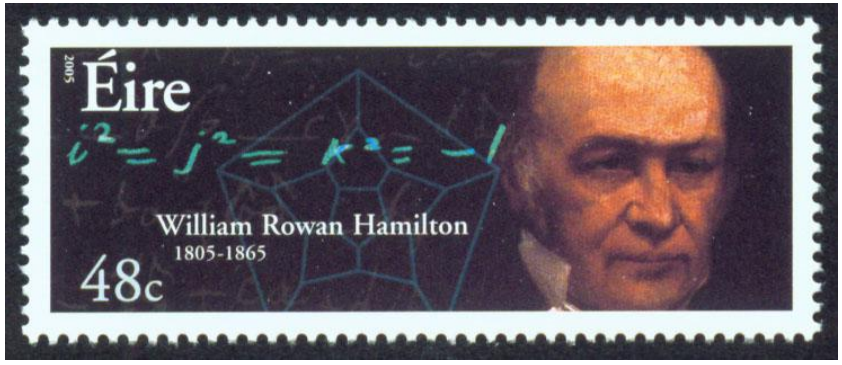
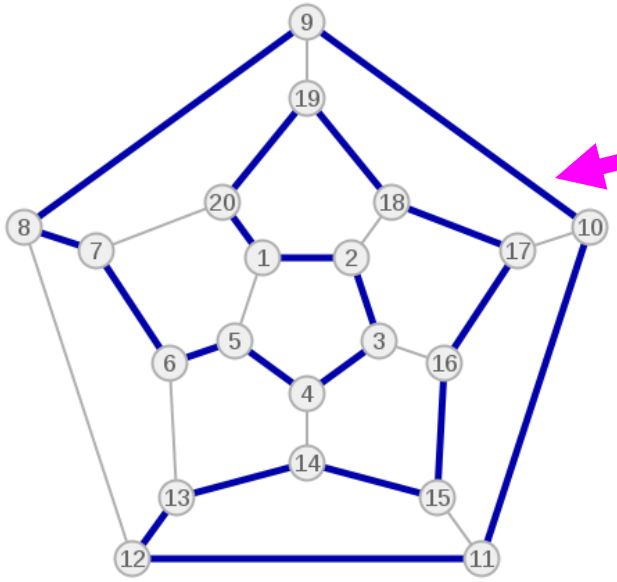
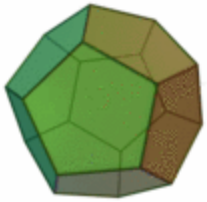
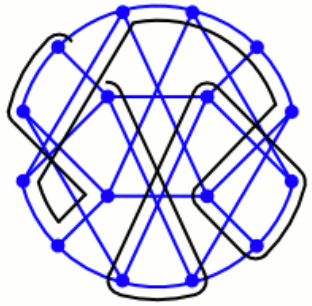
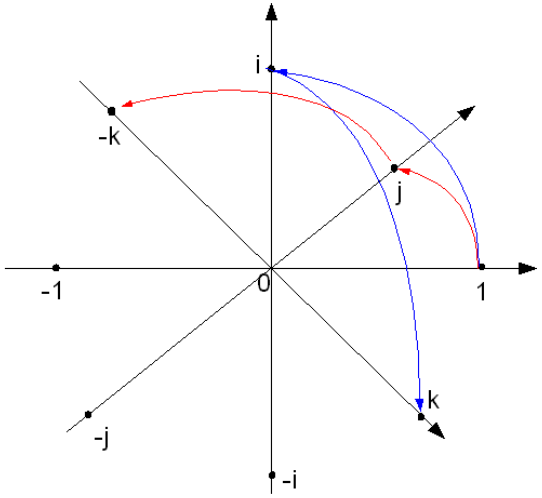
−1	−1	1	−i	i	−j	j	−k	k
i	i	−i	−1	1	k	−k	−j	j
−i	−i	i	1	−1	−k	k	j	−j
j	j	−j	−k	k	−1	1	i	−i
−j	−j	j	k	−k	1	−1	−i	i
k	k	−k	j	−j	−i	i	−1	1
−k	−k	k	−j	j	i	−i	1	−1

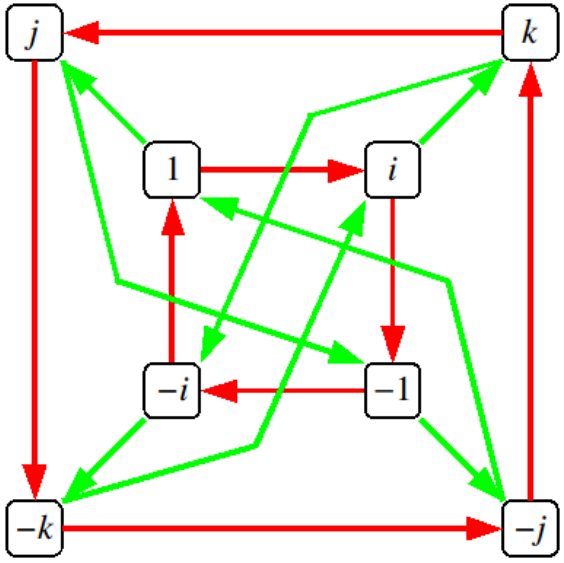












Graphical representation of quaternion units product as 90°-rotation in 4D-space

Non-commutative: ij=k ji=-k

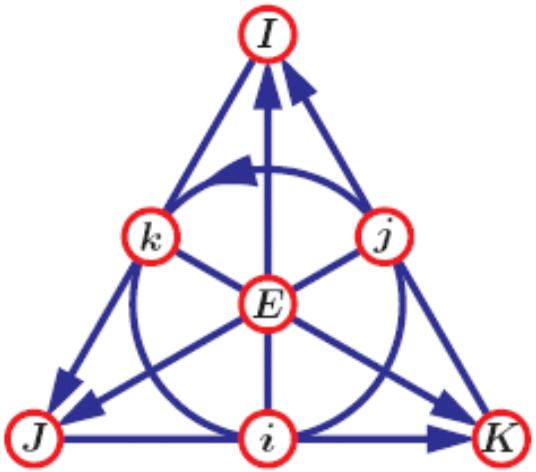
ij = k
ji = -k
ij = -ji

the Hamilton Mathematics Institute TCD

Octonions: Generalization of Quaternions

- Non-associative! (e.g., (ij)K=-E ≠ E=i(jK))
- Discovered by John Graves (1843), friend of Hamilton
- Useful in general relativity, quantum logic, string theory

×	*i*	*j*	*k*	*E*	*I*	*J*	*K*
i	−1	*k*	−*j*	*I*	−*E*	−*K*	*J*
j	−*k*	−1	*i*	*J*	*K*	−*E*	−*I*
k	*j*	−*i*	−1	*K*	−*J*	*I*	−*E*
E	−*I*	−*J*	−*K*	−1	*i*	*j*	*k*
I	*E*	−*K*	*J*	−*i*	−1	−*k*	*j*
J	*K*	*E*	−*I*	−*j*	*k*	−1	−*i*
K	−*J*	*I*	*E*	−*k*	−*j*	*i*	−1



Mnemonic diagram for unit octonions products

“The real numbers are the dependable breadwinner of the family, the complete ordered field we all rely on. The complex numbers are a slightly flashier but still respectable younger brother: not ordered, but algebraically complete. The quaternions, being noncommutative, are the eccentric cousin who is shunned at important family gatherings. But the octonions are the crazy old uncle nobody lets out of the attic: they are *nonassociative*.”

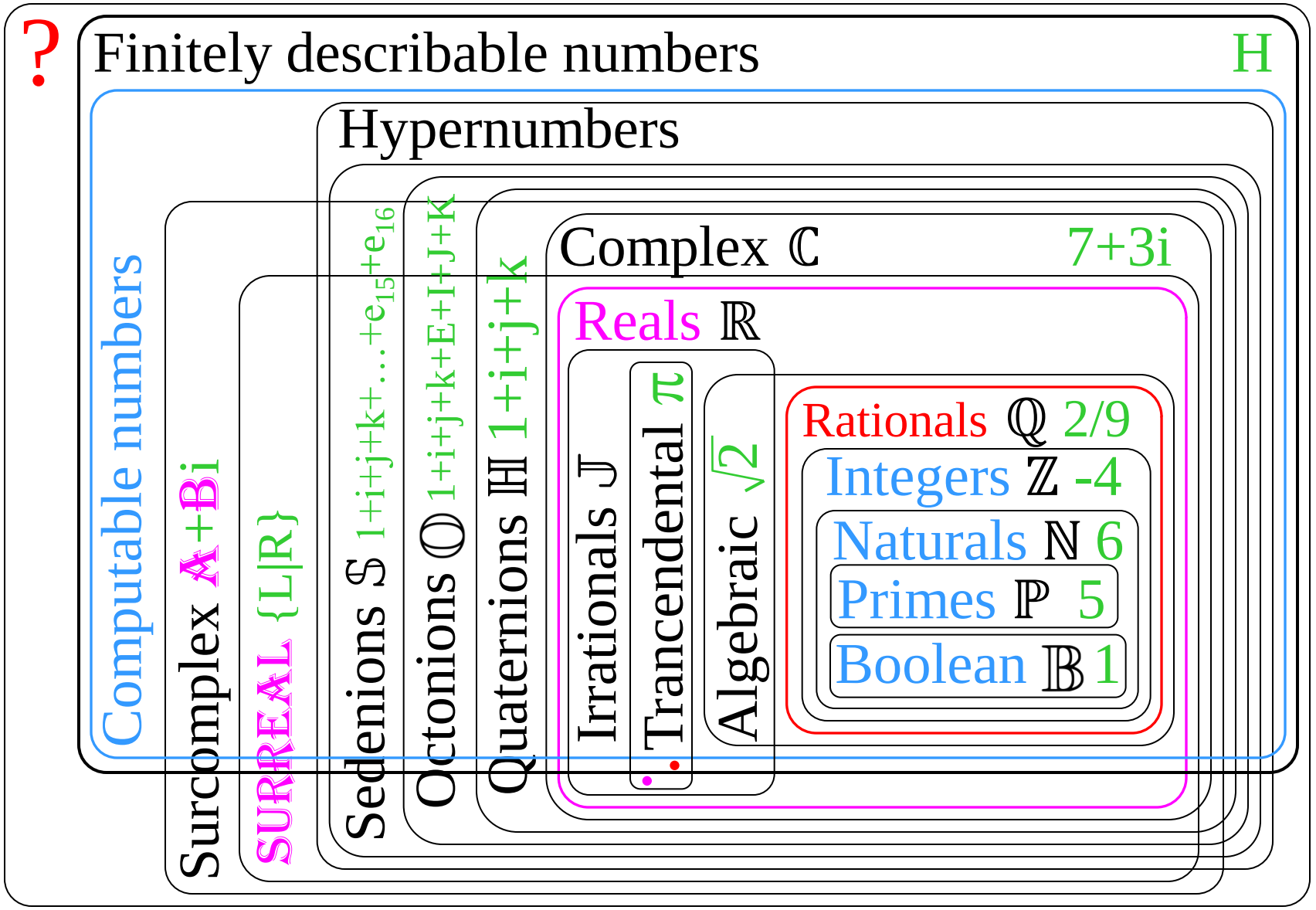
— John Baez (1961-), physicist
works on spin foams
and loop quantum gravity

Sedenions: Generalization of Octonions

- Non-alternative! (i.e., x(xy)=(xx)y doesn't hold)

×	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
1	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
e_1	e_1	-1	e_3	$-e_2$	e_5	$-e_4$	$-e_7$	e_6	e_9	$-e_8$	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$
e_2	e_2	$-e_3$	-1	e_1	e_6	e_7	$-e_4$	$-e_5$	e_{10}	e_{11}	$-e_8$	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}
e_3	e_3	e_2	$-e_1$	-1	e_7	$-e_6$	e_5	$-e_4$	e_{11}	$-e_{10}$	e_9	$-e_8$	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}
e_4	e_4	$-e_5$	$-e_6$	$-e_7$	-1	e_1	e_2	e_3	e_{12}	e_{13}	e_{14}	e_{15}	$-e_8$	$-e_9$	$-e_{10}$	$-e_{11}$
e_5	e_5	e_4	$-e_7$	e_6	$-e_1$	-1	$-e_3$	e_2	e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	$-e_8$	e_{11}	$-e_{10}$
e_6	e_6	e_7	e_4	$-e_5$	$-e_2$	e_3	-1	$-e_1$	e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	$-e_8$	e_9
e_7	e_7	$-e_6$	e_5	e_4	$-e_3$	$-e_2$	e_1	-1	e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	$-e_8$
e_8	e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_{12}$	$-e_{13}$	$-e_{14}$	$-e_{15}$	-1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_9	e_9	e_8	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$	$-e_1$	-1	$-e_3$	e_2	$-e_5$	e_4	e_7	$-e_6$
e_{10}	e_{10}	e_{11}	e_8	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}	$-e_2$	e_3	-1	$-e_1$	$-e_6$	$-e_7$	e_4	e_5
e_{11}	e_{11}	$-e_{10}$	e_9	e_8	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}	$-e_3$	$-e_2$	e_1	-1	$-e_7$	e_6	$-e_5$	e_4
e_{12}	e_{12}	e_{13}	e_{14}	e_{15}	e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_4$	e_5	e_6	e_7	-1	$-e_1$	$-e_2$	$-e_3$
e_{13}	e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	e_8	e_{11}	$-e_{10}$	$-e_5$	$-e_4$	e_7	$-e_6$	e_1	-1	e_3	$-e_2$
e_{14}	e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	e_8	e_9	$-e_6$	$-e_7$	$-e_4$	e_5	e_2	$-e_3$	-1	e_1
e_{15}	e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	e_8	$-e_7$	e_6	$-e_5$	$-e_4$	e_3	e_2	$-e_1$	-1

Generalized Numbers



Theorem: some real numbers are not finitely describable!

Theorem: some finitely describable real numbers are not computable!