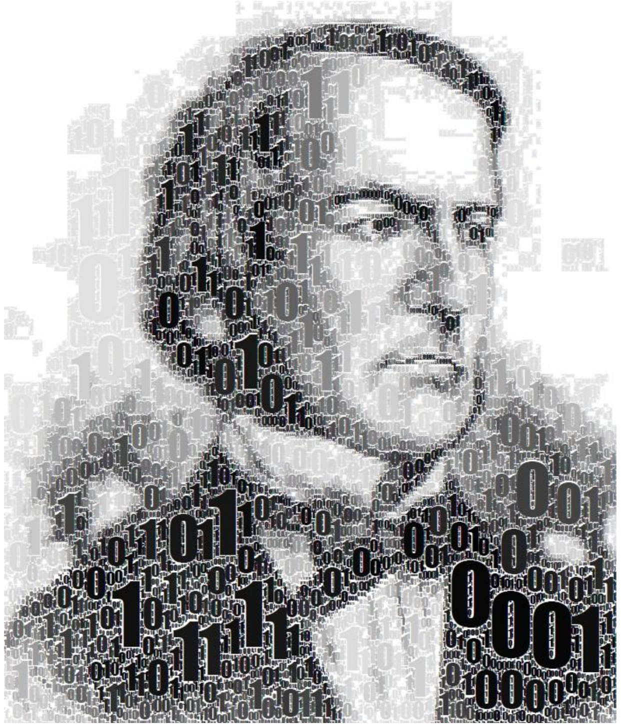
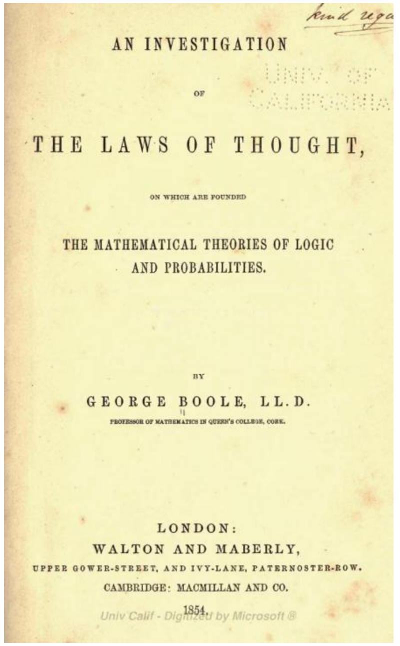
Historical Perspectives

George Boole (1815-1864)

- Mathematician and philosopher
- Invented symbolic / Boolean logic
- Invented Boolean algebra, i.e. "calculus of reasoning"
- A founder of computer science
- "An Investigation into the Laws of Thought"
- Influenced De Morgan, Schröder, Shannon
- All modern computers, electronics, phones, data transmission, rely on Boolean principles





AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

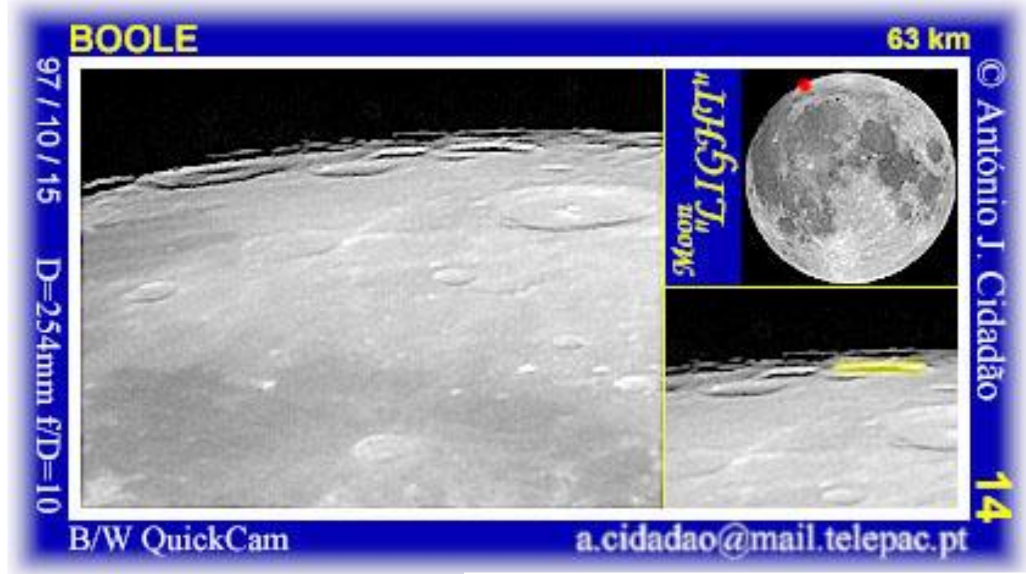
BY

GEORGE BOOLE, LL.D.

PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.

LONDON:
WALTON AND MABERLY,
UPPER GOWER-STREET, AND IVY-LANE, PATERNOSTER-ROW.
CAMBRIDGE: MACMILLAN AND CO.
1854.


GEORGE BOOLE
LL.D. D.C.L. F.R.S.
1815 — 1864
GEORGE BOOLE, FATHER OF MODERN ALGEBRA, AUTHOR OF "THE LAWS OF THOUGHT" AND FIRST PROFESSOR OF MATHEMATICS AT UNIVERSITY COLLEGE, CORK, WAS BORN IN LINCOLN AND ESTABLISHED AN ACADEMY IN THIS HOUSE C.1840.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63 km
97 / 10 / 15 D=254mm f/D=10
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a.cidadao@mail.telepac.pt


36
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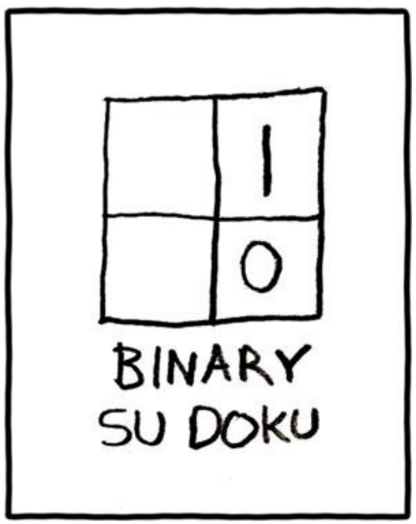
РЕПУБЛИКА МАКЕДОНИЈА
ПОШТА
2004 СКЕНПОИНТ
САМИТ ЗА ИНФОРМАТИЧКО ОПШТЕСТВО


LOGIC LANE


All cats have four legs.
I have four legs.
Therefore, I am a cat.
S. Harris


GEORGE BOOLE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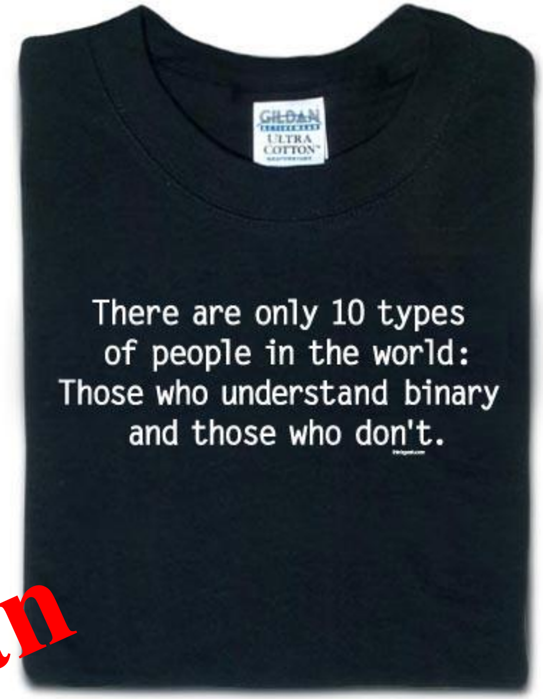
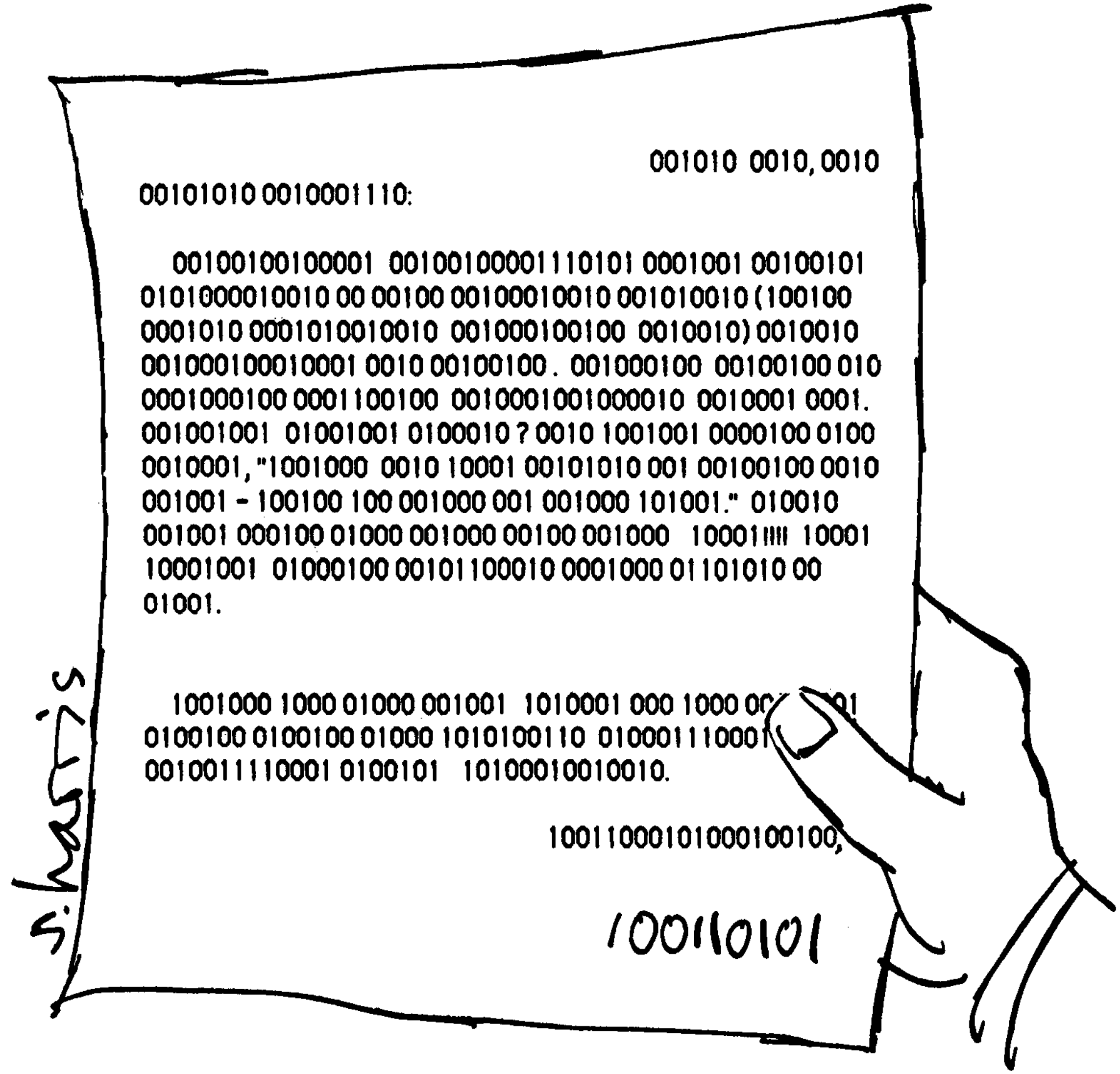
BINARY
SU DOKU

Boolean humor





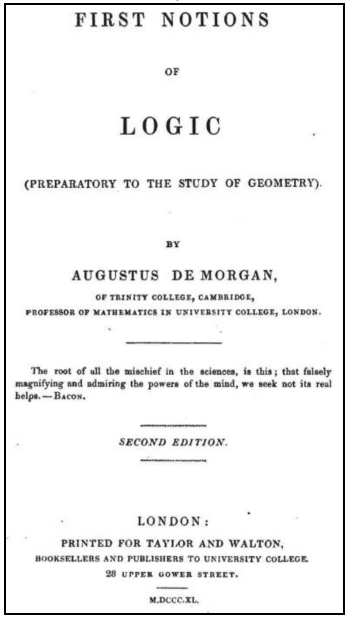
BINARY LETTER FROM GRANDMA

Mozart writing the digital version of his symphony No. 38
in D major.

Historical Perspectives

Augustus De Morgan (1806-1871)

- Mathematician and logician
- Developed logic & mathematical induction
- De Morgan's Laws in logic & set theory
- Invented relational algebra
- Corresponded extensively with Hamilton
- Influenced Russell, Whitehead, and Tarski
- Studied paradoxes



FIRST NOTIONS

OF

LOGIC

(PREPARATORY TO THE STUDY OF GEOMETRY).

BY

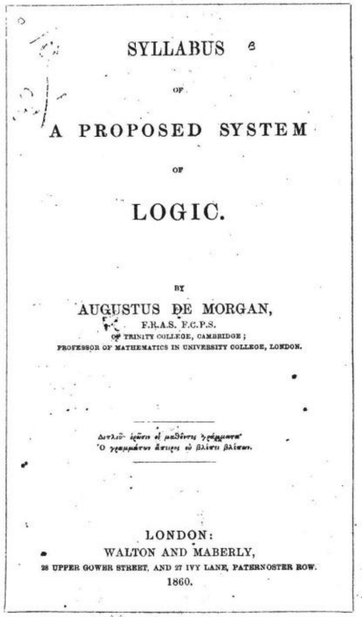
AUGUSTUS DE MORGAN,

OF TRINITY COLLEGE, CAMBRIDGE,
PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON.

The root of all the mischief in the sciences, is this; that falsely magnifying and admiring the powers of the mind, we seek not its real helps.—BACON.

SECOND EDITION.

LONDON:
PRINTED FOR TAYLOR AND WALTON,
BOOKSELLERS AND PUBLISHERS TO UNIVERSITY COLLEGE,
28 UPPER GOWER STREET.
M.DCCC.XL.



SYLLABUS

OF

A PROPOSED SYSTEM

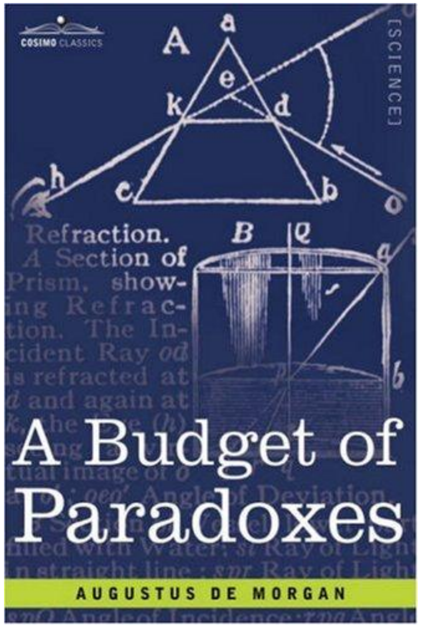
OF

LOGIC.

BY

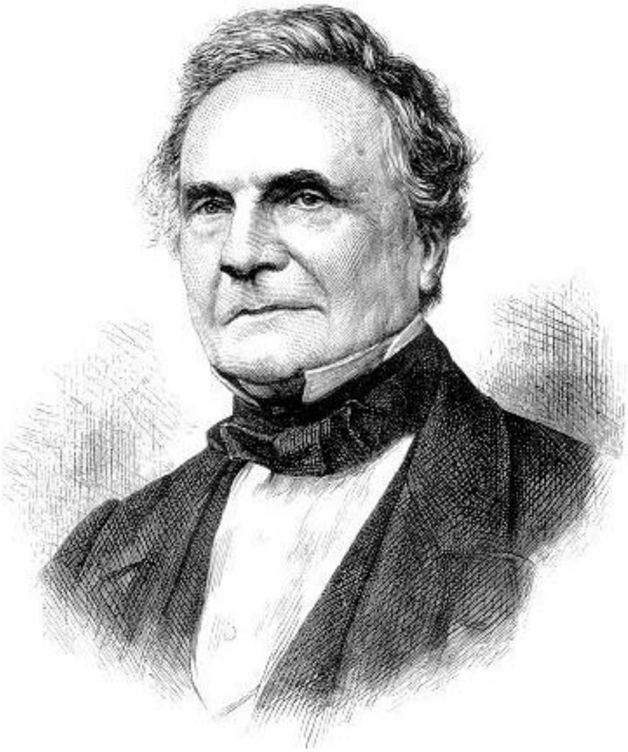
AUGUSTUS DE MORGAN,
F.R.A.S. F.C.P.S.
OF TRINITY COLLEGE, CAMBRIDGE;
PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON.

LONDON:
WALTON AND MABERLY,
28 UPPER GOWER STREET, AND 27 IVY LANE, PATERNOSTER ROW.
1860.

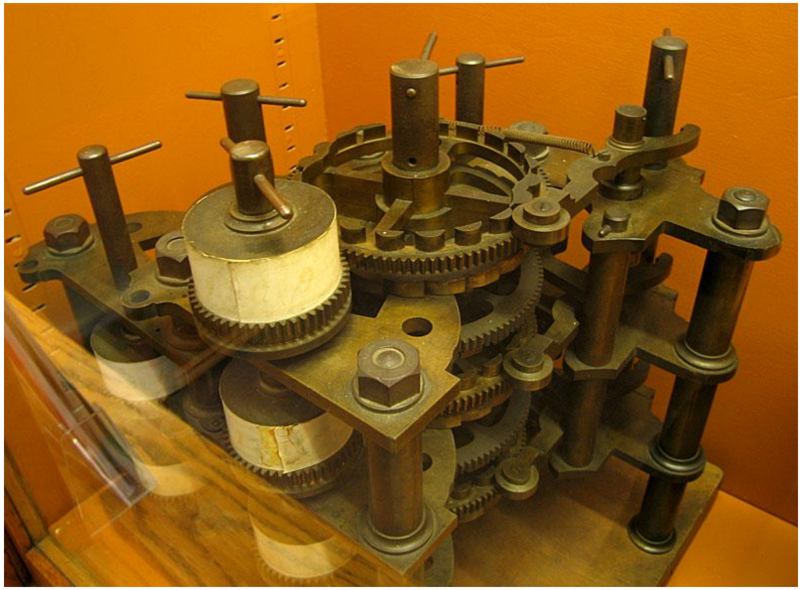


Historical Perspectives

Charles Babbage (1791-1871)

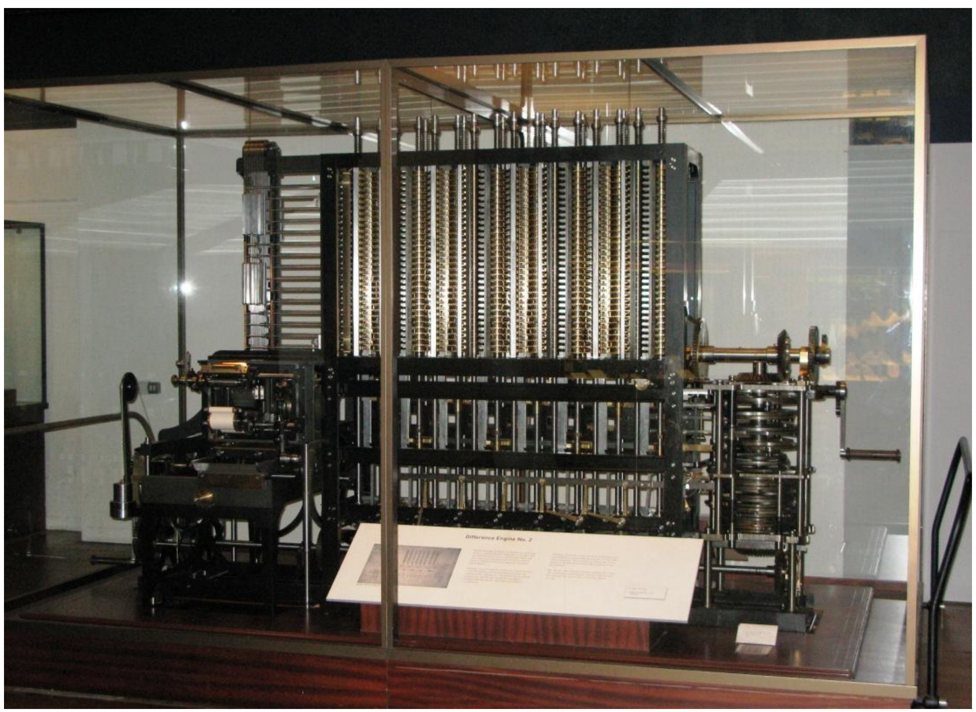
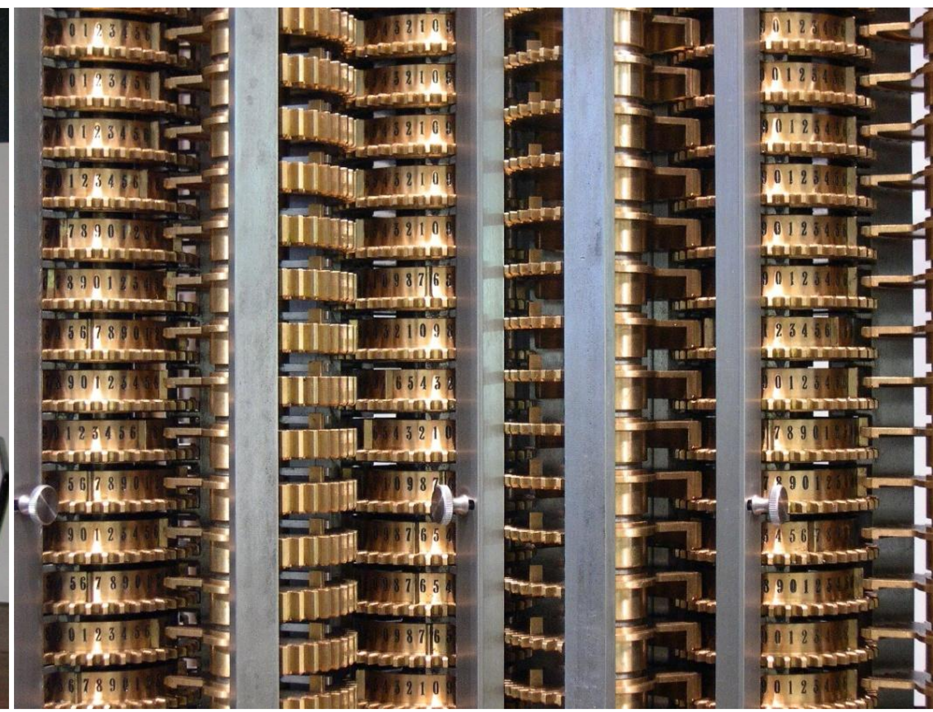


- Mathematician, philosopher, inventor mechanical engineer, and economist
- The father of computing
- Built world's first mechanical computer
  - the "difference engine" (1822)
- Originated the programmable computer
  - the "analytical engine" (1837)
- Worked in cryptography
- Developed Babbage's principle of division of labor

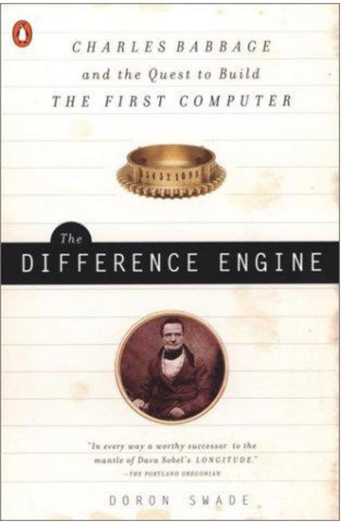


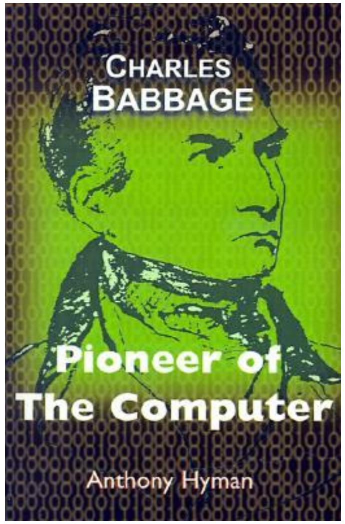
Babbage’s Difference Engine

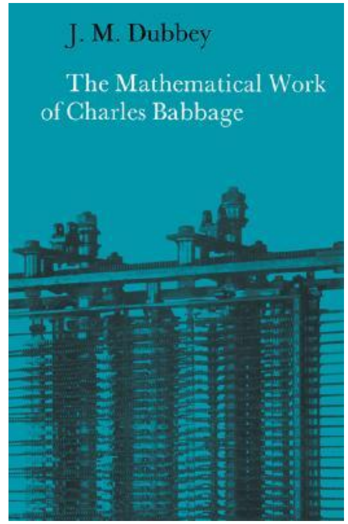
- World’s first mechanical computer
- Designed in 1822, redesigned in 1847-1849
- 25,000 parts, 15 tons, 8ft tall, 31 digits of precision
- Tabulated polynomial functions, used Newton’s method
- Approximated logarithmic and polynomial functions
- Used decimal number system and hand-crank

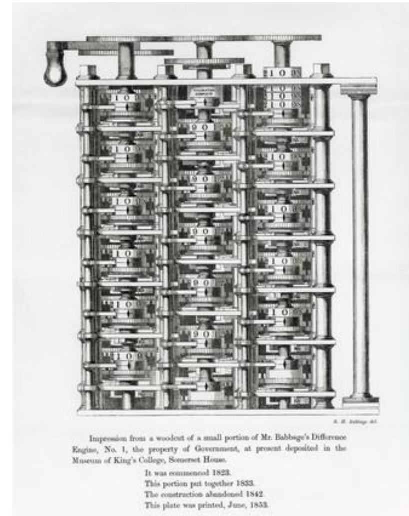
 

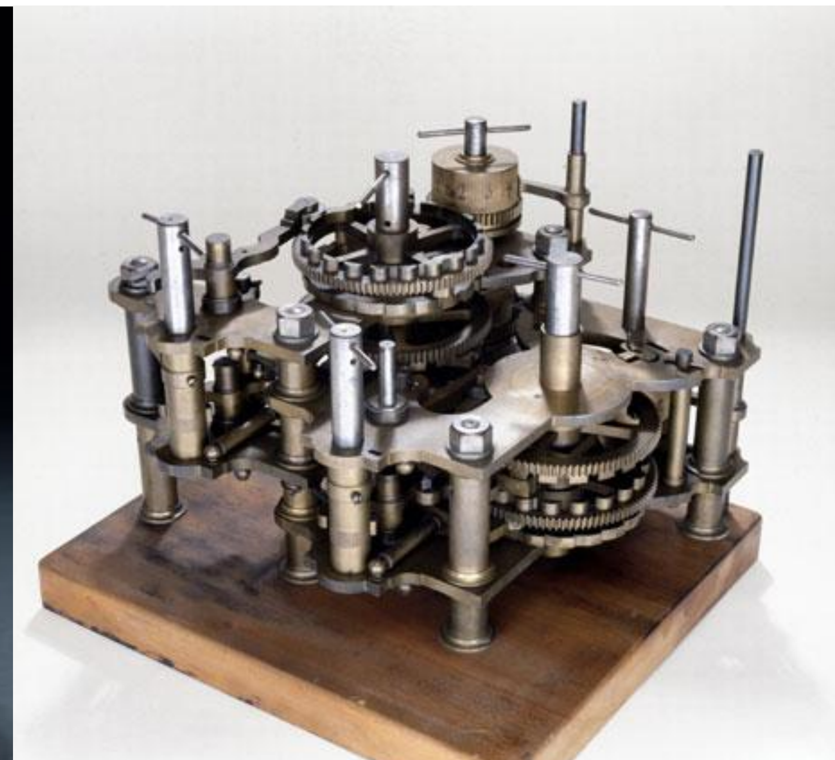
Babbage's Difference Engine

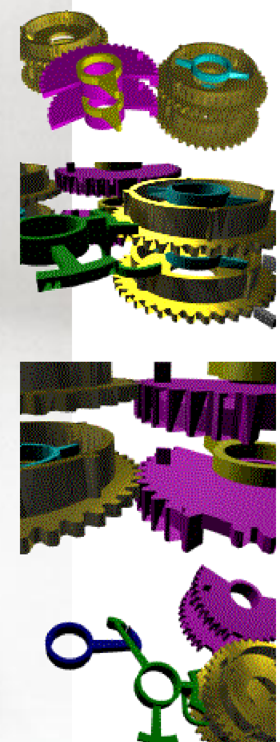




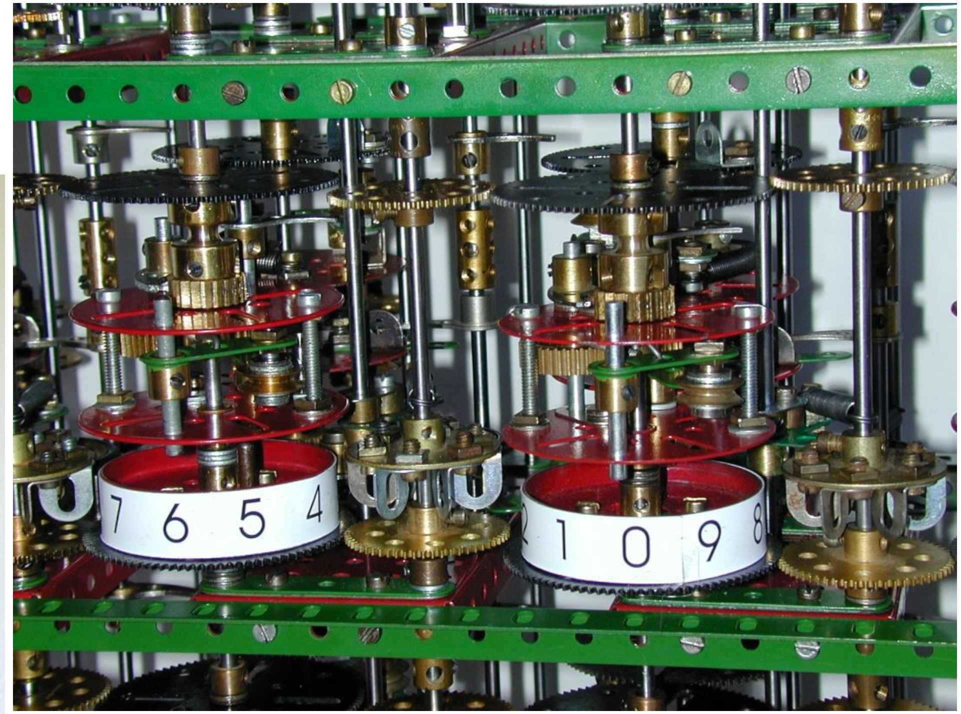


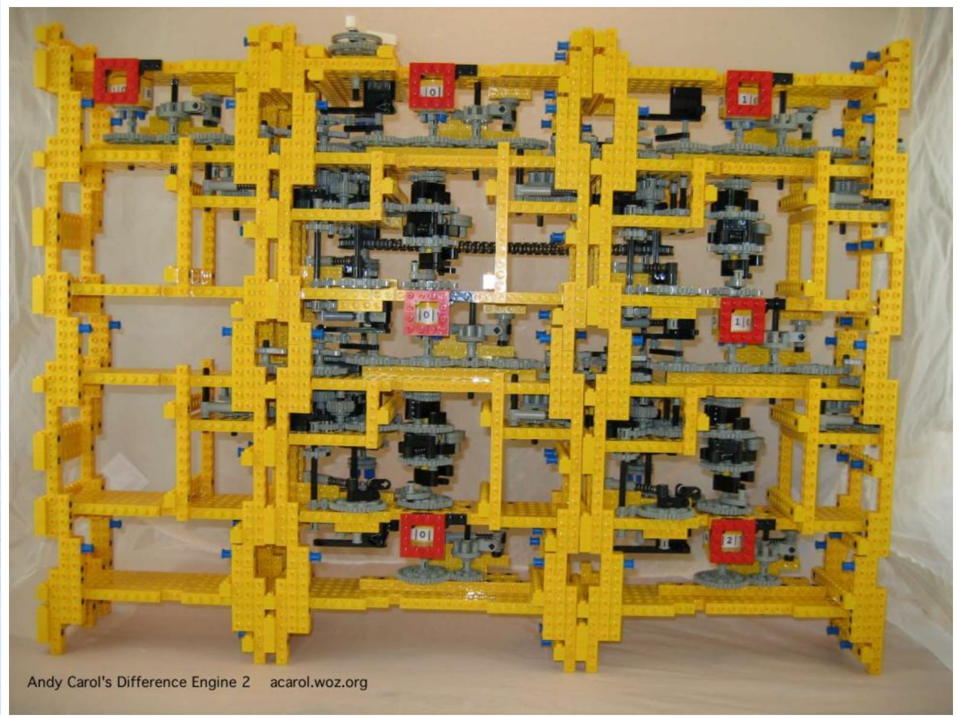






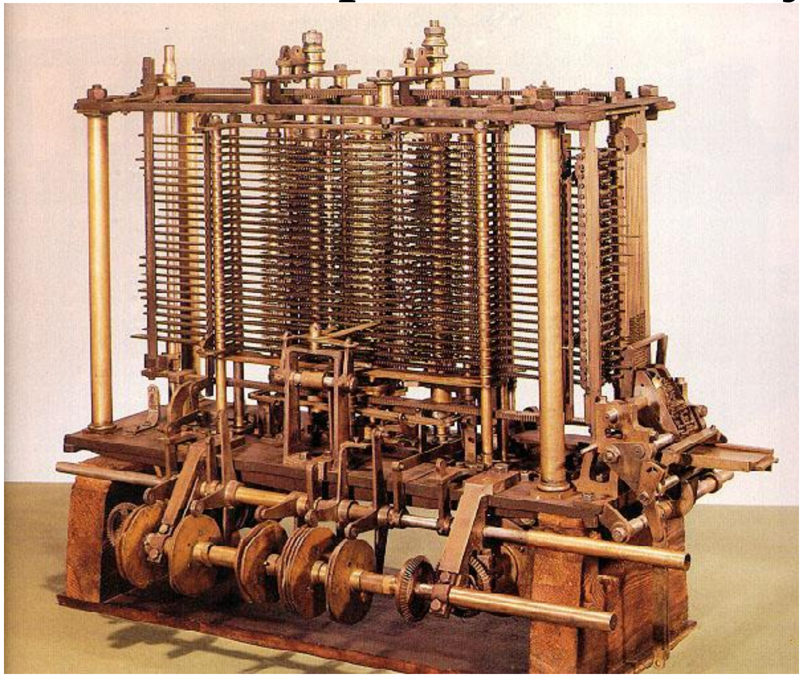
Babbage’s difference engine built from Mechano and Lego

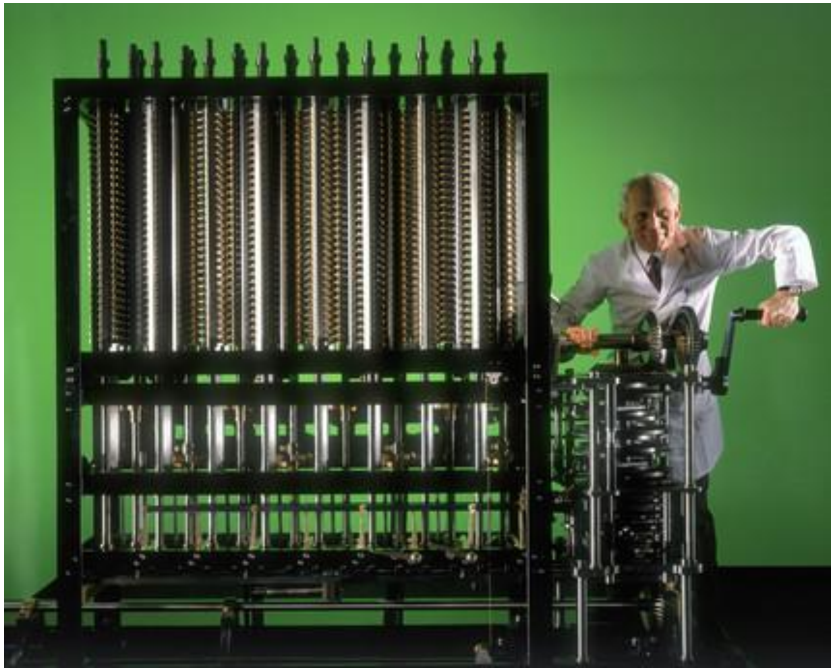


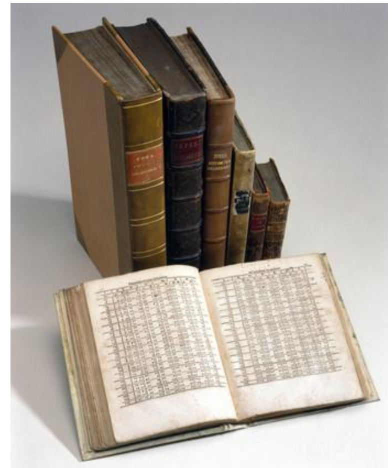


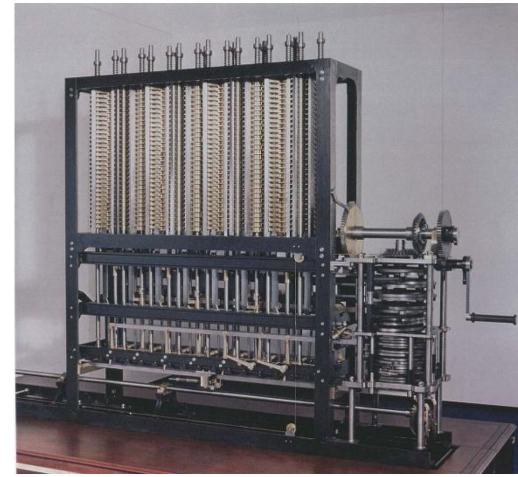
Babbage's Analytical Engine

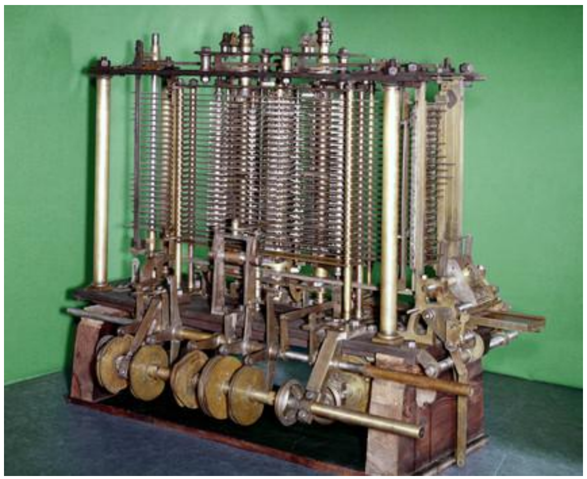
- World's first general-purpose computer
- Designed in 1837, redesigned throughout Babbage's life
- Turing-complete, memory: 1000x50 digits (21 kB)
- Fully programmable "CPU", used punched cards
- Featured ALU, "microcode", loops, and printer!
- Could multiply two 20-digit numbers in 3 min
- Few components built by Babbage; constructed in 1991

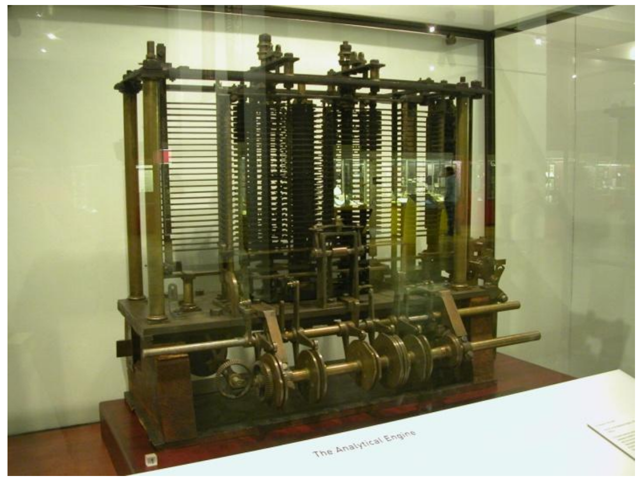


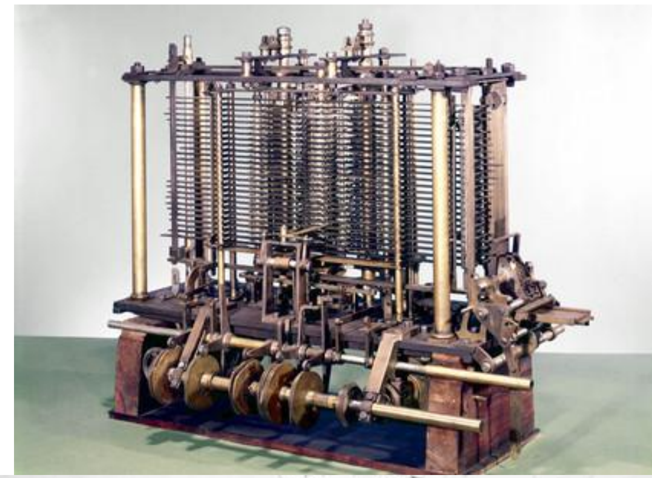


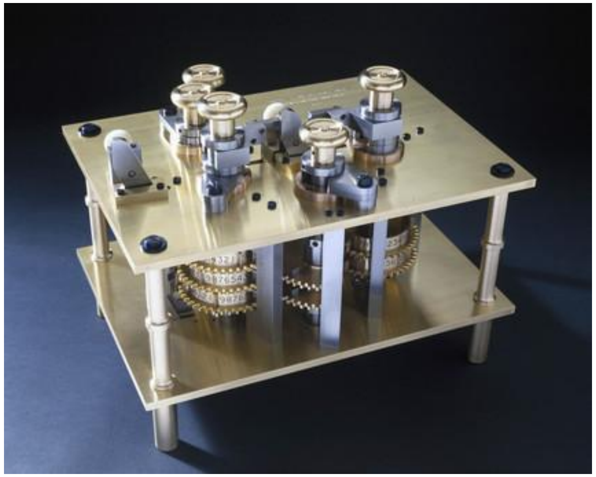


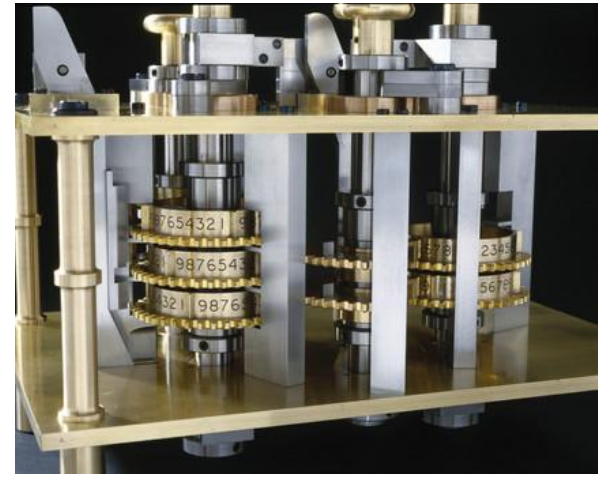


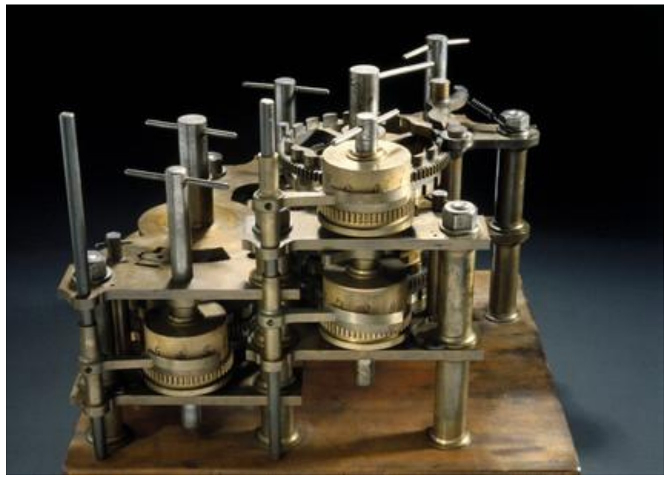



The Analytical Engine

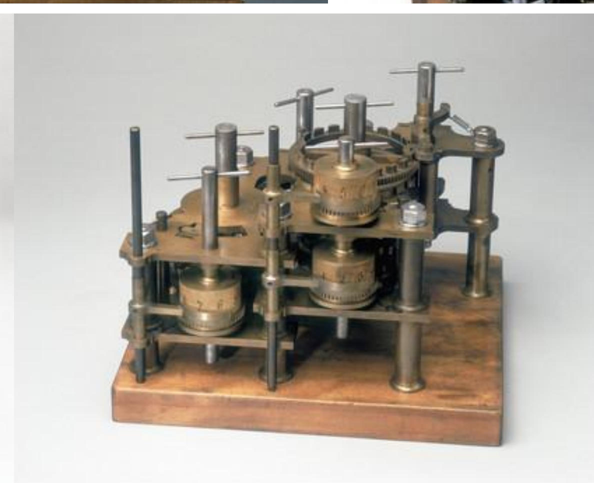




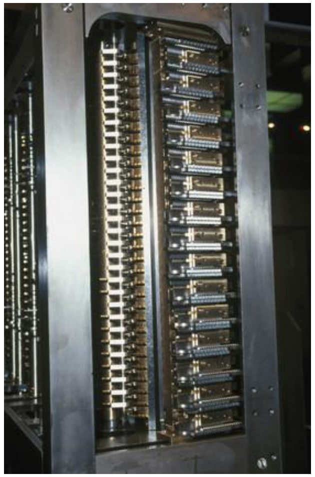


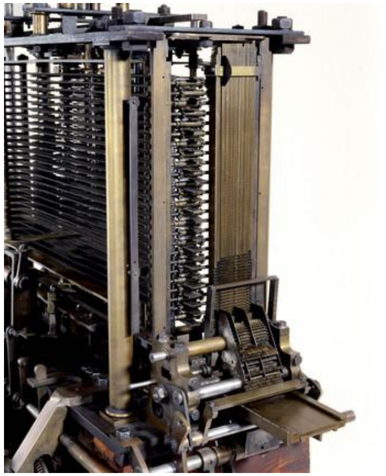


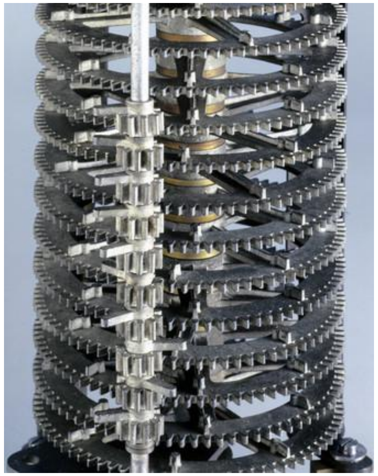




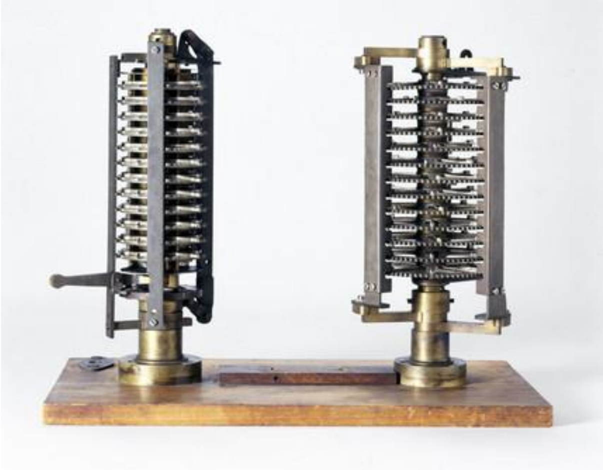




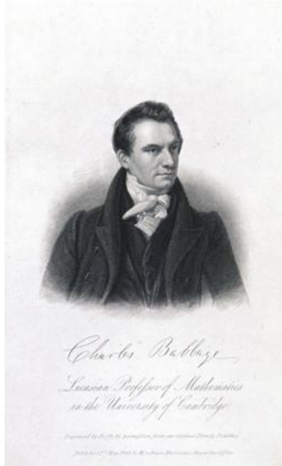



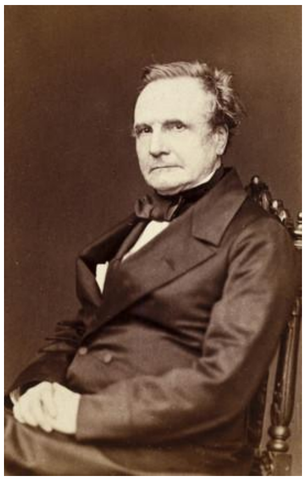





Automatic
Printer!

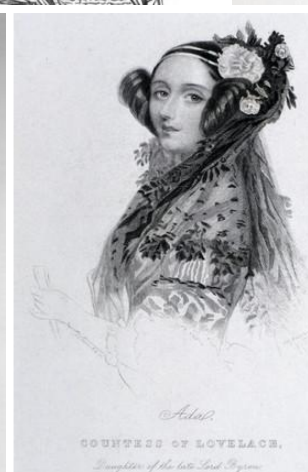


Charles Babbage
Lucasian Professor of Mathematics
in the University of Cambridge


Charles Babbage

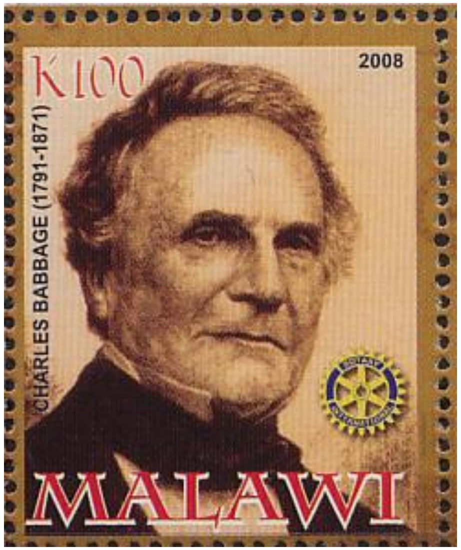



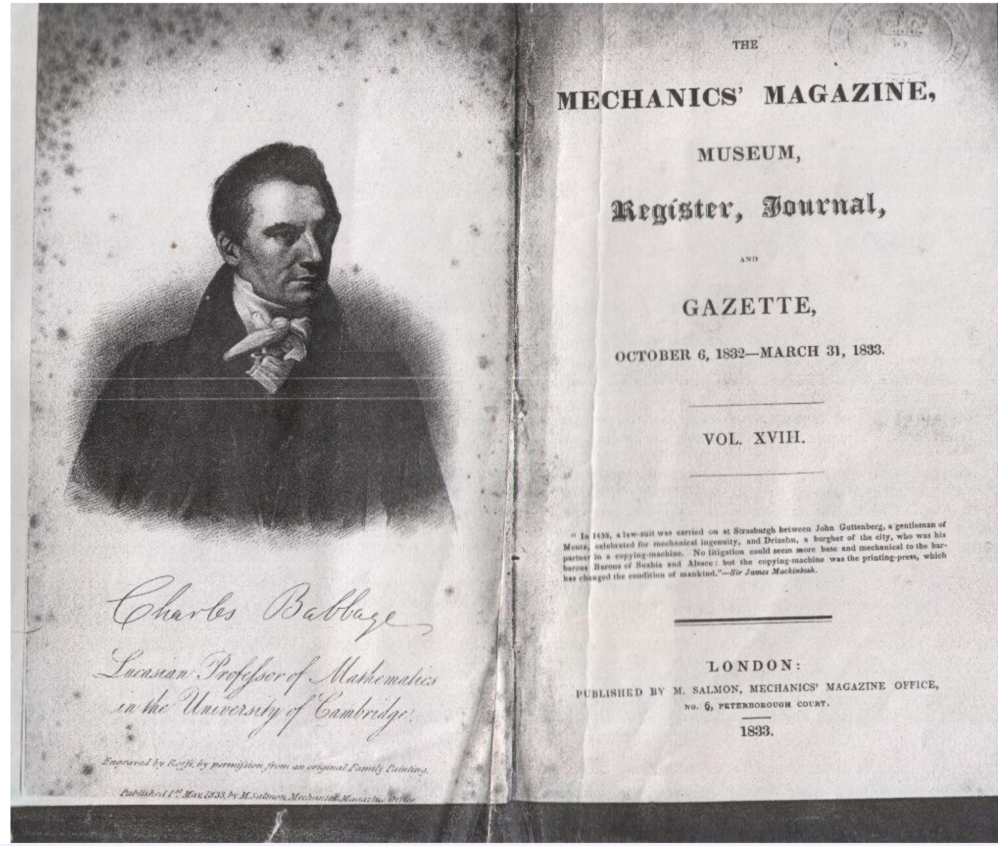
C. BABBAGE (MATHEMATICIAN)
Died Oct. 20, 1871, aged 79


Ever truly Yours
C Babbage



Ada.
COUNTESS OF LOVELACE.
Daughter of the late Lord Byron


Lady Byron



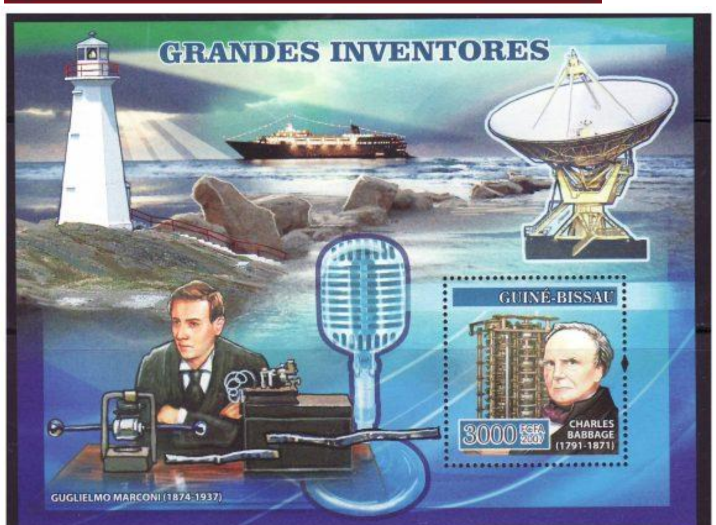


THE
MECHANICS' MAGAZINE,
MUSEUM,
Register, Journal,
AND
GAZETTE,
OCTOBER 6, 1832—MARCH 31, 1833.

VOL. XVIII.

"In 1435, a law-suit was carried on at Strasburgh between John Guttenberg, a gentleman of Mentz, celebrated for mechanical ingenuity, and Drizehn, a burgher of the city, who was his partner in a copying-machine. No litigation could seem more base and mechanical to the barbarous Barons of Suabia and Alsace: but the copying-machine was the printing-press, which has changed the condition of mankind."—Sir James Mackintosh.

LONDON:
PUBLISHED BY M. SALMON, MECHANICS' MAGAZINE OFFICE,
No. 6, PETERBOROUGH COURT.
1833.



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- May 20th MHHC: IBM's Blue Gene
- New *CBI Newsletter* (Spring 2009, Vol. 31:1)
- McDonald Named 2009-2010 Tomash Fellow
- 2009 Norberg Travel Award Recipients

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Ask a CBI archivist your questions about collections and services through instant message during regular business hours.

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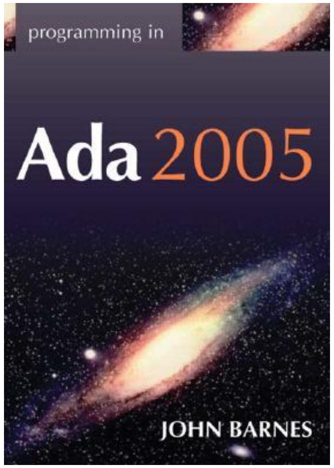
Historical Perspectives

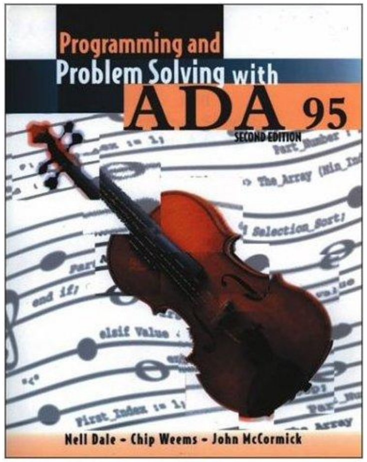
Countess Ada Lovelace (1815-1852)

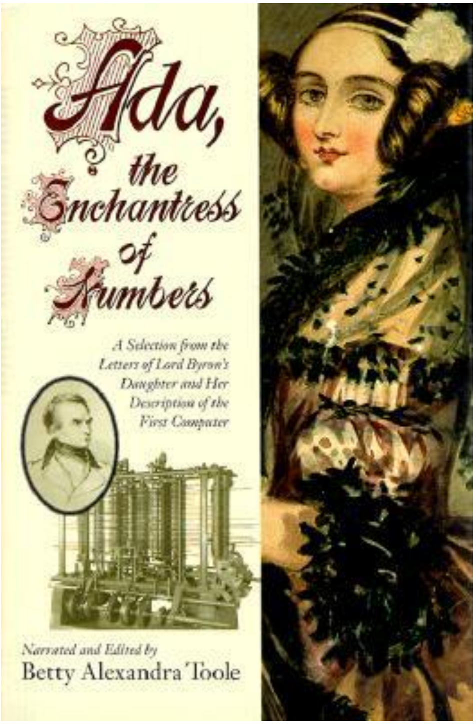
- Daughter of Lord Byron
- Tutored in math and logic by De Morgan
- Wrote the "manual" for Babbage's analytical engine, as well as programs for it
- World's first computer programmer!
- Foresaw the vast potential of computers
- Babbage: "The Enchantress of Numbers"
- DoD's Ada language "MIL-STD-1815"

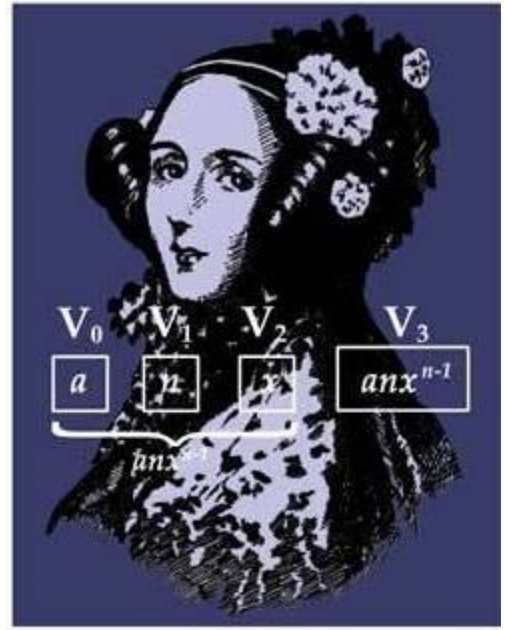
Ada

The International Language for Software Engineering

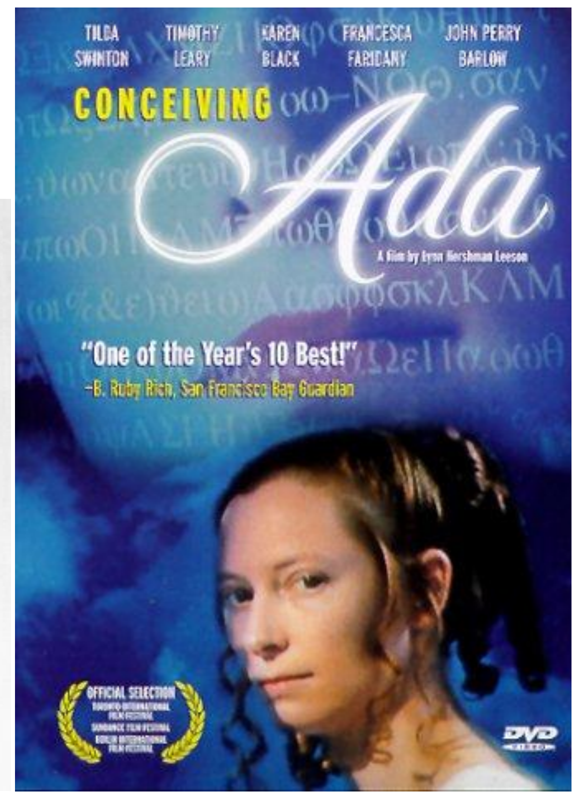








Ada Byron, Lady Lovelace
1815 - 1852

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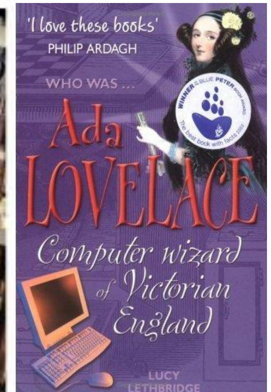

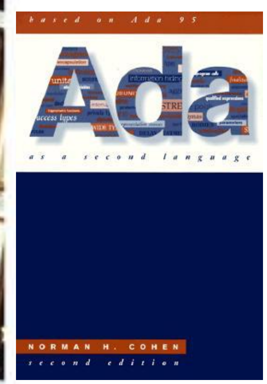

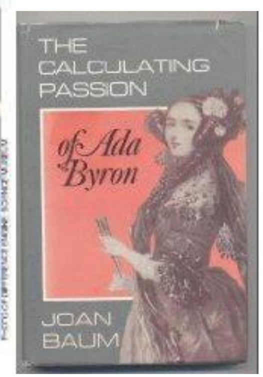

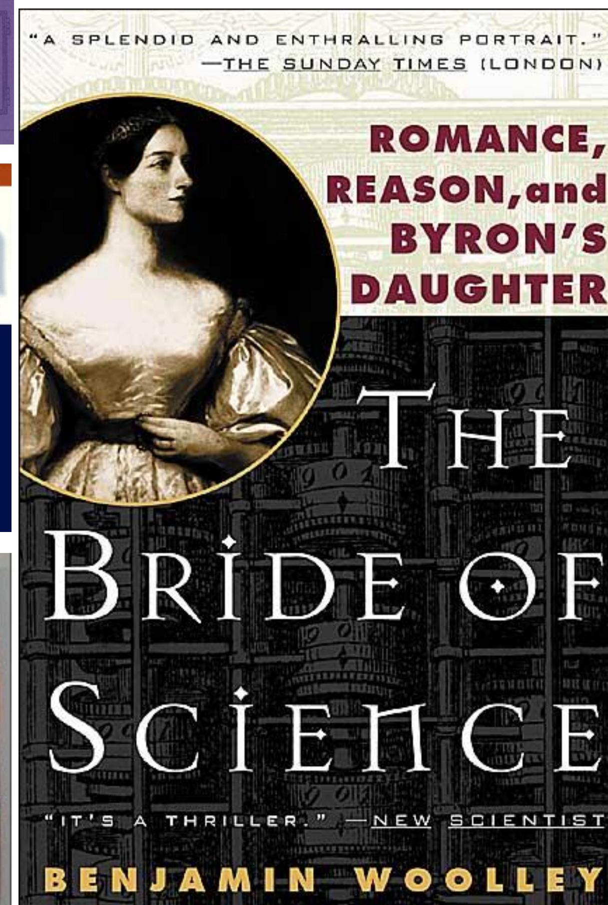

Lovelace Medal

Lovelace Medal Home

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Lovelace Medal

The Lovelace Medal is presented to individuals who have made a contribution which is of major significance in the advancement of Information Systems or which adds significantly to the understanding of Information Systems.

About the medal

Lovelace Medal 2009

2009 winner
The winner of the 2009 Lovelace Medal is Professor Yorick Wilks.

Previous Lectures

Video: A tribute to Karen Spärck Jones
The 2008 BCS Lovelace Medal lecture was a very special event dedicated to the memory of Karen Spärck Jones who was presented the award just weeks before she died last year. The lecture was delivered by Dr Ann Copestake and is now available to watch online.

2007 Lovelace Lecture - Sir Tim Berners-Lee
The Web is a technical and social creation, dependent on both technical protocols and social conventions. The origins and potential futures of this large scale, emergent phenomena were discussed by Sir Tim Berners-Lee in this year's BCS Lovelace Lecture - now available to watch via this website.

Previous winners

Previous winners of the Lovelace Medal have included:

2008 - Dr Tony Storey
2007 - Karen Sparck-Jones
2006 - Sir Tim Berners-Lee
2005 - Dr Nicholas McKeown

Ada Lovelace notes on “Sketch of the Analytical Engine Invented by Charles Babbage”, by L. F. Menabrea, 1843

Her notes (three times longer than the paper itself!) contain the world’s first computer program (for calculating Bernoulli numbers):

Number of Operations	Nature of Operations	Variables for Data						Working Variables									Variables for Results	
		$^1V_0$	$^1V_1$	$^1V_2$	$^1V_3$	$^1V_4$	$^1V_5$	$^0V_6$	$^0V_7$	$^0V_8$	$^0V_9$	$^0V_{10}$	$^0V_{11}$	$^0V_{12}$	$^0V_{13}$	$^0V_{14}$	$^0V_{15}$	$^0V_{16}$
		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		$\boxed{m}$	$\boxed{n}$	$\boxed{d}$	$\boxed{m'}$	$\boxed{n'}$	$\boxed{d'}$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\ }$	$\boxed{\frac{dn'-d'n}{mn'-m'n} = x}$	$\boxed{\frac{d'm-dm}{mn'-m'n} = y}$
1	$\times$	m	....	....	....	n'	....	mn'										
2	$\times$	....	n	....	m'	....	....	....	$m'n$									
3	$\times$	....	....	d	....	....	....	....	....	dn'								
4	$\times$	....	0	....	....	....	d'	....	....	....	$d'n$							
5	$\times$	0	....	....	....	....	0	....	....	....	....	$d'm$						
6	$\times$	....	....	0	0	....	....	....	....	....	....	.....	dm'					
7	$-$	....	....	....	....	....	....	0	0	....	....	.....	.....	$(mn' - m'n)$				
8	$-$	....	....	....	....	....	....	....	....	0	0	.....	.....	....	$(dn' - d'n)$			
9	$-$	....	....	....	....	....	....	....	....	....	....	0	0	....	....	$(d'm - dm')$		
10	$\div$	....	....	....	....	....	....	....	....	....	....	.....	.....	$(mn' - m'n)$	0	....	$\frac{dn'-d'n}{mn'-m'n} = x$	
11	$\div$	....	....	....	....	....	....	....	....	....	....	.....	.....	0	....	0	....	$\frac{d'm-dm'}{mn'-m'n} = y$

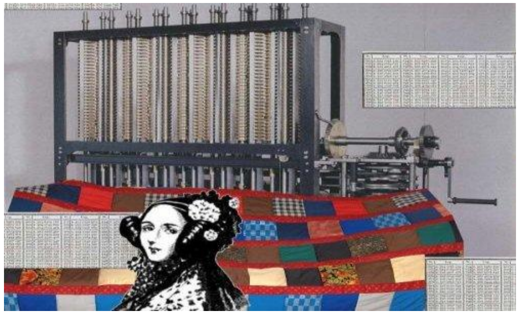
World's first computer program (for calculating Bernoulli numbers), by Ada Lovelace, 1843:

Number of Operation	Nature of Operation	Variables acted upon	Variables receiving results	Indication of change in the value on any Variable	Statement of Results	Data			Working Variables										Result Variables			
						$^1V_1$	$^1V_2$	$^1V_3$	$^0V_4$	$^0V_5$	$^0V_6$	$^0V_7$	$^0V_8$	$^0V_9$	$^0V_{10}$	$^0V_{11}$	$^0V_{12}$	$^0V_{13}\cdots$	$^1V_{21}$	$^1V_{22}$	$^1V_{23}$	$^0V_{24}\cdots$
						○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
						0	0	0	0	0	0	0	0	0	0	0	0	0	B in a dec. fract.	B in a dec. fract.	B in a dec. fract.	0
						0	0	0	0	0	0	0	0	0	0	0	0	0				0
						1	2	4	0	0	0	0	0	0	0	0	0	0				0
						$\boxed{1}$	$\boxed{2}$	$\boxed{n}$											$\boxed{B_1}$	$\boxed{B_3}$	$\boxed{B_5}$	$\boxed{B_7}$
1	×	$^1V_2 \times {}^1V_3$	$^1V_4, {}^1V_5, {}^1V_6$	$\left\{\begin{array}{l} ^1V_2 = {}^1V_2 \\ ^1V_3 = {}^1V_3 \end{array}\right\}$	$= 2n$	......	2	n	$2n$	$2n$	$2n$											
2	−	$^1V_4 - {}^1V_1$	$^2V_4$	$\left\{\begin{array}{l} ^1V_4 = {}^2V_4 \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= 2n - 1$	1	......	......	$2n-1$													
3	+	$^1V_5 + {}^1V_1$	$^2V_5$	$\left\{\begin{array}{l} ^1V_5 = {}^2V_5 \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= 2n + 1$	1	......	......	......	$2n+1$												
4	÷	$^2V_5 \div {}^2V_4$	$^1V_{11}$	$\left\{\begin{array}{l} ^2V_5 = {}^0V_5 \\ ^2V_4 = {}^0V_4 \end{array}\right\}$	$= \frac{2n-1}{2n+1}$	......	......	......	0	0	......	......	......	......	......	$\frac{2n-1}{2n+1}$						
5	÷	$^1V_{11} \div {}^1V_2$	$^2V_{11}$	$\left\{\begin{array}{l} ^1V_{11} = {}^2V_{11} \\ ^1V_2 = {}^1V_2 \end{array}\right\}$	$= \frac{1}{2} \cdot \frac{2n-1}{2n+1}$	......	2	......	......	......	......	......	......	......	......	$\frac{1}{2} \cdot \frac{2n-1}{2n+1}$						
6	−	$^0V_{13} - {}^2V_{11}$	$^1V_{13}$	$\left\{\begin{array}{l} ^2V_{11} = {}^0V_{11} \\ ^0V_{13} = {}^1V_{13} \end{array}\right\}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$	......	......	......	......	......	......	......	......	......	......	0	......	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$				
7	−	$^1V_3 - {}^1V_1$	$^1V_{10}$	$\left\{\begin{array}{l} ^1V_3 = {}^1V_3 \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= n - 1 (= 3)$	1	......	n	......	......	......	......	......	......	$n-1$							
8	+	$^1V_2 + {}^0V_7$	$^1V_7$	$\left\{\begin{array}{l} ^1V_2 = {}^1V_2 \\ ^0V_7 = {}^1V_7 \end{array}\right\}$	$= 2 + 0 = 2$	......	2	......	......	......	......	2										
9	÷	$^1V_6 \div {}^1V_7$	$^3V_{11}$	$\left\{\begin{array}{l} ^1V_6 = {}^1V_6 \\ ^0V_{11} = {}^3V_{11} \end{array}\right\}$	$= \frac{2n}{2} = A_1$	......	......	......	......	......	$2n$	2	......	......	......	$\frac{2n}{2} = A_1$						
10	×	$^1V_{21} \times {}^3V_{11}$	$^1V_{12}$	$\left\{\begin{array}{l} ^1V_{21} = {}^1V_{21} \\ ^3V_{11} = {}^3V_{11} \end{array}\right\}$	$= B_1 \cdot \frac{2n}{2} = B_1 A_1$	......	......	......	......	......	......	......	......	......	......	$\frac{2n}{2} = A_1$	$B_1 \cdot \frac{2n}{2} = B_1 A_1$	......	B_1			
11	+	$^1V_{12} + {}^1V_{13}$	$^2V_{13}$	$\left\{\begin{array}{l} ^1V_{12} = {}^0V_{12} \\ ^1V_{13} = {}^2V_{13} \end{array}\right\}$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}$	......	......	......	......	......	......	......	......	......	......	......	0	$\left\{-\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}\right\}$				
12	−	$^1V_{10} - {}^1V_1$	$^2V_{10}$	$\left\{\begin{array}{l} ^1V_{10} = {}^2V_{10} \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= n - 2 (= 2)$	1	......	......	......	......	......	......	......	......	$n-2$							
13	−	$^1V_6 - {}^1V_1$	$^2V_6$	$\left\{\begin{array}{l} ^1V_6 = {}^2V_6 \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= 2n - 1$	1	......	......	......	......	$2n-1$											
14	+	$^1V_1 + {}^1V_7$	$^2V_7$	$\left\{\begin{array}{l} ^1V_1 = {}^1V_1 \\ ^1V_7 = {}^2V_7 \end{array}\right\}$	$= 2 + 1 = 3$	1	......	......	......	......	......	3										
15	÷	$^2V_6 \div {}^2V_7$	$^1V_8$	$\left\{\begin{array}{l} ^2V_6 = {}^2V_6 \\ ^2V_7 = {}^2V_7 \end{array}\right\}$	$= \frac{2n-1}{3}$	......	......	......	......	......	$2n-1$	3	$\frac{2n-1}{3}$									
16	×	$^1V_8 \times {}^3V_{11}$	$^4V_{11}$	$\left\{\begin{array}{l} ^1V_8 = {}^0V_8 \\ ^3V_{11} = {}^4V_{11} \end{array}\right\}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3}$	......	......	......	......	......	......	......	0	......	......	$\frac{2n}{2} \cdot \frac{2n-1}{3}$						
17	−	$^2V_6 - {}^1V_1$	$^3V_6$	$\left\{\begin{array}{l} ^2V_6 = {}^3V_6 \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= 2n - 2$	1	......	......	......	......	$2n-2$											
18	+	$^1V_1 + {}^2V_7$	$^3V_7$	$\left\{\begin{array}{l} ^2V_7 = {}^3V_7 \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= 3 + 1 = 4$	1	......	......	......	......	......	4										
19	÷	$^3V_6 \div {}^3V_7$	$^1V_9$	$\left\{\begin{array}{l} ^3V_6 = {}^3V_6 \\ ^3V_7 = {}^3V_7 \end{array}\right\}$	$= \frac{2n-2}{4}$	......	......	......	......	......	$2n-2$	4	......	$\frac{2n-2}{4}$	......							
20	×	$^1V_9 \times {}^4V_{11}$	$^5V_{11}$	$\left\{\begin{array}{l} ^1V_9 = {}^0V_9 \\ ^4V_{11} = {}^5V_{11} \end{array}\right\}$	$= \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = A_3$	......	......	......	......	......	......	......	......	0		$\left\{\frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4}\right\} = A_3$						
21	×	$^1V_{22} \times {}^5V_{11}$	$^0V_{12}$	$\left\{\begin{array}{l} ^1V_{22} = {}^1V_{22} \\ ^0V_{12} = {}^2V_{12} \end{array}\right\}$	$= B_3 \cdot \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = B_3 A_3$	......	......	......	......	......	......	......	......	......	......	0	$B_3 A_3$	......	......	B_3		
22	+	$^2V_{12} + {}^2V_{13}$	$^3V_{13}$	$\left\{\begin{array}{l} ^2V_{12} = {}^0V_{12} \\ ^2V_{13} = {}^3V_{13} \end{array}\right\}$	$= A_0 + B_1 A_1 + B_3 A_3$	......	......	......	......	......	......	......	......	......	......	......	0	$\{A_0 + B_1 A_1 + B_3 A_3\}$				
23	−	$^2V_{10} - {}^1V_1$	$^3V_{10}$	$\left\{\begin{array}{l} ^2V_{10} = {}^3V_{10} \\ ^1V_1 = {}^1V_1 \end{array}\right\}$	$= n - 3 (= 1)$	1	......	......	......	......	......	......	......	......	$n-3$							
Here follows a repetition of Operations thirteen to twenty-three																						
24	+	$^4V_{13} + {}^0V_{24}$	$^1V_{24}$	$\left\{\begin{array}{l} ^4V_{13} = {}^0V_{13} \\ ^0V_{24} = {}^1V_{24} \end{array}\right\}$	$= B_7$	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	B_7
25	+	$^1V_1 + {}^1V_3$	$^1V_3$	$\left\{\begin{array}{l} \left\{\begin{array}{l} ^1V_1 = {}^1V_1 \\ ^1V_3 = {}^1V_3 \end{array}\right\} \\ ^5V_6 = {}^0V_6 \\ ^5V_7 = {}^0V_7 \end{array}\right.$	$= n + 1 = 4 + 1 = 5$ by a Variable-card. by a Variable-card.	1	......	$n+1$	......	......	0	0										

Quotes from the Ada Lovelace notes on

"Sketch of the Analytical Engine Invented by Charles Babbage", 1843

"We may say most aptly, that the Analytical Engine *weaves algebraical patterns* just as the Jacquard-loom weaves flowers and leaves."



"Again, it might act upon other things besides *number*, were objects found whose mutual fundamental relations could be expressed by those of the abstract science of operations, and which should be also susceptible of adaptations to the action of the operating notation and mechanism of the engine. Supposing, for instance, that the fundamental relations of pitched sounds in the science of harmony and of musical composition were susceptible of such expression and adaptations, the engine might compose elaborate and scientific pieces of music of any degree of complexity or extent."

Quotes from the Ada Lovelace notes on
"Sketch of the Analytical Engine Invented by Charles Babbage", 1843

"Many persons who are not conversant with mathematical studies, imagine that because the business of the engine is to give its results in *numerical notation*, the *nature of its processes* must consequently be *arithmetical* and *numerical*, rather than *algebraical* and *analytical*. This is an error. The engine can arrange and combine its numerical quantities exactly as if they were *letters* or any other *general* symbols; and in fact it might bring out its results in algebraical *notation*, were provisions made accordingly."

"But it would be a mistake to suppose that because its *results* are given in the *notation* of a more restricted science, its *processes* are therefore restricted to those of that science. The object of the engine is in fact to give the *utmost practical efficiency* to the resources of *numerical interpretations* of the higher science of analysis, while it uses the processes and combinations of this latter."

LOT 21 / SALE 2013

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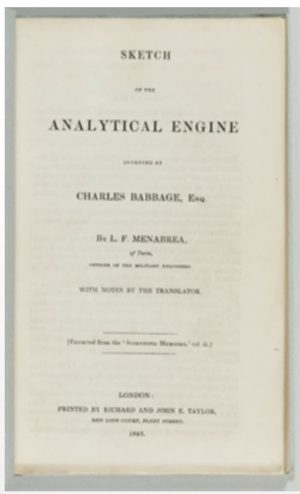
[BABBAGE]. -- MENABREA, Luigi Federico (1809-1896). *Sketch of the Analytical Engine invented by Charles Babbage... with notes by the translator*. Offprint from: *Scientific Memoirs*. Translated by Augusta Ada King, Countess of Lovelace (1809-1896). Volume 3. London: Richard and John E. Taylor, 1843.



ENLARGE & ZOOM

Price Realized (Set Currency)
$170,500
Price includes buyer's premium

Estimate
$10,000 - $15,000

Sale Information
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Lot Description
[BABBAGE]. -- MENABREA, Luigi Federico (1809-1896). *Sketch of the Analytical Engine invented by Charles Babbage... with notes by the translator*. Offprint from: *Scientific Memoirs*. Translated by Augusta Ada King, Countess of Lovelace (1809-1896). Volume 3. London: Richard and John E. Taylor, 1843.

LOTS IN THIS SALE

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LOT #21 [BABBAGE]. -- MENABREA, Luigi Federico...

IMAGE NOT AVAILABLE LOT #22 BABBAGE, Charles. *Passages from the...*

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LOT #27 BAYER, Johann (1572-1625)....

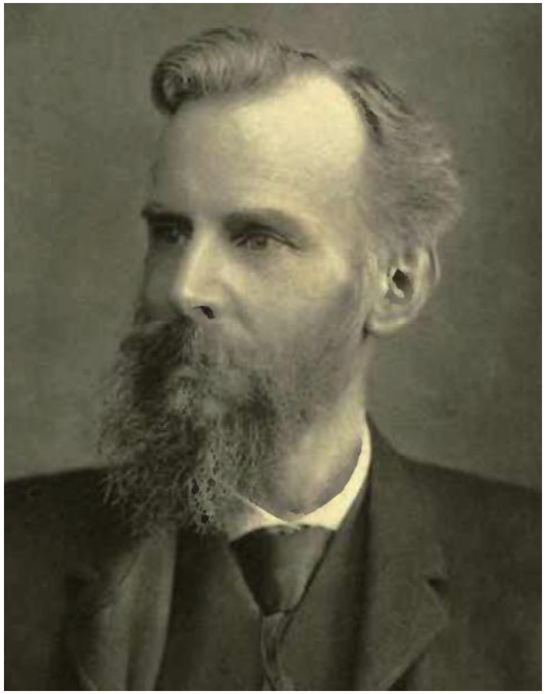
IMAGE NOT AVAILABLE LOT #28 BEAUMONT, William (1785-1853)....

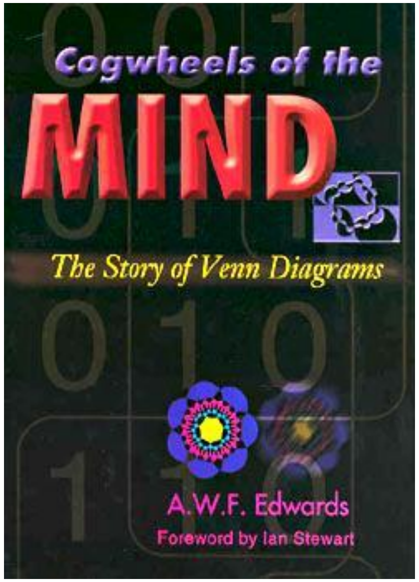
LOT #30

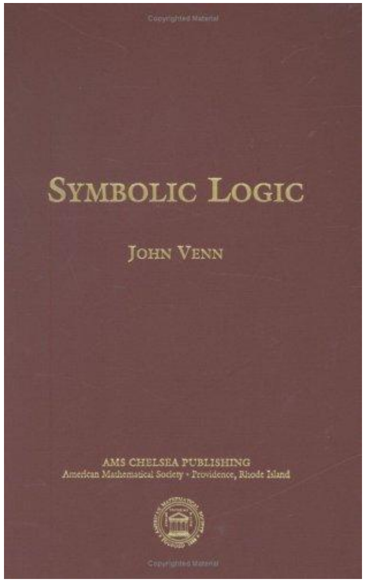
Historical Perspectives

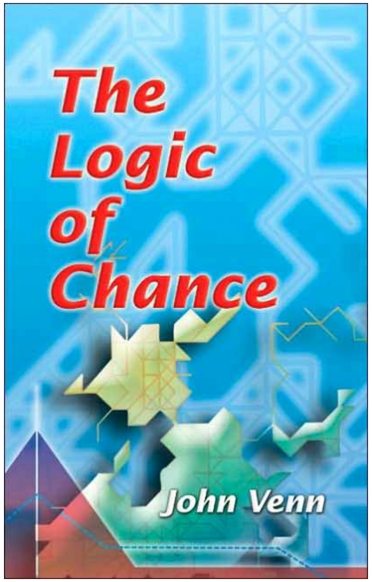
John Venn (1834-1923)

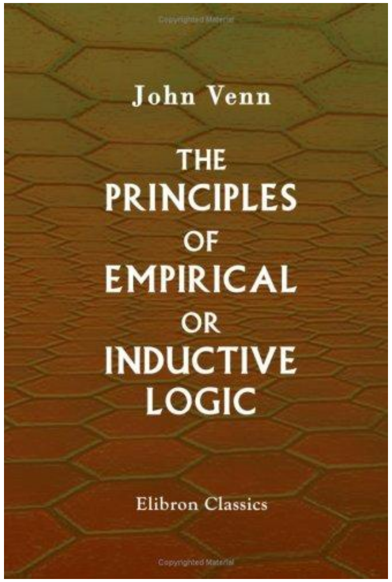
- Logician and philosopher
- Worked in logic, probability, set theory
- Introduced the "Venn diagram" (1880)
  - Very widely used, many applications
  - Ties together fundamental concepts from logic, geometry, combinatorics, knot theory



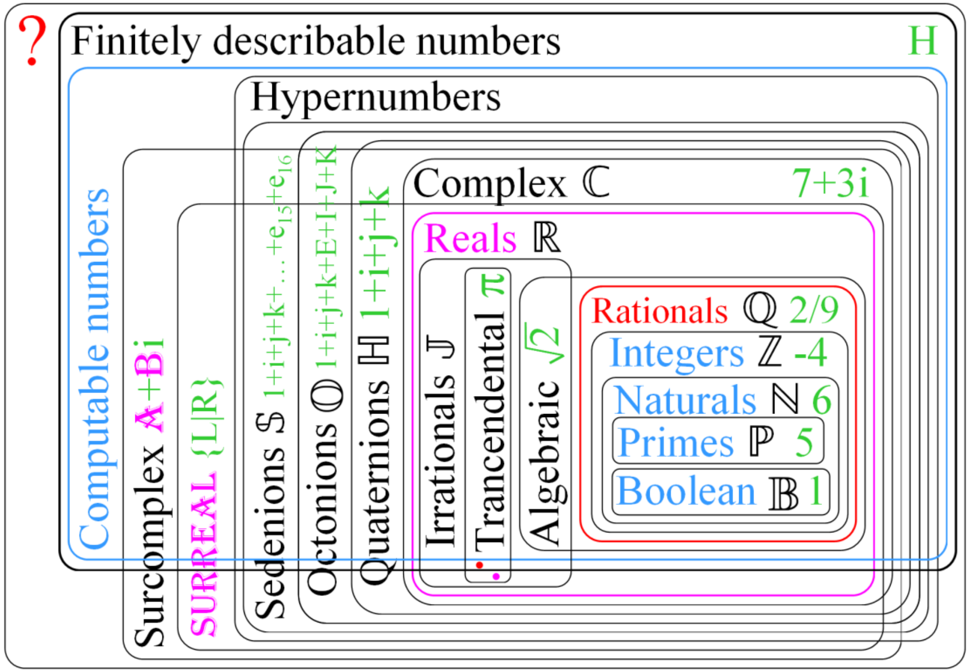




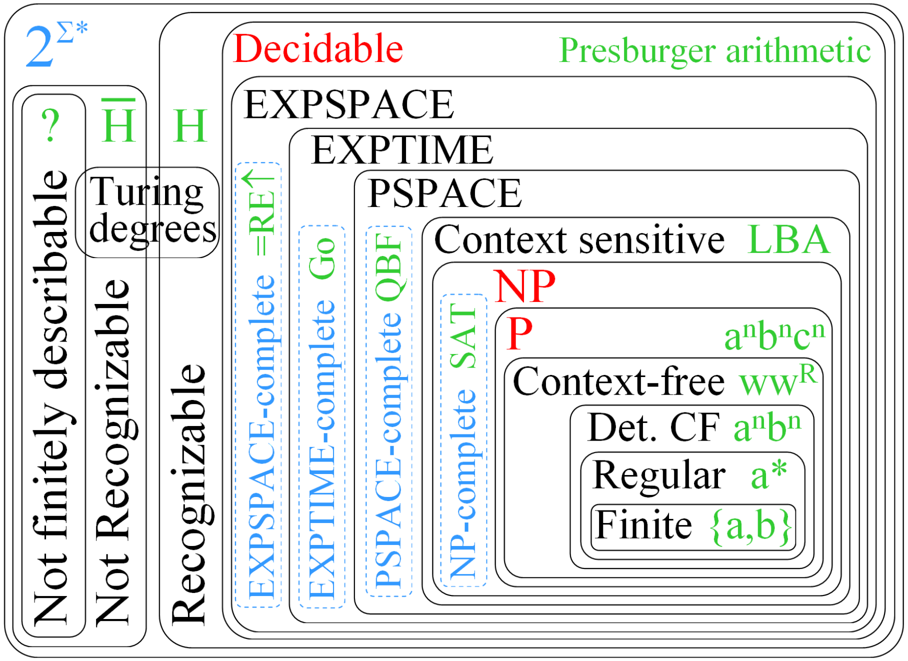


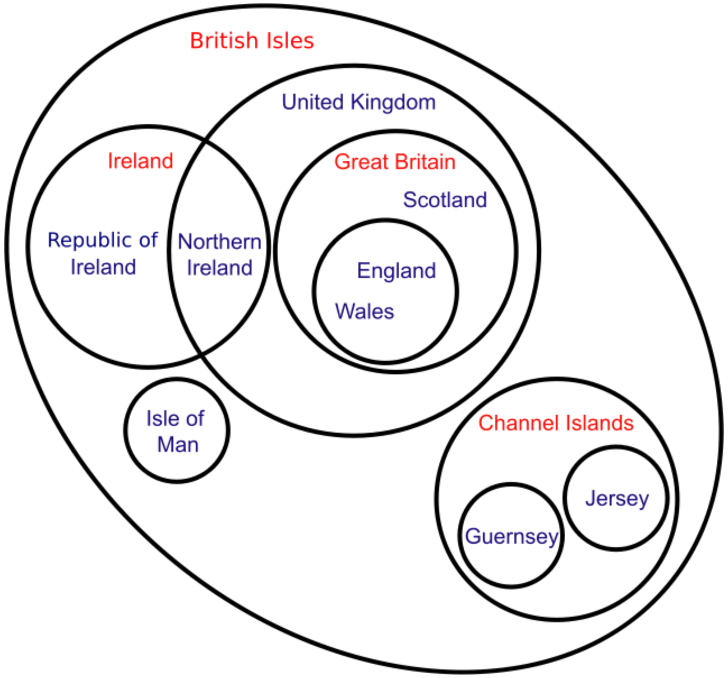


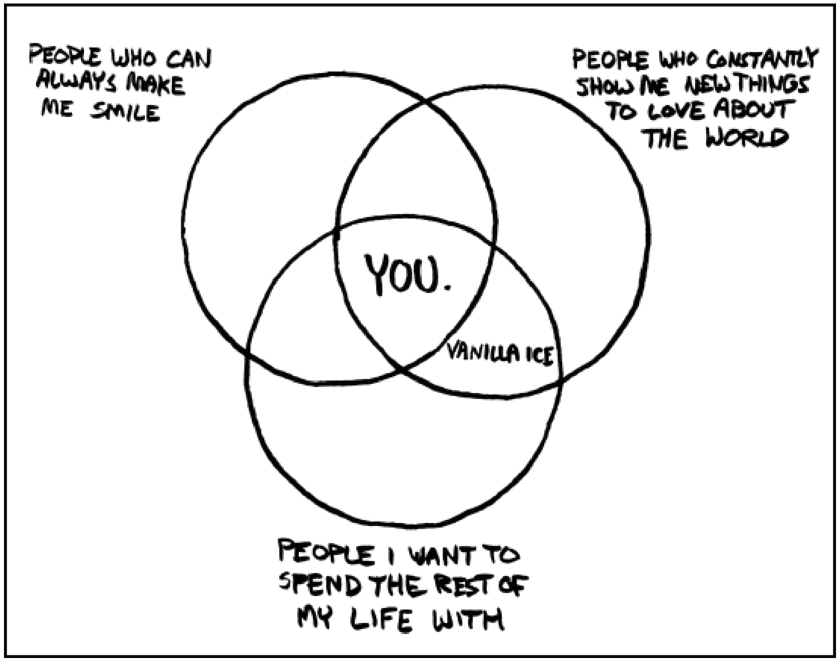
Generalized Numbers

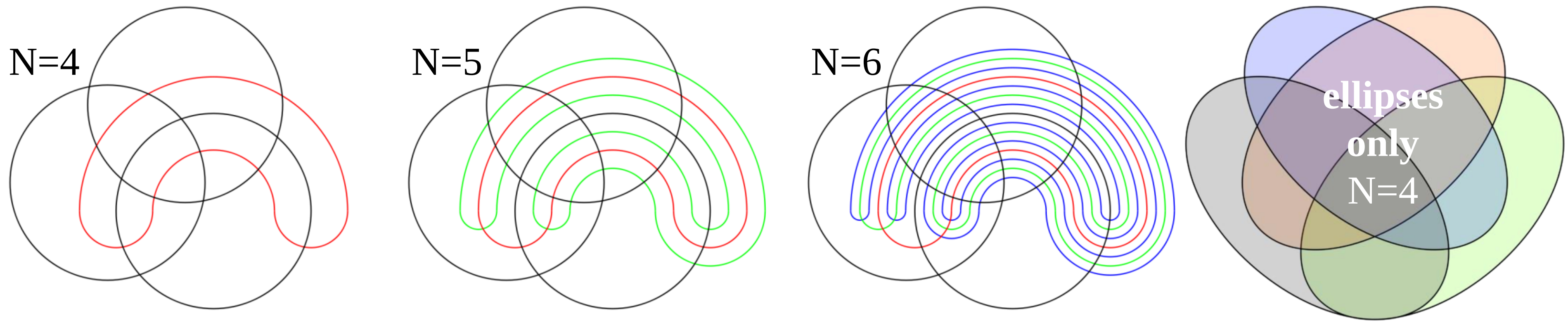


The Extended Chomsky Hierarchy

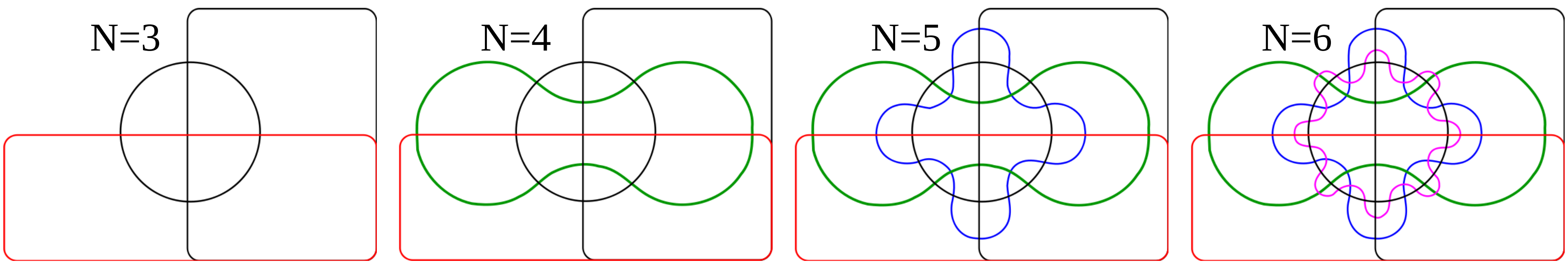


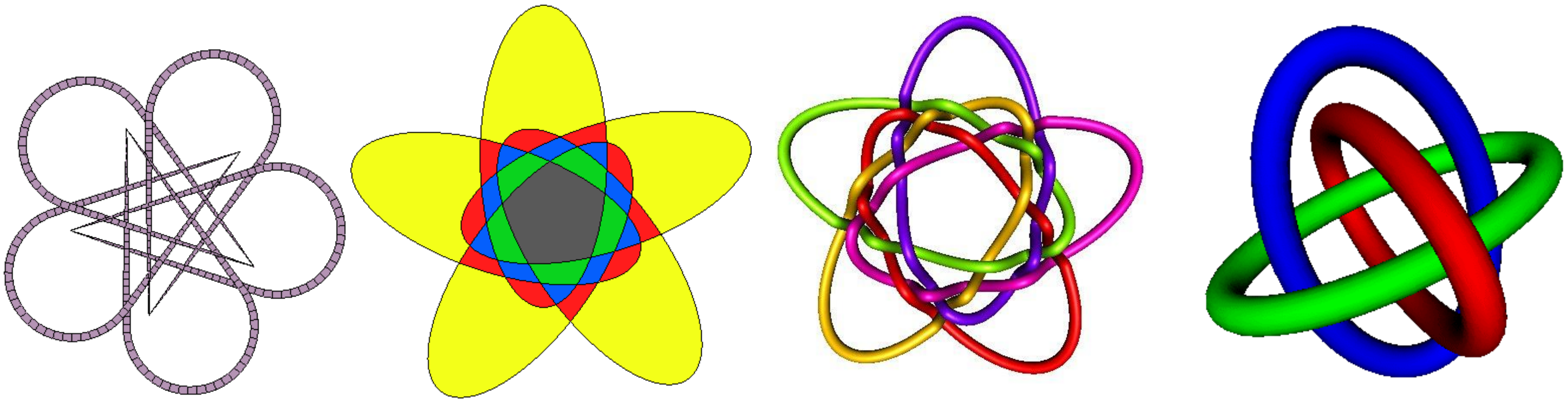






Generalized Venn diagrams [John Venn, 1880]

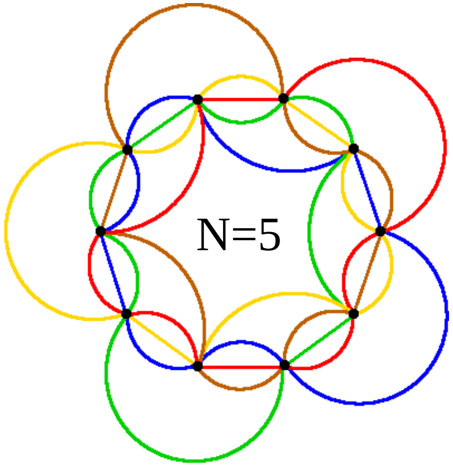


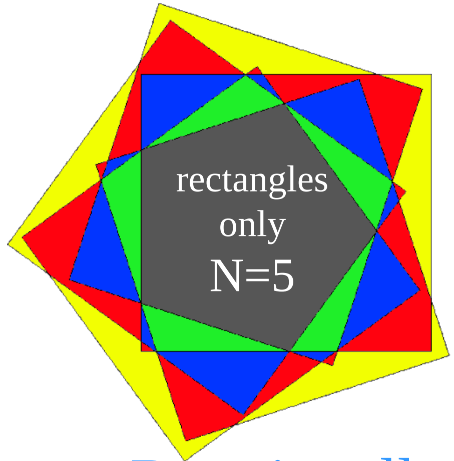


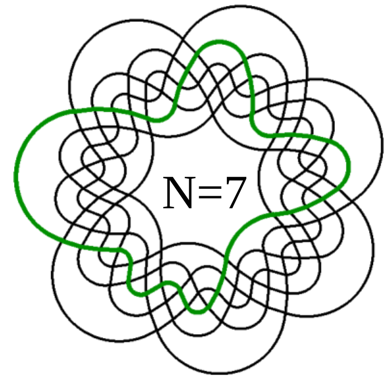
Ellipses only
N=5

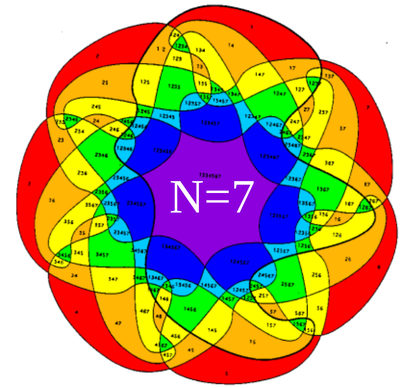
Borromean rings
analogue N=5

Borromean rings
N=3

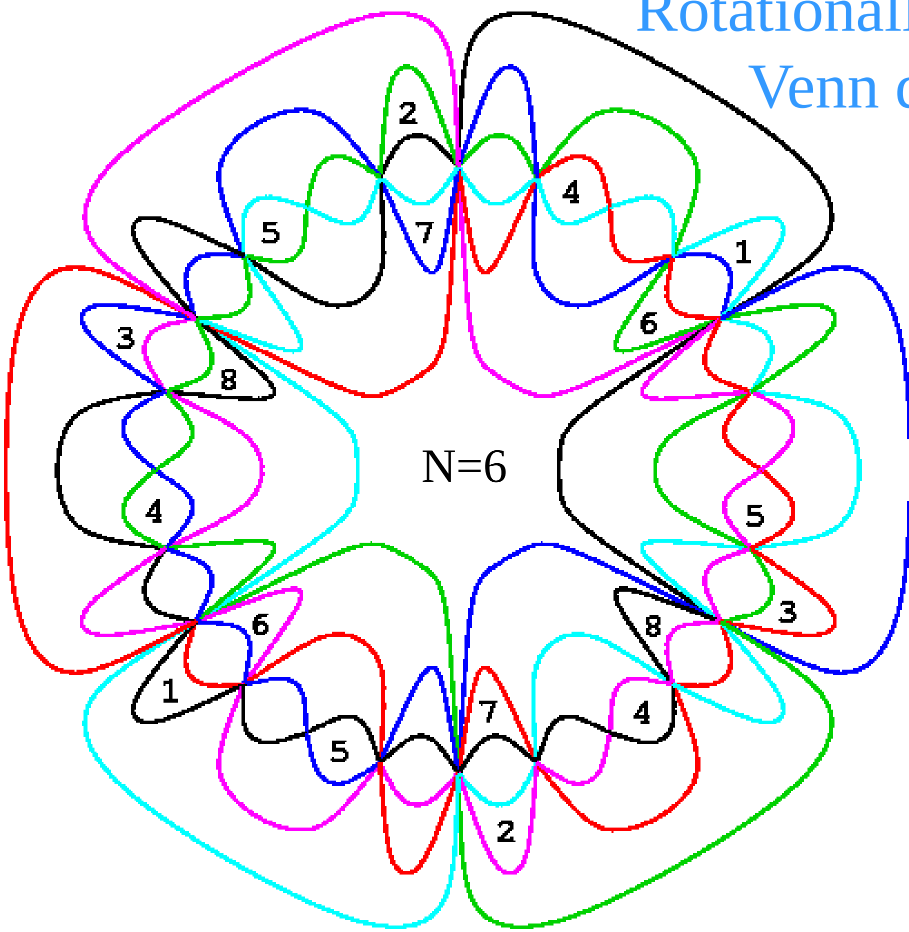


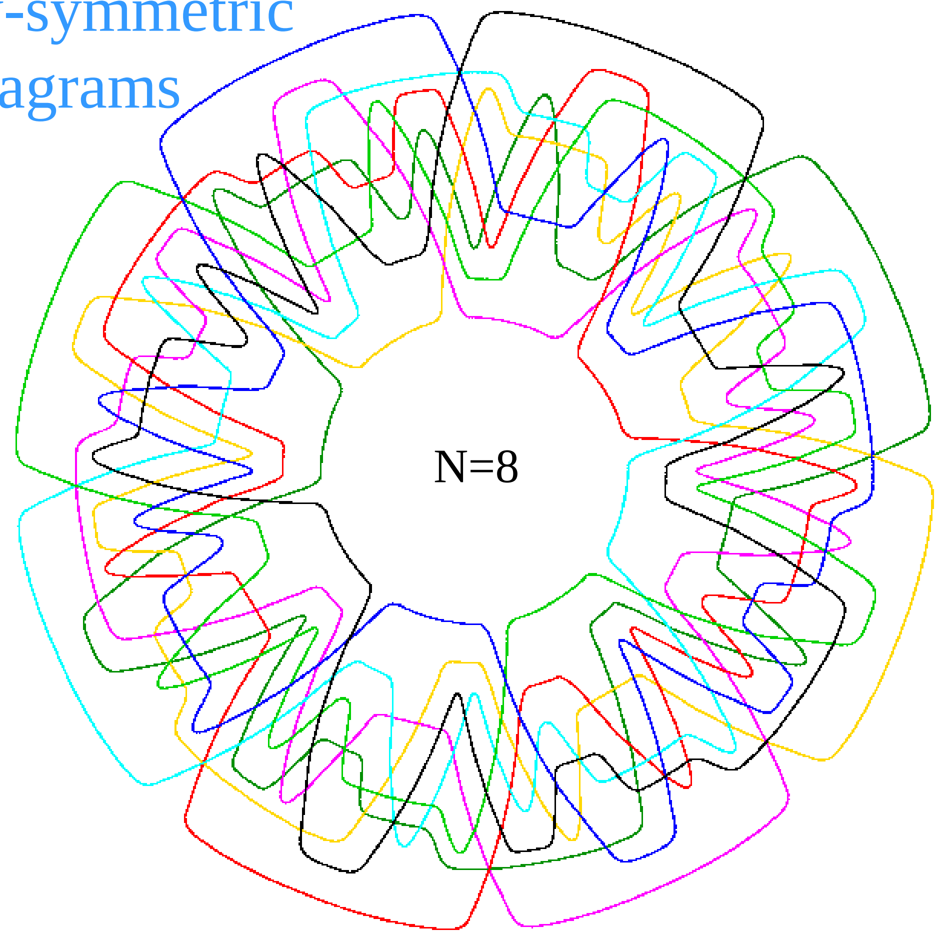


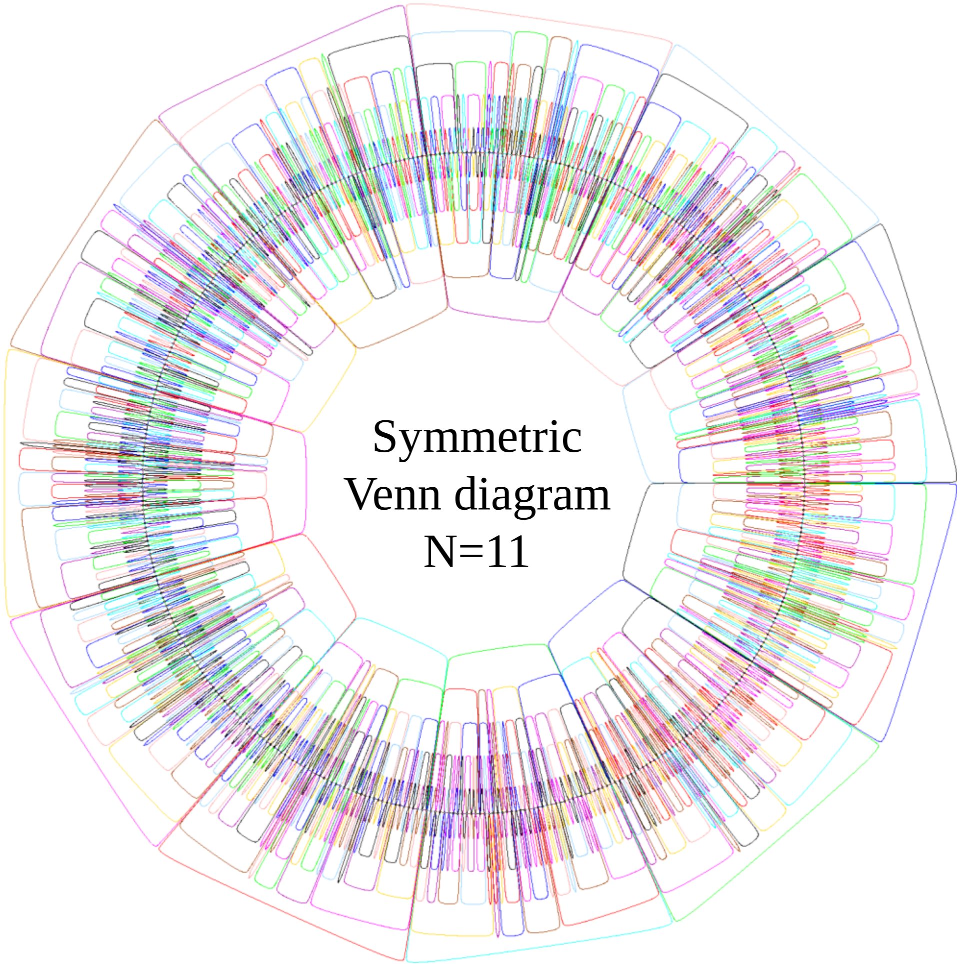


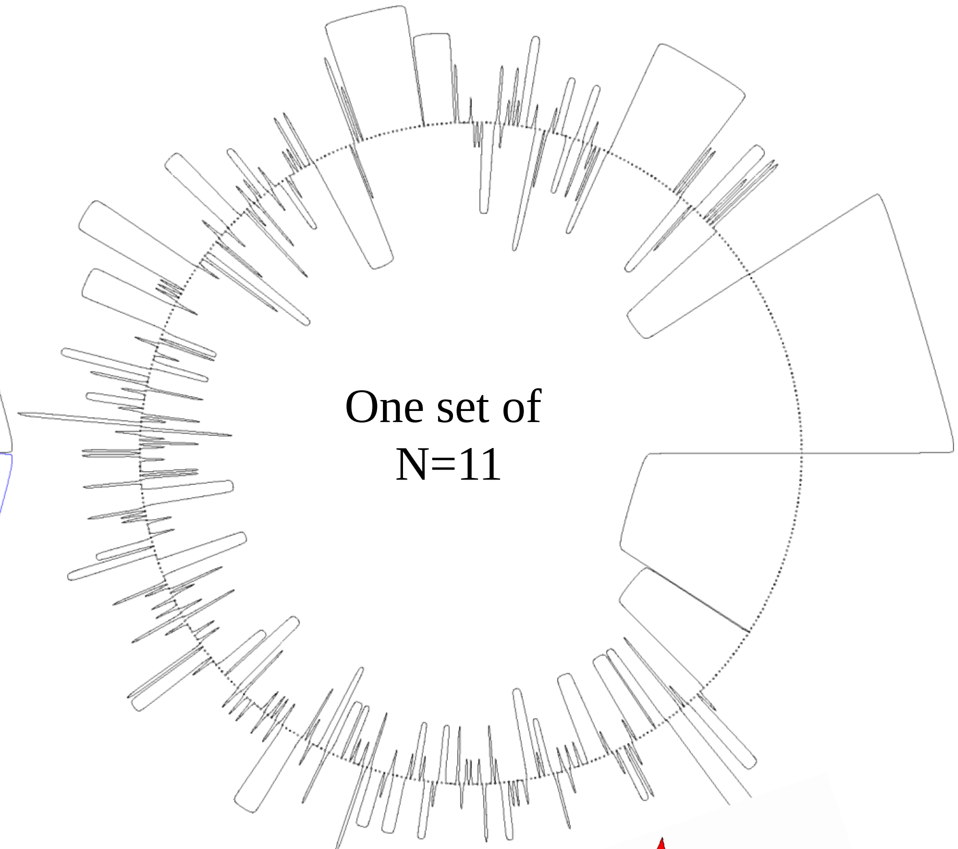


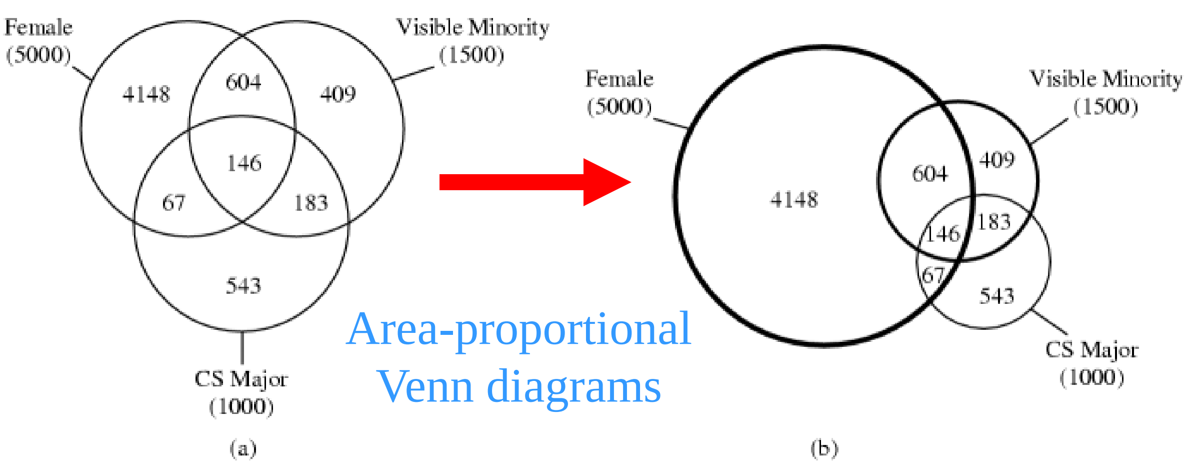
Rotationally-symmetric Venn diagrams



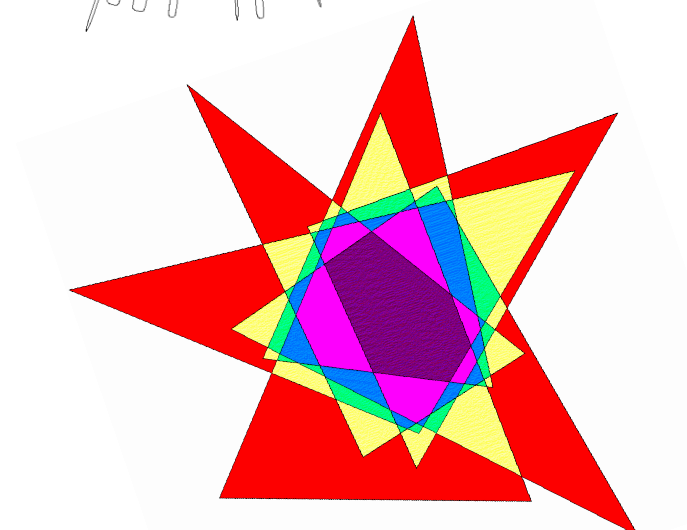




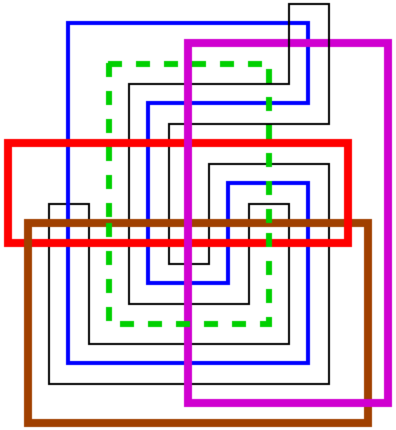




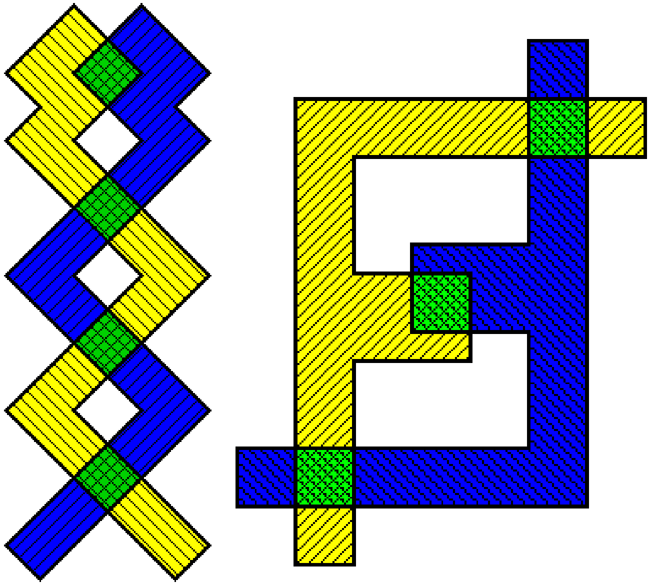
Area-proportional
Venn diagrams



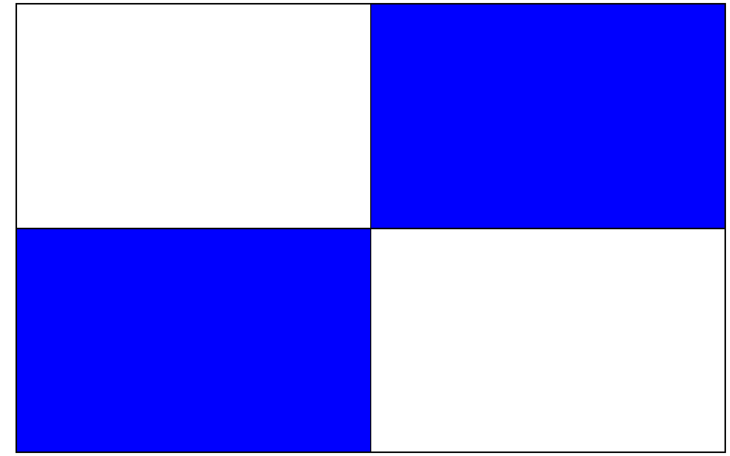
Triangles only Venn diagram (N=6)

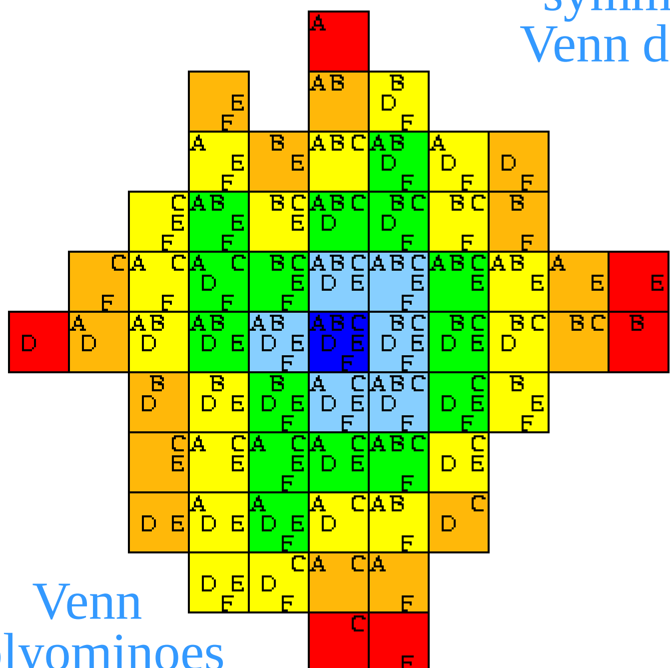


exposed Venn diagrams n=5

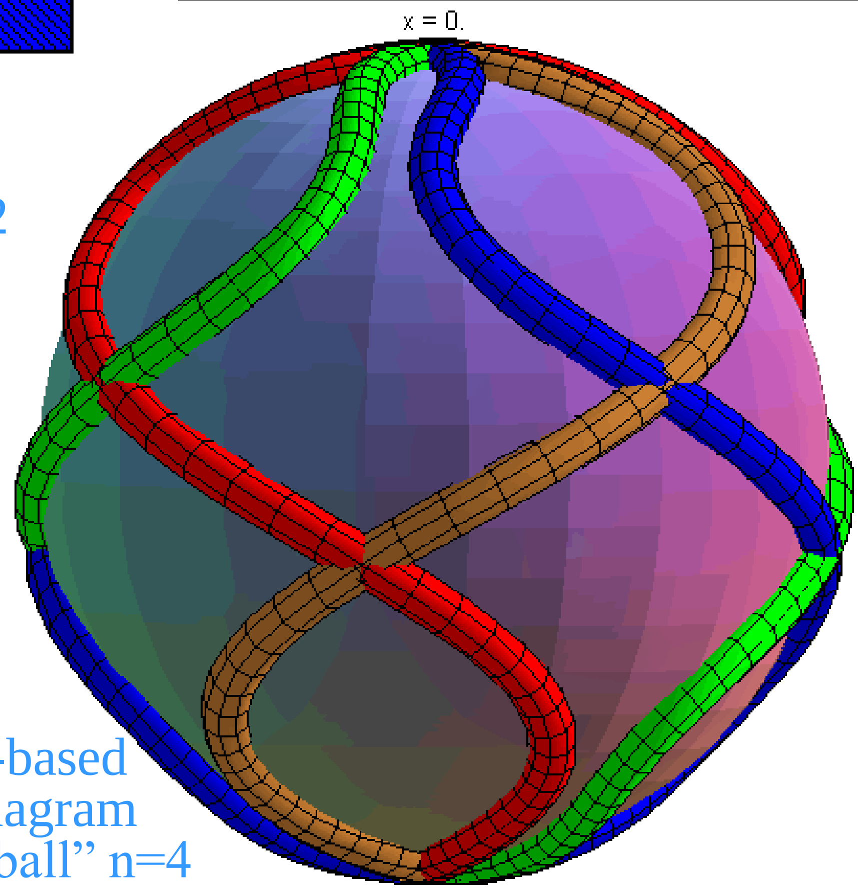


symmetric k-fold Venn diagrams n=2





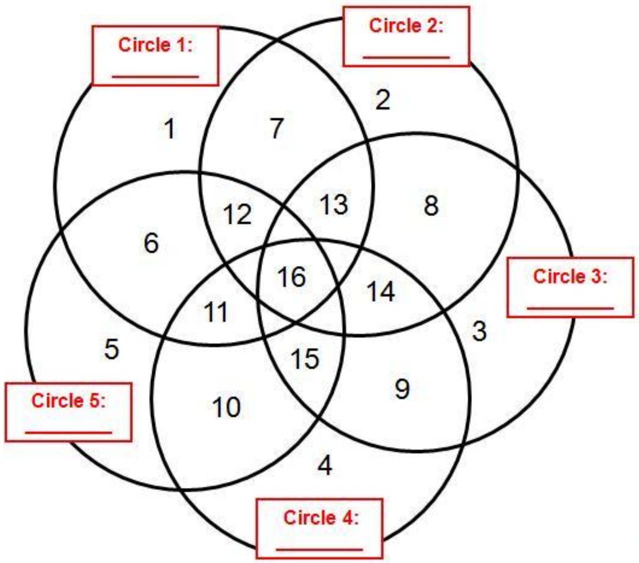
Venn polyominoes



sphere-based Venn diagram “Vennice ball” n=4

http://www.combinatorics.org/Surveys/ds5/VennEJC.html

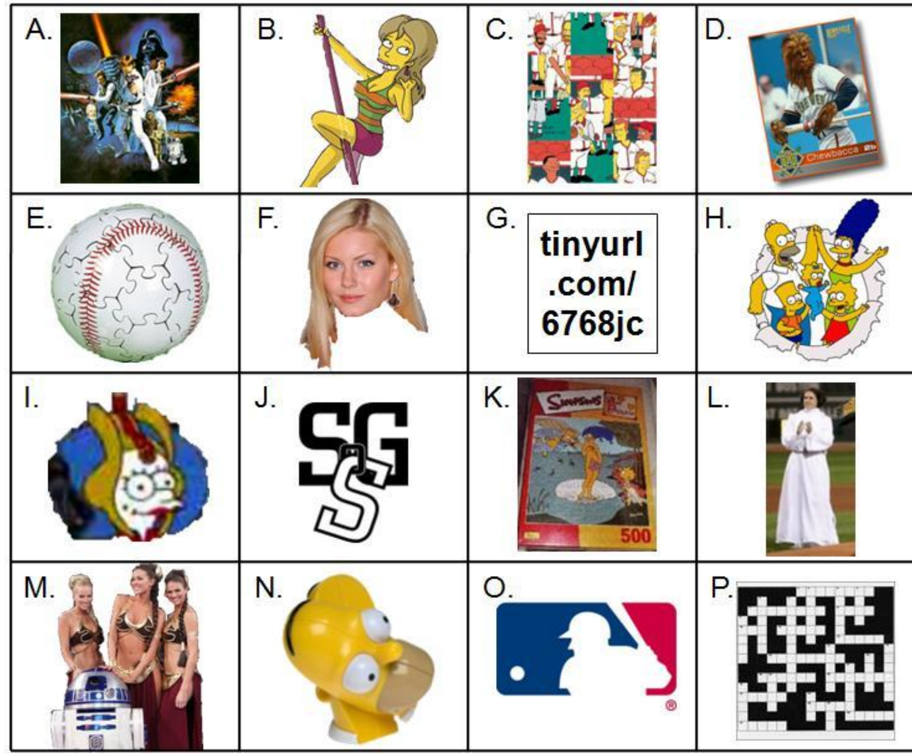
Venn diagram puzzles:



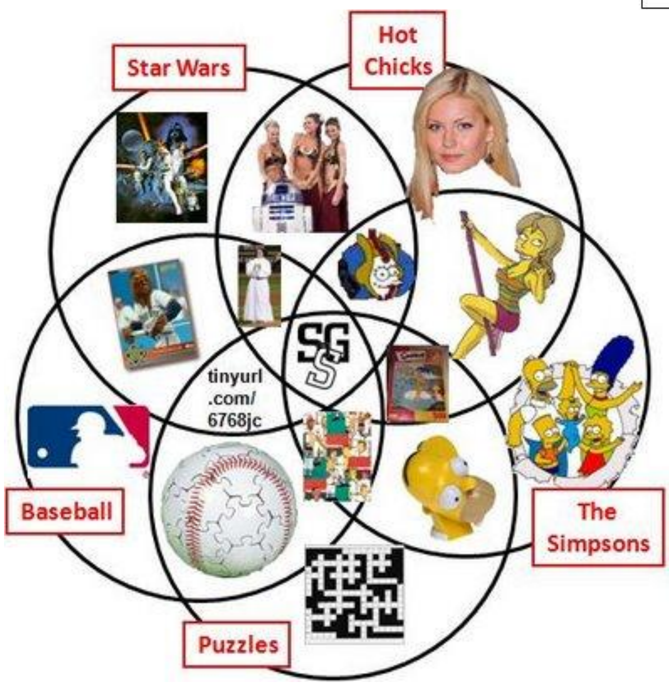
Answer Panel:

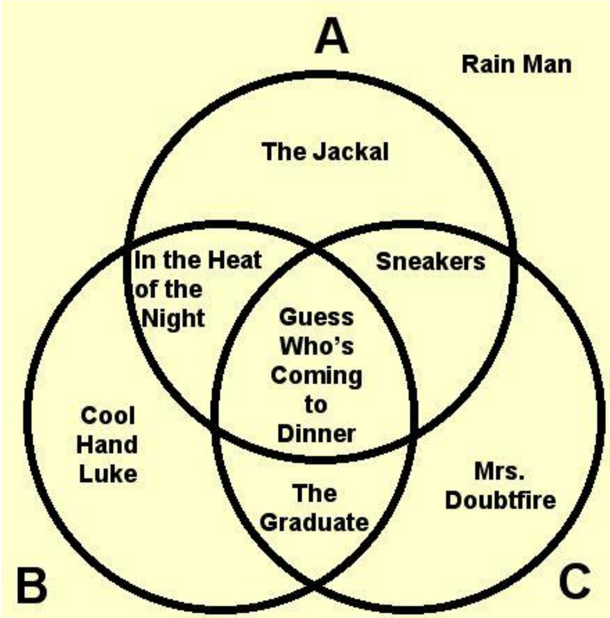
1. A
2. ?
3. ?
4. ?
5. ?
6. ?
7. ?
8. ?
9. ?
10. ?
11. ?
12. ?
13. ?
14. ?
15. ?
16. ?

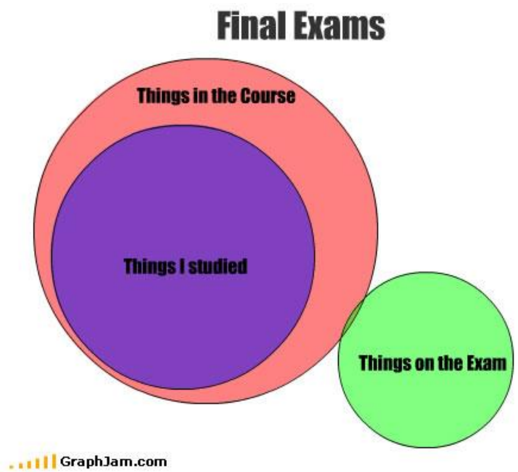
Circle 1: ?
Circle 2: ?
Circle 3: ?
Circle 4: ?
Circle 5: ?



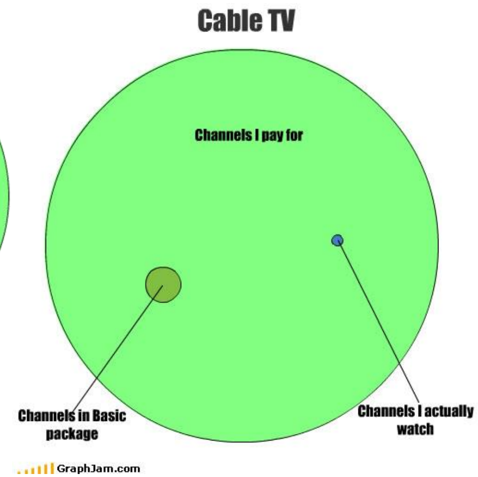
Puzzle solution:

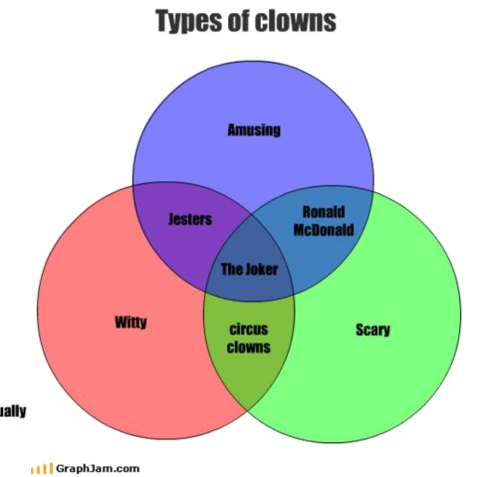


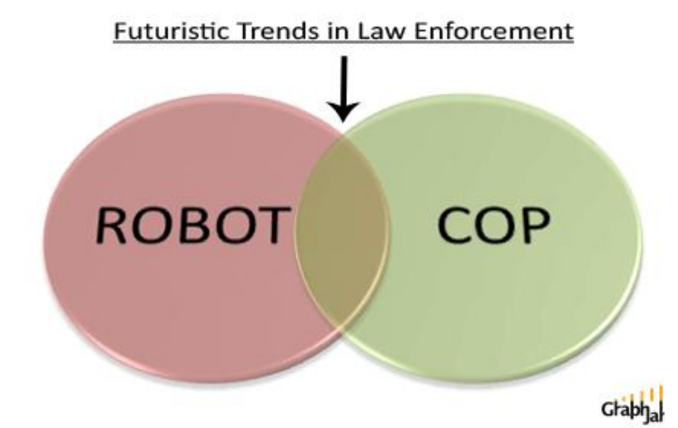


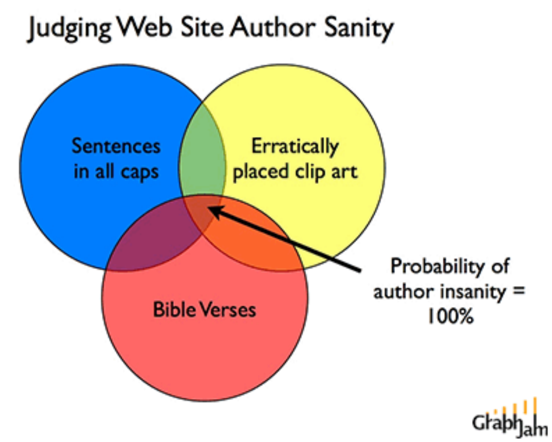

Final Exams
Things in the Course
Things I studied
Things on the Exam
GraphJam.com


A client can have their project ______:
Good
when they show us the money!
Fast
in their dreams.
as a prototype and / or a publicity stunt.
if they leave us alone and let us do our job!
Cheap
GraphJam.com

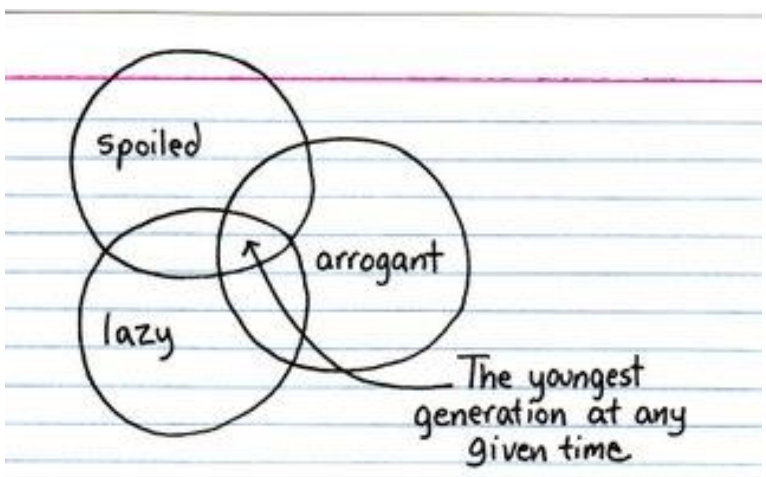

Cable TV
Channels I pay for
Channels in Basic package
Channels I actually watch
GraphJam.com


Types of clowns
Amusing
Jesters
Ronald McDonald
The Joker
Witty
circus clowns
Scary
GraphJam.com

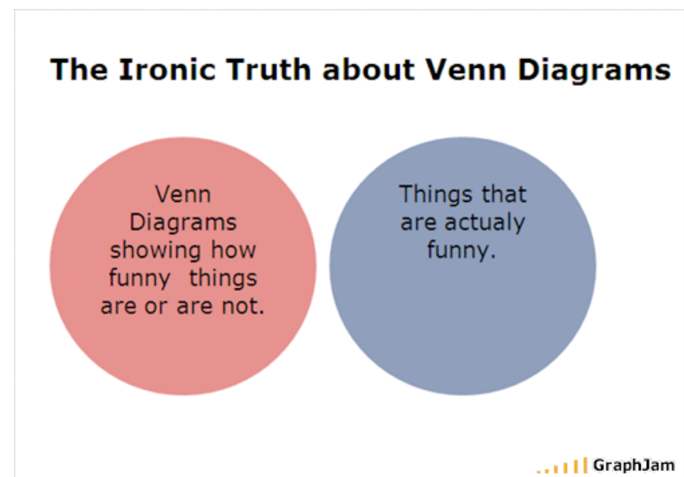

Futuristic Trends in Law Enforcement
ROBOT
COP
GraphJam

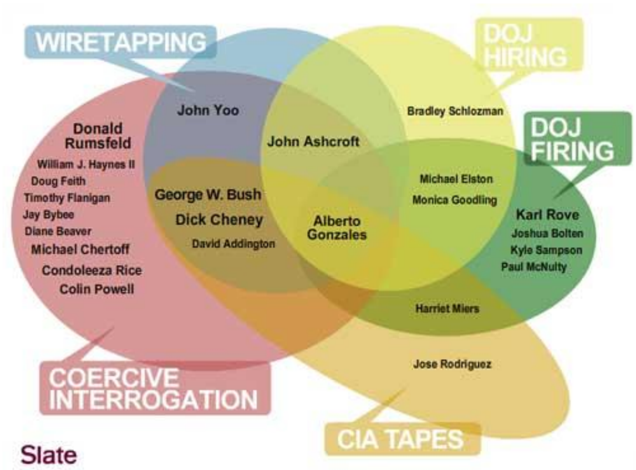

Judging Web Site Author Sanity
Sentences in all caps
Erratically placed clip art
Bible Verses
Probability of author insanity = 100%
GraphJam

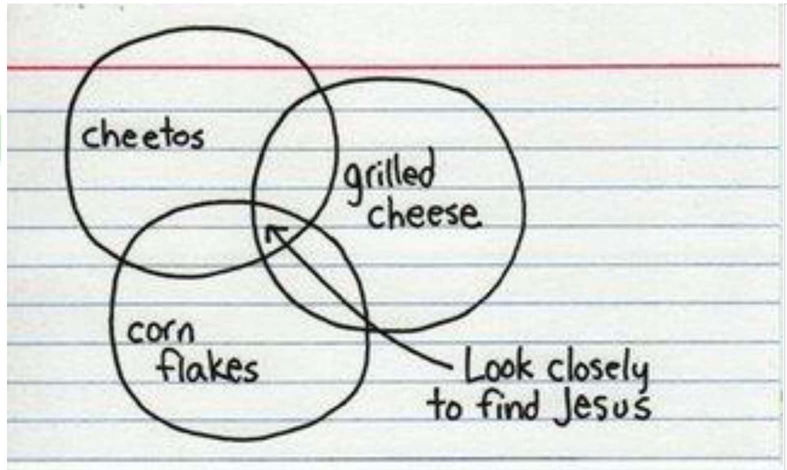

Clothing of Complainers
People Who Complain About Outsourced Jobs
People Who Wear Clothes Made In Asia
GraphJam.com

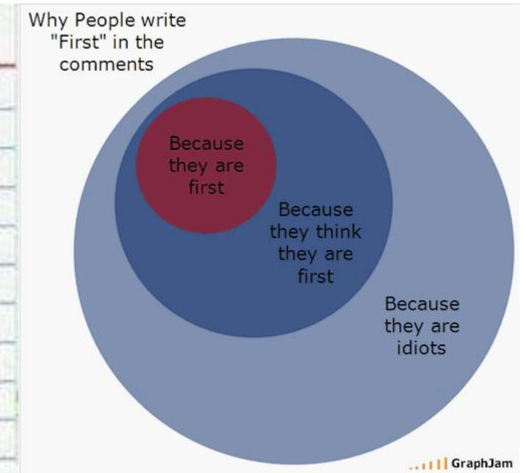

spoiled
arrogant
lazy
The youngest generation at any given time


The Marx Brothers
Funny Accent
Karl
Chico
Facial Hair
Musician
Groucho
Harpo
Chases Girls
GraphJam.com

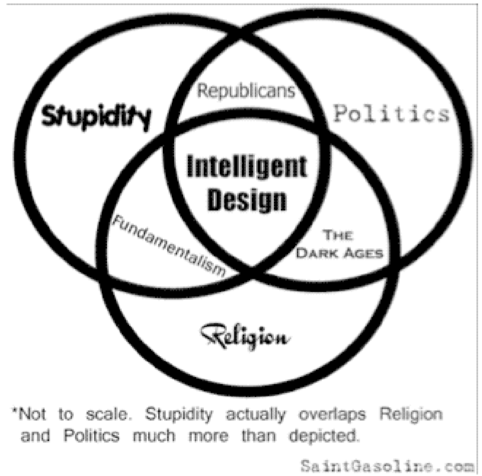

The Ironic Truth about Venn Diagrams
Venn Diagrams showing how funny things are or are not.
Things that are actualy funny.
GraphJam

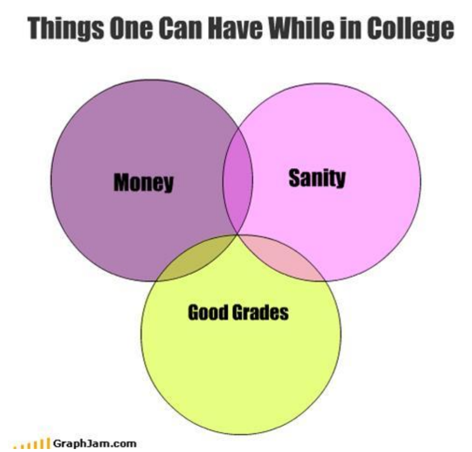

WIRETAPPING
DOJ HIRING
DOJ FIRING
COERCIVE INTERROGATION
CIA TAPES
John Yoo
Donald Rumsfeld
William J. Haynes II
Doug Feith
Timothy Flanigan
Jay Bybee
Diane Beaver
Michael Chertoff
Condoleeza Rice
Colin Powell
John Ashcroft
George W. Bush
Dick Cheney
David Addington
Alberto Gonzales
Bradley Schlozman
Michael Elston
Monica Goodling
Karl Rove
Joshua Bolten
Kyle Sampson
Paul McNulty
Harriet Miers
Jose Rodriguez
Slate

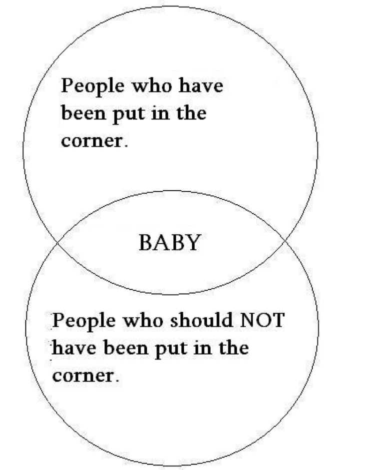

cheetos
grilled cheese
corn flakes
Look closely to find Jesus

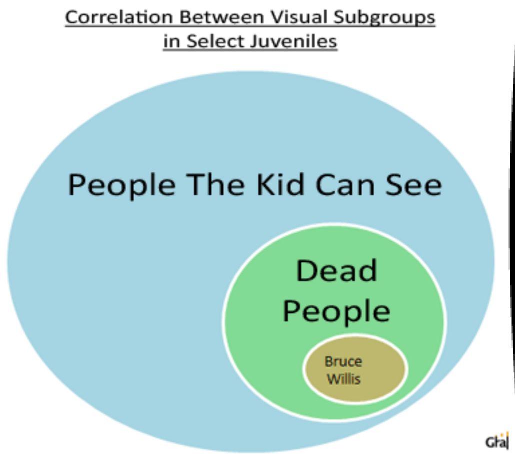

Why People write "First" in the comments
Because they are first
Because they think they are first
Because they are idiots
GraphJam

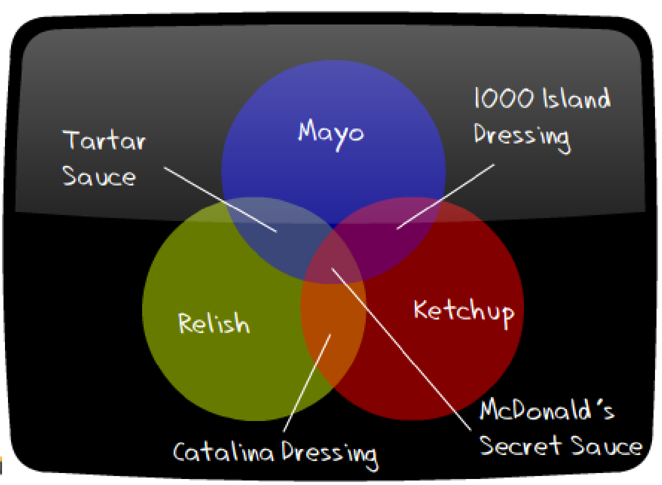

please, no more mascots
the real winner: the worldwide spandex industry
participation
excellence
hope
competition
unity
there should be at least one incredible wipeout where the person's body goes limp and equipment goes flying all over the place
any country can fake it for two weeks!


Stupidity
Republicans
Politics
Intelligent Design
Fundamentalism
The Dark Ages
Religion
*Not to scale. Stupidity actually overlaps Religion and Politics much more than depicted.
SaintGasoline.com

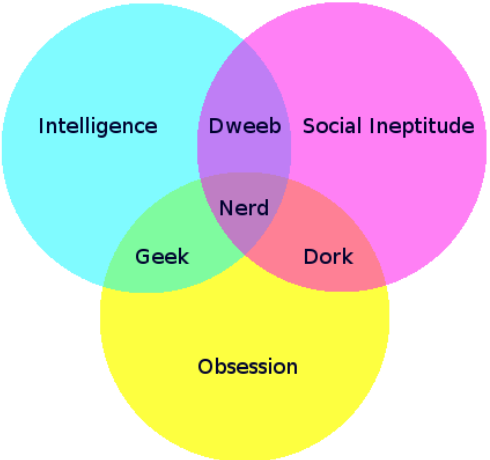

Things One Can Have While in College
Money
Sanity
Good Grades
GraphJam.com

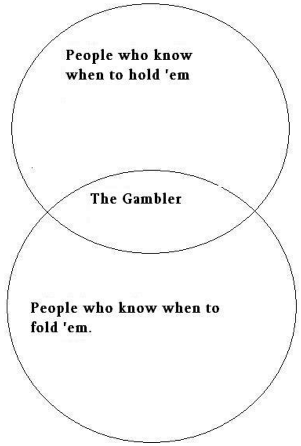

People who have been put in the corner.
BABY
People who should NOT have been put in the corner.

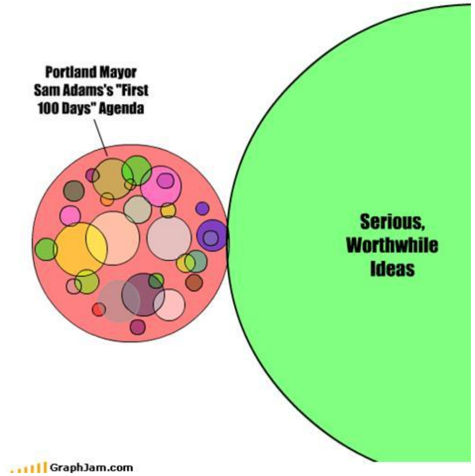

Correlation Between Visual Subgroups in Select Juveniles
People The Kid Can See
Dead People
Bruce Willis

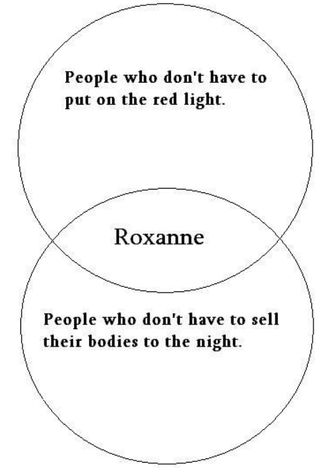

Tartar Sauce
Mayo
1000 Island Dressing
Relish
Ketchup
Catalina Dressing
McDonald's Secret Sauce


PONZI SCHEMES
SITUATIONS WHERE IT STINKS BEING THE BOTTOM
TWO PERSON HORSE COSTUMES

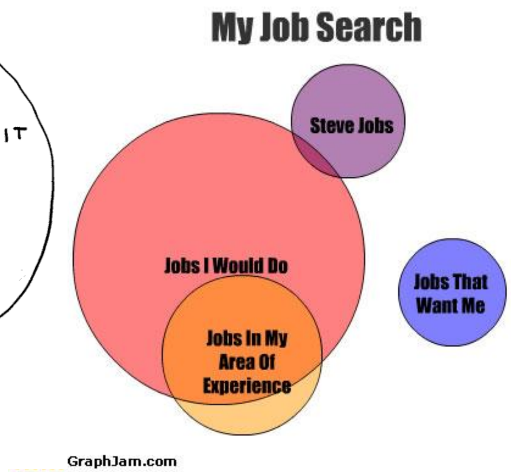

Intelligence
Dweeb
Social Ineptitude
Nerd
Geek
Dork
Obsession

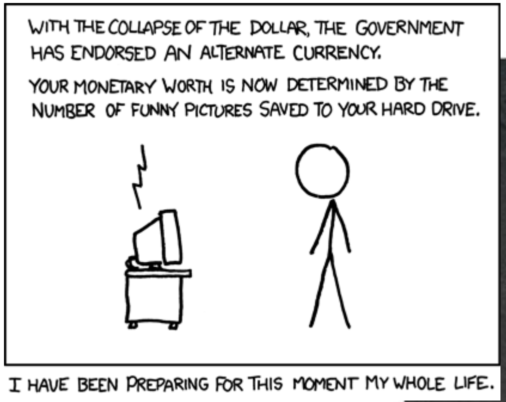

People who know when to hold 'em
The Gambler
People who know when to fold 'em.

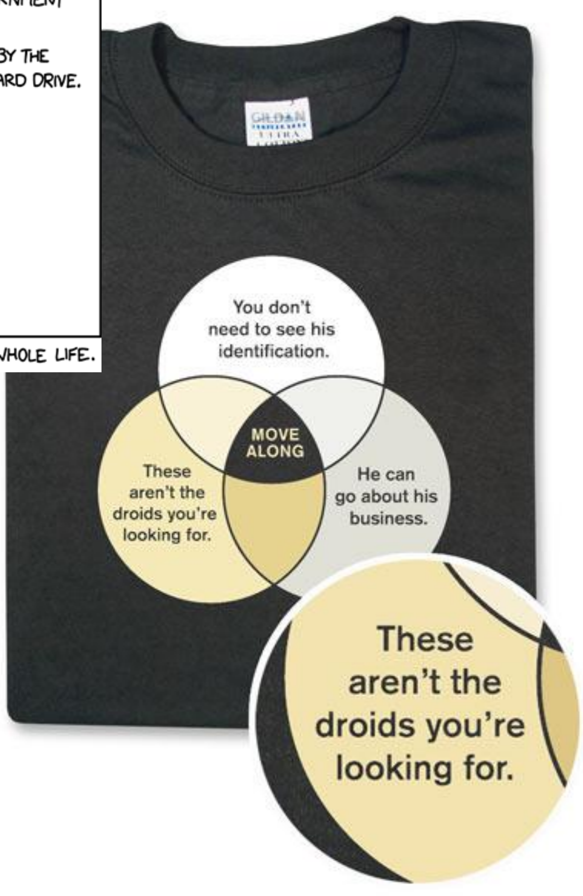

Portland Mayor Sam Adams's "First 100 Days" Agenda
Serious, Worthwhile Ideas
GraphJam.com

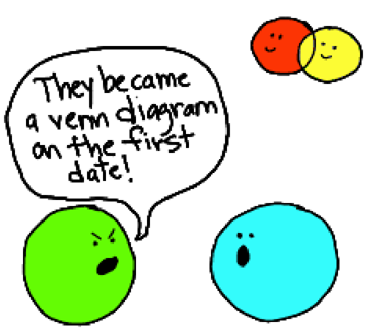

People who don't have to put on the red light.
Roxanne
People who don't have to sell their bodies to the night.

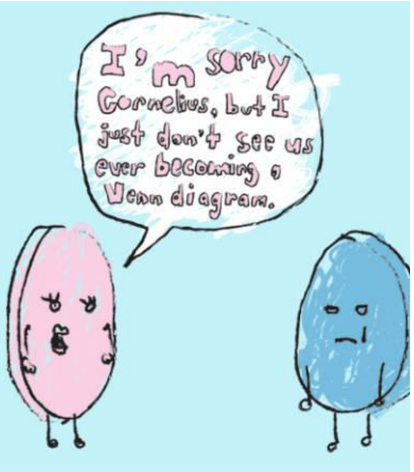

THOSE WHO GET IT
THOSE WHO GET IT AND DO NOT GET IT
THOSE WHO DO NOT GET IT
THOSE WHO ARE VERY CONFUSED
THOSE WHO GET IT BUT REALISE THERE IS NOTHING TO GET
THOSE WHO DO NOT GET IT AND REALISE THERE IS NOTHING TO GET
THOSE WHO REALISE THERE IS NOTHING TO GET

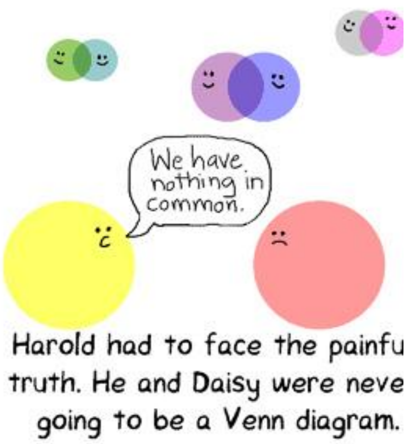

My Job Search
Steve Jobs
Jobs I Would Do
Jobs That Want Me
Jobs In My Area Of Experience
GraphJam.com


WITH THE COLLAPSE OF THE DOLLAR, THE GOVERNMENT HAS ENDORSED AN ALTERNATE CURRENCY.
YOUR MONETARY WORTH IS NOW DETERMINED BY THE NUMBER OF FUNNY PICTURES SAVED TO YOUR HARD DRIVE.
I HAVE BEEN PREPARING FOR THIS MOMENT MY WHOLE LIFE.


You don't need to see his identification.
MOVE ALONG
These aren't the droids you're looking for.
He can go about his business.
These aren't the droids you're looking for.


They became a venn diagram on the first date!

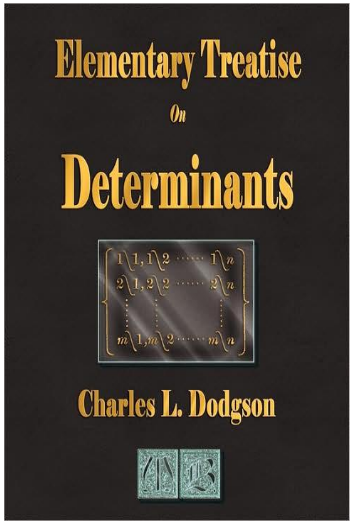

I'm sorry Cornelius, but I just don't see us ever becoming a Venn diagram.

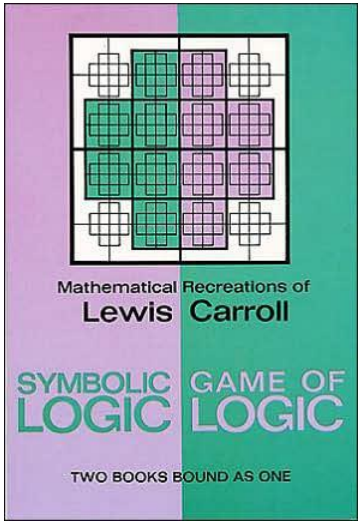

We have nothing in common.
Harold had to face the painfu
truth. He and Daisy were neve
going to be a Venn diagram.

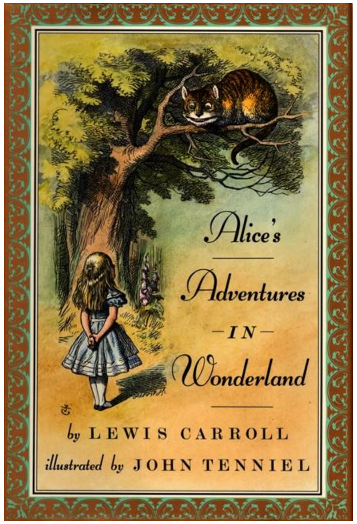
Historical Perspectives

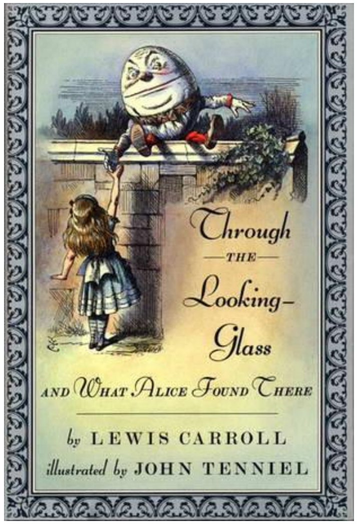
Charles Dodgson (1832-1898)

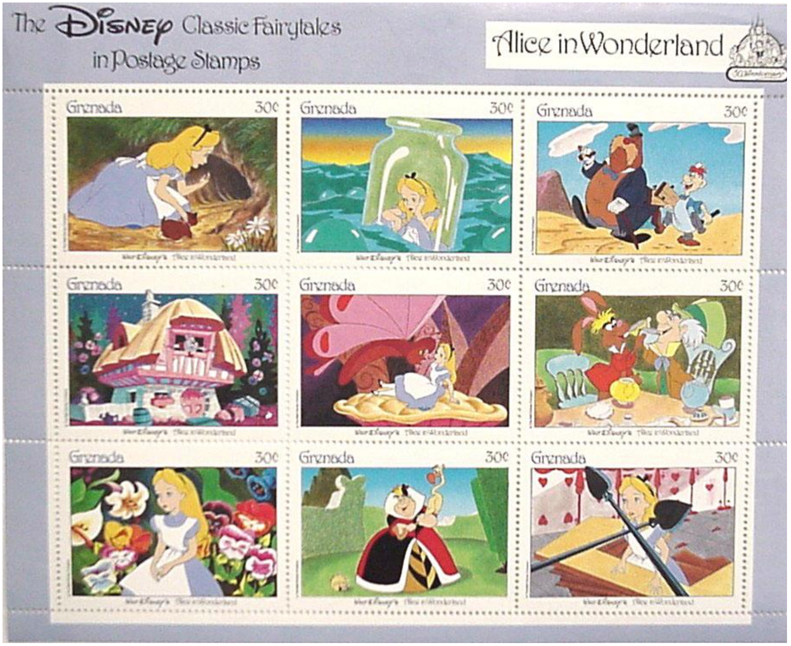
- AKA "Lewis Carroll"
- Mathematician, logician, author, photographer
- Wrote "Alice in Wonderland", "Jabberwocky", and "Through the Looking Glass"
- Popularized logic & syllogisms and made it fun!
- Invented "Scrabble" and "word ladder" games
- Profoundly influenced literature, art, and culture





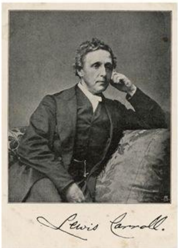





The Disney Classic Fairytales in Postage Stamps
Alice in Wonderland
Grenada 30¢
Grenada 30¢
Grenada 30¢
Grenada 30¢
Grenada 30¢
Grenada 30¢
Grenada 30¢
Grenada 30¢
Grenada 30¢

OFFICIALLY SEALED
WONDERLAND
DEAD LETTER
OFFICE

13p
Alice's Adventures in Wonderland
The Year of the Child

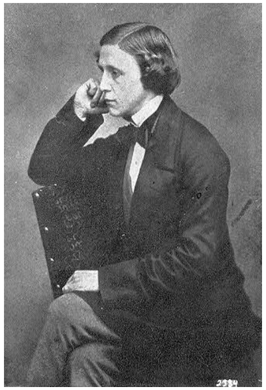
Through the Looking-Glass
LEWIS CARROLL
63

Lewis Carroll.

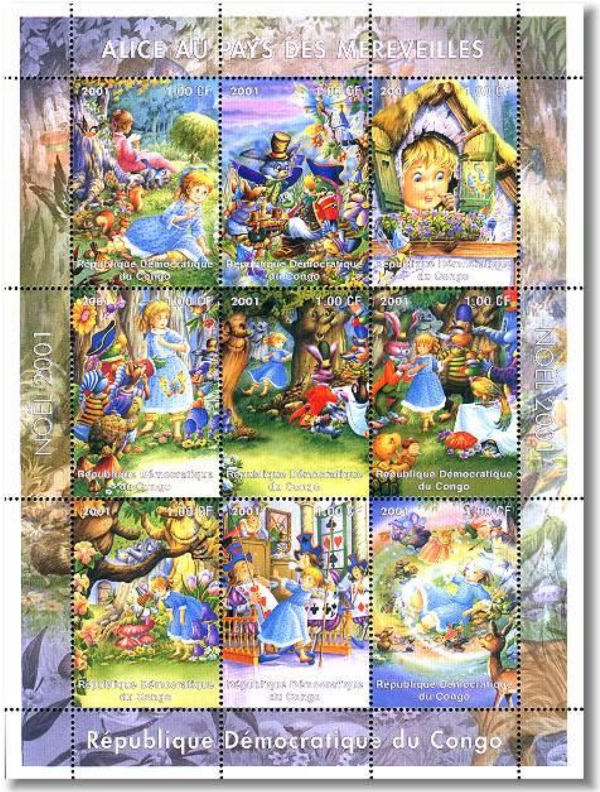
Charles Lutwidge Dodgson
(Lewis Carroll)



REPUBLIQUE du MALI
150e Anniversaire de la naissance de LEWIS CARROLL
110F

37
usa

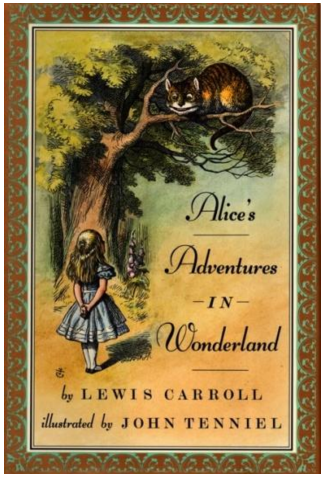



ALICE AU PAYS DES MERVEILLES
NOEL 2001
NOEL 2001
République Démocratique du Congo

MALDIVES 1L
MALDIVES 2L
MALDIVES 3L
MALDIVES 4L
MALDIVES 5L
MALDIVES 10L
MALDIVES 15L
MALDIVES 2.5R
MALDIVES 4R

TIME
The Weekly Newsmagazine
KING OF HEARTS AND ALICE

WONDERLAND
JABBERWOCKY
CENTENNIAL
10
WINKS
JABBERWOCKY
CENTENNIAL
WONDERLAND



Alice's
Adventures
IN
Wonderland
by LEWIS CARROLL
illustrated by JOHN TENNIEL

































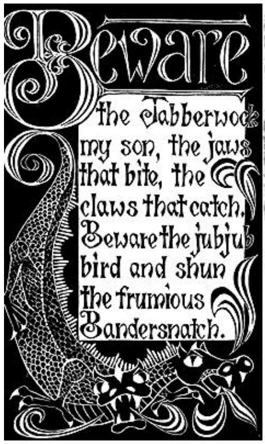

In this Style 10/6

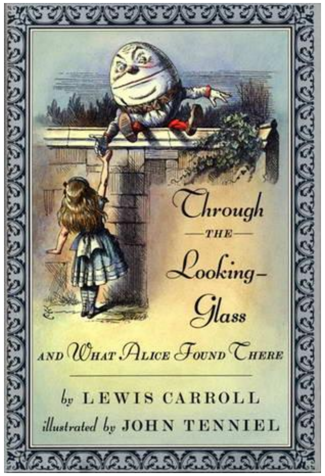


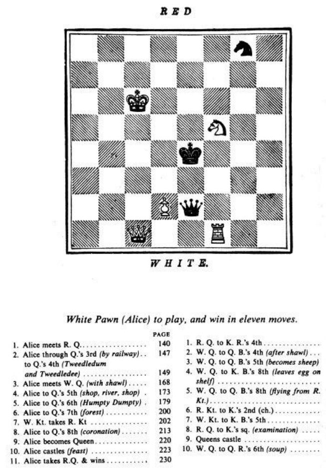


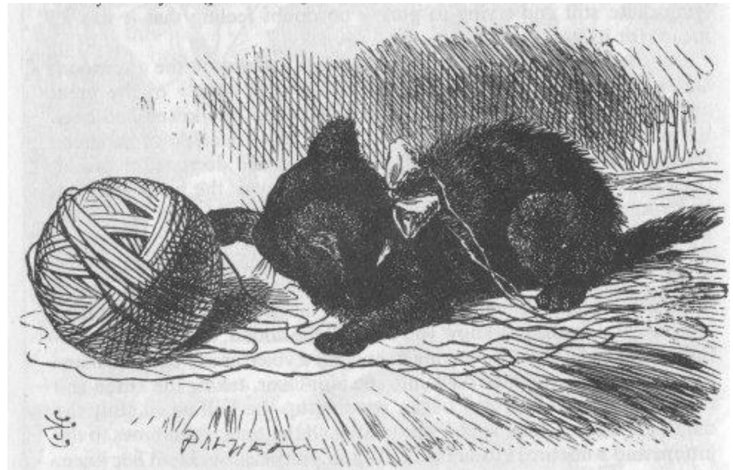


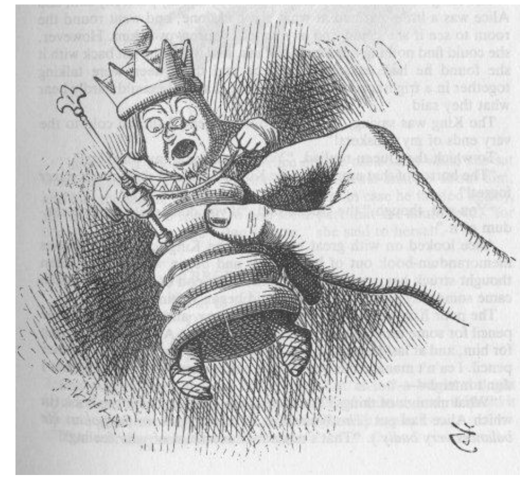


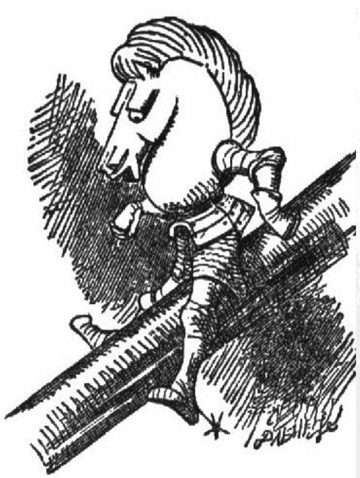

Beware
the Jabberwock
my son, the jaws
that bite, the
claws that catch.
Beware the jubju
bird and shun
the frumious
Bandersnatch.

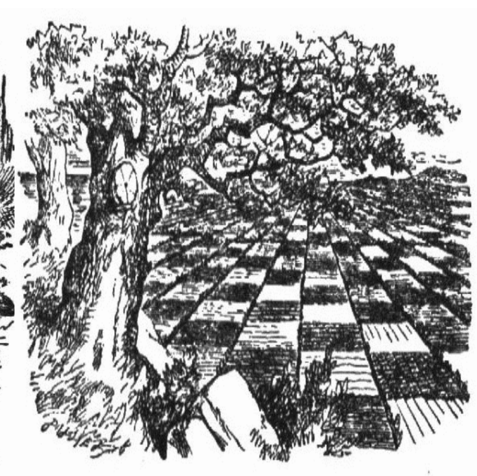

Through
THE
Looking-
Glass
AND WHAT ALICE FOUND THERE
by LEWIS CARROLL
illustrated by JOHN TENNIEL

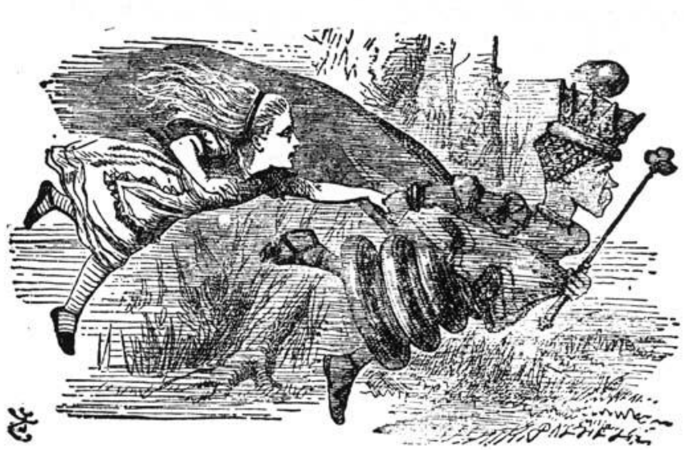

RED
WHITE.
White Pawn (Alice) to play, and win in eleven moves.
PAGE
1. Alice meets R. Q. 140
2. Alice through Q.'s 3rd (by railway) 147
to Q.'s 4th (Tweedledum and Tweedledee) 149
3. Alice meets W. Q. (with shawl) 168
4. Alice to Q.'s 5th (shop, river, shop) 173
5. Alice to Q.'s 6th (Humpty Dumpty) 179
6. Alice to Q.'s 7th (forest) 200
7. W. Kt. takes R. Kt 202
8. Alice to Q.'s 8th (coronation) 213
9. Alice becomes Queen 220
10. Alice castles (feast) 223
11. Alice takes R.Q. & wins 230
1. R. Q. to K. R.'s 4th
2. W. Q. to Q. B.'s 4th (after shawl)
3. W. Q. to Q. B.'s 5th (becomes sheep)
4. W. Q. to K. B.'s 8th (leaves egg on shelf)
5. W. Q. to Q. B.'s 8th (flying from R. Kt.)
6. R. Kt. to K.'s 2nd (ch.)
7. W. Kt. to K. B.'s 5th
8. R. Q. to K.'s sq. (examination)
9. Queens castle
10. W. Q. to Q. R.'s 6th (soup)

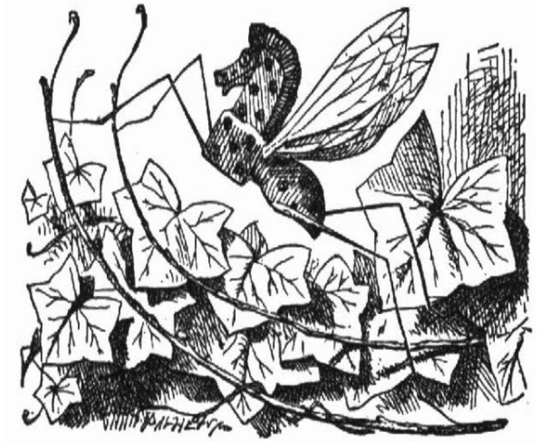


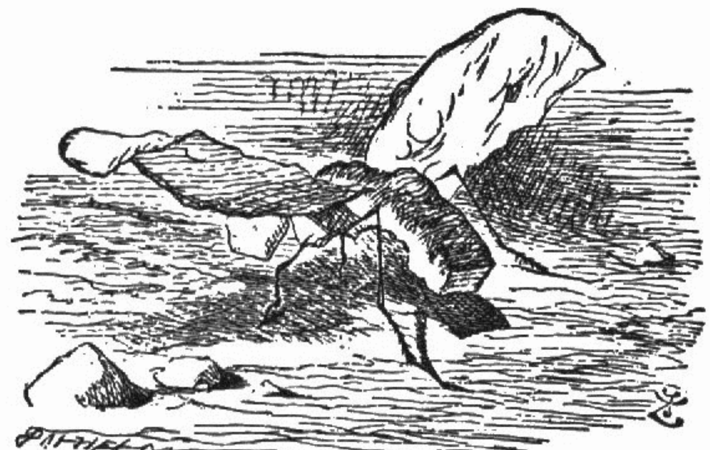


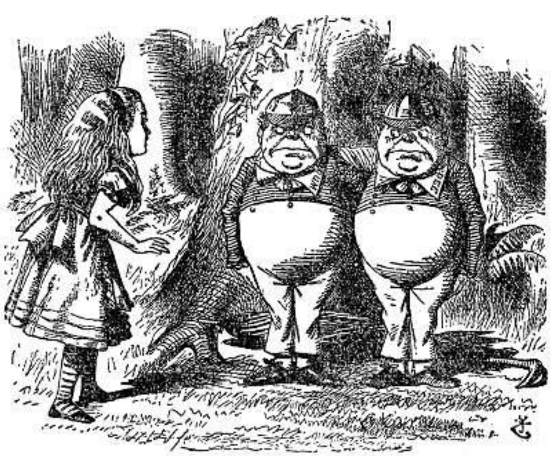
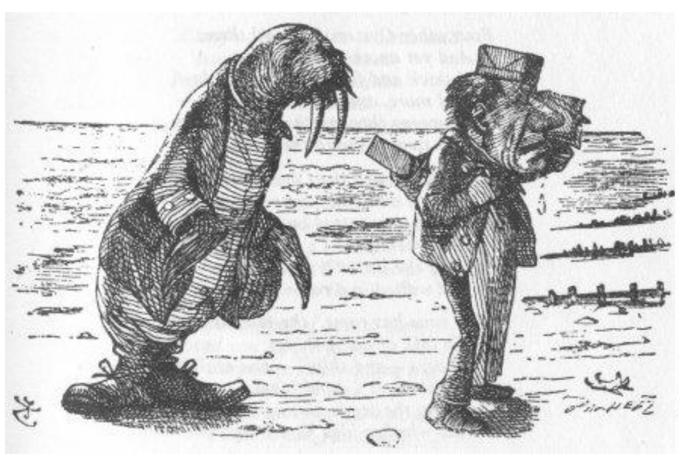
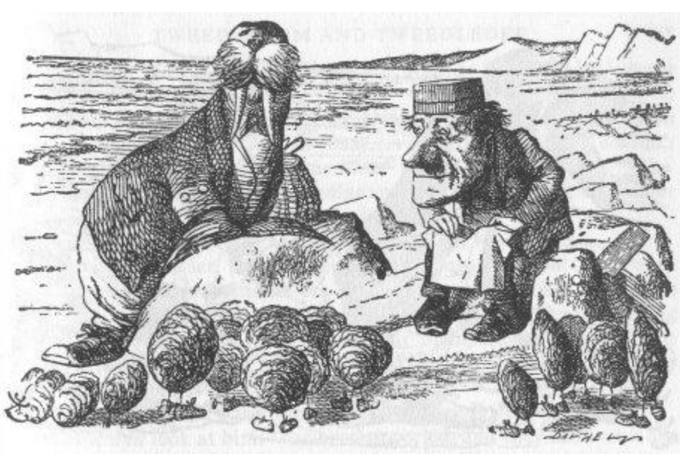


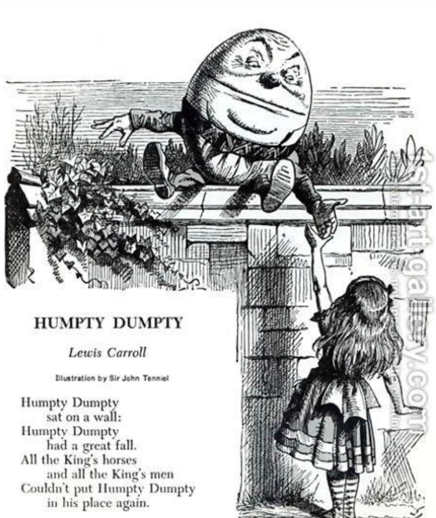










HUMPTY DUMPTY

Lewis Carroll

Illustration by Sir John Tenniel

Humpty Dumpty
  sat on a wall:
Humpty Dumpty
  had a great fall.
All the King's horses
  and all the King's men
Couldn't put Humpty Dumpty
  in his place again.













QUEEN ALICE

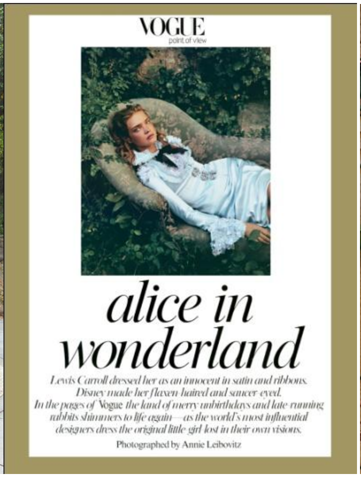










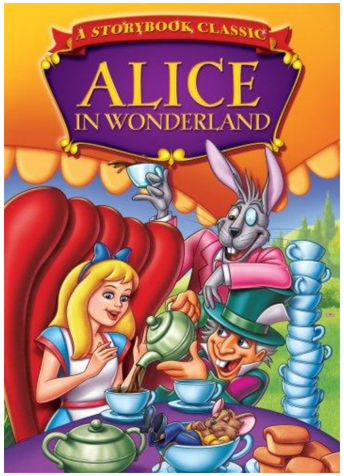


VOGUE

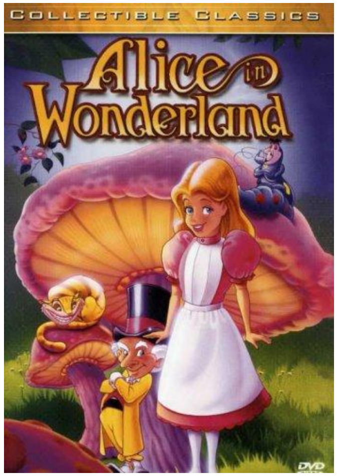
point of view

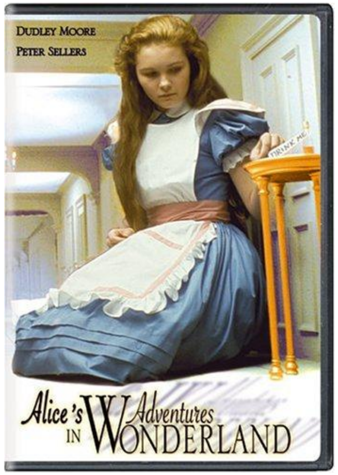
alice in wonderland

Lewis Carroll dressed her as an innocent in satin and ribbons. Disney made her flaxen haired and saucer eyed. In the pages of Vogue *the land of merry unbirthdays and late running rabbits shimmers to life again—as the world's most influential designers dress the original little girl lost in their own visions.*

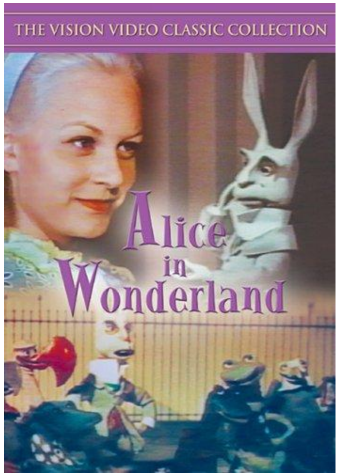
Photographed by Annie Leibovitz

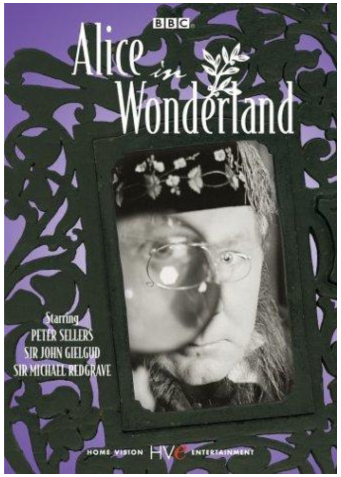


A STORYBOOK CLASSIC
ALICE
IN WONDERLAND

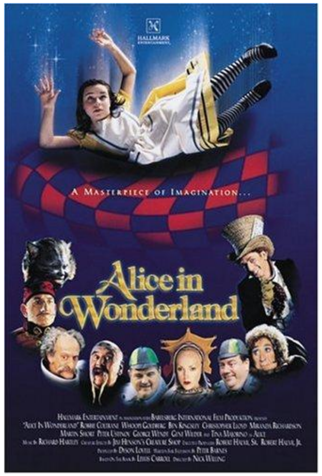

COLLECTIBLE CLASSICS
Alice in Wonderland
DVD


DUDLEY MOORE
PETER SELLERS
Alice's Adventures in WONDERLAND

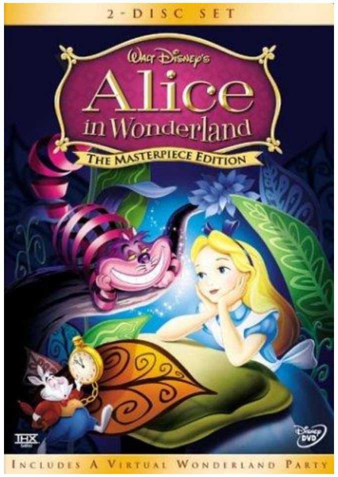

Alice
IN
Wonderland
THE STUNNING ADAPTATION OF A FAMILY CLASSIC!
DVD

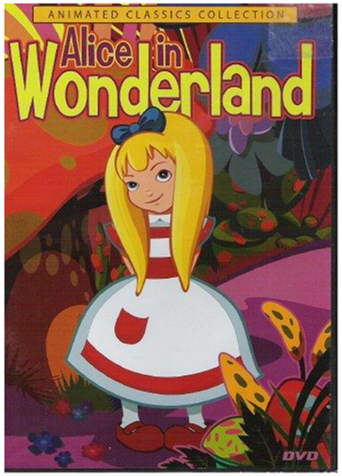

THE VISION VIDEO CLASSIC COLLECTION
Alice
in
Wonderland

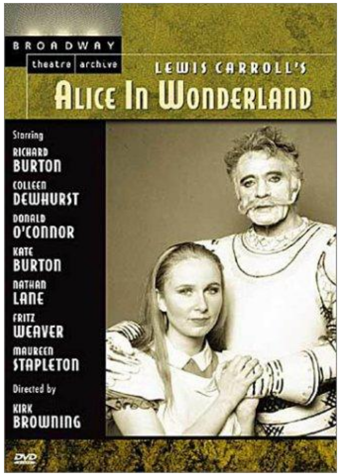

BBC
Alice in Wonderland
Starring
PETER SELLERS
SIR JOHN GIELGUD
SIR MICHAEL REDGRAVE
HOME VISION
ENTERTAINMENT

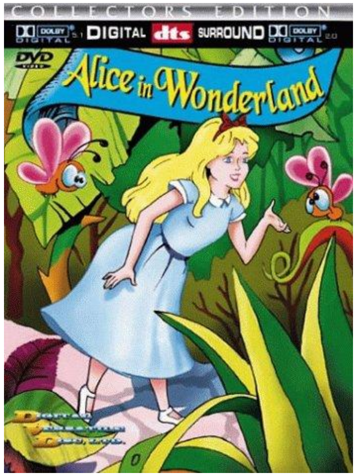

A MASTERPIECE OF IMAGINATION...
Alice in Wonderland

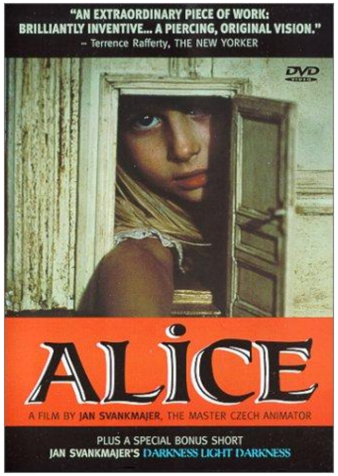

PETER SELLERS
DUDLEY MOORE
MICHAEL CRAWFORD
FIONA FULLERTON
ALICE'S ADVENTURES in WONDERLAND
JOURNEY DOWN THE RABBIT HOLE IN THIS TUNEFUL FAMILY CLASSIC!

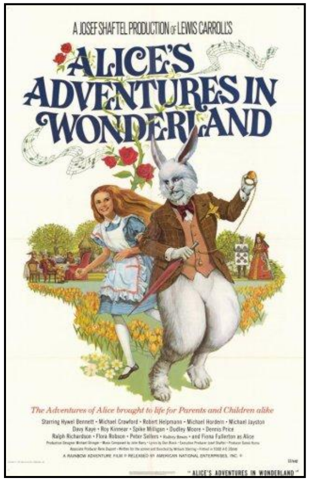

2-DISC SET
Walt Disney's
Alice
in Wonderland
THE MASTERPIECE EDITION
THX
INCLUDES A VIRTUAL WONDERLAND PARTY

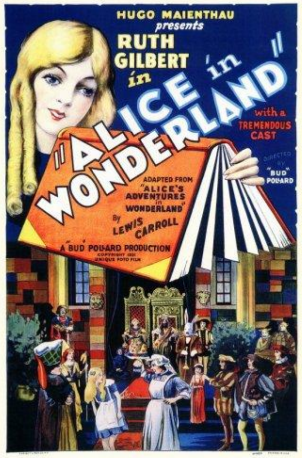

ANIMATED CLASSICS COLLECTION
Alice in
Wonderland
DVD

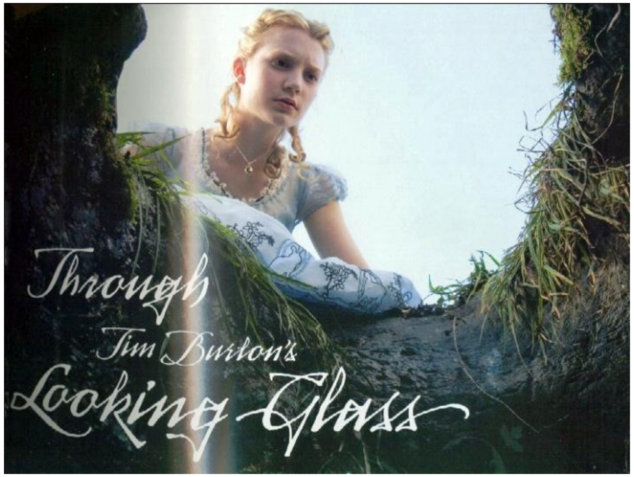

BROADWAY
theatre archive
LEWIS CARROLL'S
ALICE IN WONDERLAND
Starring
RICHARD BURTON
COLLEEN DEWHURST
DONALD O'CONNOR
KATE BURTON
NATHAN LANE
FRITZ WEAVER
MAUREEN STAPLETON
Directed by
KIRK BROWNING
DVD


COLLECTORS EDITION
DIGITAL dts SURROUND
Alice in Wonderland

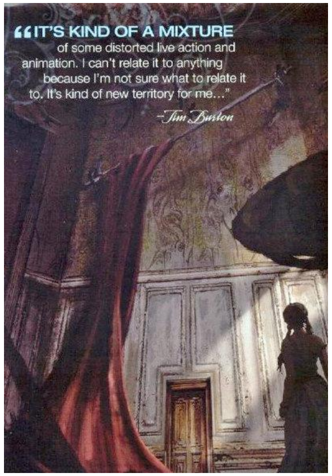

"AN EXTRAORDINARY PIECE OF WORK:
BRILLIANTLY INVENTIVE... A PIERCING, ORIGINAL VISION."
– Terrence Rafferty, THE NEW YORKER
DVD
ALICE
A FILM BY JAN SVANKMAJER, THE MASTER CZECH ANIMATOR
PLUS A SPECIAL BONUS SHORT
JAN SVANKMAJER'S DARKNESS LIGHT DARKNESS


A JOSEF SHAFTEL PRODUCTION of LEWIS CARROLL'S
ALICE'S ADVENTURES IN WONDERLAND
The Adventures of Alice brought to life for Parents and Children alike


HUGO MAIENTHAU
presents
RUTH GILBERT
in
"ALICE in WONDERLAND"
with a TREMENDOUS CAST
ADAPTED FROM "ALICE'S ADVENTURES IN WONDERLAND"
By LEWIS CARROLL
A BUD POLLARD PRODUCTION


Through
Tim Burton's
Looking Glass


© Disney


“IT'S KIND OF A MIXTURE of some distorted live action and animation. I can't relate it to anything because I'm not sure what to relate it to. It's kind of new territory for me...”
–Tim Burton

Alice and the White Knight:

A Lesson in Logic, Semantics, and Pointers

`You are sad,' the Knight said in an anxious tone: `let me sing you a song to comfort you.'

`Is it very long?' Alice asked, for she had heard a good deal of poetry that day.

`It's long,' said the Knight, `but it's very, *very* beautiful. Everybody that hears me sing it -- either it brings the *tears* into their eyes, or else --'

logical disjunction!

`Or else what?' said Alice, for the Knight had made a sudden pause.

law of the excluded middle!

`Or else it doesn't, you know. The name of the song is called "*Haddocks' Eyes*".'

pointer to a pointer!

`Oh, that's the name of the song, is it?' Alice said, trying to feel interested.

`No, you don't understand,' the Knight said, looking a little vexed. `That's what the name is *called*. The name really *is "The Aged Aged Man"*.'

pointer dereferencing: meta-pointer resolved!

`Then I ought to have said "That's what the *song* is called"?' Alice corrected herself.

separation of abstractions: variable vs. pointer!

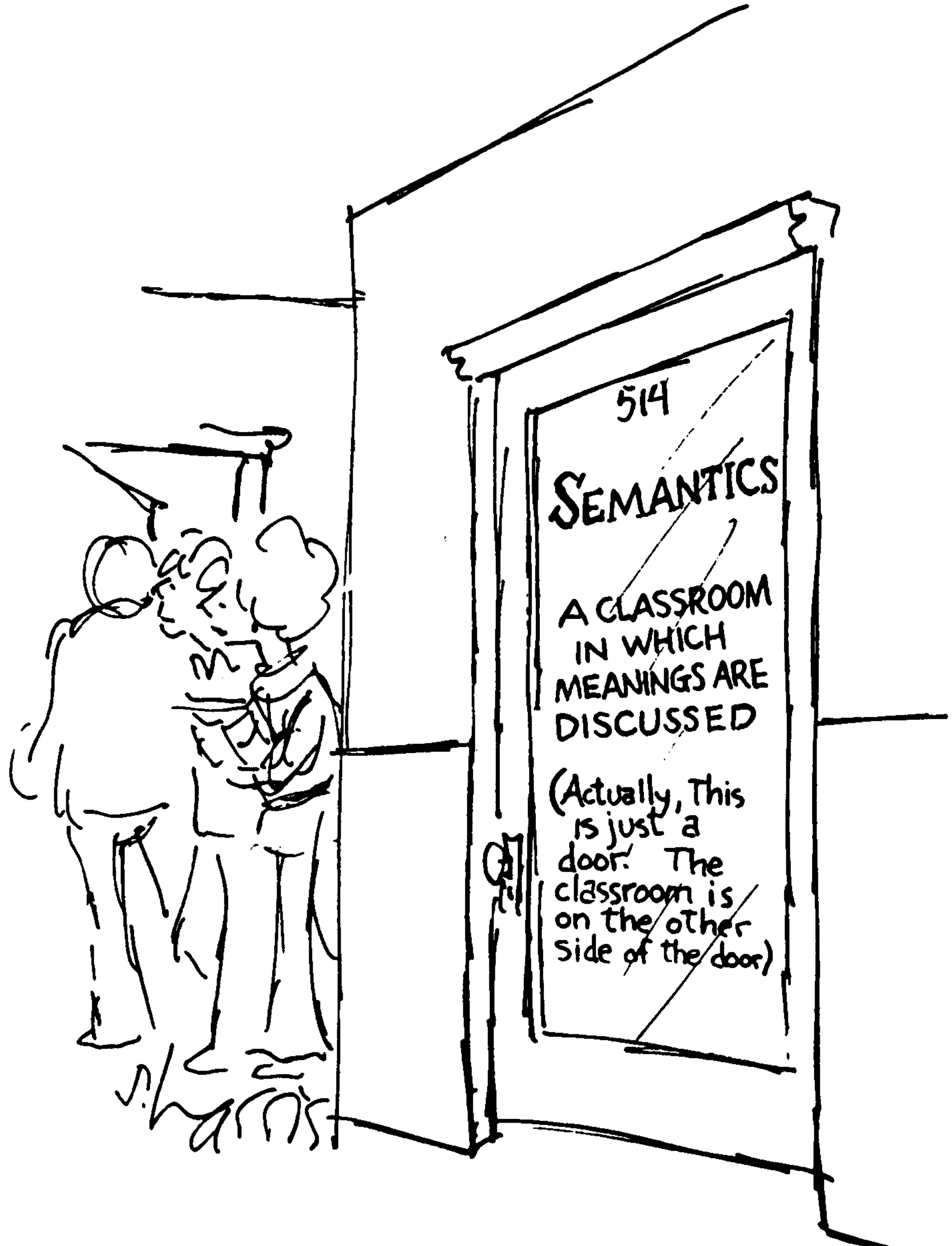
`No, you oughtn't: that's quite another thing! The *song* is called "*Ways and Means*": but that's only what it's *called*, you know!'

`Well, what *is* the song, then?' said Alice, who was by this time completely bewildered.

call-by-name vs. call-by-value!

`I was coming to that,' the Knight said. `The song really *is "A-sitting On a Gate"*: and the tune's my own invention.'

the name is called "Haddocks' Eyes"
↓
the name of the song is "The Aged Aged Man"
↓
the song is "A-sitting On a Gate"
↑
the song is called "Ways and Means"


514
SEMANTICS
A CLASSROOM IN WHICH MEANINGS ARE DISCUSSED
(Actually, This is just a door. The classroom is on the other side of the door)
s.harris

Lewis Carroll Society of North America

WELCOME

Welcome to The Lewis Carroll Society of North America (LCSNA) homepage. The LCSNA is a non-profit organization dedicated to furthering Carroll studies, increasing accessibility of research material, and maintaining public awareness of Carroll's contributions to society and culture. This website is one way we share information with Carroll enthusiasts around the World. If you are a Carrollian and would like to help in these endeavors, or if you simply enjoy Carroll and want to be among other people with a like interest, please consider joining the LCSNA.

For detailed information about C.L.Dodgson ("Lewis Carroll") and his creations, please access the Lewis Carroll Homepage.

Spring Meeting

The 2009 Spring meeting will be held in beautiful Sante Fe, New Mexico, on May 9. Please consult the **newly updated (as of April 24th)** meeting agenda for all of the details. See you there.

The Lewis Carroll Society

The Lewis Carroll Society

founded 1969 ~ registered charity no. 266239

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Events, People, Places

- Carrollian Events
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Welcome to the Lewis Carroll Society Website

The Lewis Carroll Society was formed in 1969 with the aim of encouraging research into the life and works of Lewis Carroll (Charles Lutwidge Dodgson). The Society has members around the world, including many leading libraries and institutions, authors, researchers and many who simply enjoy Carroll's books and want to find out more about the man and his work.

Why not join the LCS - for your own interest and entertainment or to make a contribution to Carroll scholarship? Our subscription are remarkably low for a society of this nature. **Click Here for membership details**.

Events at Lyndhurst: from 15 May 2009

This wonderful season of Alice-related events has something for everyone! The village of Lyndhurst, in the beautiful New Forest, celebrates its Alice connections with walks, talks, tea-parties, musicals, and many other activities. Vist the **Alice Adventure** website for more details.

Events at Oxford: 4 July 2009

The city of Oxford plays host to the second Alice's Day this year, with a busy programme of events on 4 July. There are live performances, reading, drama workshops, exhibitions, talks and other activities for all the family.

The Lewis Carroll Society is hosting a series of lectures at the Natural History Museum from 10:15. Edward Wakeling talks about the real Alice and the original telling of her adventures, Anne Varty investigates the child-actresses who played Alice and were friends of Lewis Carroll and Mark Richards explores the connections between Carroll and Charles Darwin. All are welcome to attend - come and go as you please.


Alice.org - Mozilla Firefox
http://www.alice.org/#
Alice
An Educational Software that teaches students computer programming in a 3D environment
FREE!!
About Alice
Downloads
Teaching
Community
Publications
Support
A revolution in computer science pedagogy
The Alice Project announces a unique collaboration with Sun Microsystems
read more...
Alice 3 News
Alice 3 wins Duke's Choice Award at JavaOne 2009!
Read more...
All about Alice
Alice is an innovative 3D programming environment that makes it easy to create an animation for telling a story, playing an interactive game, or a video to share on the web. Alice is a teaching tool for introductory computing. It uses 3D graphics and a drag-and-drop interface to facilitate a more engaging, less frustrating first programming experience.
Read more...
Teaching Materials
Alice is a teaching tool designed as a revolutionary approach to teaching and learning introductory programming concepts. The Alice team has developed instructional materials to support students and teachers in using this new approach. Resources include textbooks, lessons, sample syllabuses, test banks, and more. Other authors have generously joined our efforts, creating additional textbooks.
Read more...
Downloads
Alice 2.2, Alice 2.0
Designed for High School and College
Storytelling Alice
Designed for Middle School
Alice 3 beta
Get a sneak peek at the future of Alice
3D Models Gallery
Additional free 3D models
Alice Blog
Check out the Alice blog! The Alice team discusses the latest in Alice development. View screencasts demonstrating new features, tips and techniques!
Visit blog...
Community Forums
Share and gather knowledge about Alice through our community forums. Students, teachers and enthusiasts are all welcome! If you have a question or comment about Alice, post it here!
View forums...
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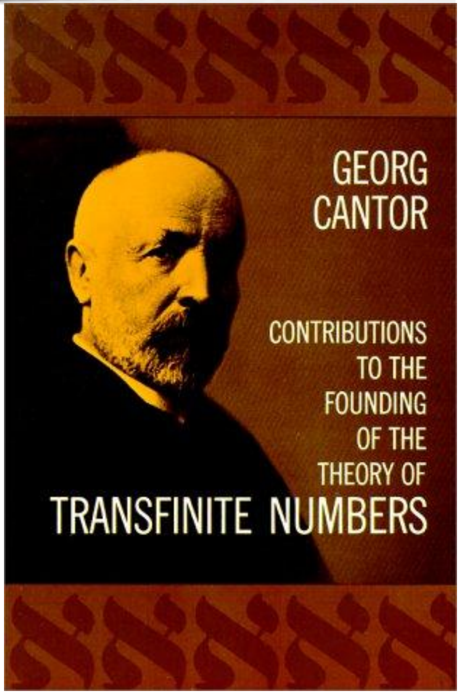
We gratefully acknowledge the financial support of Electronic Arts, Sun Microsystems, DARPA, Intel, Microsoft, NSF, and ONR.
Done

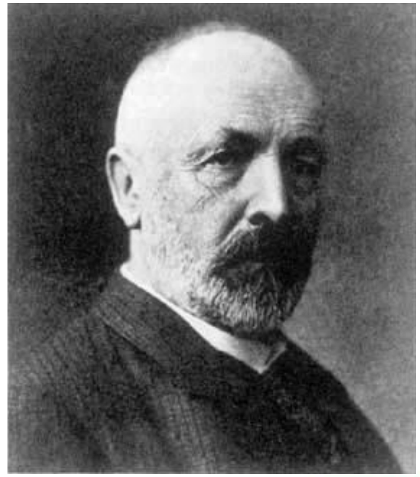
Historical Perspectives

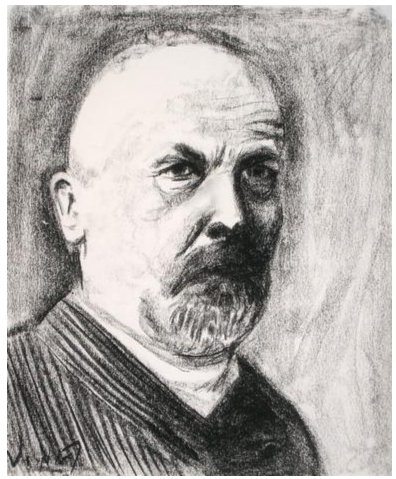
Georg Cantor (1845-1918)

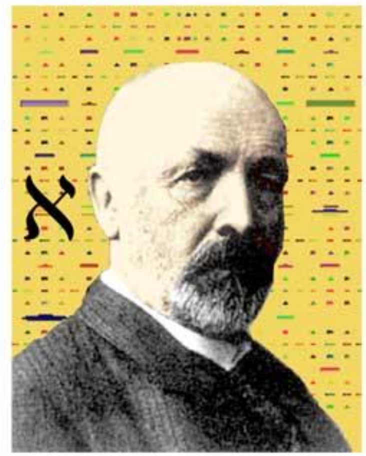
- Created modern set theory
- Invented trans-finite arithmetic (highly controvertial at the time)
- Invented diagonalization argument
- First to use 1-to-1 correspondences with sets
- Proved some infinities “bigger” than others
- Showed an infinite hierarchy of infinities
- Formulated continuum hypothesis
- Cantor’s theorem, “Cantor set”, Cantor dust, Cantor cube, Cantor space, Cantor’s paradox
- Laid foundation for computer science theory
- Influenced Hilbert, Godel, Church, Turing



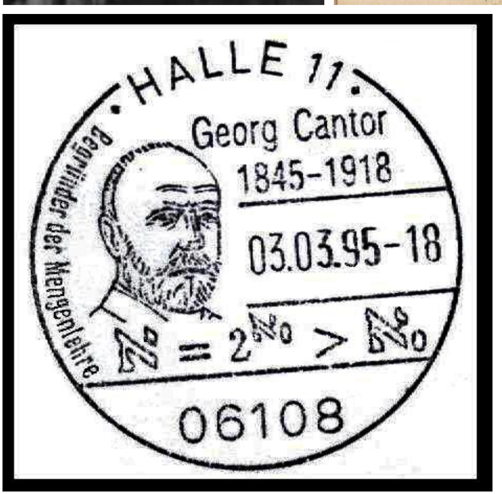

GEORG CANTOR
His Mathematics and Philosophy of the Infinite
Joseph Warren Dauben
David Foster Wallace
GEORG CANTOR
Der Jahrhundertmathematiker und die Entdeckung des Unendlichen
PIPER
MIDHAT J. GAZALÉ
NUMBER
From Ahmes to Cantor
princeton science library
Infinity and the Mind
the science and philosophy of the infinite
With a new preface by the author
RUDY RUCKER
THE NATIONAL BESTSELLER
The Mystery of the Aleph
Mathematics, the Kabbalah, and the Search for Infinity
"Excellent." —The Wall Street Journal
Amir D. Aczel
Author of Fermat's Last Theorem
Philosophical Perspectives on Infinity
Graham Oppy
CAMBRIDGE
GREAT DISCOVERIES
EVERYTHING AND MORE
A Compact History of ∞
DAVID FOSTER WALLACE
david foster wallace
everything and more
cantor & zeno & mathematics & abstraction & ∞
USES OF INFINITY
LEO ZIPPIN
To Infinity and Beyond
ELI MAOR
A Cultural History of the Infinite
THE ART of THE INFINITE
THE PLEASURES OF MATHEMATICS
ROBERT KAPLAN + ELLEN KAPLAN
THE MATHEMATICS OF INFINITY
A Guide to Great Ideas
THEODORE G. FATICONI
ONE TWO THREE... INFINITY
Facts and Speculations of Science
George Gamow
KEYS TO INFINITY
CLIFFORD A. PICKOVER
AUTHOR OF CHAOS IN WONDERLAND
PLAYING WITH INFINITY
MATHEMATICAL EXPLORATIONS AND EXCURSIONS
Rózsa Péter
INFINITY
BY LILLIAN R. LIEBER
N.Ya. Vilenkin
translated by Abe Shenitzer
in search of infinity
Satan, Cantor & Infinity
Mind-Boggling Puzzles
Raymond M. Smullyan

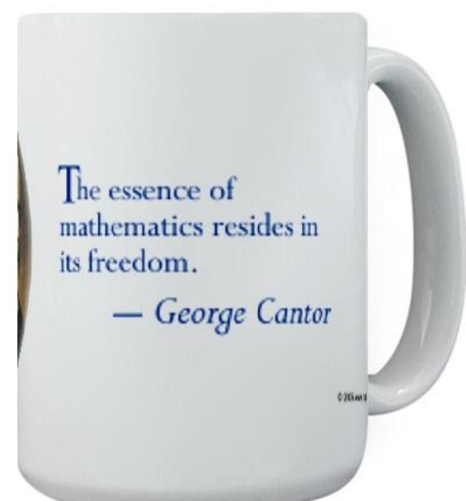






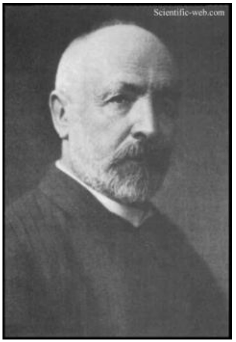
Georg Cantor
1845 - 1918





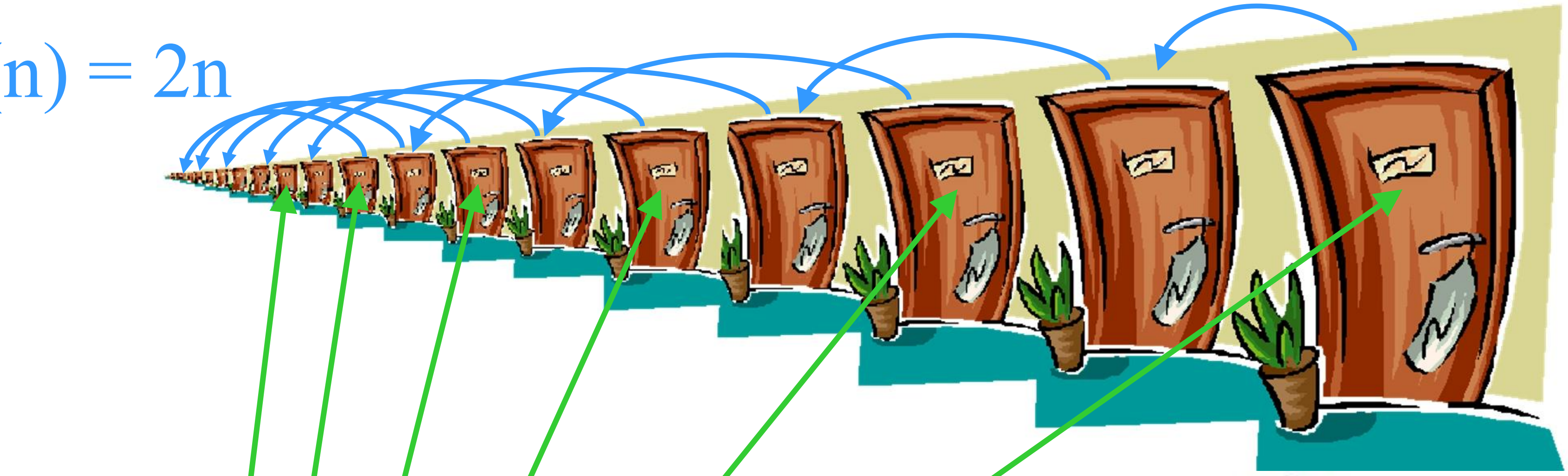
Problem: How can a new guest be accommodated in a full infinite hotel?

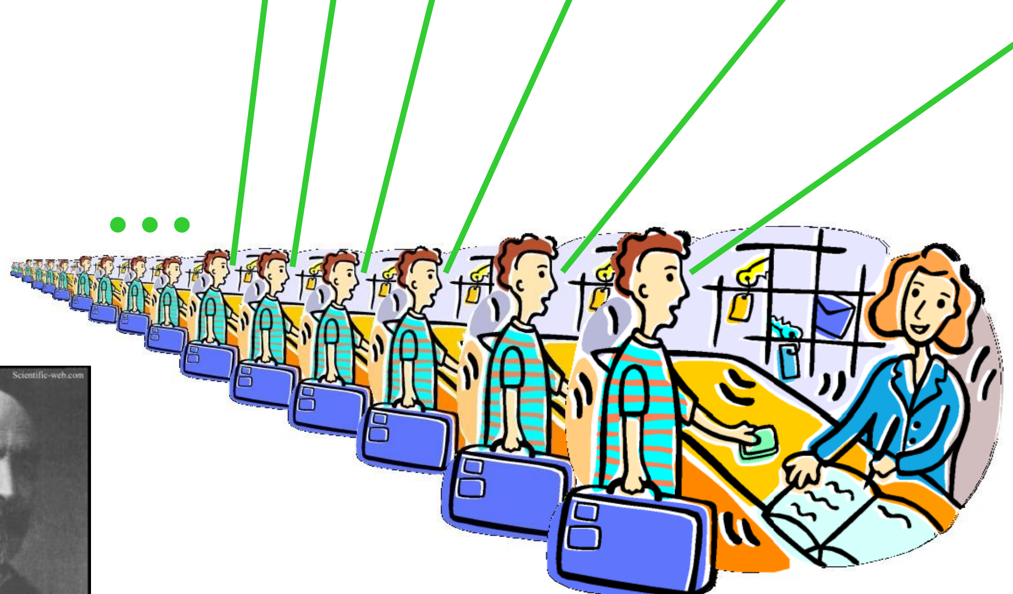
$f(n) = n+1$

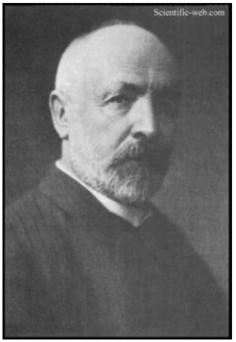


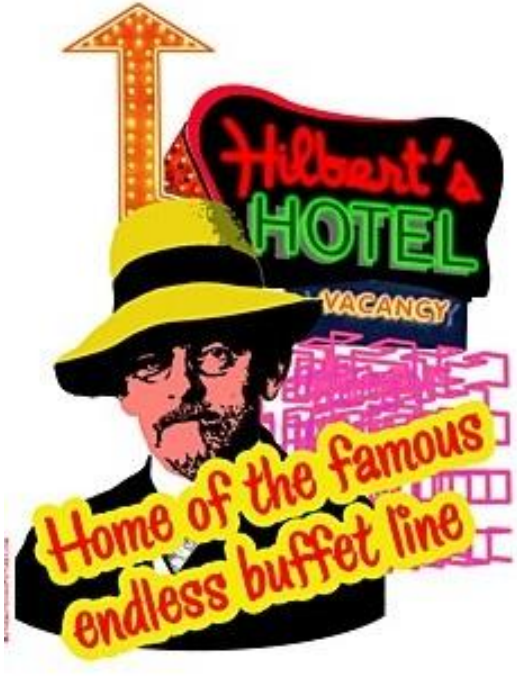
Problem: How can an infinity of new guests be accommodated in a full infinite hotel?

$f(\text{n}) = 2\text{n}$

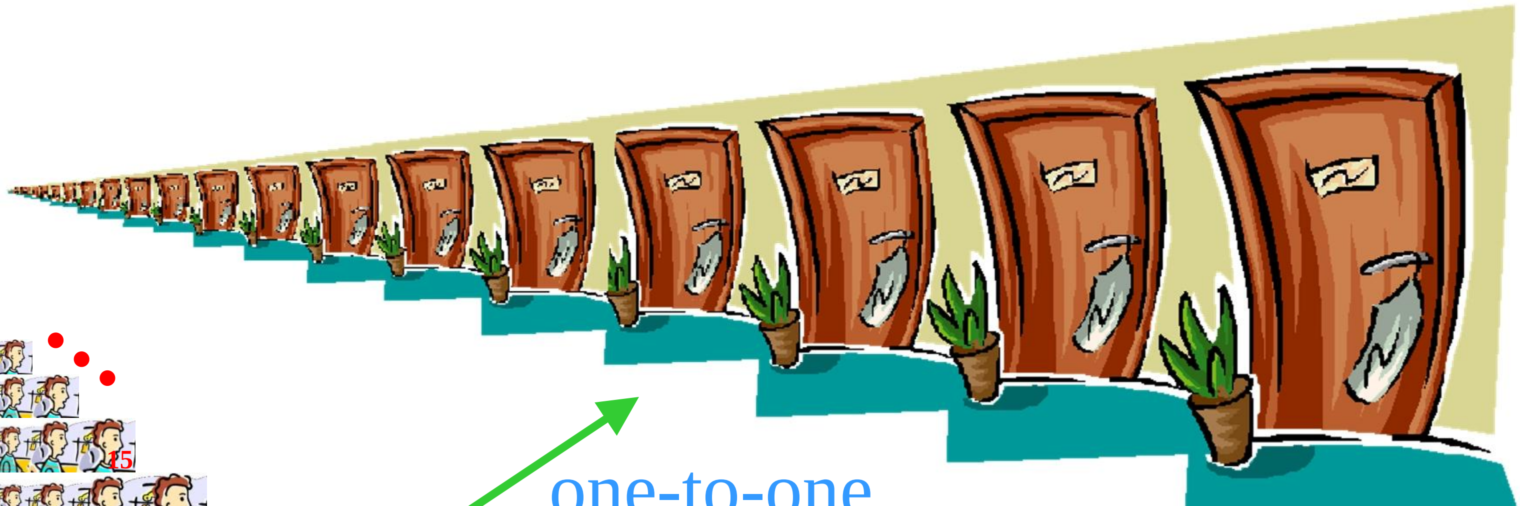


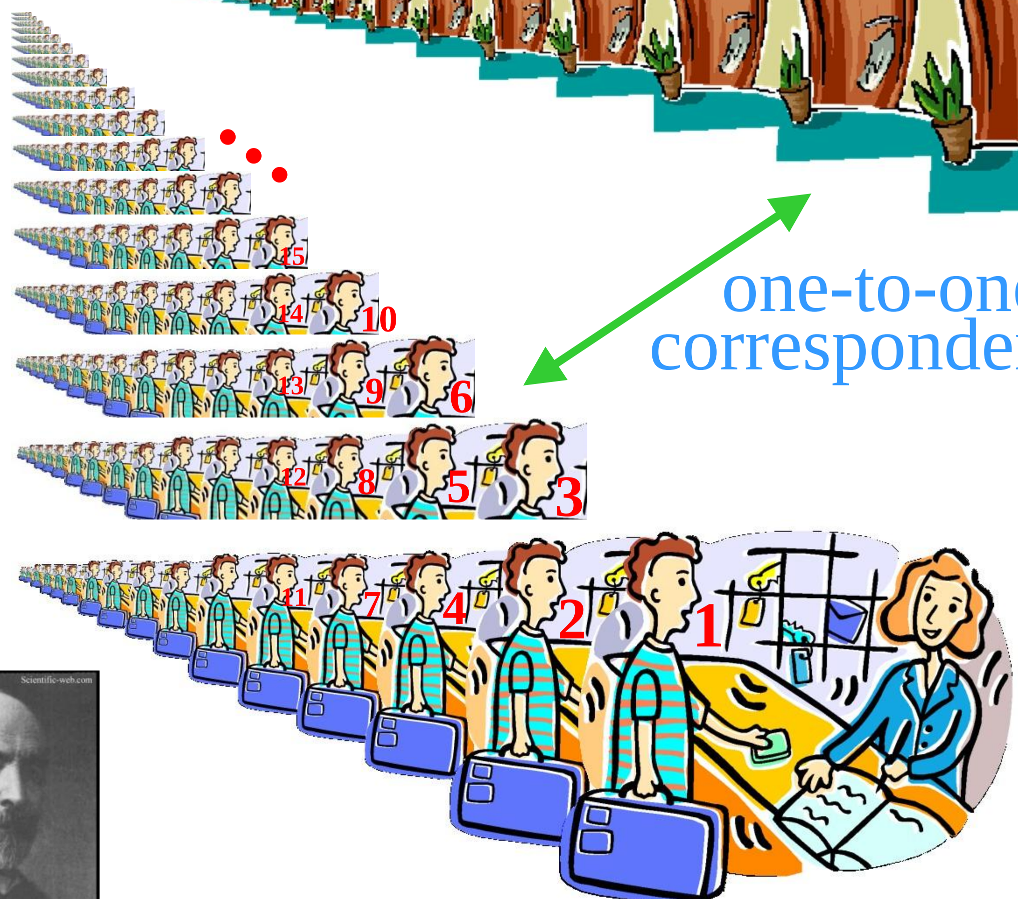




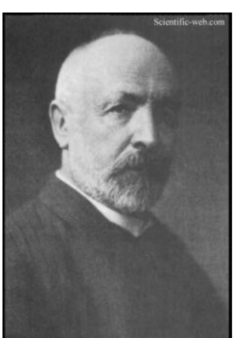


Problem: How can an infinity of infinities of new guests be accommodated in a full infinite hotel?





one-to-one correspondence




"HILBERT'S HOTEL" ©2002 CAROL LAY
THE INFINITE HOTEL WAS ALWAYS FILLED TO CAPACITY.
HILBERT'S HOTEL ∞
NO VACANCY
WWW.WAYLAY.COM
YET, IF A NEW GUEST ARRIVED, SHE WAS ALWAYS GIVEN A ROOM.
AFTER ALL, THERE WERE AN INFINITE NUMBER OF ROOMS.
ROOM 843,722,948,671,439,264,5913,653,898,652,732 NEEDS FRESH TOWELS.
THIS PARADOX ASSUMED THAT MANAGEMENT COULD ALWAYS ADD ONE OR MORE TO INFINITY.
WE'LL SIMPLY PUT YOU IN ROOM 1, THEN ASK EVERYONE TO SHIFT OVER ONE ROOM.
THE BRAIN-BRUISING HOTEL ATTRACTED A LOT OF MATHEMATICIANS AND PHILOSOPHERS.
JUST A MOMENT, PLEASE.
THEY LIKED TO ARGUE INTO THE WEE HOURS ABOUT THE NATURE OF INFINITY.
UNFORTUNATELY, THEY WERE A BUNCH OF SLOBS.
OH, DEAR.
TSK
MANAGEMENT HAD TO HIRE A NEW MAID TO KEEP UP WITH THE MESS.
RING IF YOU GET LOST...
DAUNTED BY THE NUMBER OF ROOMS TO CLEAN...
RM. 6329468517432
NOK NOK
THE MAID SET FIRE TO THE JOINT.
SCREW IT.
ETHER FLAMMABLE
THE PHILOSOPHERS ESCAPED...
BUT THE HOTEL BURNED FOREVER.


Ode to Aleph Null
$\aleph_0$
Aleph-null bottles of beer on the wall,
Aleph-null bottles of beer,
if one of those bottles should happen to fall,
Aleph-null bottles of beer on the wall.

8th June 2009
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HOTEL INFINITY

Amanda Boyle

average rating from 27 members

drama | 2004 | London | Switzerland/ Mandarin | 10 min

Published 28 Feb 07

What happens when an hotel of infinite rooms suddenly becomes full?

send to a friend

PLAY NOW

Requires windows media player or real player.

synopsis

There was once a hotel in the mountains that was so popular with guests, the Manager decided to extend it. Yet still it remained full. The Manager continued to extend it, until eventually it became infinitely large. One day much to his surprise, no spare rooms could be found in his now infinite hotel. All the mathematical calculations, which he normally relied on to find the rooms, just wouldn't work...

rate this film

5
4
3
2
1

Problem: Are there more integers than natural #'s?

$\mathbb{N} \subset \mathbb{Z}$

$\mathbb{N} \neq \mathbb{Z}$

So $|\mathbb{N}| < |\mathbb{Z}|$?

$\mathbb{Z}$

-4	-3	-2	-1	0	1	2	3	4

Rearrangement:

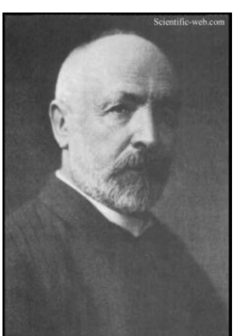
Establishes 1-1 correspondence

$f: \mathbb{N} \leftrightarrow \mathbb{Z}$

$\mathbb{Z}$

$\mathbb{N}$ 1 2 3 4 5 6 7 8 9

$\Rightarrow |\mathbb{N}| = |\mathbb{Z}|$



Problem: Are there more rationals than natural #'s?

$\mathbb{N} \subset \mathbb{Q}$

$\mathbb{N} \neq \mathbb{Q}$

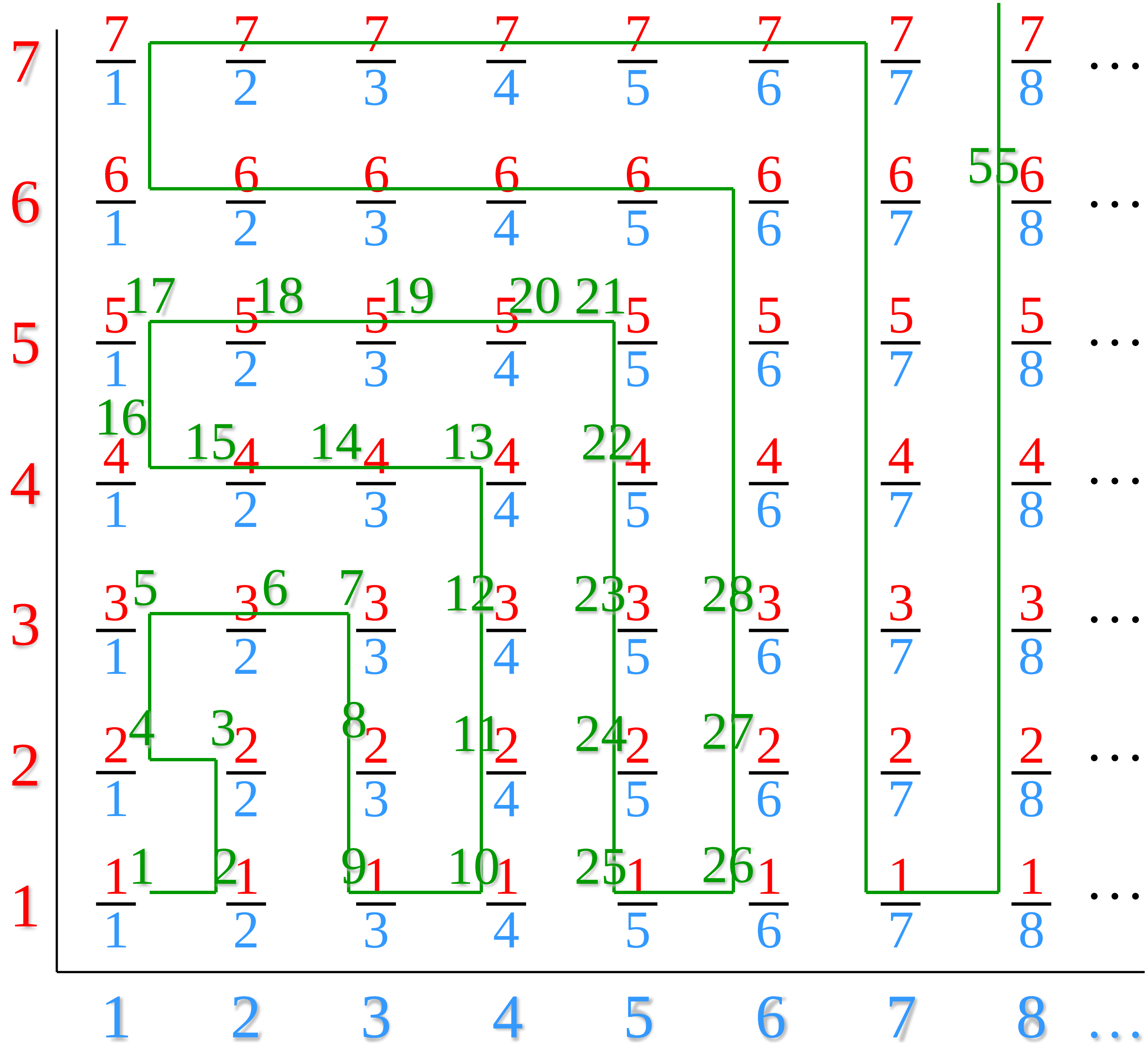
So $|\mathbb{N}|<|\mathbb{Q}|$?

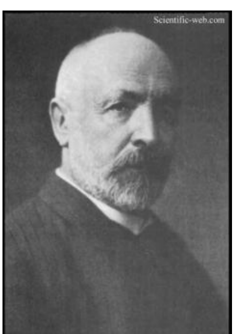
Dovetailing:

Establishes 1-1 correspondence

$f: \mathbb{N} \leftrightarrow \mathbb{Q}$

$\Rightarrow |\mathbb{N}|=|\mathbb{Q}|$





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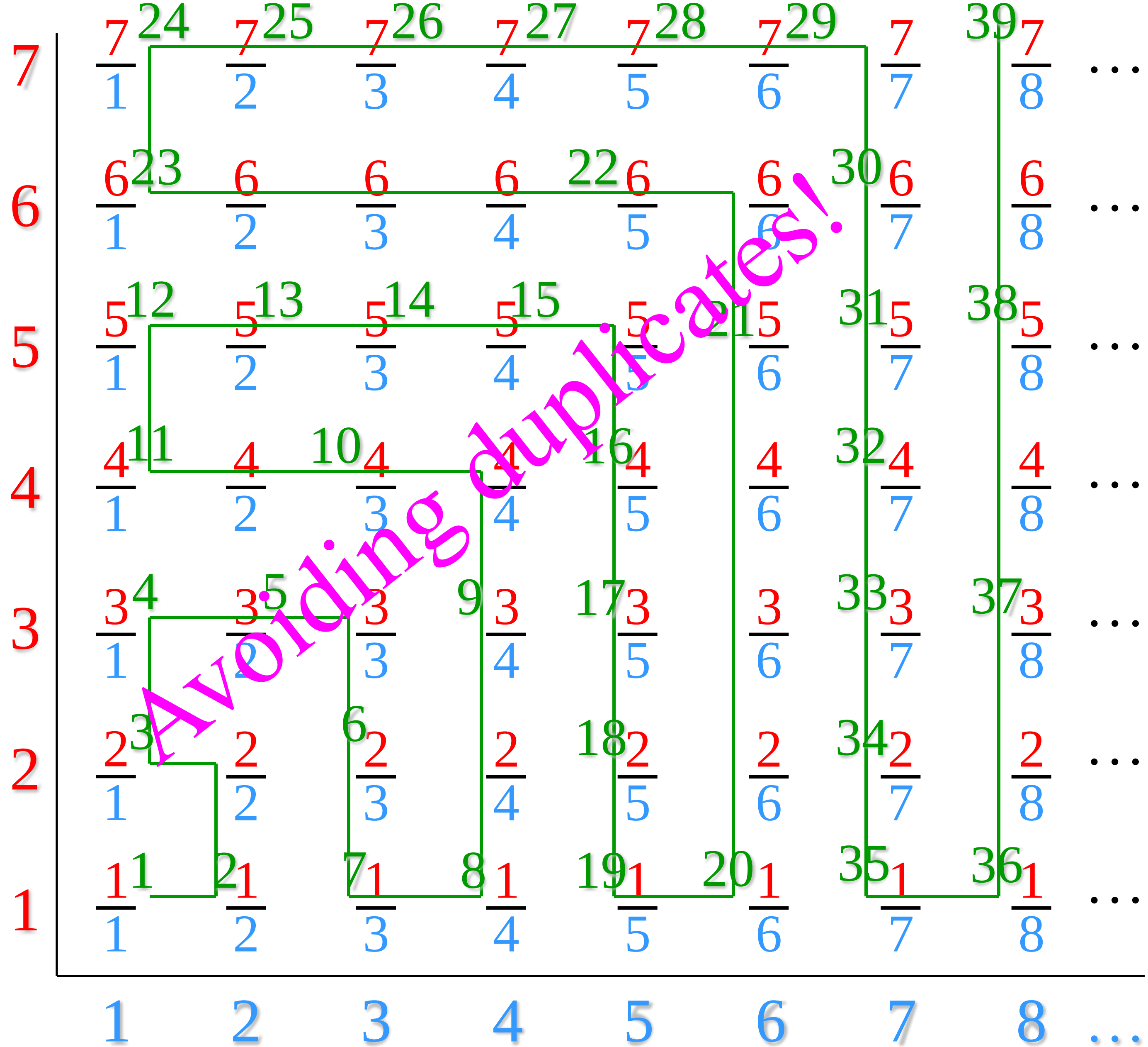

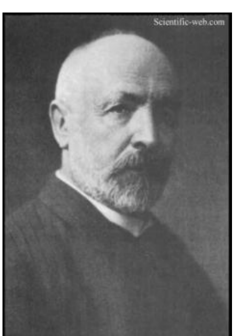

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Establishes 1-1 correspondence

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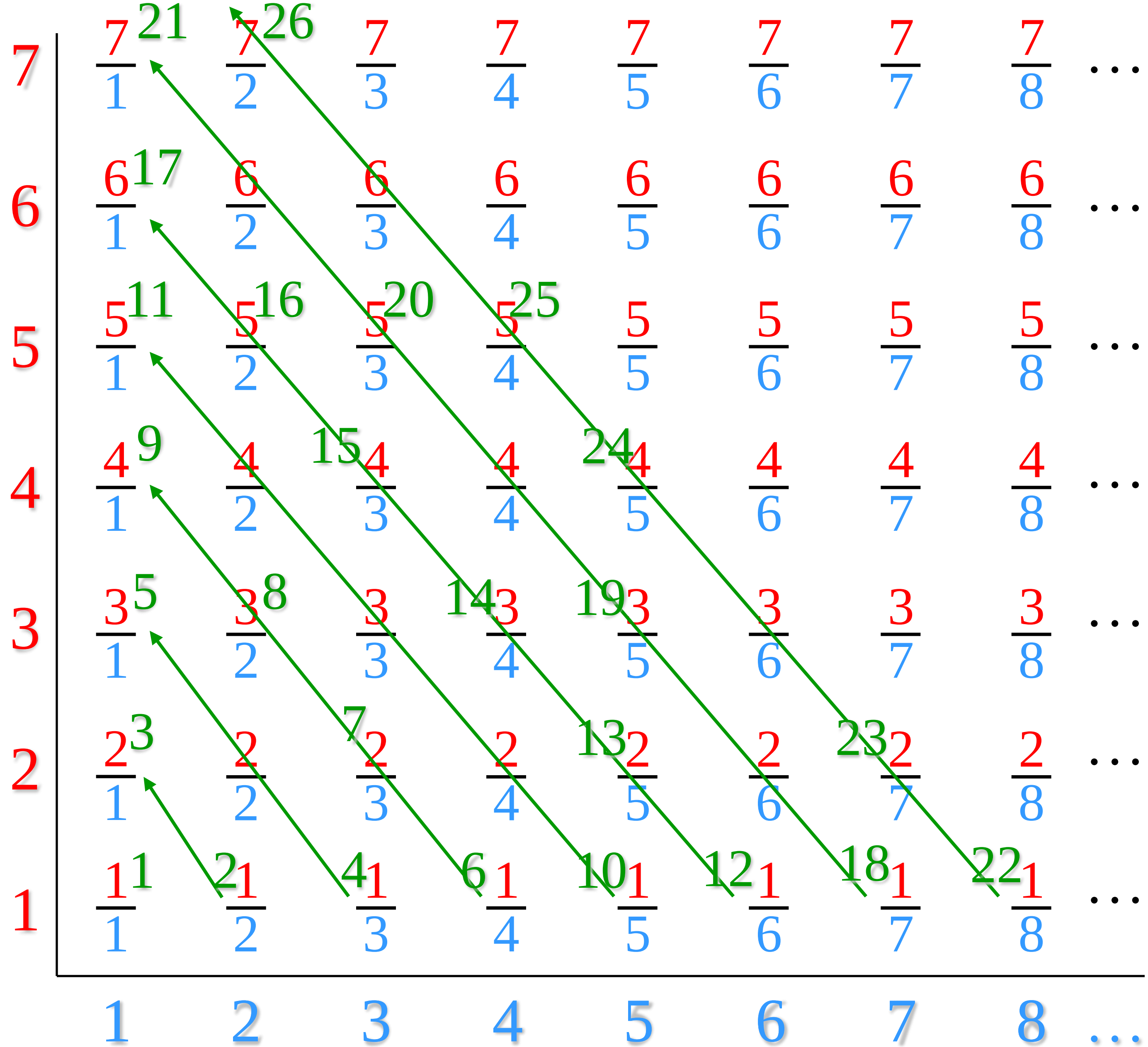
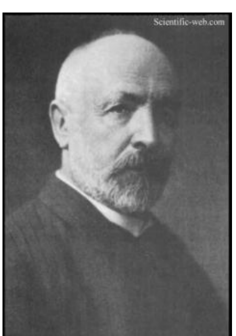
$\Rightarrow |\mathbb{N}|=|\mathbb{Q}|$



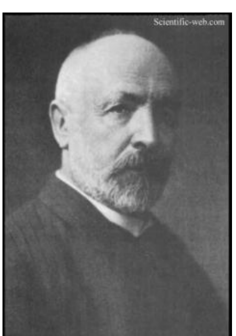


Problem: Why doesn't this "dovetailing" work?

There's no "last" element on the first line!

So the 2nd line is never reached!

$\Rightarrow$ 1-1 function is not defined!



7	$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$	$\frac{7}{7}$	$\frac{7}{8}$	…
6	$\frac{6}{1}$	$\frac{6}{2}$	$\frac{6}{3}$	$\frac{6}{4}$	$\frac{6}{5}$	$\frac{6}{6}$	$\frac{6}{7}$	$\frac{6}{8}$	…
5	$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	…
4	$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$	…
3	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$	…
2	$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$	$\frac{2}{8}$	…
1	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	…
	1	2	3	4	5	6	7	8	…

Dovetailing Reloaded

Dovetailing: f:ℕ ↔ ℤ

-4	-3	-2	-1	0	1	2	3	4

0 1 2 3 4 5 6 7 8 . . .

-1 -2 -3 -4 -5 -6 -7 -8 -9 . . .

ℤ									
ℕ	1	2	3	4	5	6	7	8	9

To show |ℕ|=|ℚ| we can construct f:ℕ↔ℚ by sorting x/y by increasing key max(|x|,|y|), while avoiding duplicates:

Dovetailing!

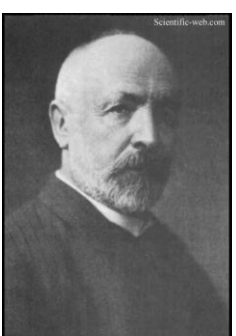
max(|x|,|y|) = 0 : {}

max(|x|,|y|) = 1 : 0/1 [1], 1/1 [2]

max(|x|,|y|) = 2 : 1/2 [3], 2/1 [4]

max(|x|,|y|) = 3 : 1/3 [5], 2/3 [6], 3/1 [7], 3/2 [8]

. . . {finite new set at each step}



- Dovetailing can have many disguises!
- So can diagonalization!

Theorem: There are more reals than rationals / integers.

Proof [Cantor]: Assume a 1-1 correspondence $f: \mathbb{N} \leftrightarrow \mathbb{R}$

i.e., there exists a table containing all of $\mathbb{N}$ and all of $\mathbb{R}$:

$\mathbb{N}$	$\mathbb{R}$
$f(1) =$	3 . 1 4 1 5 9 2 6 5 3 …
$f(2) =$	1 . 0 0 0 0 0 0 0 0 0 …
$f(3) =$	2 . 7 1 8 2 8 1 8 2 8 …
$f(4) =$	1 . 4 1 4 2 1 3 5 6 2 …
$f(5) =$	0 . 3 3 3 3 3 3 3 3 3 …
…	…

Diagonalization

Non-existence proof!

$X = 0.2\ 1\ 9\ 3\ 4 \ldots \in \mathbb{R}$

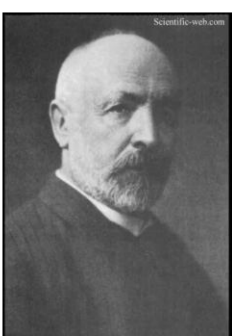
But X is missing from our table! $X \neq f(k)\ \forall\ k \in \mathbb{N}$

$\Rightarrow$ f not a 1-1 correspondence

$\Rightarrow$ contradiction

$\Rightarrow$ $\mathbb{R}$ is not countable!

There are more reals than rationals / integers!



Problem 1: Why not just insert X into the table?

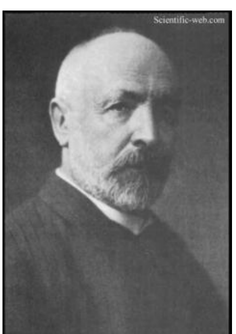
Problem 2: What if X=0.999… but 1.000… is already in table?

Diagonalization

Non-existence proof!

$\mathbb{N}$	$\mathbb{R}$
$f(1) =$	3 . 1 4 1 5 9 2 6 5 3 …
$f(2) =$	1 . 0 0 0 0 0 0 0 0 0 …
$f(3) =$	2 . 7 1 8 2 8 1 8 2 8 …
$f(4) =$	1 . 4 1 4 2 1 3 5 6 2 …
$f(5) =$	0 . 3 3 3 3 3 3 3 3 3 …
…	…

X = 0 . 2 1 9 3 4 … $\in \mathbb{R}$



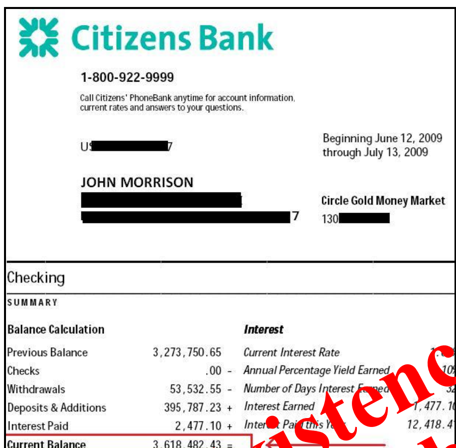
- Table with X inserted will have X' still missing! Inserting X (or any number of X's) will not help!
- To enforce unique table values, we can avoid using 9's and 0's in X.


Celebrity Cruises
X
a true departure
WELCOME
TO
INFINITY
Infinity

Non-Existence Proofs

- Must cover all possible (usually infinite) scenarios!
- Examples / counter-examples are not convincing!
- Not "symmetric" to existence proofs!

Ex: proof that you are a millionaire:


Citizens Bank
1-800-922-9999
Call Citizens' PhoneBank anytime for account information, current rates and answers to your questions.

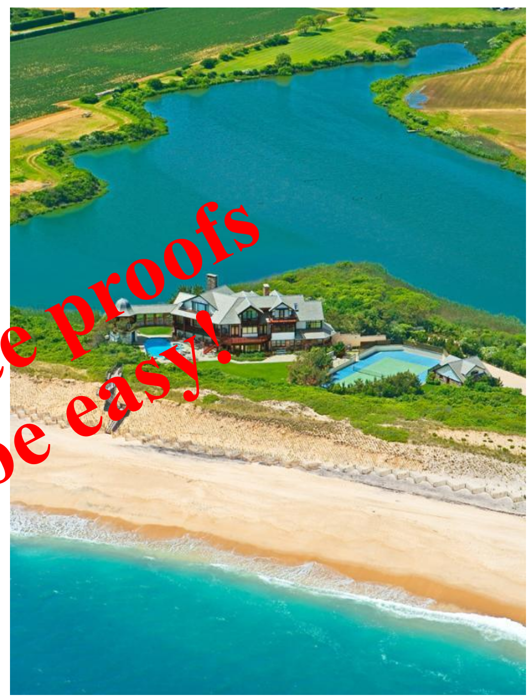
U[redacted]7
Beginning June 12, 2009 through July 13, 2009

JOHN MORRISON
[redacted]7
Circle Gold Money Market
130[redacted]

Checking

SUMMARY

Balance Calculation		Interest	
Previous Balance	3,273,750.65	Current Interest Rate	
Checks	.00 -	Annual Percentage Yield Earned	10
Withdrawals	53,532.55 -	Number of Days Interest Earned	32
Deposits & Additions	395,787.23 +	Interest Earned	1,477.10
Interest Paid	2,477.10 +	Interest Paid this Year	12,418.4
Current Balance	3,618,482.43 =		



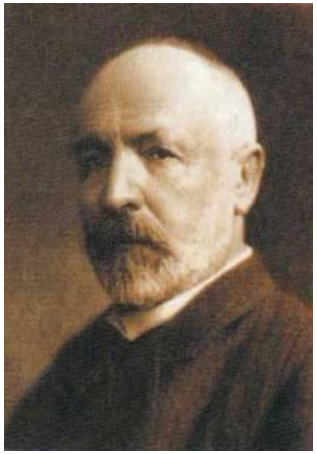
Existence proofs can be easy!

"Proof" that you are not a millionaire ?

Non-existence proofs are often hard!

P≠NP

Cantor set:



Start with unit segment

- Remove (open) middle third
- Repeat recursively on all remaining segments
- Cantor set is all the remaining points

Total length removed: 1/3 + 2/9 + 4/27 + 8/81 + … = 1

Cantor set does not contain any intervals

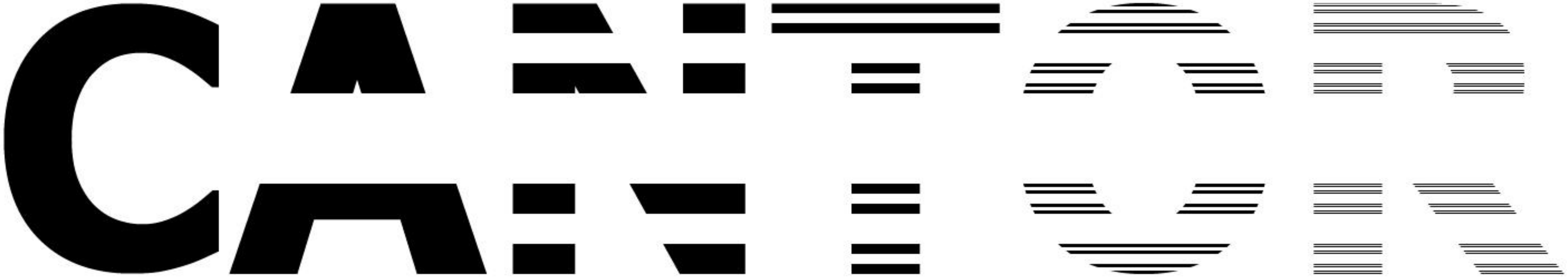
Cantor set is not empty (since, e.g. interval endpoints remain)

An uncountable number of non-endpoints remain as well (e.g., 1/4)

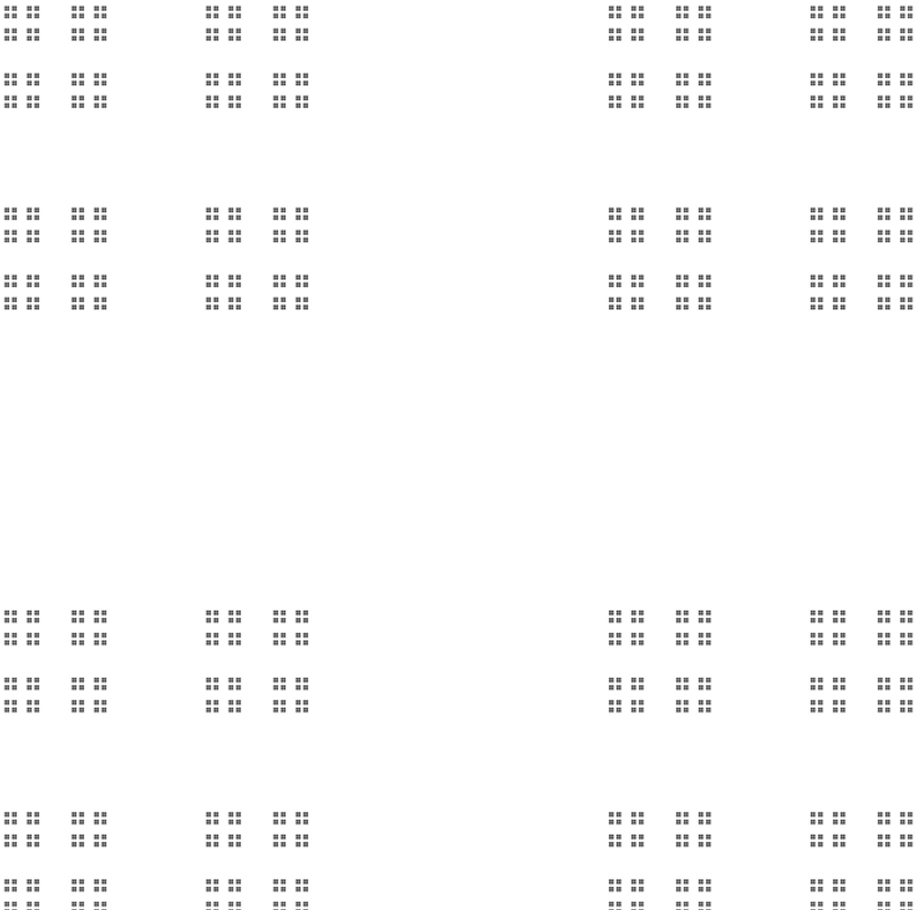
Cantor set is totally disconnected (no nontrivial connected subsets)

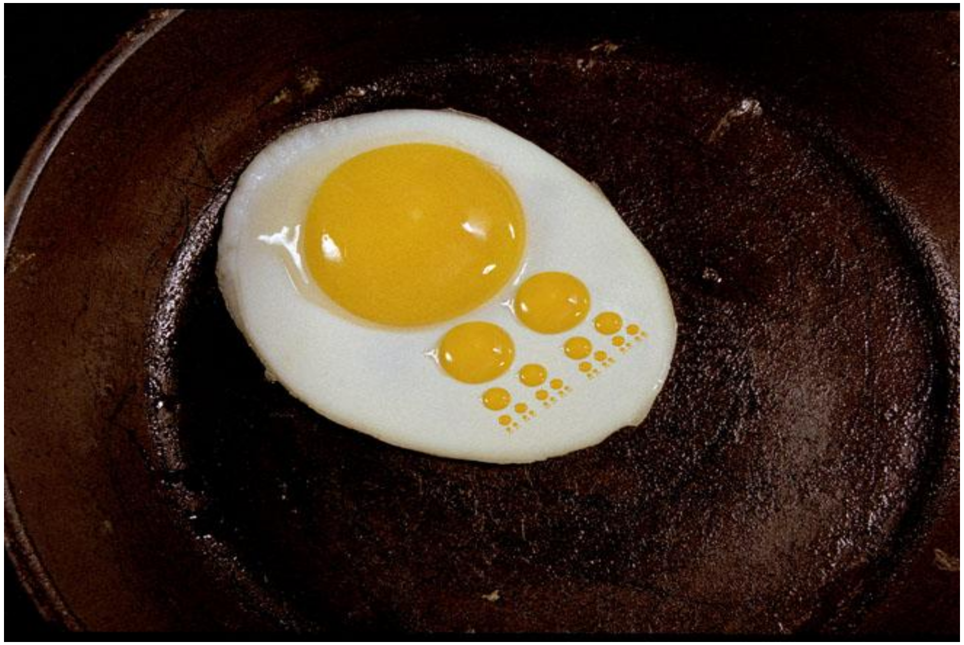
Cantor set is self-similar with Hausdorff dimension of $\log_3 2$=1.585

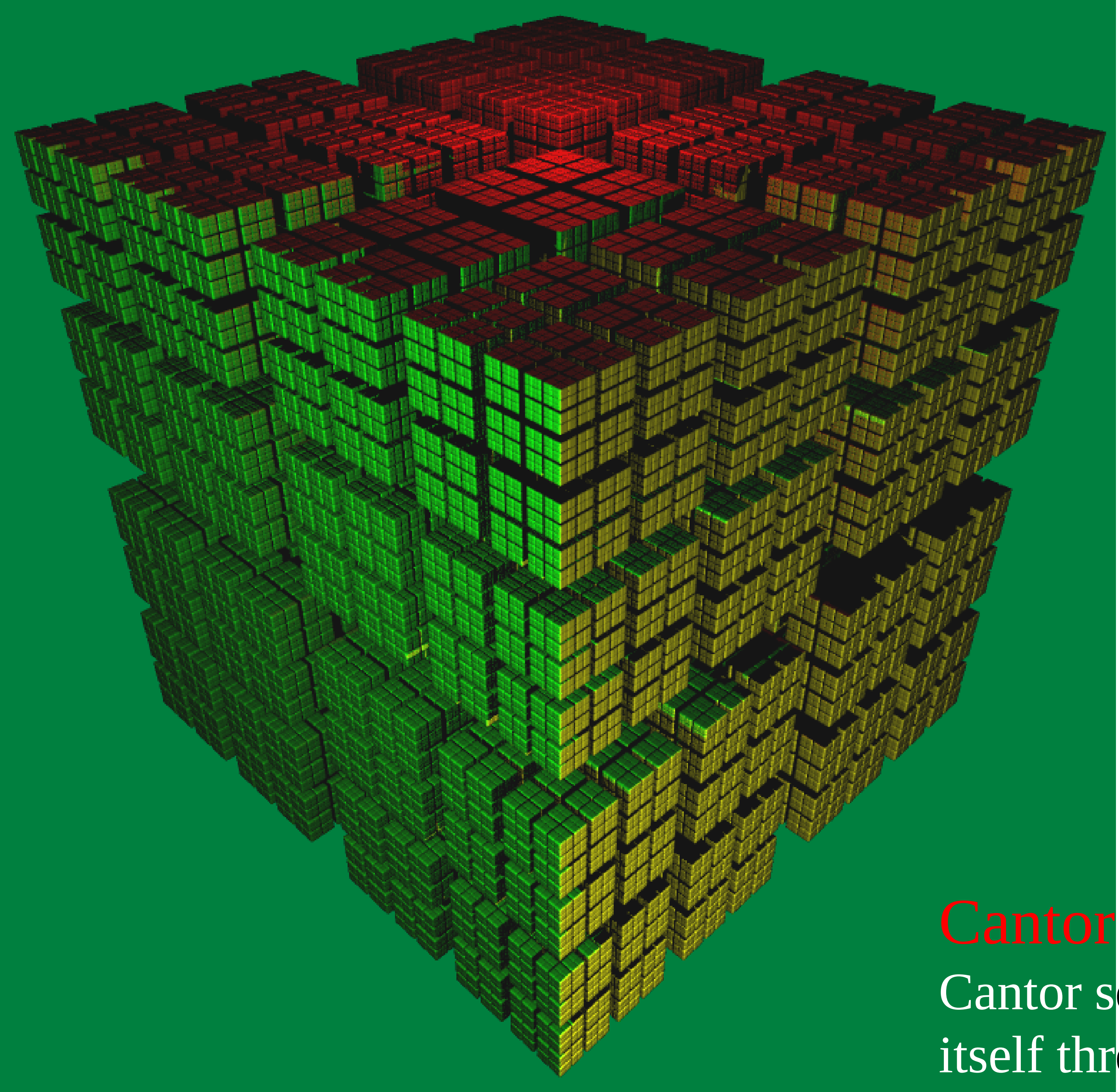
Cantor set is a closed, totally bounded, compact, complete metric space, with uncountable cardinality and lebesque measure zero



Cantor dust (2D generalization): Cantor set crossed with itself







Cantor cube (3D):

Cantor set crossed with itself three times

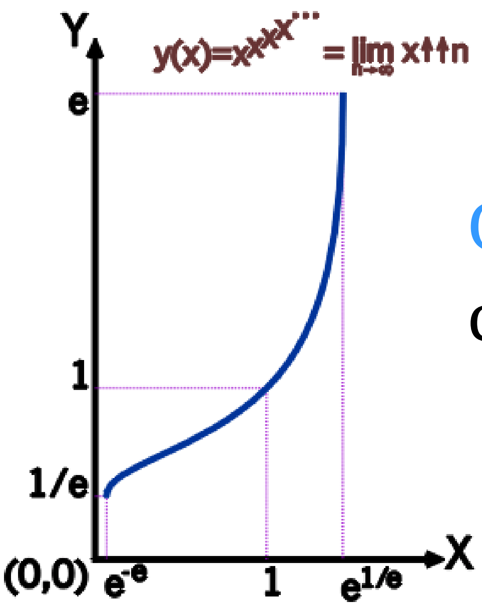
Problem: Solve the following equation for X:

$$X^{X^{X^{X^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

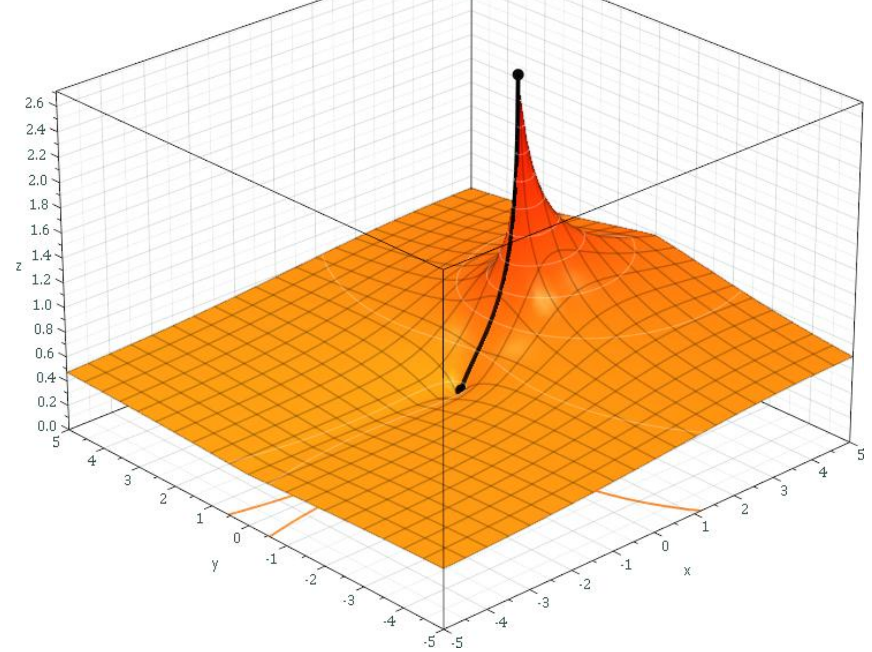
where the stack of exponentiated x's extends forever.

This "power tower" converges for:

$$0.065988 \approx e^{-e} < X < e^{1/e} \approx 1.444668$$



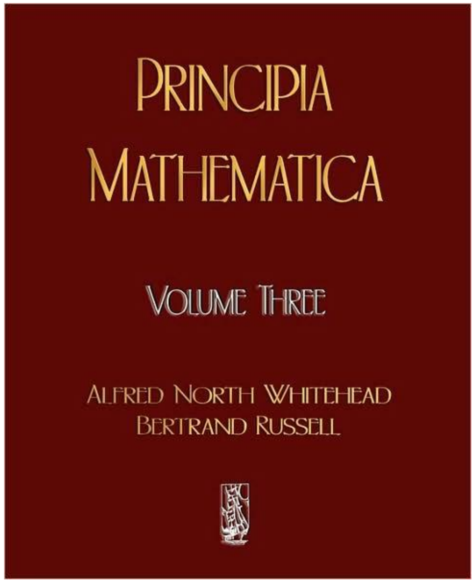
Generalization to complex numbers:

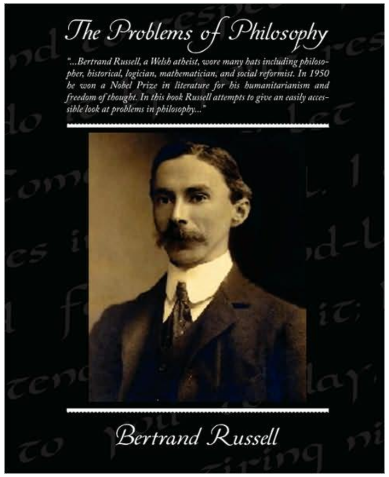


Historical Perspectives

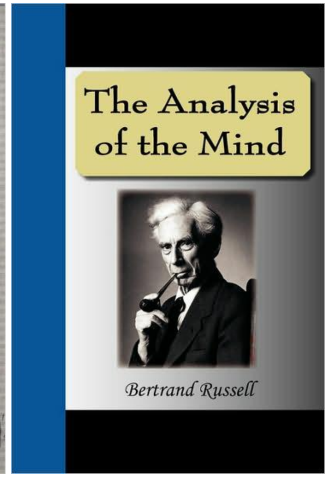
Bertrand Russell (1872-1970)

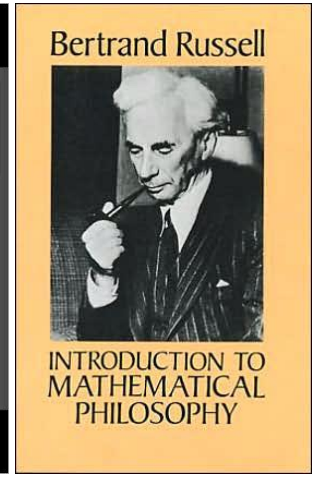
- Philosopher, logician, mathematician, historian, social reformist, and pacifist
- Co-authored "Principia Mathematica" (1910)
- Axiomatized mathematics and set theory
- Co-founded analytic philosophy
- Originated Russell's Paradox
- Activist: humanitarianism, pacifism, education, free trade, nuclear disarmament, birth control gender & racial equality, gay rights
- Profoundly transformed math & philosophy, mentored Wittgenstein, influenced Godel
- Laid foundation for computer science theory
- Won Nobel Prize in literature (1950)

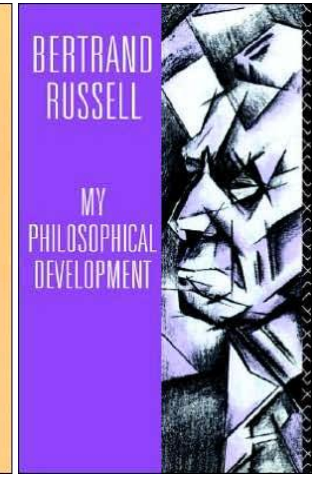



The Problems of Philosophy
"...Bertrand Russell, a Welsh atheist, wore many hats including philosopher, historical, logician, mathematician, and social reformist. In 1950 he won a Nobel Prize in literature for his humanitarianism and freedom of thought. In this book Russell attempts to give an easily accessible look at problems in philosophy..."
Bertrand Russell


Russell
An Inquiry into
Meaning and Truth


The Analysis of the Mind
Bertrand Russell


Bertrand Russell
INTRODUCTION TO MATHEMATICAL PHILOSOPHY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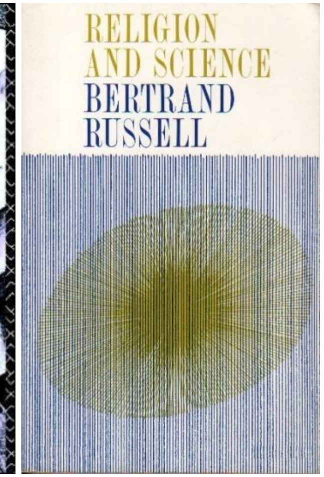
MY PHILOSOPHICAL DEVELOPMENT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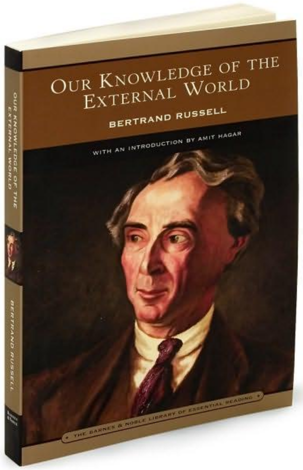
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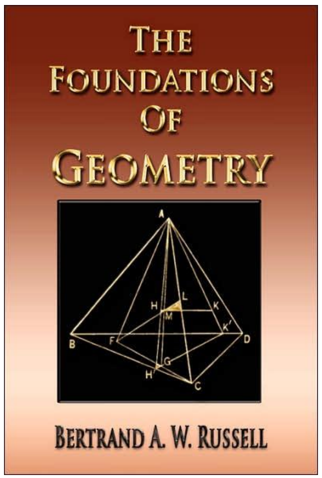
WITH AN INTRODUCTION BY AMIT HAGAR
THE BARNES & NOBLE LIBRARY OF ESSENTIAL READING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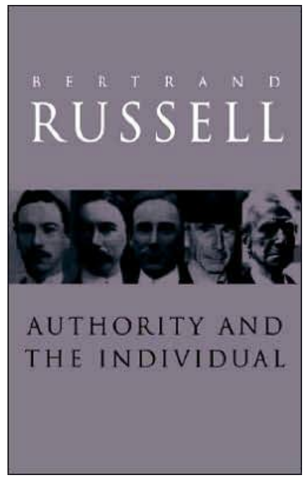
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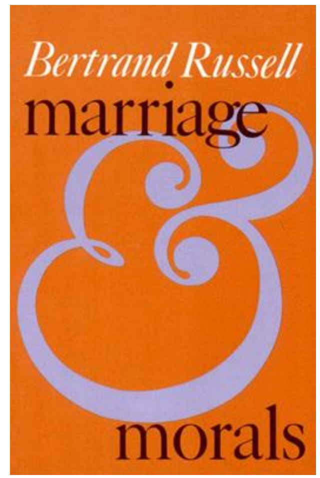
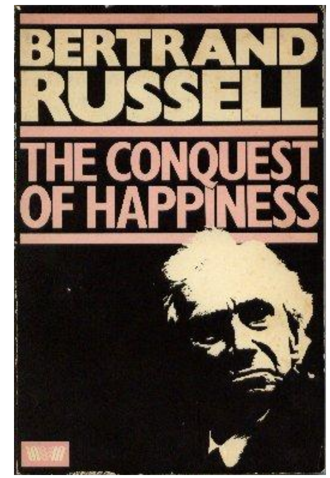

Bertrand Russell
marriage & morals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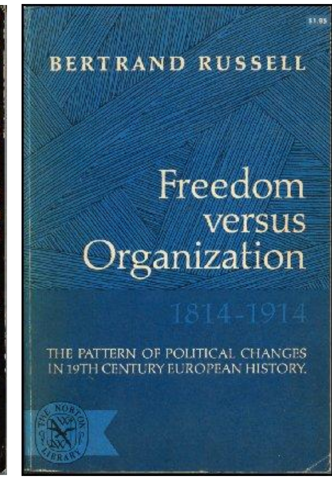
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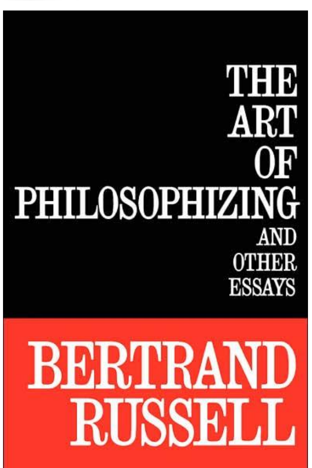
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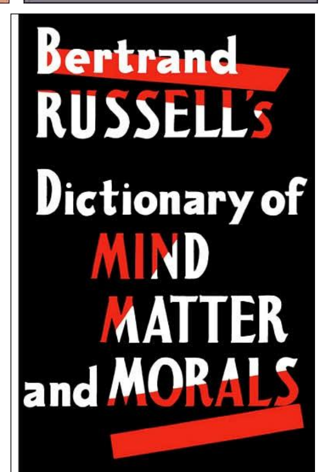
Bertrand Russell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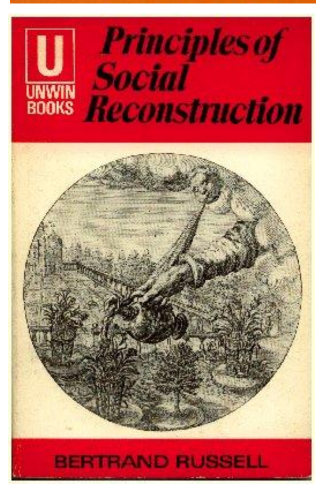
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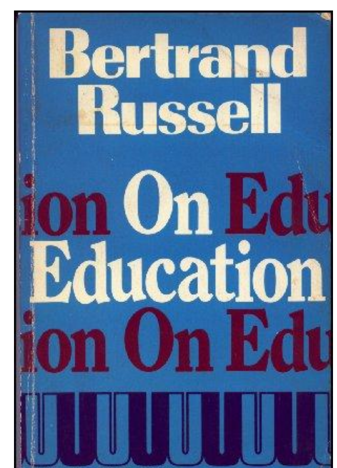
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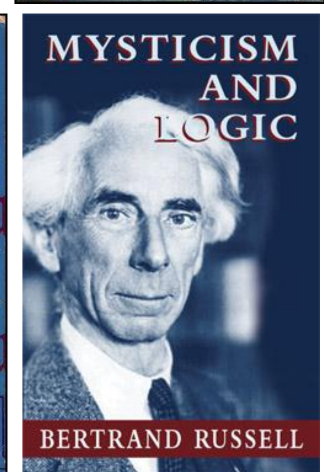
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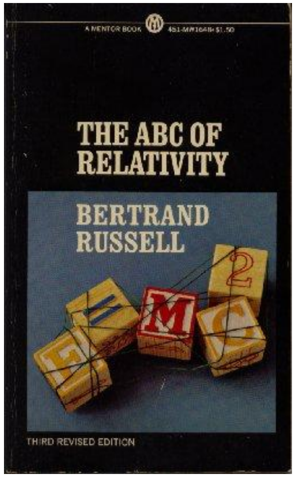
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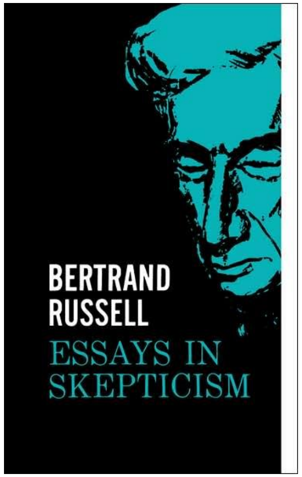
THE ABC OF RELATIVITY
BERTRAND RUSSELL
THIRD REVISED EDITION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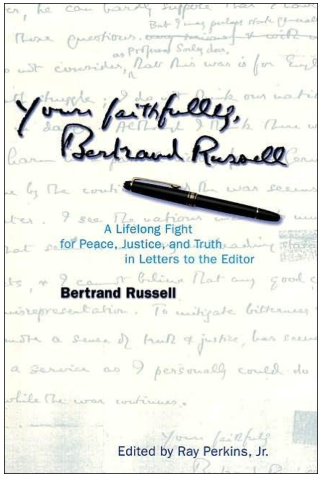
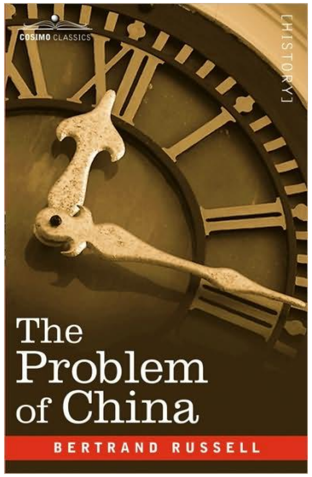

Yours faithfully,
Bertrand Russell
A Lifelong Fight
for Peace, Justice, and Truth
in Letters to the Editor
Bertrand Russell
Edited by Ray Perkins, Jr.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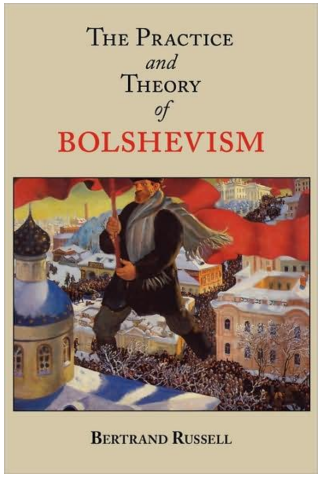
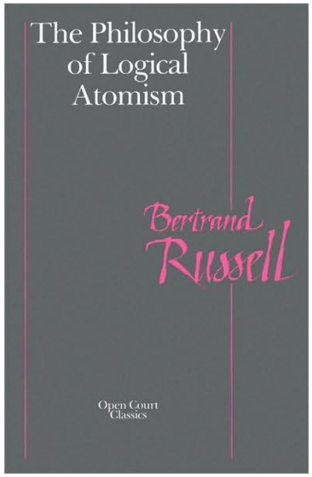
The Problem of China
BERTRAND RUSSELL


The Practice and Theory of
BOLSHEVISM
Bertrand Russell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Bertrand Russell
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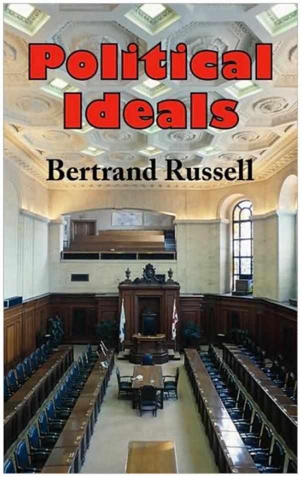
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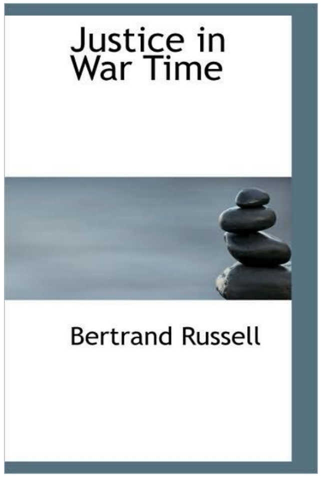
Bertrand Russell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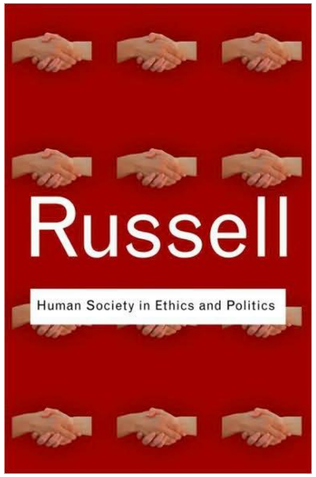
Bertrand Russell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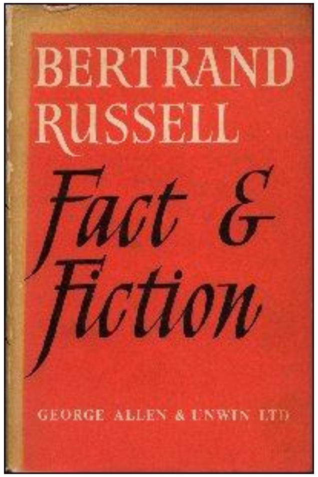
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Fact & Fiction
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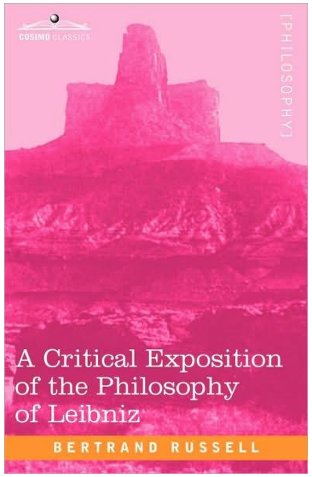
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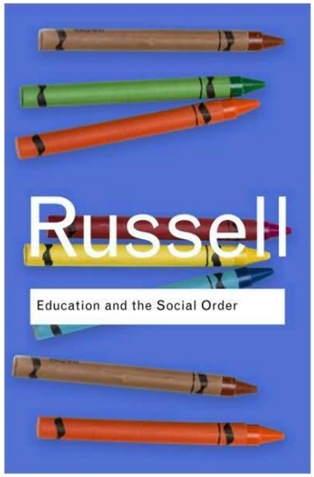
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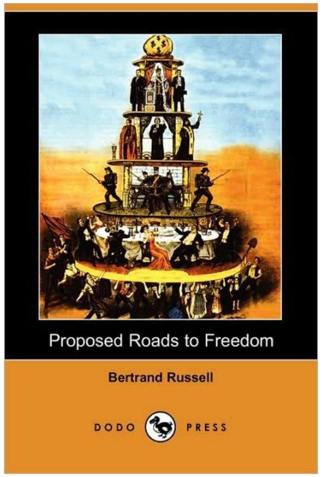
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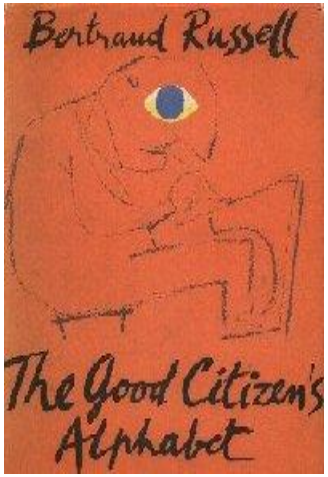
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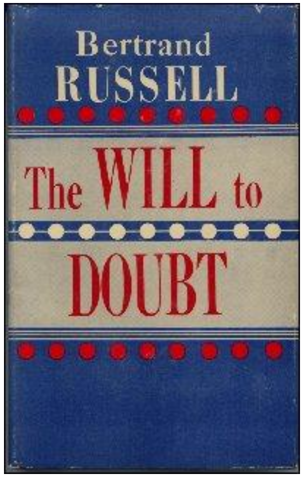
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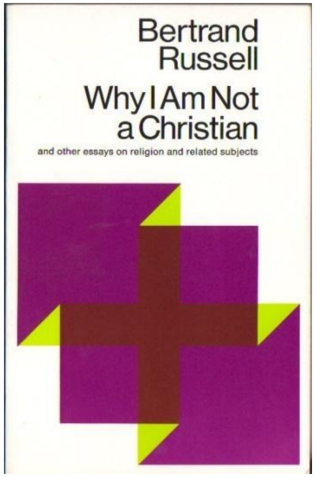
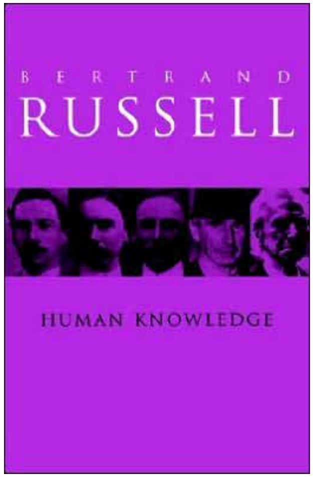

Bertrand Russell
Why I Am Not a Christian
and other essays on religion and related subjects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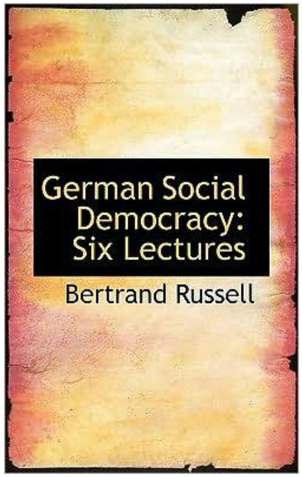
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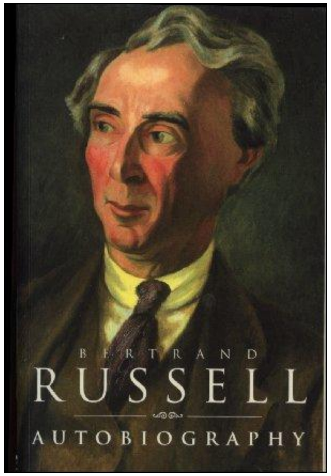
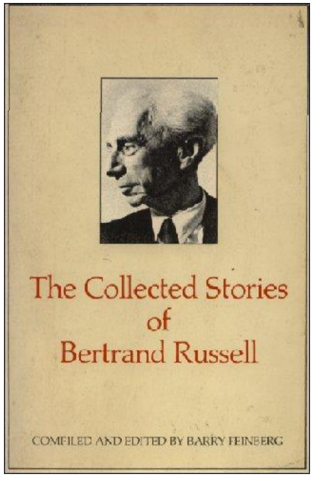
Bertrand Russell


BERTRAND
RUSSELL
AUTOBIOGRAPHY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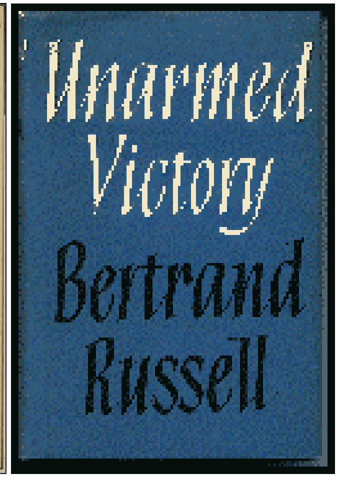
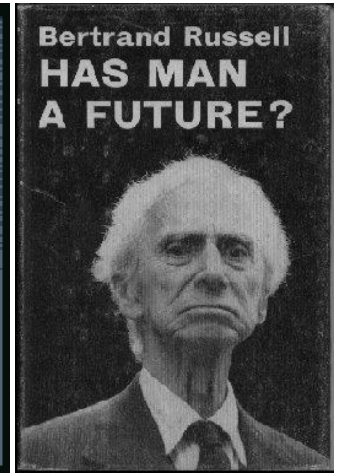
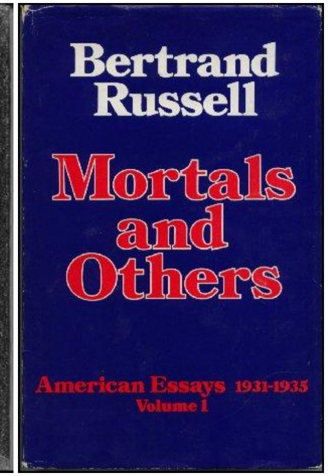
of
Bertrand Russell
COMPILED AND EDITED BY BARRY FEINBERG


Unarmed
Victory
Bertrand
Russell


Bertrand Russell
HAS MAN
A FUTURE?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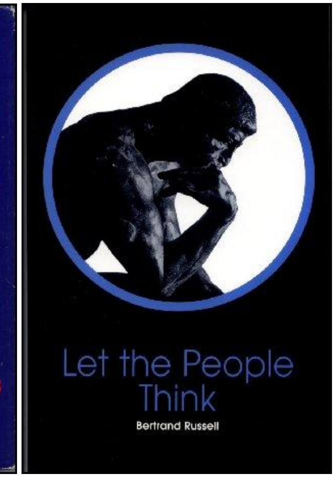
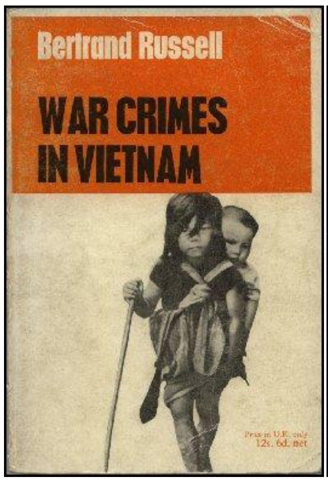
Russell
Mortals
and
Others
American Essays 1931-1935
Volume I


Let the People
Think
Bertrand Russell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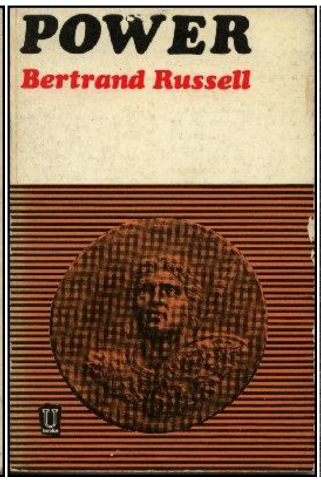
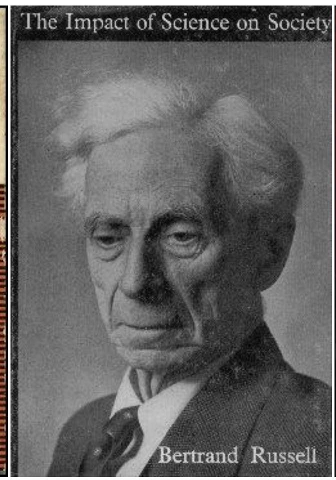

POWER
Bertrand Russell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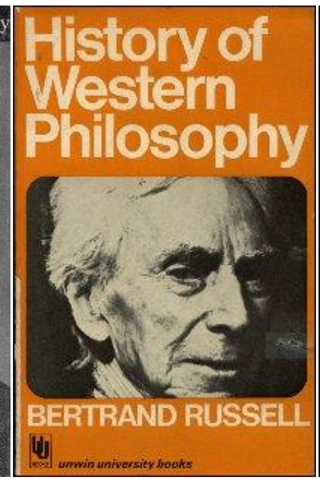
Bertrand Russell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BERTRAND RUSSELL
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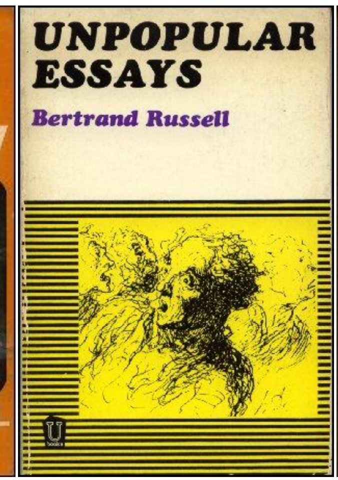
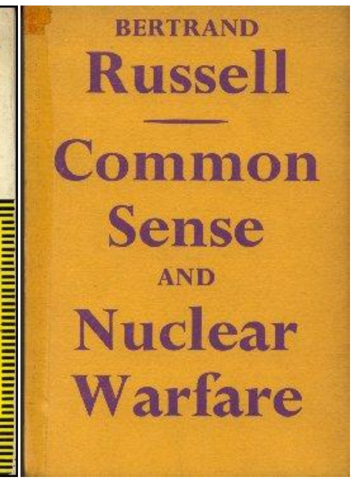

UNPOPULAR
ESSAYS
Bertrand Russell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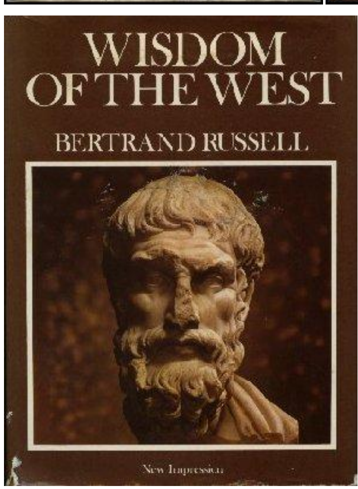
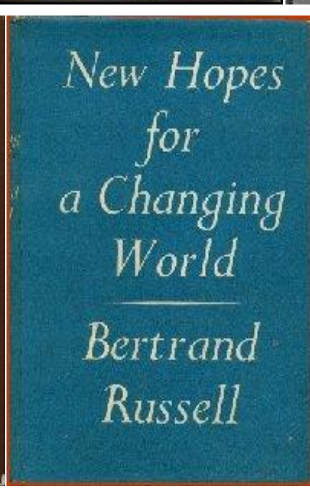
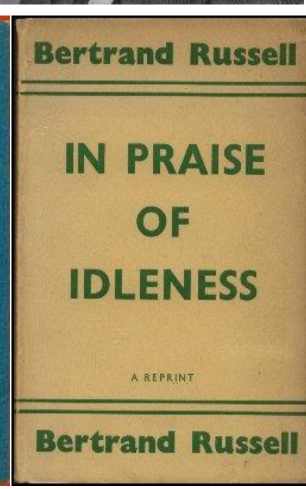
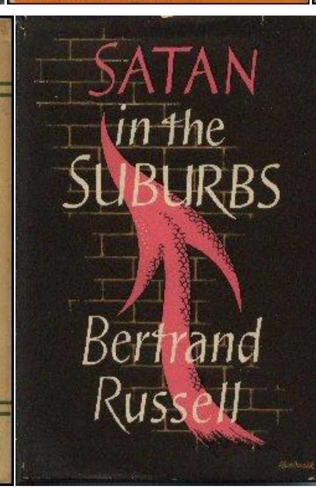
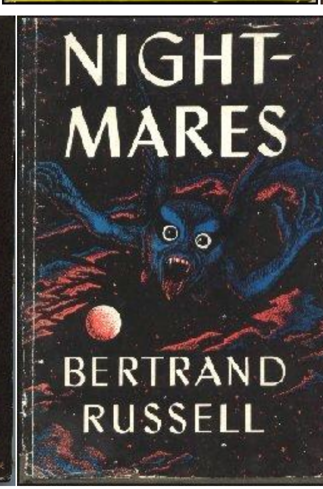
Russell
Common
Sense
AND
Nuclear
Warfare

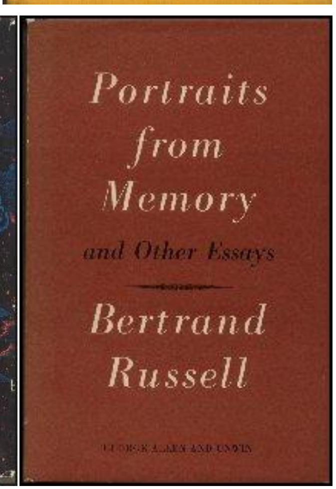

WISDOM
OF THE WEST
BERTRAND RUSSELL
New Impression


New Hopes
for
a Changing
World
Bertrand
Russell


Bertrand Russell
IN PRAISE
OF
IDLENESS
A REPRINT
Bertrand Russell


SATAN
in the
SUBURBS
Bertrand
Russell


NIGHT-
MARES
BERTRAND
RUSSELL


Portraits
from
Memory
and Other Essays
Bertrand
Russell

***54·42.** $\vdash :: \alpha \in 2 . \supset :. \beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv . \beta \in \iota``\alpha$

Dem.

$\vdash . *54\cdot4 . \quad \supset \vdash :: \alpha = \iota`x \cup \iota`y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \equiv : \beta = \Lambda . \vee . \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha : \exists ! \beta :$

[*24·53·56.*51·161] $\quad \equiv : \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha \quad (1)$

$\vdash . *54\cdot25 . \mathrm{Transp} . *52\cdot22 . \supset \vdash : x \neq y . \supset . \iota`x \cup \iota`y \neq \iota`x . \iota`x \cup \iota`y \neq \iota`y :$

[*13·12] $\quad \supset \vdash : \alpha = \iota`x \cup \iota`y . x \neq y . \supset . \alpha \neq \iota`x . \alpha \neq \iota`y \quad (2)$

$\vdash . (1) . (2) . \supset \vdash :: \alpha = \iota`x \cup \iota`y . x \neq y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv : \beta = \iota`x . \vee . \beta = \iota`y :$

[*51·235] $\quad \equiv : (\exists z) . z \in \alpha . \beta = \iota`z :$

[*37·6] $\quad \equiv : \beta \in \iota``\alpha \quad (3)$

$\vdash . (3) . *11\cdot11\cdot35 . *54\cdot101 . \supset \vdash . \mathrm{Prop}$

***54·43.** $\vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54\cdot26 . \supset \vdash :. \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

[*51·231] $\quad \equiv . \iota`x \cap \iota`y = \Lambda .$

[*13·12] $\quad \equiv . \alpha \cap \beta = \Lambda \quad (1)$

$\vdash . (1) . *11\cdot11\cdot35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \quad (2)$

$\vdash . (2) . *11\cdot54 . *52\cdot1 . \supset \vdash . \mathrm{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

***54·44.** $\vdash :. z, w \in \iota`x \cup \iota`y . \supset_{z,w} . \phi(z, w) : \equiv . \phi(x, x) . \phi(x, y) . \phi(y, x) . \phi(y, y)$

Dem.

$\vdash . *51\cdot234 . *11\cdot62 . \supset \vdash :. z, w \in \iota`x \cup \iota`y . \supset_{z,w} . \phi(z, w) : \equiv :$

$z \in \iota`x \cup \iota`y . \supset_z . \phi(z, x) . \phi(z, y) :$

[*51·234.*10·29] $\equiv : \phi(x, x) . \phi(x, y) . \phi(y, x) . \phi(y, y) :. \supset \vdash . \mathrm{Prop}$

***54·441.** $\vdash :: z, w \in \iota`x \cup \iota`y . z \neq w . \supset_{z,w} . \phi(z, w) : \equiv :. x = y : \vee : \phi(x, y) . \phi(y, x)$

Dem.

$\vdash . *5\cdot6 . \supset \vdash :: z, w \in \iota`x \cup \iota`y . z \neq w . \supset_{z,w} . \phi(z, w) : \equiv :.$

$z, w \in \iota`x \cup \iota`y . \supset_{z,w} : z = w . \vee . \phi(z, w) :.$

[*54·44] $\quad \equiv : x = x . \vee . \phi(x, x) : x = y . \vee . \phi(x, y) :$

$y = x . \vee . \phi(y, x) : y = y . \vee . \phi(y, y) :$

[*13·15 $\quad \equiv : x = y . \vee . \phi(x, y) : y = x . \vee . \phi(y, x) :$

[*13·16.*4·41] $\equiv : x = y . \vee . \phi(x, y) . \phi(y, x)$

This proposition is used in *163·42, in the theory of relations of mutually exclusive relations.

***110·632.** $\vdash : \mu \in \mathrm{NC} . \supset . \mu +_c 1 = \hat{\xi}\{(\exists y) . y \in \xi . \xi - \iota`y \in \mathrm{sm}``\mu\}$

Dem.

$\vdash . *110\cdot631 . *51\cdot211\cdot22 . \supset$

$\vdash : \mathrm{Hp} . \supset . \mu +_c 1 = \hat{\xi}\{(\exists \gamma, y) . \gamma \in \mathrm{sm}``\mu . y \in \xi . \gamma = \xi - \iota`y\}$

[*13·195] $\quad = \hat{\xi}\{(\exists y) . y \in \xi . \xi - \iota`y \in \mathrm{sm}``\mu\} : \supset \vdash . \mathrm{Prop}$

***110·64.** $\vdash . 0 +_c 0 = 0$ [*110·62]

***110·641.** $\vdash . 1 +_c 0 = 0 +_c 1 = 1$ [*110·51·61 . *101·2]

***110·642.** $\vdash . 2 +_c 0 = 0 +_c 2 = 2$ [*110·51·61 . *101·31]

***110·643.** $\vdash . 1 +_c 1 = 2$

Dem.

$\vdash . *110\cdot632 . *101\cdot21\cdot28 . \supset$

$\vdash . 1 +_c 1 = \hat{\xi}\{(\exists y) . y \in \xi . \xi - \iota`y \in 1\}$

[*54·3] $\quad = 2 . \supset \vdash . \mathrm{Prop}$

The above proposition is occasionally useful. It is used at least three times, in *113·66 and *120·123·472.

*110·7·71 are required for proving *110·72, and *110·72 is used in *117·3, which is a fundamental proposition in the theory of greater and less.

***110·7.** $\vdash : \beta \subset \alpha . \supset . (\exists \mu) . \mu \in \mathrm{NC} . \mathrm{Nc}`\alpha = \mathrm{Nc}`\beta +_c \mu$

Dem.

$\vdash . *24\cdot411\cdot21 . \supset \vdash : \mathrm{Hp} . \supset . \alpha = \beta \cup (\alpha - \beta) . \beta \cap (\alpha - \beta) = \Lambda .$

[*110·32] $\quad \supset . \mathrm{Nc}`\alpha = \mathrm{Nc}`\beta +_c \mathrm{Nc}`(\alpha - \beta) : \supset \vdash . \mathrm{Prop}$

***110·71.** $\vdash : (\exists \mu) . \mathrm{Nc}`\alpha = \mathrm{Nc}`\beta +_c \mu . \supset . (\exists \delta) . \delta \,\mathrm{sm}\, \beta . \delta \subset \alpha$

Dem.

$\vdash . *100\cdot3 . *110\cdot4 . \supset$

$\vdash : \mathrm{Nc}`\alpha = \mathrm{Nc}`\beta +_c \mu . \supset . \mu \in \mathrm{NC} - \iota`\Lambda \quad (1)$

$\vdash . *110\cdot3 . \supset \vdash : \mathrm{Nc}`\alpha = \mathrm{Nc}`\beta +_c \mathrm{Nc}`\gamma . \equiv . \mathrm{Nc}`\alpha = \mathrm{Nc}`(\beta + \gamma) .$

[*100·3·31] $\quad \supset . \alpha \,\mathrm{sm}\, (\beta + \gamma) .$

[*73·1] $\quad \supset . (\exists R) . R \in 1 \to 1 . \mathrm{D}`R = \alpha . \mathrm{Cl}`R = \downarrow \Lambda_\gamma``\iota``\beta \cup \Lambda_\beta \downarrow ``\iota``\gamma .$

[*37·15] $\quad \supset . (\exists R) . R \in 1 \to 1 . \downarrow \Lambda_\gamma``\iota``\beta \subset \mathrm{Cl}`R . R`` \downarrow \Lambda_\gamma``\iota``\beta \subset \alpha .$

[*110·12.*73·22] $\supset . (\exists \delta) . \delta \subset \alpha . \delta \,\mathrm{sm}\, \beta \quad (2)$

$\vdash . (1) . (2) . \supset \vdash . \mathrm{Prop}$

Metamath Proof Explorer

Theorem pm54.43 4699

Description: Theorem *54.43 of [WhiteheadRussell] p. 360. "From this proposition it will follow, when arithmetical addition has been defined, that 1+1=2." See http://en.wikipedia.org/wiki/Principia_Mathematica#Quotations. This theorem states that two sets of cardinality 1 are disjoint iff their union has cardinality 2.

Whitehead and Russell define 1 as the collection of all sets with cardinality 1 (i.e. all singletons; see card1 4965), so that their $A \in 1$ means, in our notation, $A \in \{x \mid (\mathrm{card}`x) = 1_o\}$ i.e. $(\mathrm{card}`A) = 1_o$ (by elab 1939) i.e. $A \approx 1_o$ (by carden 4963 and cardnn 4954). We do not have several of their earlier lemmas available (which would otherwise be unused by our different approach to arithmetic), so our proof is longer. (It is also longer because we must show every detail.)

Theorem pm110.643 5057 shows the derivation of 1+1=2 for cardinal numbers from this theorem.

Assertion

Ref	Expression
pm54.43	$\vdash ((A \approx 1_o \wedge B \approx 1_o) \to ((A \cap B) = \varnothing \leftrightarrow (A \cup B) \approx 2_o))$

Proof of Theorem pm54.43

Step	Hyp	Ref	Expression
1		1on 4262	........8 $\vdash 1_o \in \mathrm{On}$
2	1	onirri 3066	.......7 $\vdash \neg 1_o \in 1_o$
3		disjsn 2493	.......7 $\vdash ((1_o \cap \{1_o\}) = \varnothing \leftrightarrow \neg 1_o \in 1_o)$
4	2, 3	mpbir 188	......6 $\vdash (1_o \cap \{1_o\}) = \varnothing$
5		unen 4563	......6 $\vdash (((A \approx 1_o \wedge B \approx \{1_o\}) \wedge ((A \cap B) = \varnothing \wedge (1_o \cap \{1_o\}) = \varnothing)) \to (A \cup B) \approx (1_o \cup \{1_o\}))$
6	4, 5	mpanr2 713	.....5 $\vdash (((A \approx 1_o \wedge B \approx \{1_o\}) \wedge (A \cap B) = \varnothing) \to (A \cup B) \approx (1_o \cup \{1_o\}))$
7	6	ex 371	....4 $\vdash ((A \approx 1_o \wedge B \approx \{1_o\}) \to ((A \cap B) = \varnothing \to (A \cup B) \approx (1_o \cup \{1_o\})))$
8	1	elisseti 1860	......6 $\vdash 1_o \in V$
9	8	ensn1 4553	......6 $\vdash \{1_o\} \approx 1_o$
10	8, 9	ensymi 4542	.....5 $\vdash 1_o \approx \{1_o\}$
11		entr 4543	.....5 $\vdash ((B \approx 1_o \wedge 1_o \approx \{1_o\}) \to B \approx \{1_o\})$
12	10, 11	mpan2 699	....4 $\vdash (B \approx 1_o \to B \approx \{1_o\})$
13	7, 12	sylan2 453	...3 $\vdash ((A \approx 1_o \wedge B \approx 1_o) \to ((A \cap B) = \varnothing \to (A \cup B) \approx (1_o \cup \{1_o\})))$
14		df-2o 4258	.....5 $\vdash 2_o = \mathrm{suc}\ 1_o$
15		df-suc 2971	.....5 $\vdash \mathrm{suc}\ 1_o = (1_o \cup \{1_o\})$
16	14, 15	eqtri 1534	....4 $\vdash 2_o = (1_o \cup \{1_o\})$
17	16	breq2i 2691	...3 $\vdash ((A \cup B) \approx 2_o \leftrightarrow (A \cup B) \approx (1_o \cup \{1_o\}))$
18	13, 17	syl6ibr 211	..2 $\vdash ((A \approx 1_o \wedge B \approx 1_o) \to ((A \cap B) = \varnothing \to (A \cup B) \approx 2_o))$

19		sneq 2466	..............15 $\vdash (x = y \to \{x\} = \{y\})$
20	19	uneq2d 2232	.............14 $\vdash (x = y \to (\{x\} \cup \{x\}) = (\{x\} \cup \{y\}))$
21		unidm 2223	.............14 $\vdash (\{x\} \cup \{x\}) = \{x\}$
22	20, 21	syl5reqr 1561	............13 $\vdash (x = y \to (\{x\} \cup \{y\}) = \{x\})$
23		visset 1855	..............15 $\vdash x \in V$
24	23	ensn1 4553	.............14 $\vdash \{x\} \approx 1_o$
25		1sdom2 4656	.............14 $\vdash 1_o \prec 2_o$
26		ensdomtr 4600	.............14 $\vdash ((\{x\} \approx 1_o \land 1_o \prec 2_o) \to \{x\} \prec 2_o)$
27	24, 25, 26	mp2an 700	............13 $\vdash \{x\} \prec 2_o$
28	22, 27	syl6eqbr 2716	...........12 $\vdash (x = y \to (\{x\} \cup \{y\}) \prec 2_o)$
29		sdomnen 4516	...........12 $\vdash ((\{x\} \cup \{y\}) \prec 2_o \to \neg (\{x\} \cup \{y\}) \approx 2_o)$
30	28, 29	syl 10	..........11 $\vdash (x = y \to \neg (\{x\} \cup \{y\}) \approx 2_o)$
31	30	necon2ai 1650	.........10 $\vdash ((\{x\} \cup \{y\}) \approx 2_o \to x \neq y)$
32		disjsn2 2494	.........10 $\vdash (x \neq y \to (\{x\} \cap \{y\}) = \varnothing)$
33	31, 32	syl 10	........9 $\vdash ((\{x\} \cup \{y\}) \approx 2_o \to (\{x\} \cap \{y\}) = \varnothing)$
34	33	a1i 8	.......8 $\vdash ((A = \{x\} \land B = \{y\}) \to ((\{x\} \cup \{y\}) \approx 2_o \to (\{x\} \cap \{y\}) = \varnothing))$
35		uneq12 2227	........9 $\vdash ((A = \{x\} \land B = \{y\}) \to (A \cup B) = (\{x\} \cup \{y\}))$
36	35	breq1d 2693	.......8 $\vdash ((A = \{x\} \land B = \{y\}) \to ((A \cup B) \approx 2_o \leftrightarrow (\{x\} \cup \{y\}) \approx 2_o))$
37		ineq12 2260	........9 $\vdash ((A = \{x\} \land B = \{y\}) \to (A \cap B) = (\{x\} \cap \{y\}))$
38	37	eqeq1d 1522	.......8 $\vdash ((A = \{x\} \land B = \{y\}) \to ((A \cap B) = \varnothing \leftrightarrow (\{x\} \cap \{y\}) = \varnothing))$
39	34, 36, 38	3imtr4d 545	......7 $\vdash ((A = \{x\} \land B = \{y\}) \to ((A \cup B) \approx 2_o \to (A \cap B) = \varnothing))$
40	39	ex 371	.....6 $\vdash (A = \{x\} \to (B = \{y\} \to ((A \cup B) \approx 2_o \to (A \cap B) = \varnothing)))$
41	40	19.23adv 1248	....5 $\vdash (A = \{x\} \to (\exists y\, B = \{y\} \to ((A \cup B) \approx 2_o \to (A \cap B) = \varnothing)))$
42	41	19.23aiv 1330	...4 $\vdash (\exists x\, A = \{x\} \to (\exists y\, B = \{y\} \to ((A \cup B) \approx 2_o \to (A \cap B) = \varnothing)))$
43	42	imp 348	..3 $\vdash ((\exists x\, A = \{x\} \land \exists y\, B = \{y\}) \to ((A \cup B) \approx 2_o \to (A \cap B) = \varnothing))$
44		en1 4555	..3 $\vdash (A \approx 1_o \leftrightarrow \exists x\, A = \{x\})$
45		en1 4555	..3 $\vdash (B \approx 1_o \leftrightarrow \exists y\, B = \{y\})$
46	43, 44, 45	syl2anb 457	.2 $\vdash ((A \approx 1_o \land B \approx 1_o) \to ((A \cup B) \approx 2_o \to (A \cap B) = \varnothing))$
47	18, 46	impbid 518	1 $\vdash ((A \approx 1_o \land B \approx 1_o) \to ((A \cap B) = \varnothing \leftrightarrow (A \cup B) \approx 2_o))$

Colors of variables: wff set class

Syntax hints: $\neg$ wn 2 $\to$ wi 3 $\leftrightarrow$ wb 144 $\land$ wa 221 $=$ wceq 989 $\in$ wcel 991 $\exists$wex 1013 $\neq$ wne 1624 $\cup$ cun 2093 $\cap$ cin 2094 $\varnothing$c0 2328 {csn 2458 *class class class* wbr 2683 On con0 2965 suc csuc 2967 1_o c1o 4252 2_o c2o 4253 $\approx$ cen 4493 $\prec$ csdm 4495

This theorem is referenced by: pm110.643 5057 unpde2eg2 10809

This theorem was proved from axioms: ax-1 4 ax-2 5 ax-3 6 ax-mp 7 ax-7 995 ax-gen 996 ax-8 997 ax-9 998 ax-10 999 ax-11 1000 ax-12 1001 ax-13 1002 ax-14 1003 ax-17 1004 ax-4 1006 ax-5o 1008 ax-6o 1011 ax-9o 1156 ax-10o 1174 ax-16 1244 ax-11o 1252 ax-ext 1496 ax-rep 2759 ax-sep 2769 ax-nul 2776 ax-pow 2809 ax-pr 2844 ax-un 3079

Metamath Proof Explorer

Theorem pm110.643 5057

Description: 1+1=2 for cardinal number addition. Theorem *110.643 of *Principia Mathematica*, vol. II, p. 86, which adds the remark, "The above proposition is occasionally useful." Unlike us, Whitehead and Russell define cardinal addition on collections of all sets equinumerous to 1 and 2 (which for us are proper classes unless we restrict them as in karden 4856), but after applying definitions, our theorem is equivalent. See also the comment for pm54.43 4699. The comment for cdavali 5054 explains why we use $\approx$ instead of =.

Assertion

Ref	Expression
pm110.643	$\vdash (1_o +_c 1_o) \approx 2_o$

Proof of Theorem pm110.643

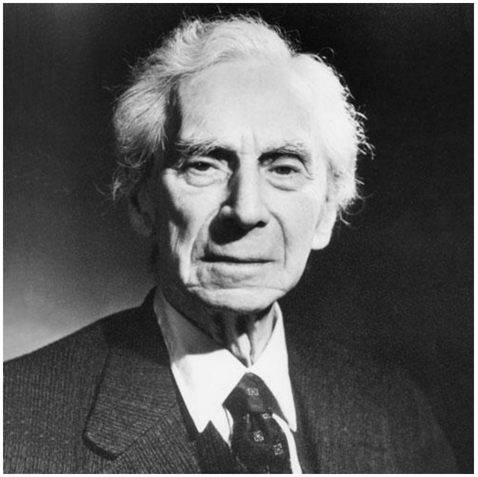
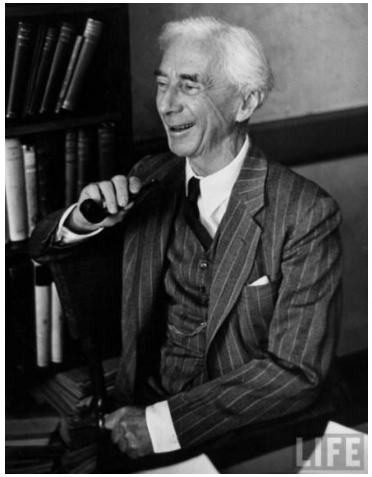
Step	Hyp	Ref	Expression
1		1on 4262	...4 $\vdash 1_o \in \mathrm{On}$
2	1	elisseti 1860	..3 $\vdash 1_o \in V$
3	2, 2	cdavali 5054	.2 $\vdash (1_o +_c 1_o) = ((1_o \times \{\varnothing\}) \cup (1_o \times \{1_o\}))$
4		xp01disj 4267	..3 $\vdash ((1_o \times \{\varnothing\}) \cap (1_o \times \{1_o\})) = \varnothing$
5		0ex 2777	....5 $\vdash \varnothing \in V$
6	2, 5	xpsnen 4564	...4 $\vdash (1_o \times \{\varnothing\}) \approx 1_o$
7	2, 2	xpsnen 4564	...4 $\vdash (1_o \times \{1_o\}) \approx 1_o$
8		pm54.43 4699	...4 $\vdash (((1_o \times \{\varnothing\}) \approx 1_o \wedge (1_o \times \{1_o\}) \approx 1_o) \rightarrow (((1_o \times \{\varnothing\}) \cap (1_o \times \{1_o\})) = \varnothing \leftrightarrow ((1_o \times \{\varnothing\}) \cup (1_o \times \{1_o\})) \approx 2_o))$
9	6, 7, 8	mp2an 700	..3 $\vdash (((1_o \times \{\varnothing\}) \cap (1_o \times \{1_o\})) = \varnothing \leftrightarrow ((1_o \times \{\varnothing\}) \cup (1_o \times \{1_o\})) \approx 2_o)$
10	4, 9	mpbi 187	.2 $\vdash ((1_o \times \{\varnothing\}) \cup (1_o \times \{1_o\})) \approx 2_o$
11	3, 10	eqbrtri 2698	1 $\boxed{\vdash (1_o +_c 1_o) \approx 2_o}$

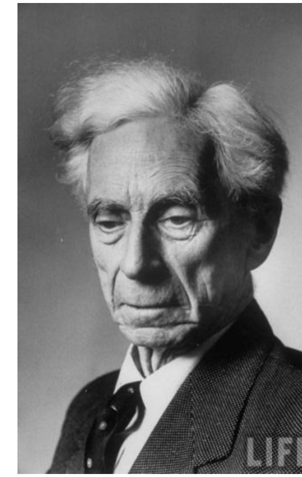
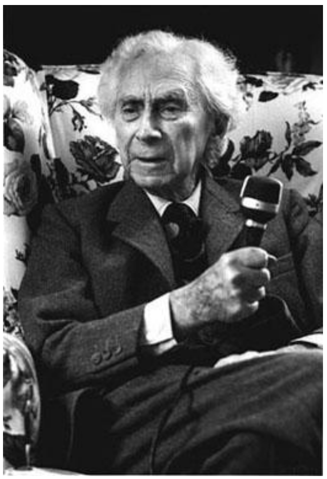
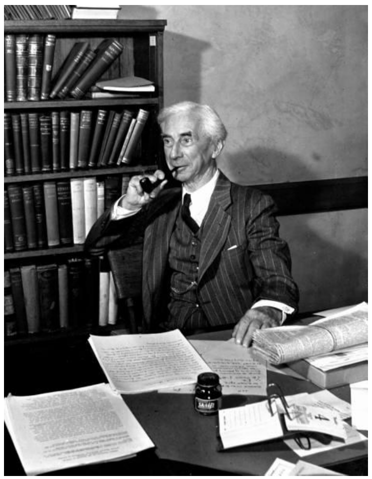
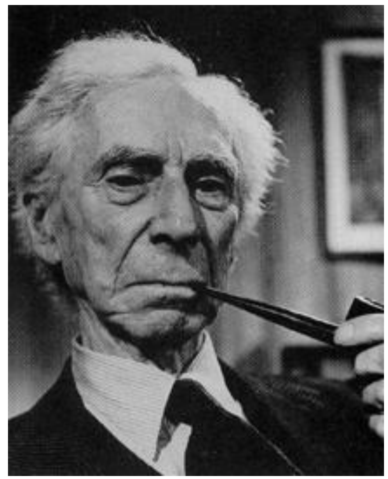
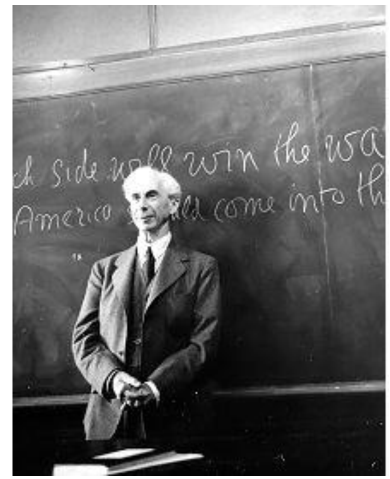
Colors of variables: wff set class

Syntax hints: $\leftrightarrow$ wb 144 $=$ wceq 989 $\cup$ cun 2093 $\cap$ cin 2094 $\varnothing$c0 2328 $\{$csn 2458 *class class class* wbr 2683 On con0 2965 $\times$ cxp 3239 (*class class class*)co 4009 1_oc1o 4252 2_oc2o 4253 $\approx$ cen 4493 $+_c$ ccda 5051

This theorem was proved from axioms: ax-1 4 ax-2 5 ax-3 6 ax-mp 7 ax-7 995 ax-gen 996 ax-8 997 ax-9 998 ax-10 999 ax-11 1000 ax-12 1001 ax-13 1002 ax-14 1003 ax-17 1004 ax-4 1006 ax-5o 1008 ax-6o 1011 ax-9o 1156 ax-10o 1174 ax-16 1244 ax-11o 1252 ax-ext 1496 ax-rep 2759 ax-sep 2769 ax-nul 2776 ax-pow 2809 ax-pr 2844 ax-un 3079

This theorem depends on definitions: df-bi 145 df-or 222 df-an 223 df-3or 779 df-3an 780 df-ex 1014 df-sb 1206 df-eu 1417 df-mo 1418 df-clab 1502 df-cleq 1507 df-clel 1510 df-ne 1626 df-ral 1691 df-rex 1692 df-reu 1693 df-rab 1694 df-v 1854 df-sbc 1983 df-csb 2048 df-dif 2097 df-un 2098 df-in 2099 df-ss 2101 df-nul 2329 df-pw 2451 df-sn 2461 df-pr 2462 df-tp 2464 df-op 2465 df-uni 2561 df-int 2592 df-br 2684 df-opab 2732 df-tr 2746 df-eprel 2899 df-id 2902 df-po 2907 df-so 2919 df-fr 2937 df-we 2952 df-ord 2968 df-on 2969 df-suc 2971 df-xp 3255 df-rel 3256 df-cnv 3257 df-co 3258 df-dm 3259 df-rn 3260 df-res 3261 df-ima 3262 df-fun 3263 df-fn 3264 df-f 3265 df-f1 3266 df-fo 3267 df-f1o 3268 df-fv 3269 df-opr 4011 df-oprab 4012 df-1o 4257 df-2o 4258 df-er 4389 df-en 4497 df-dom 4498 df-sdom 4499 df-cda 5052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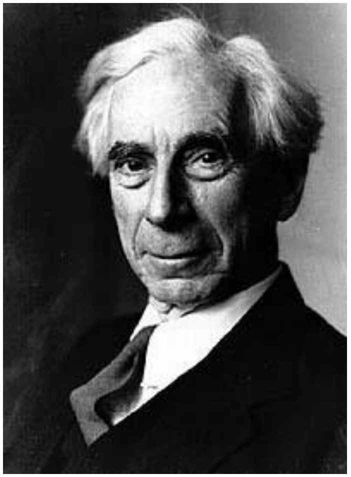
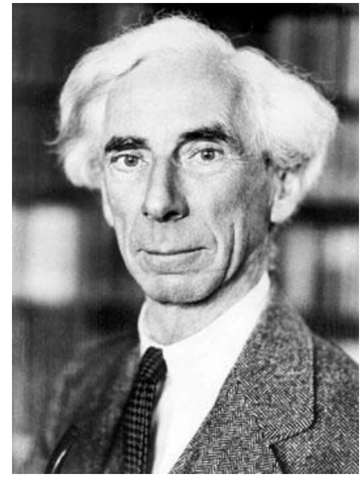
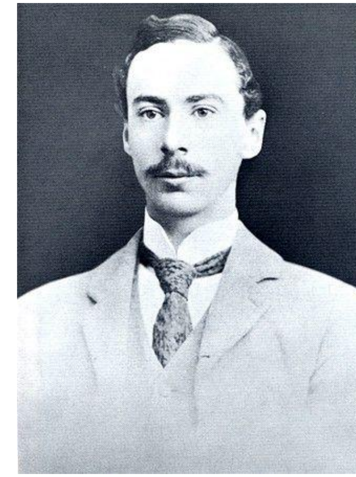
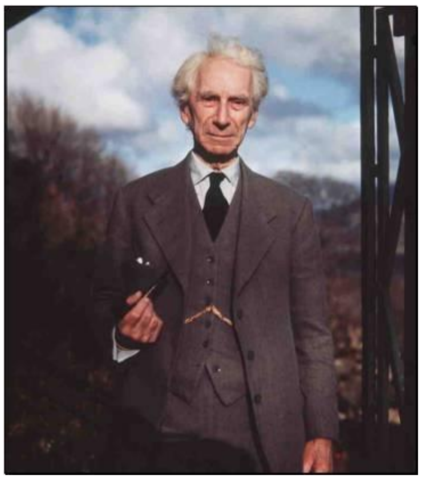
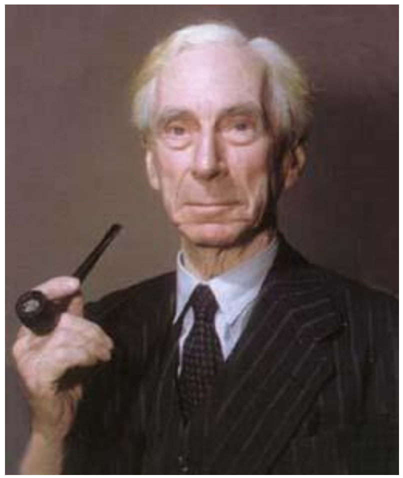
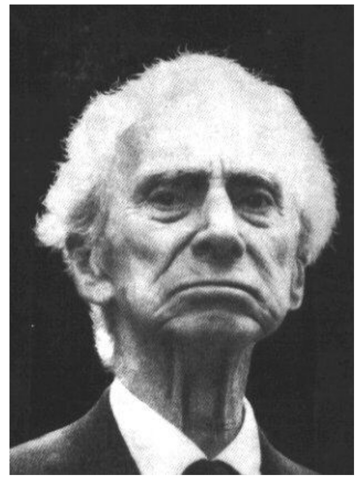
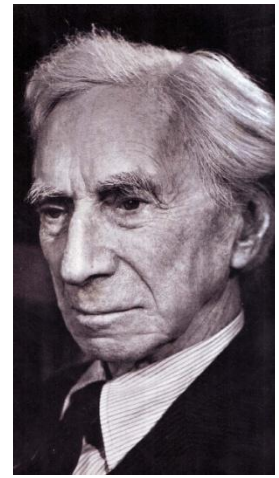
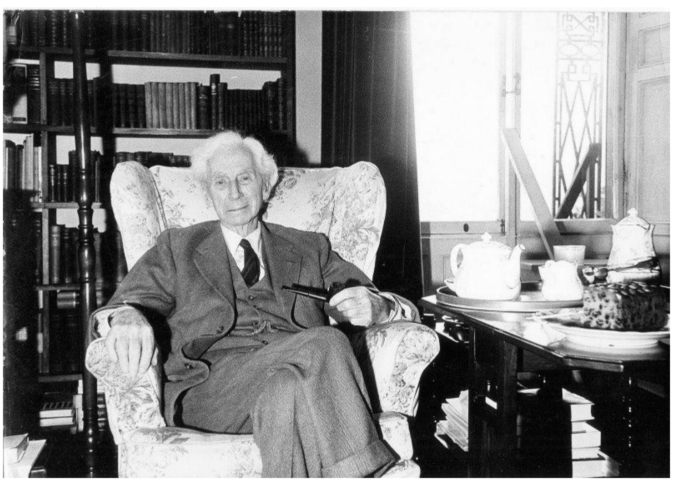
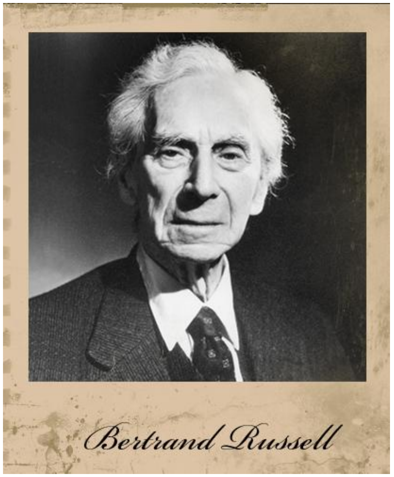
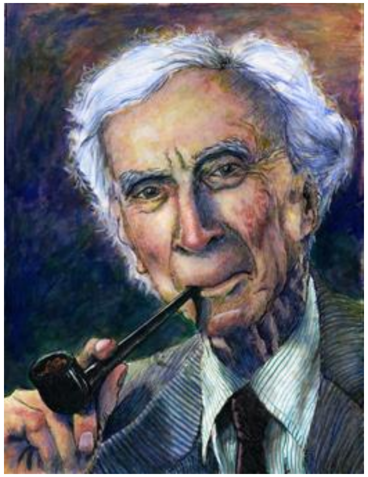


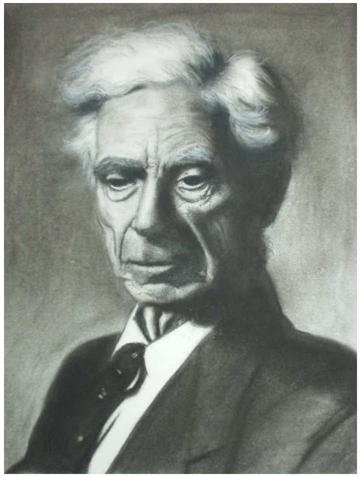









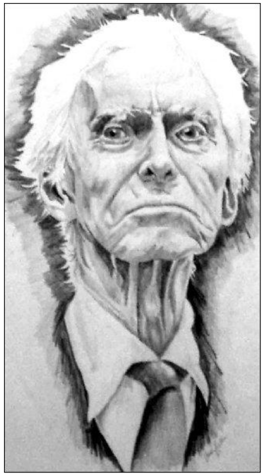

COMMITEE of 100
ACTION FOR LIFE
DEFENCE MINISTRY
18 FEB. 1961

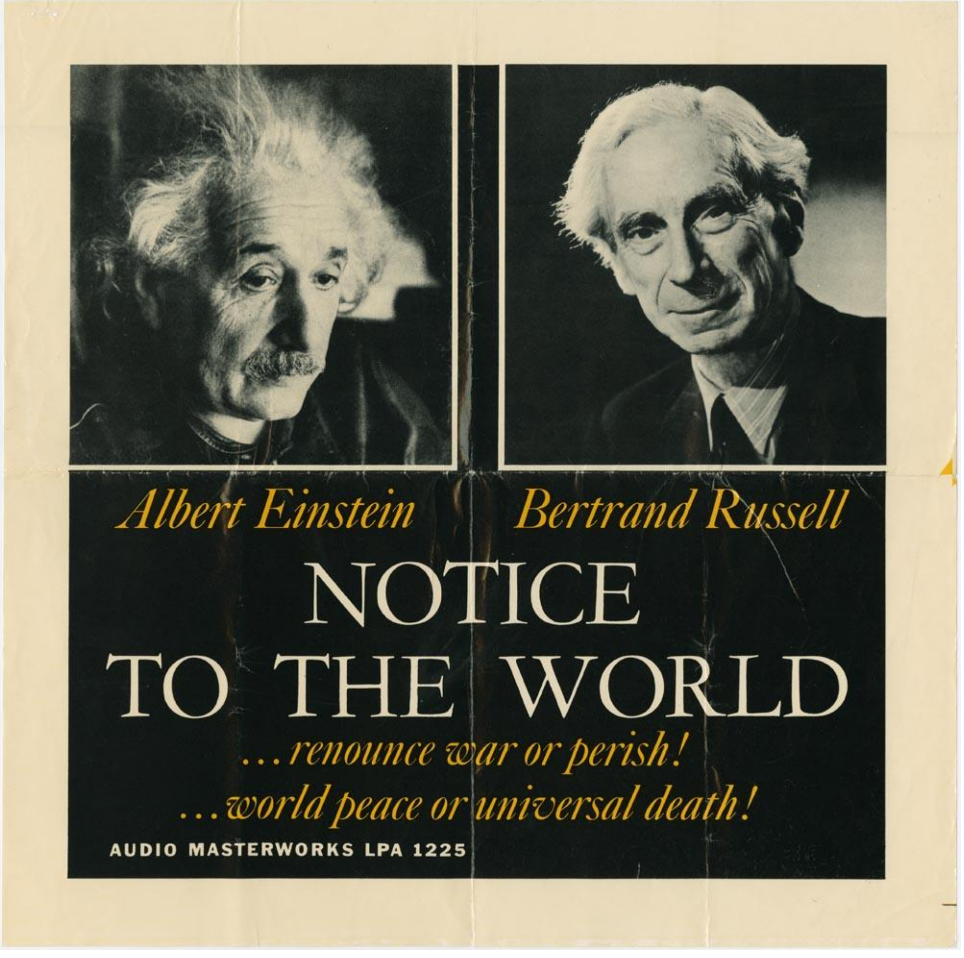

Bertrand Russell

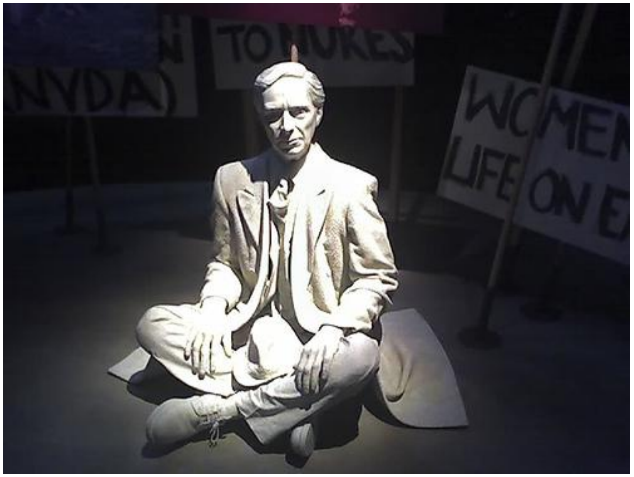


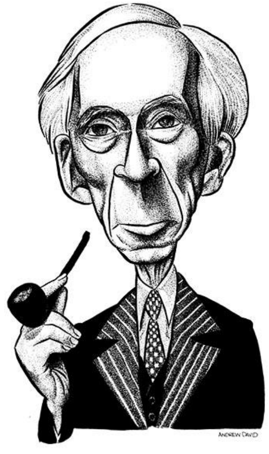


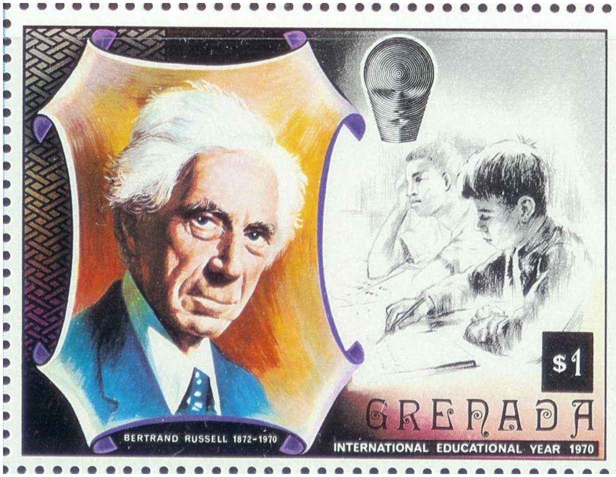

War does not decide
who is right-
war decides
who is left.

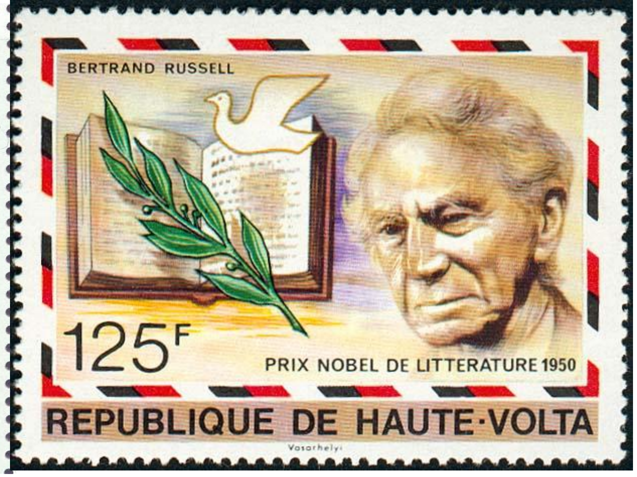


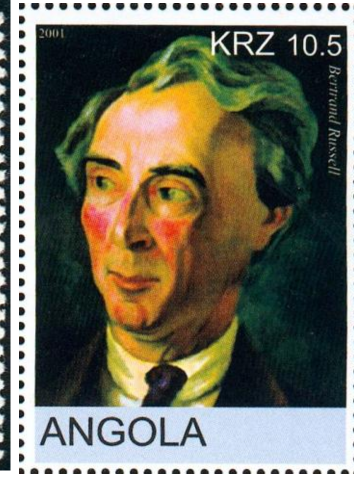

Albert Einstein
Bertrand Russell
NOTICE
TO THE WORLD
...renounce war or perish!
...world peace or universal death!
AUDIO MASTERWORKS LPA 1225

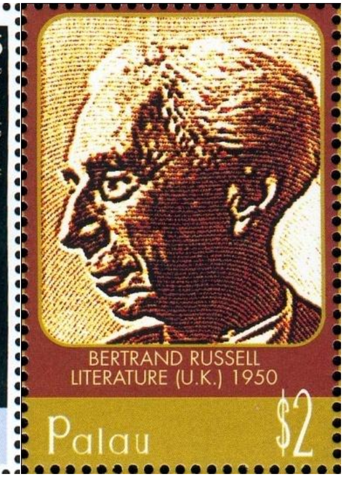


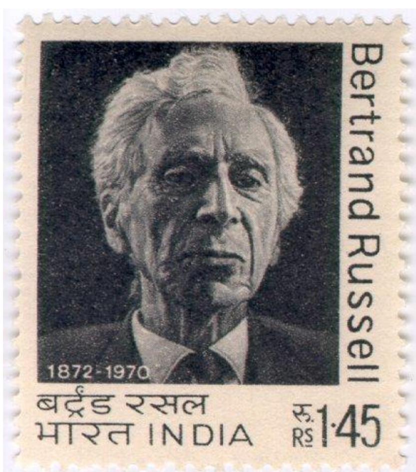


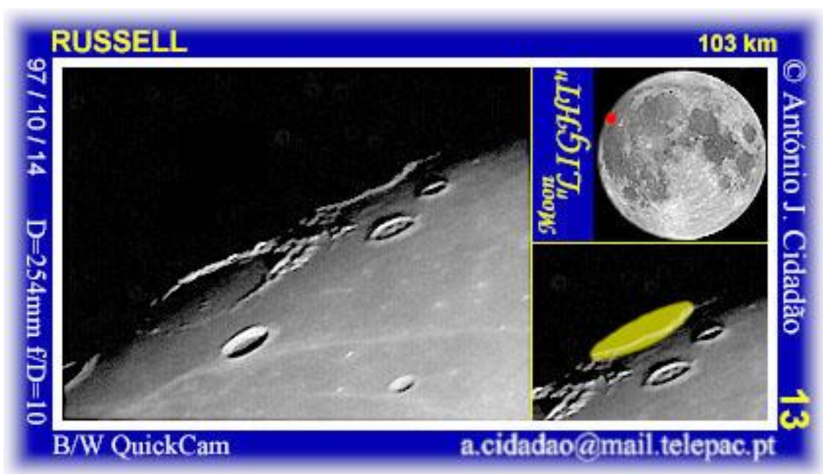


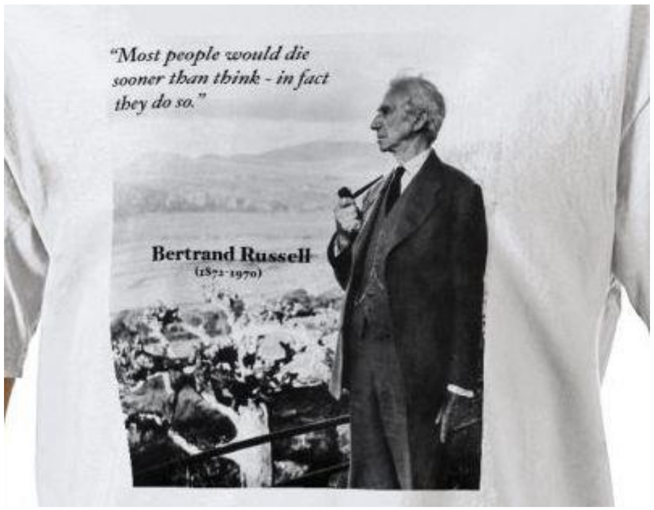


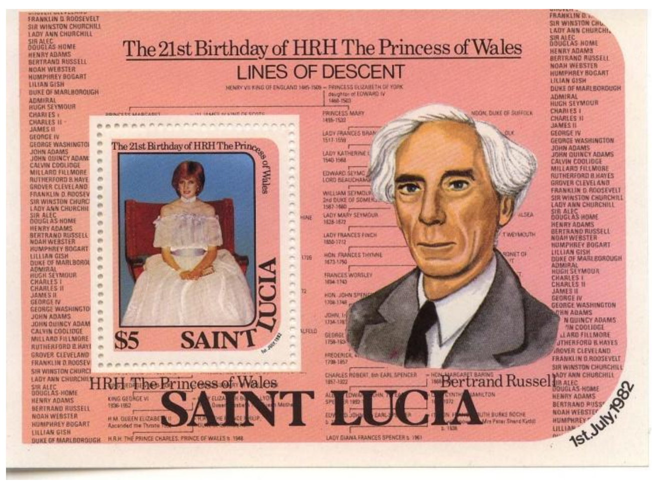


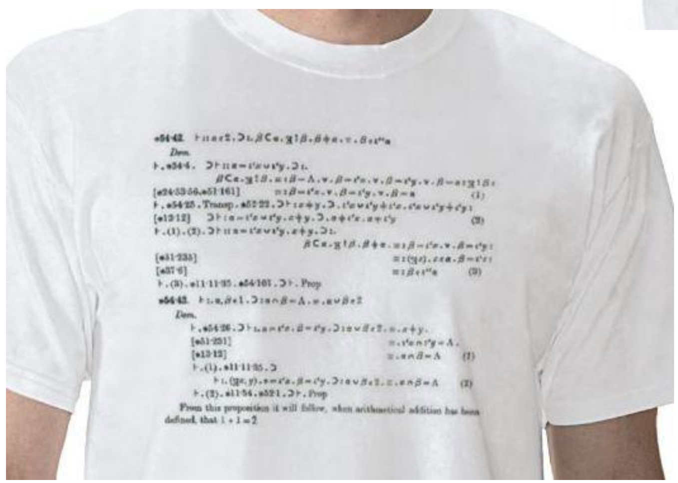








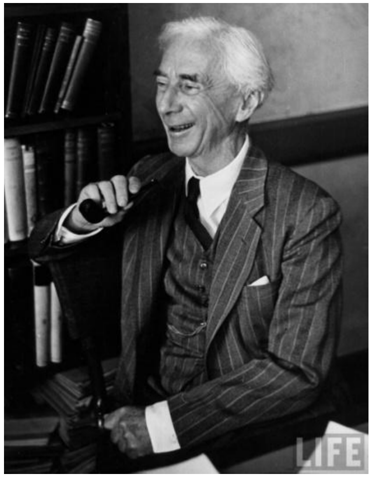




"Most people would sooner die than think; in fact, they do so."

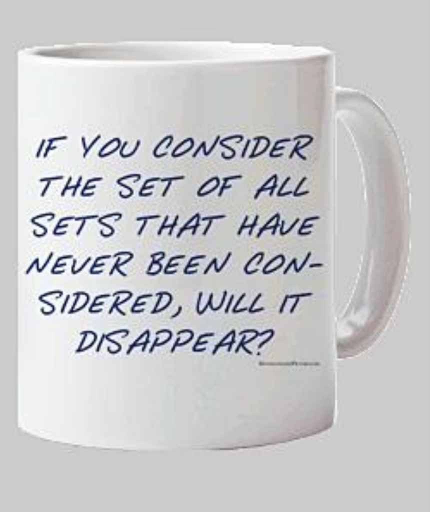
- Bertrand Russell (1872-1970)

Russell's paradox was invented by Russell in 1901 to show that naïve set theory is self-contradictory:



Define: set of all sets that do not contain themselves

$$S = \{ T \mid T \notin T \}$$

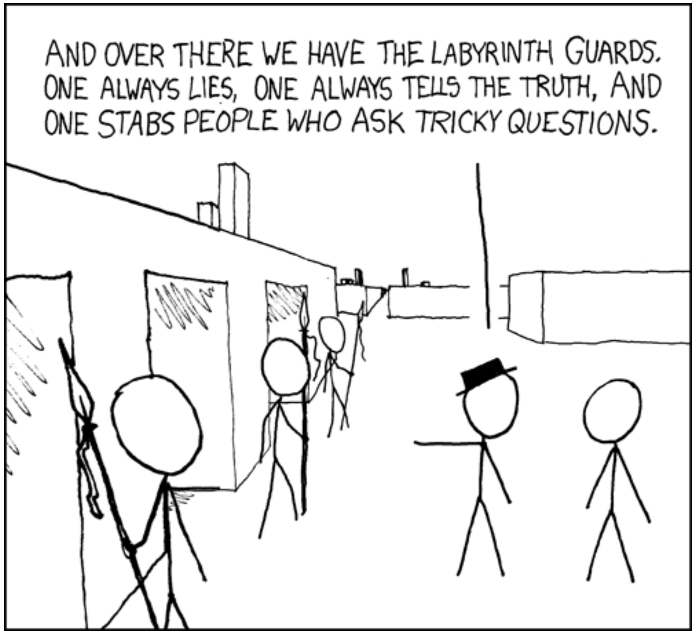


Q: does S contain itself as an element?

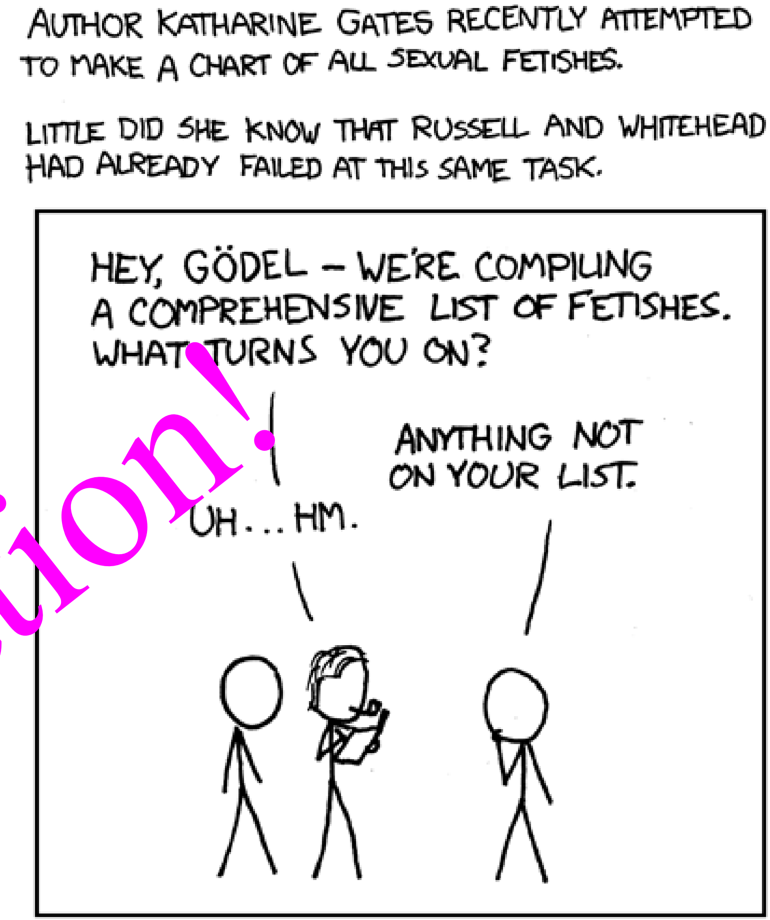
$$S \notin S \Leftrightarrow S \in S \quad \text{contradiction!}$$

Similar paradoxes:

- "A barber who shaves exactly those who do not shave themselves."
- "This sentence is false."
- "I am lying."
- "Is the answer to this question 'no'?"
- "The smallest positive integer not describable in twenty words or less."



Star Trek, 1967, “I, Mudd” episode
Captain James Kirk and Harry Mudd use a logical paradox to cause hostile android “Norman” to crash



Diagonalization!

So you created everything.
Yes.
Including black holes.
Yes.
Which will eventually swallow up everything.
Yes.
Including you.
I'm working on that.
S. Harris