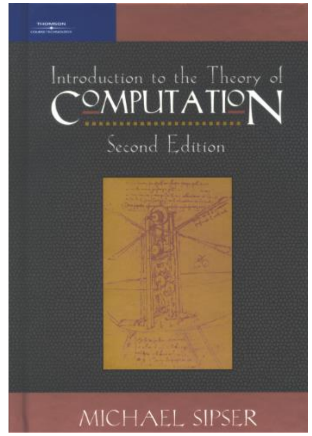
3

THE CHURCH–TURING THESIS



So far in our development of the theory of computation we have presented several models of computing devices. Finite automata are good models for devices that have a small amount of memory. Pushdown automata are good models for devices that have an unlimited memory that is usable only in the last in, first out manner of a stack. We have shown that some very simple tasks are beyond the capabilities of these models. Hence they are too restricted to serve as models of general purpose computers.

3.1 TURING MACHINES

We turn now to a much more powerful model, first proposed by Alan Turing in 1936, called the ***Turing machine***. Similar to a finite automaton but with an unlimited and unrestricted memory, a Turing machine is a much more accurate model of a general purpose computer. A Turing machine can do everything that a real computer can do. Nonetheless, even a Turing machine cannot solve certain problems. In a very real sense, these problems are beyond the theoretical limits of computation.

The Turing machine model uses an infinite tape as its unlimited memory. It has a tape head that can read and write symbols and move around on the tape.

constant called a ***coefficient***. For example,

$$6 \cdot x \cdot x \cdot x \cdot y \cdot z \cdot z = 6x^3yz^2$$

is a term with coefficient 6, and

$$6x^3yz^2 + 3xy^2 - x^3 - 10$$

is a polynomial with four terms over the variables x, y, and z. For this discussion, we consider only coefficients that are integers. A ***root*** of a polynomial is an assignment of values to its variables so that the value of the polynomial is 0. This polynomial has a root at $x = 5$, $y = 3$, and $z = 0$. This root is an ***integral root*** because all the variables are assigned integer values. Some polynomials have an integral root and some do not.

Hilbert's tenth problem was to devise an algorithm that tests whether a polynomial has an integral root. He did not use the term *algorithm* but rather "a process according to which it can be determined by a finite number of operations."[4] Interestingly, in the way he phrased this problem, Hilbert explicitly asked that an algorithm be "devised." Thus he apparently assumed that such an algorithm must exist—someone need only find it.

As we now know, no algorithm exists for this task; it is algorithmically unsolvable. For mathematicians of that period to come to this conclusion with their intuitive concept of algorithm would have been virtually impossible. The intuitive concept may have been adequate for giving algorithms for certain tasks, but it was useless for showing that no algorithm exists for a particular task. Proving that an algorithm does not exist requires having a clear definition of algorithm. Progress on the tenth problem had to wait for that definition.

The definition came in the 1936 papers of Alonzo Church and Alan Turing. Church used a notational system called the λ-calculus to define algorithms. Turing did it with his "machines." These two definitions were shown to be equivalent. This connection between the informal notion of algorithm and the precise definition has come to be called the ***Church–Turing thesis***.

The Church–Turing thesis provides the definition of algorithm necessary to resolve Hilbert's tenth problem. In 1970, Yuri Matijasevič, building on work of Martin Davis, Hilary Putnam, and Julia Robinson, showed that no algorithm exists for testing whether a polynomial has integral roots. In Chapter 4 we develop the techniques that form the basis for proving that this and other problems are algorithmically unsolvable.

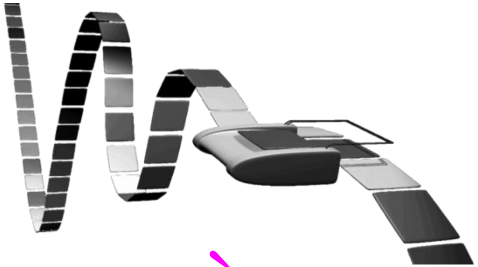
Intuitive notion of algorithms	equals	*Turing machine algorithms*

FIGURE 3.22
The Church–Turing Thesis

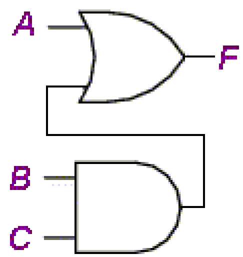
[4]Translated from the original German.

Computational Universality

Theorem: Many other systems are equivalent to Turing machines.

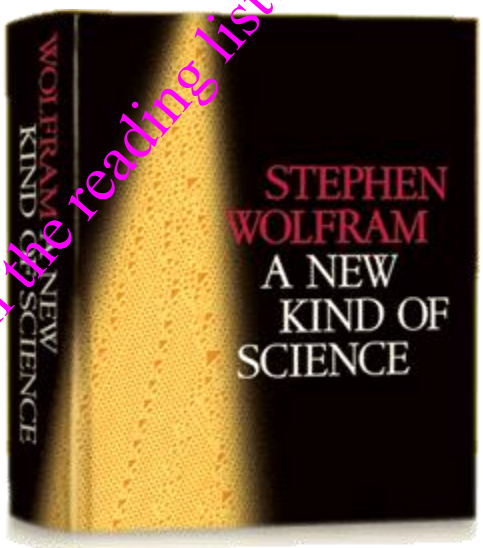
$\equiv$ 

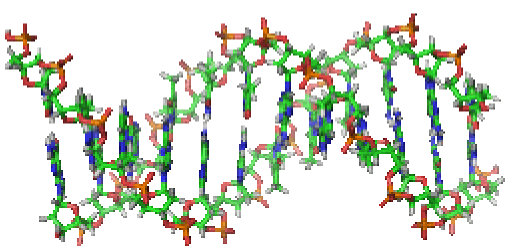
- Grammars $cS \rightarrow aNbc \mid S$
- λ-calculus $(\lambda X . X + 1)$
- Post tag systems $A \rightarrow bc$
- μ-recursive functions $\mu(f)(x,y) = z$
- Cellular automata
- Boolean circuits
- Diophantine equations
- DNA
- Billiards!

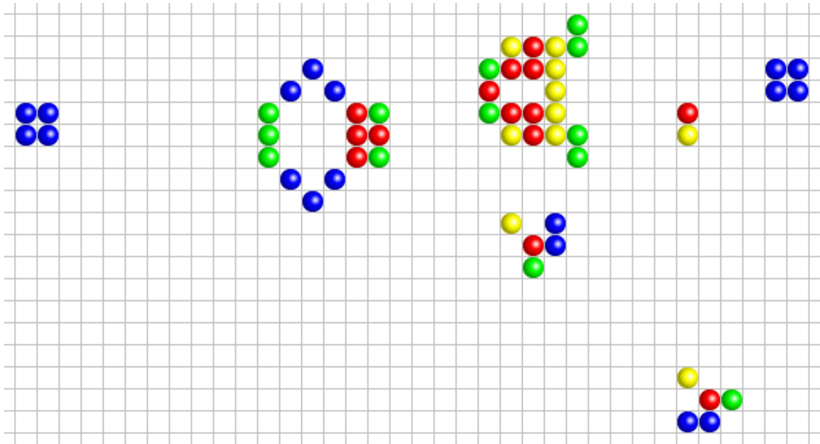


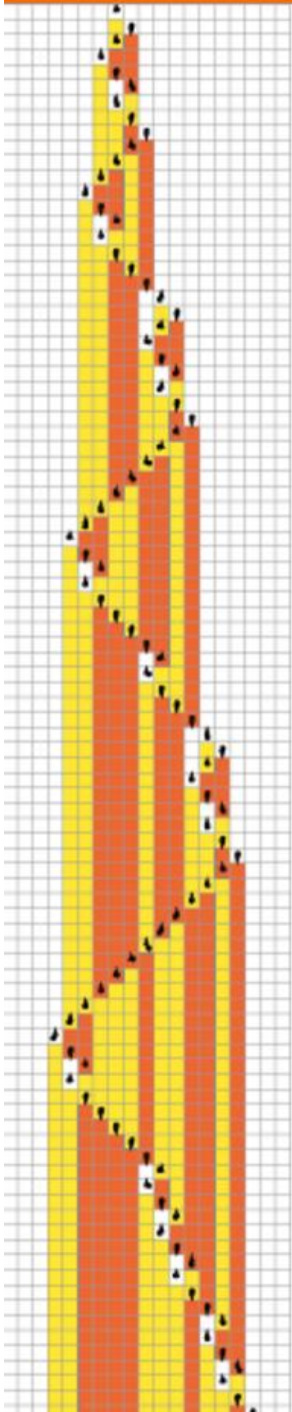
$x^3 + y^3 + z^3 = 33$

On the reading list!









Announced May 14th, 2007: 5th Anniversary of the Publication of *A New Kind of Science*

THE WOLFRAM 2,3 TURING MACHINE RESEARCH PRIZE

Oct 24, 2007

We have the solution!

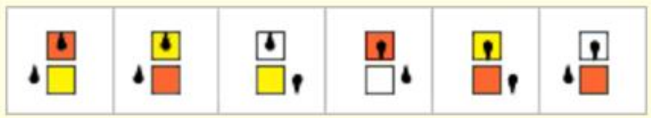
Wolfram's 2,3 Turing machine is universal

Congratulations Alex Smith.

Find out more »

$25,000 prize

Is this Turing machine universal, or not?



The machine has 2 states and 3 colors, and is 596440 in Wolfram's numbering scheme.
If it is universal then it is the smallest universal Turing machine that exists.

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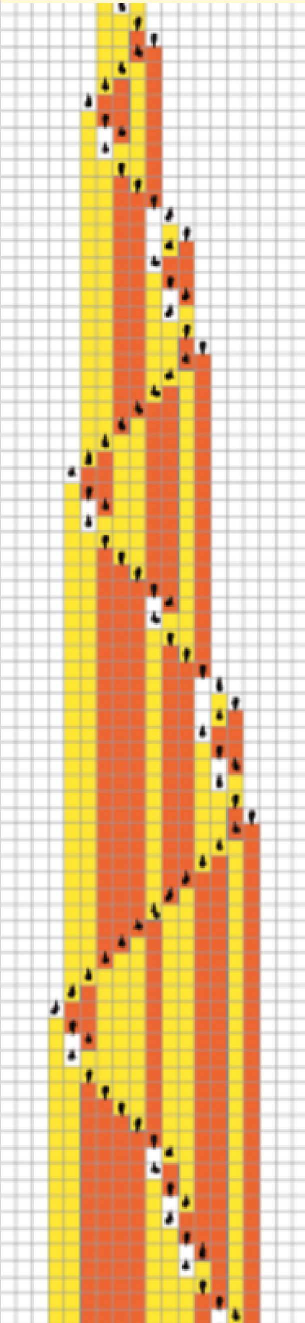
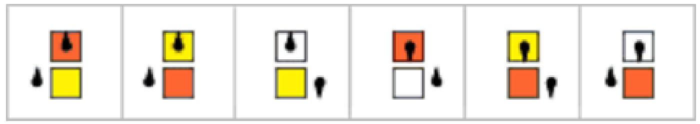
A universal Turing machine is powerful enough to emulate any standard computer.
The question is: how simple can the rules for a universal Turing machine be?

Since the 1960s it has been known that there is a universal 7,4 machine. In A New Kind of Science, Stephen Wolfram found a universal 2,5 machine, and suggested that the particular 2,3 machine that is the subject of this prize might be universal.

The prize is for determining whether or not the 2,3 machine is in fact universal.



Wolfram's 2,3 Turing machine is universal!



The lower limit on Turing machine universality is proved—

providing new evidence for Wolfram's Principle of Computational Equivalence.

The Wolfram 2,3 Turing Machine Research Prize has been won by 20-year-old Alex Smith of Birmingham, UK.

Smith's Proof (to be published in Complex Systems):

Prize Submission » *Mathematica* Programs »

News Release » Technical Commentary »

Stephen Wolfram's Blog Post »

Media Enquiries »

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What is a Turing Machine? » | Notable Universal Turing Machines »

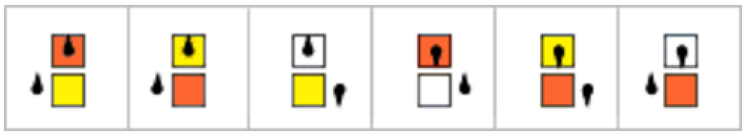
The Rules for the Machine

The rules for the Turing machine that is the subject of this prize are:

{{1, 2} -> {1, 1, -1}, {1, 1} -> {1, 2, -1}, {1, 0} -> {2, 1, 1},
{2, 2} -> {1, 0, 1}, {2, 1} -> {2, 2, 1}, {2, 0} -> {1, 2, -1}}

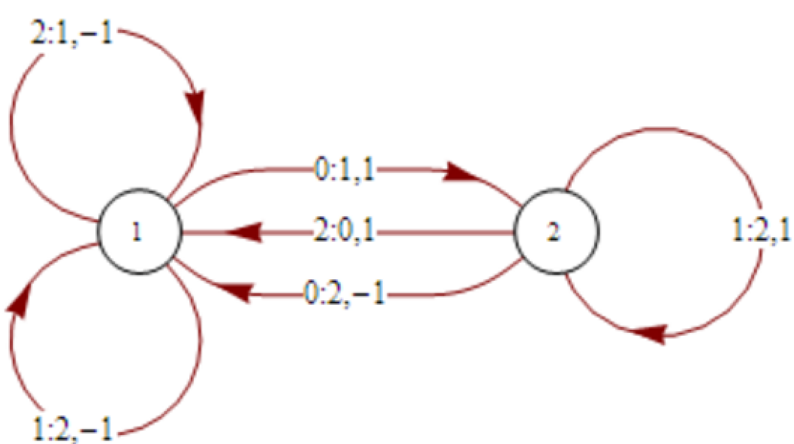
where this means {state, color} -> {state, color, offset}. (Colors of cells on the tape are sometimes instead thought of as "symbols" written to the tape.)

These rules can be represented pictorially by:



where the orientation of each arrow represents the state.

The rules can also be represented by the state transition diagram:

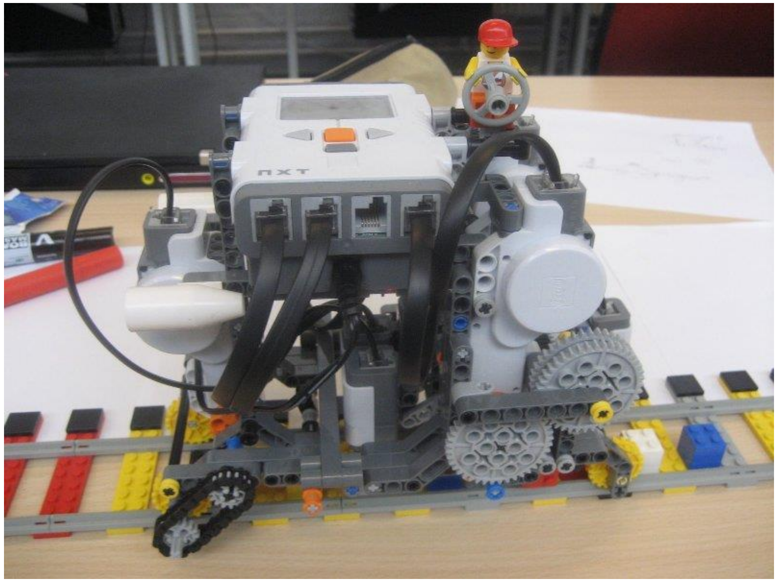


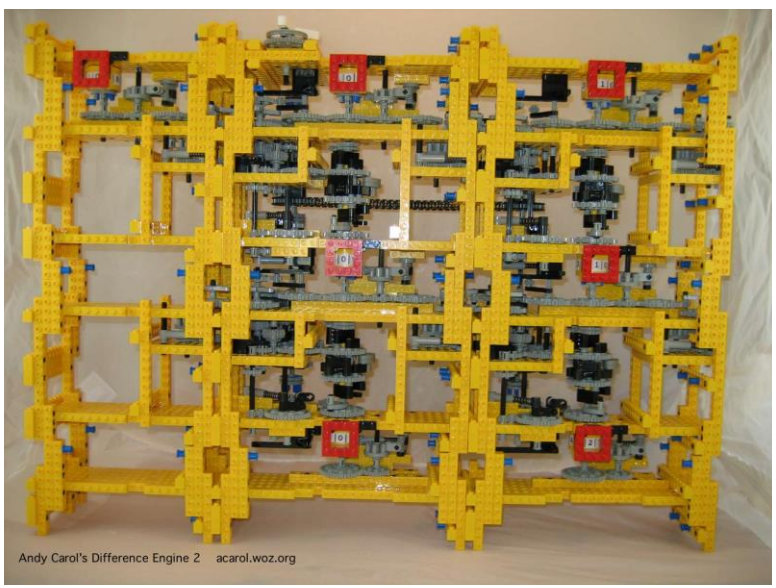
A 2-state 3-symbol universal Turing machine! (the smallest possible)

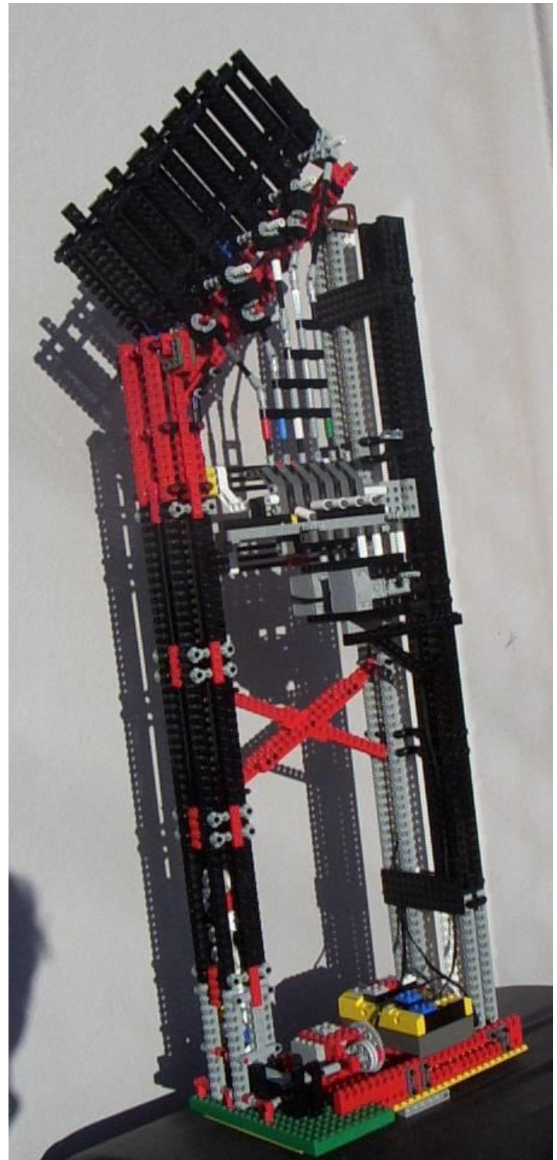
In Wolfram's numbering scheme for Turing machines, this is machine 596440. There are a total of (2 3 2)^(2 3)=12^6=2985984 machines with 2 states and 3 colors.

Note that there is no halt state for this Turing machine.

Computational Universality



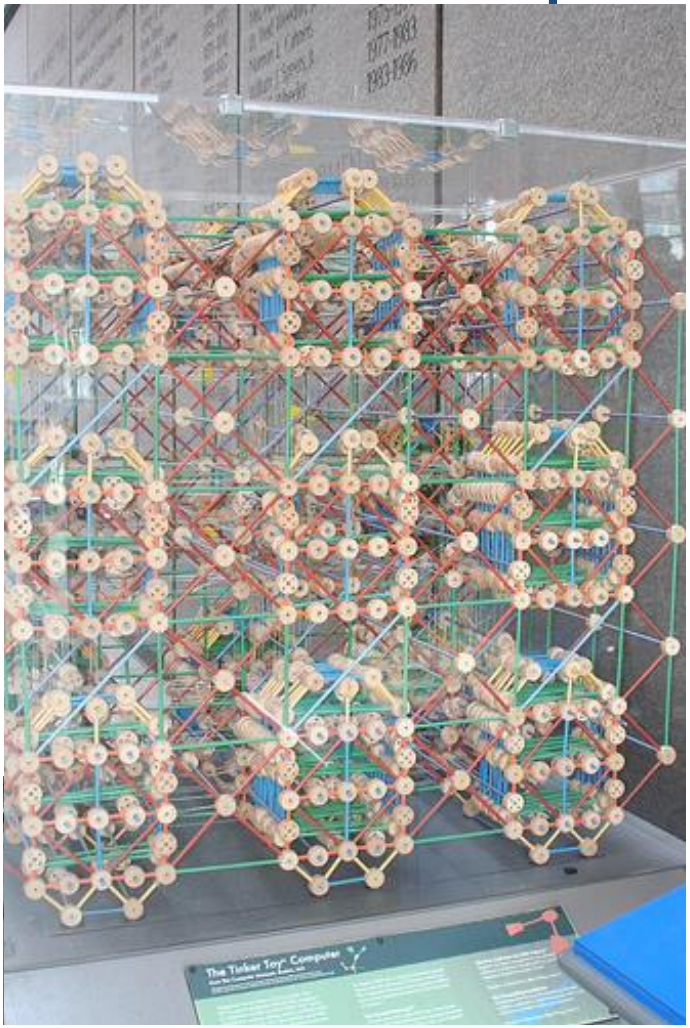




Lego Turing machines

Mechano computers

Computational Universality



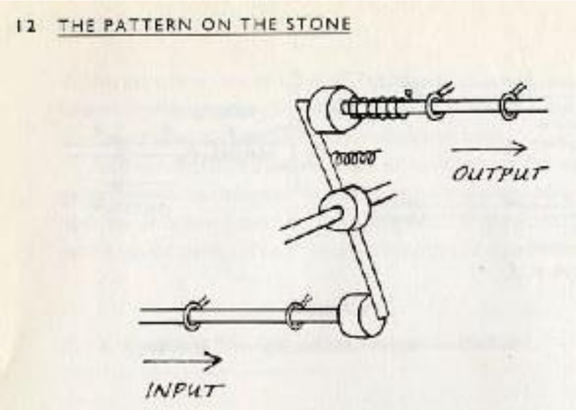


FIGURE 5
Mechanical inverter

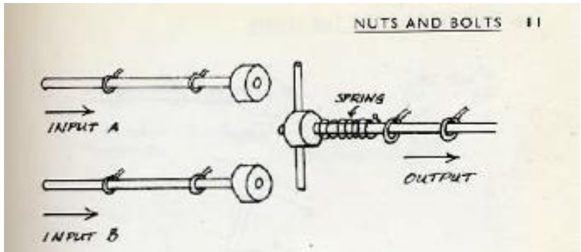


FIGURE 4
Mechanical implementation of the OR function

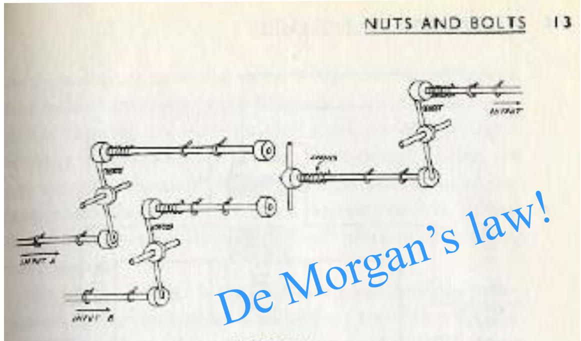
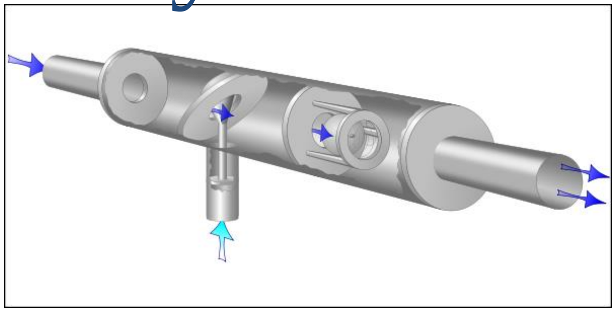
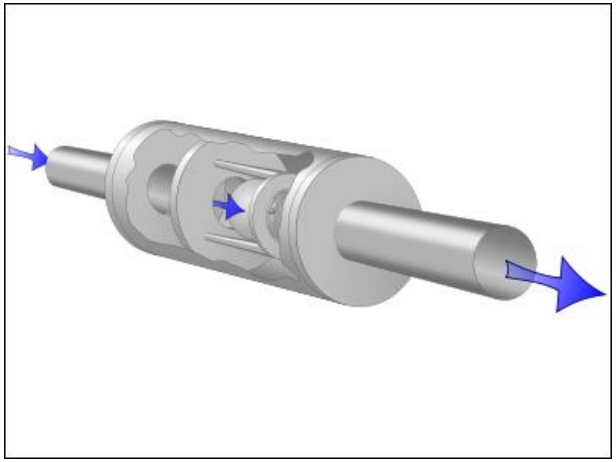
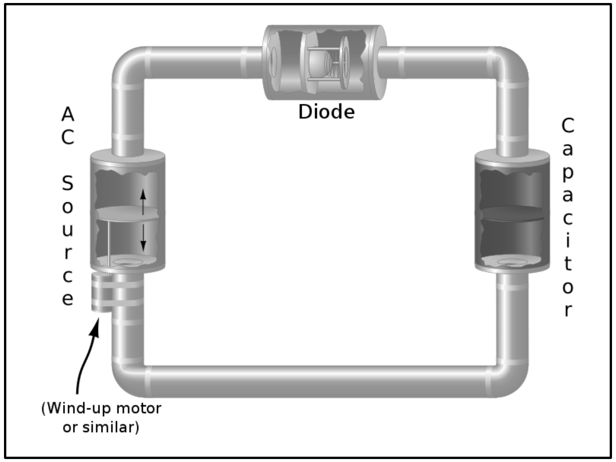


FIGURE 6
An And block constructed by connecting an Or block to inverters

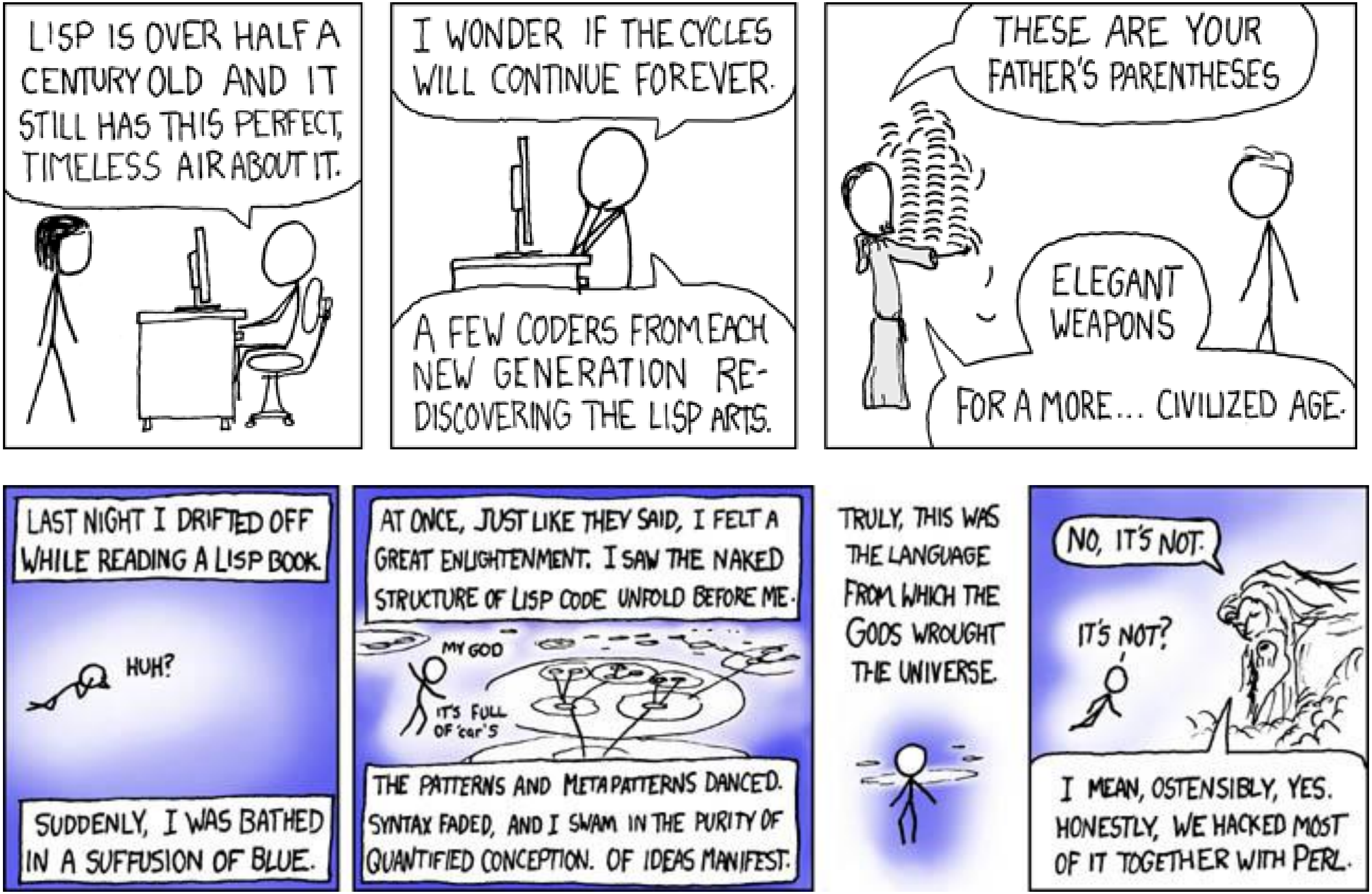




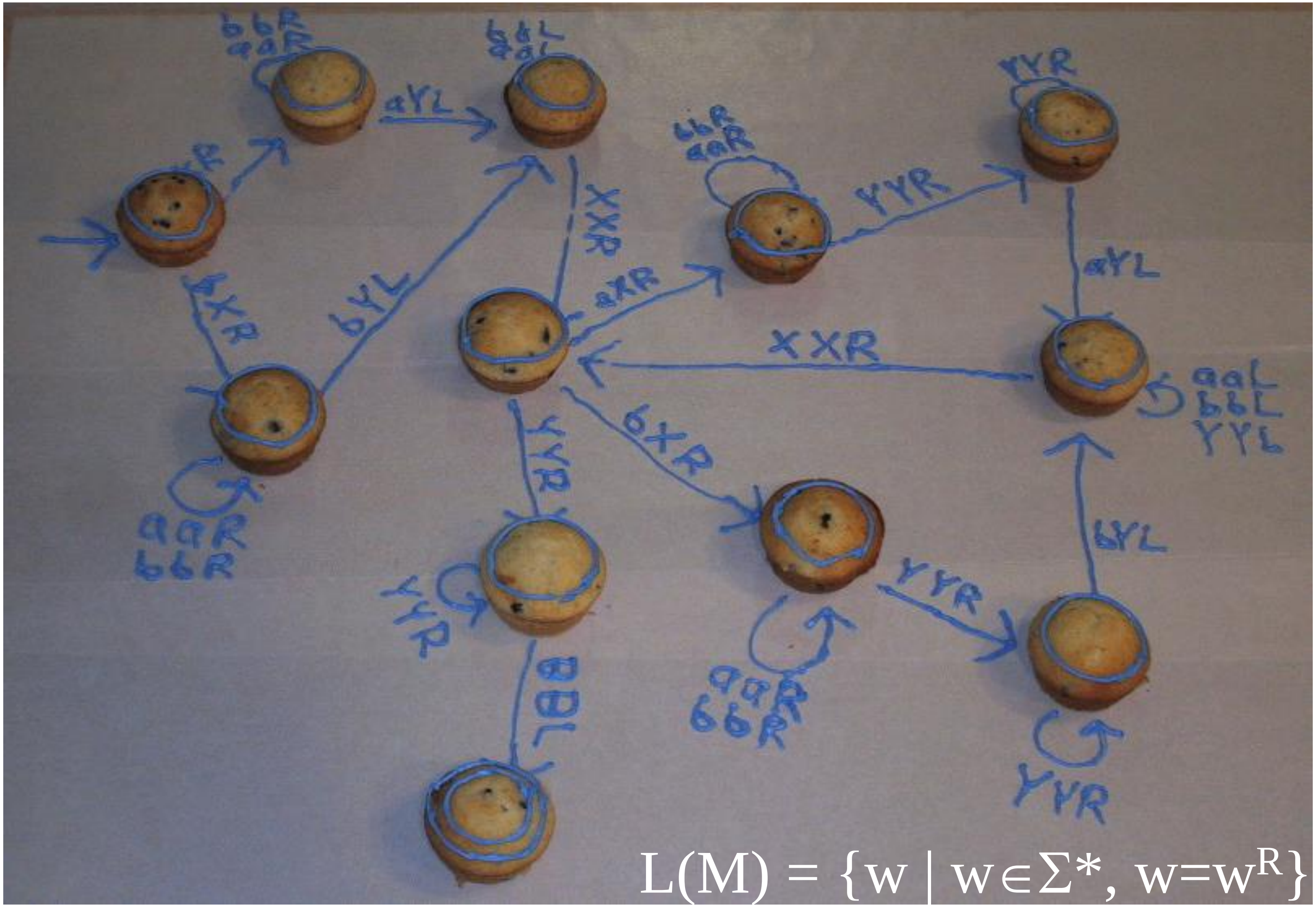


Tinker toy computers | Nuts-and-bolts computers | Hydraulic computers

λ-Calculus and LISP

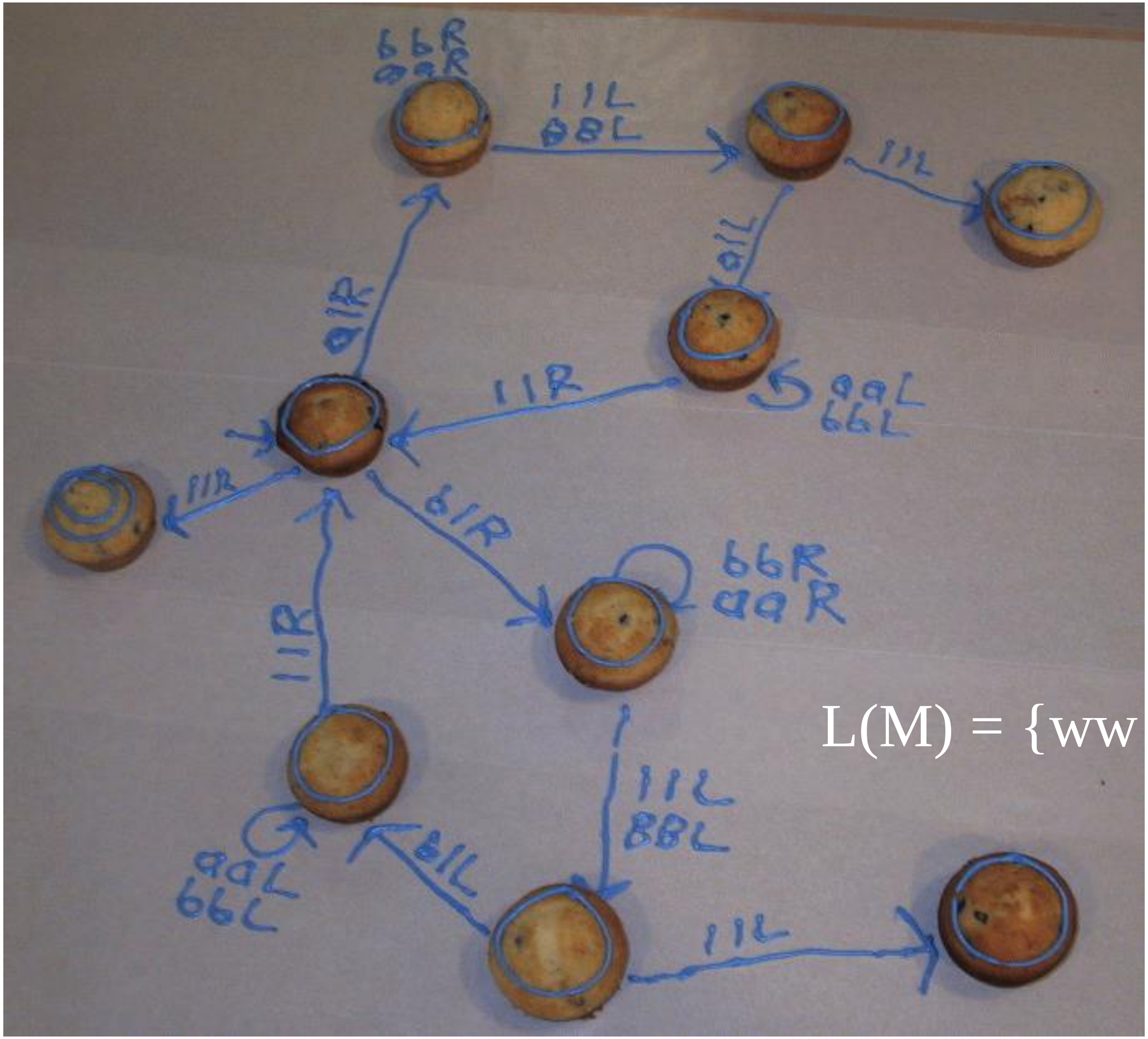


Blueberry Muffin Turing Machines



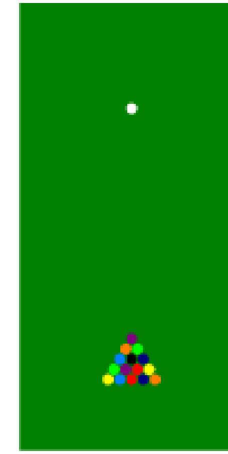
$L(M) = \{w \mid w \in \Sigma^*, w = w^R\}$

Blueberry Muffin Turing Machines

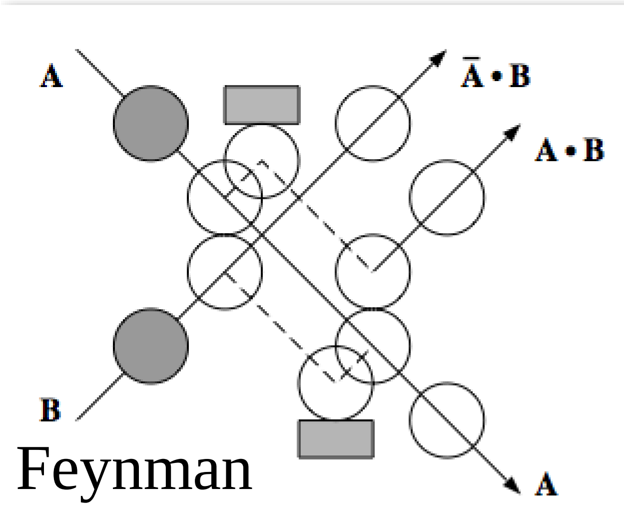


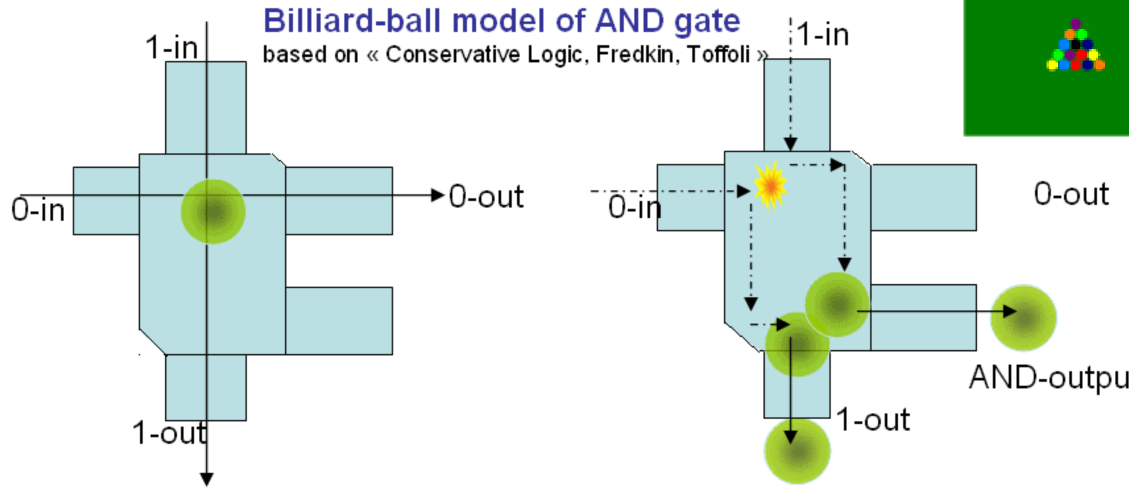
L(M) = {ww | w∈Σ*}

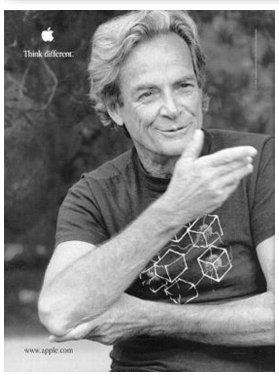
Universality of Billiards



Theorem: Billiards is computationally universal!







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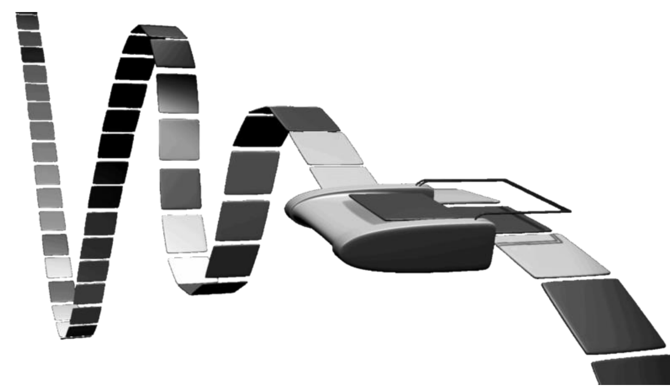
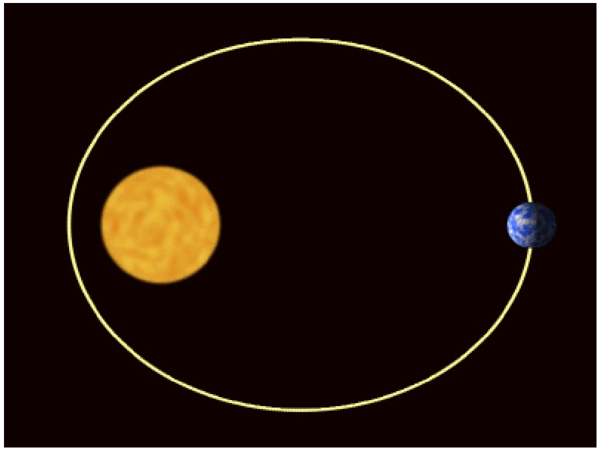


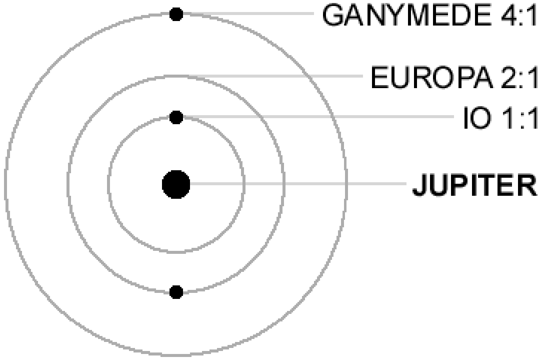
Corollary: Pool is “undecidable”!

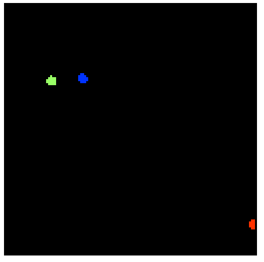
Corollary: Newtonian mechanics is universal!

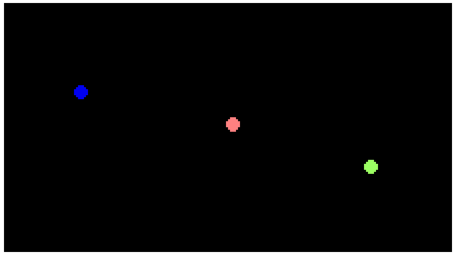
Orbital Mechanics

New solutions to gravitational N-body problems:









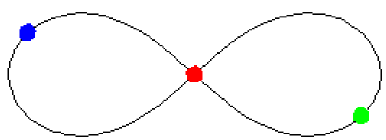
Observation: Planetary systems are like "3D billiards".

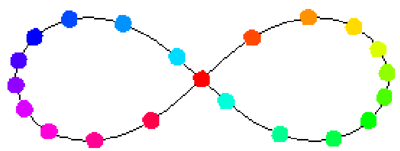
Theorem: Gravitational systems are chaotic & undecidable!

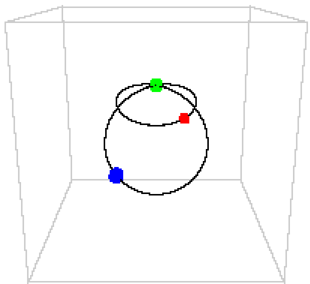
JOHANNES KEPLER'S UPHILL BATTLE

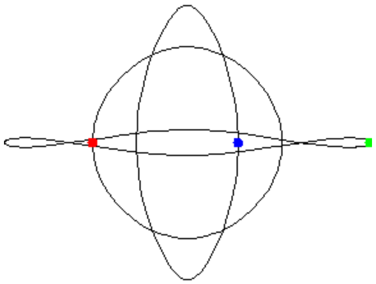
Orbital Mechanics

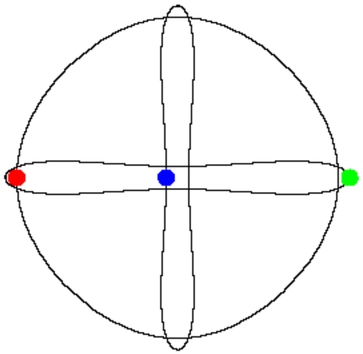
New solutions to the gravitational N-body problem:









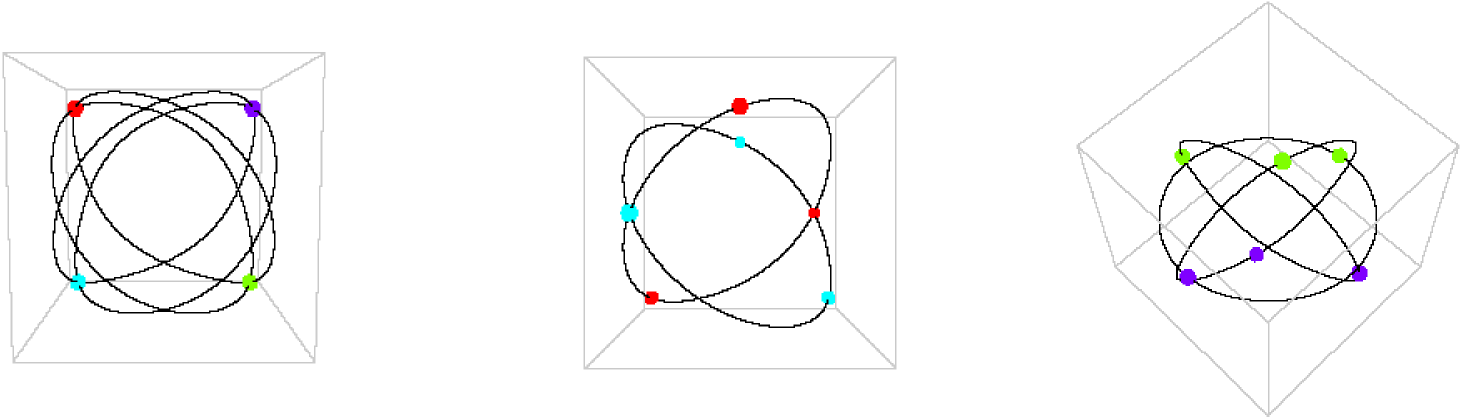


Theorem: These orbits are stable!

Chris Moore: http://www.santafe.edu/~moore/gallery.html

Orbital Mechanics

New solutions to the gravitational N-body problem:



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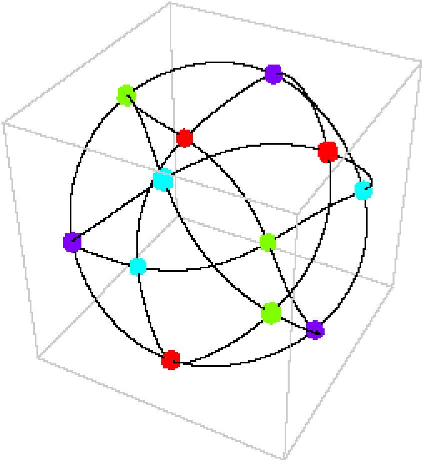
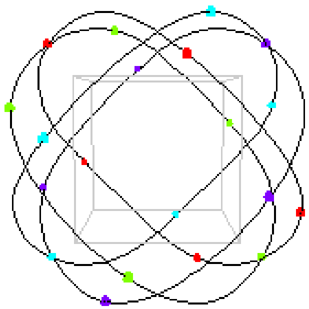
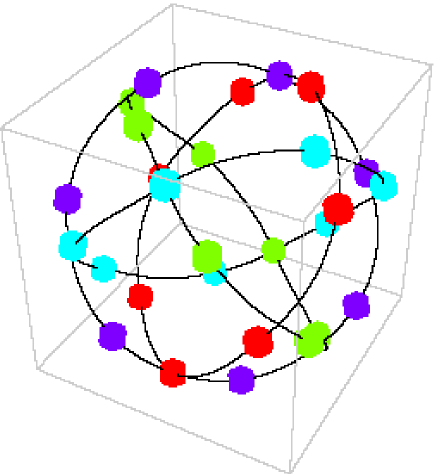

Theorem: These orbits are stable!

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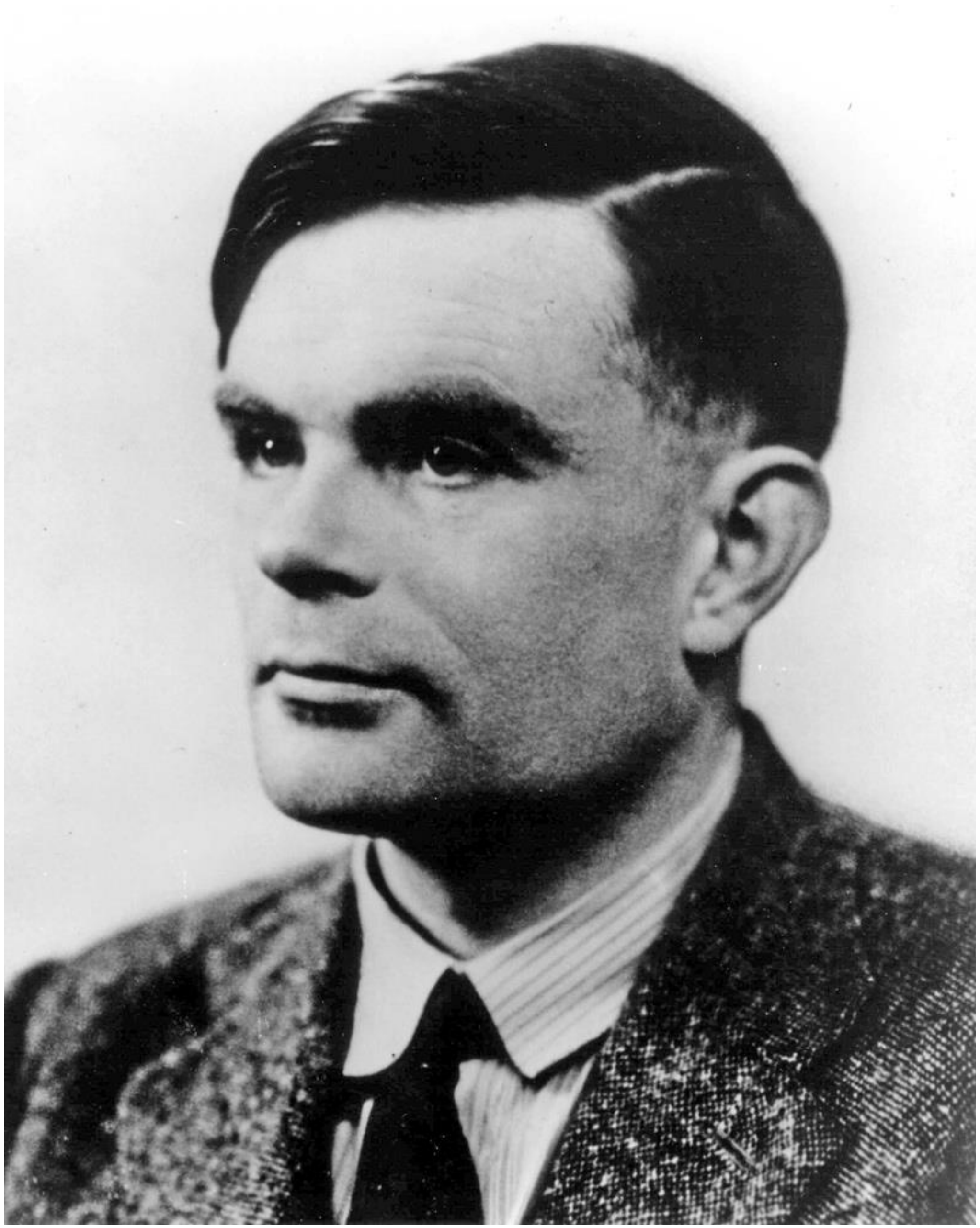
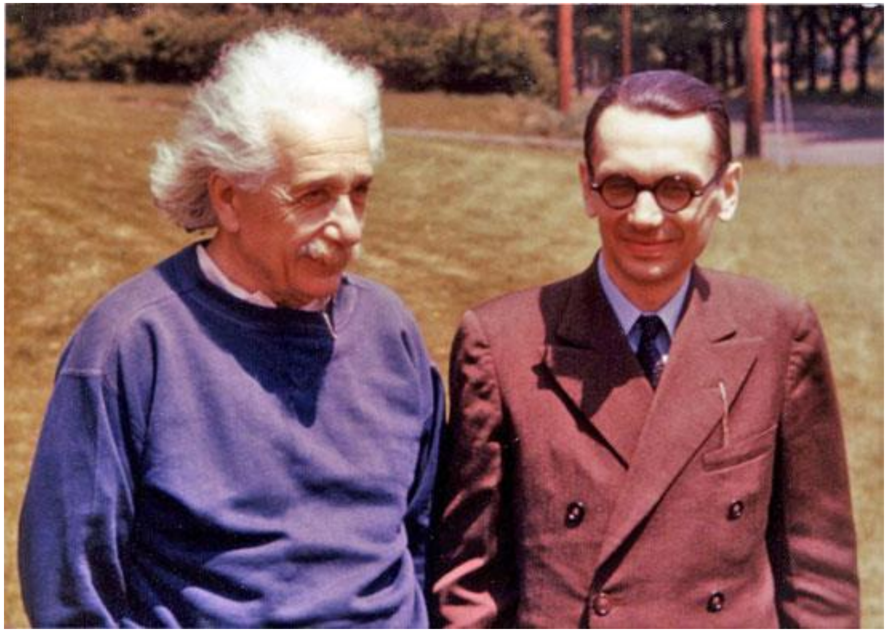
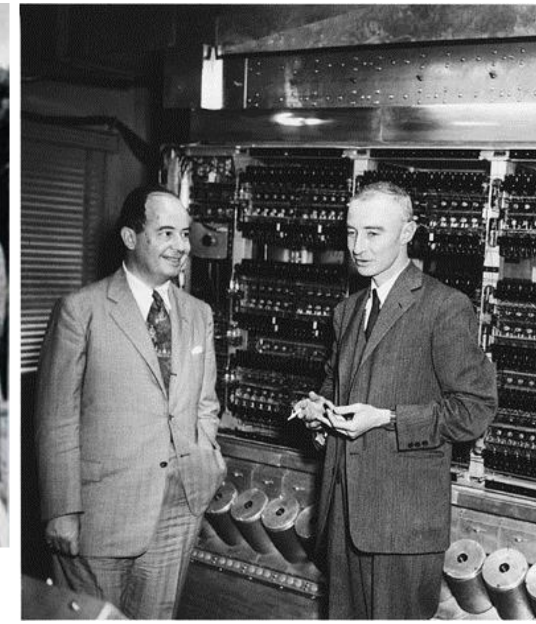
HIGH-GRAVITY BASEBALL

The Church-Turing Thesis



Princeton / Los Alamos / ENIAC

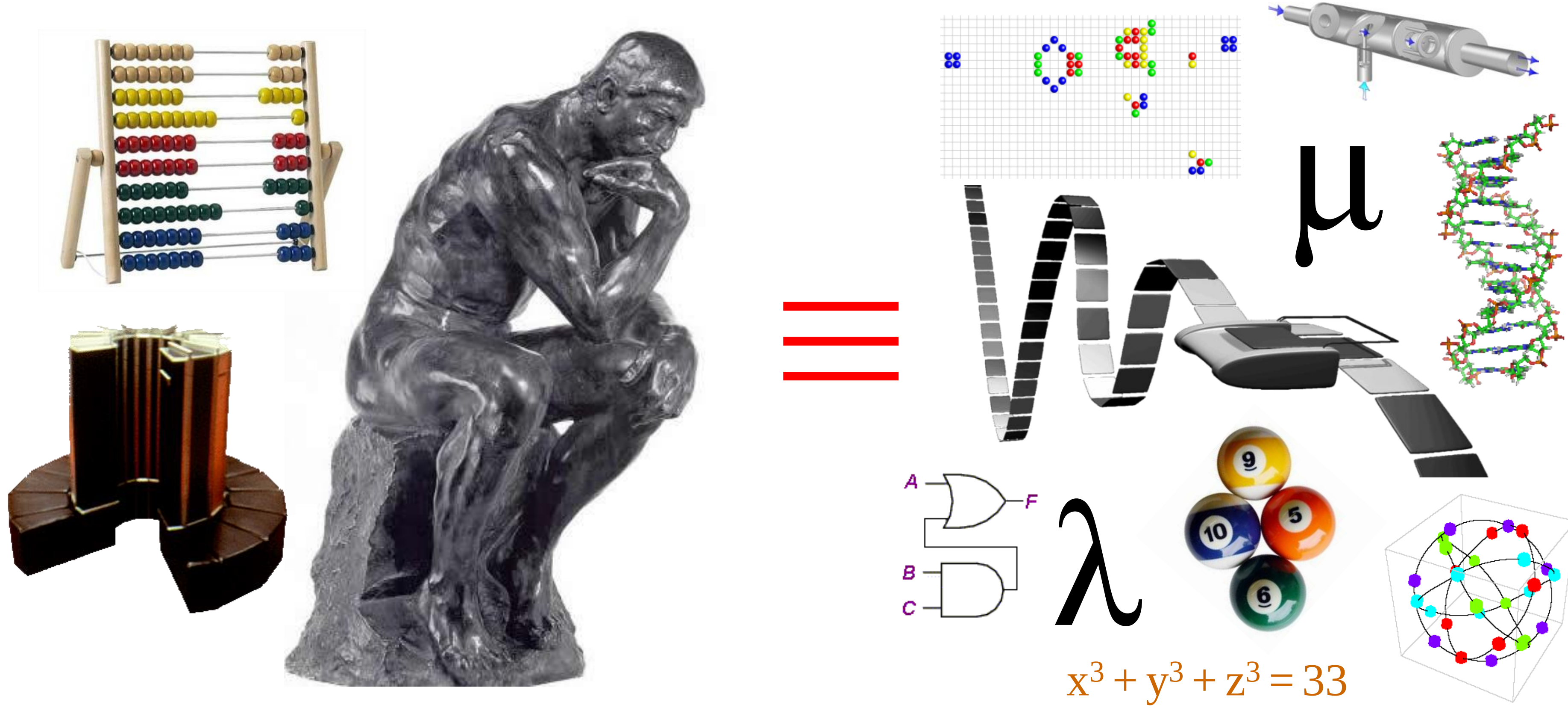




Church - Turing - Gödel - Einstein - von Neumann - Ulam - Oppenheimer - Feynman

The Church-Turing Thesis

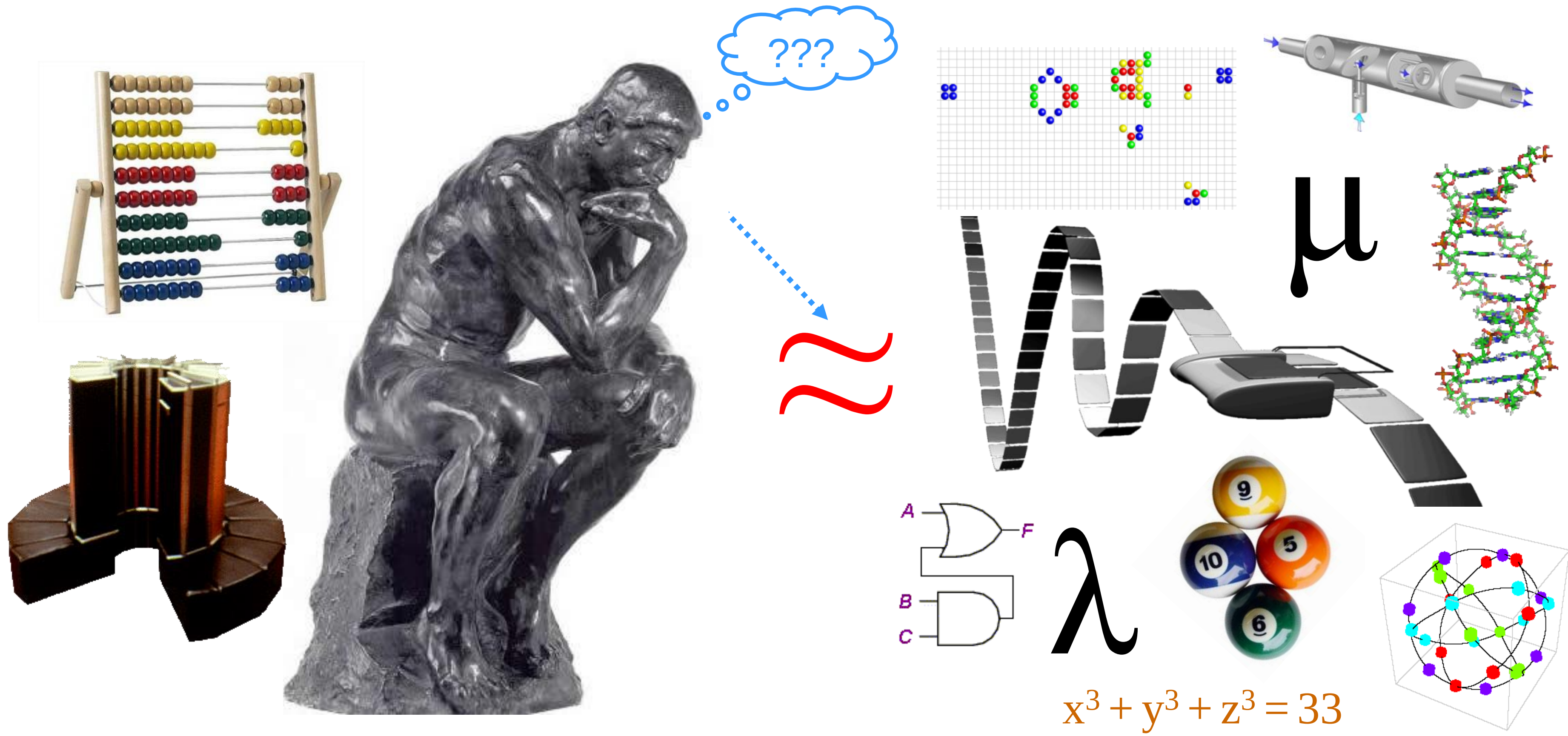
Q: What does it mean "to be computable"?



The Church-Turing Thesis: Anything that is "intuitively computable" is also Turing-machine computable.

The Church-Turing Thesis

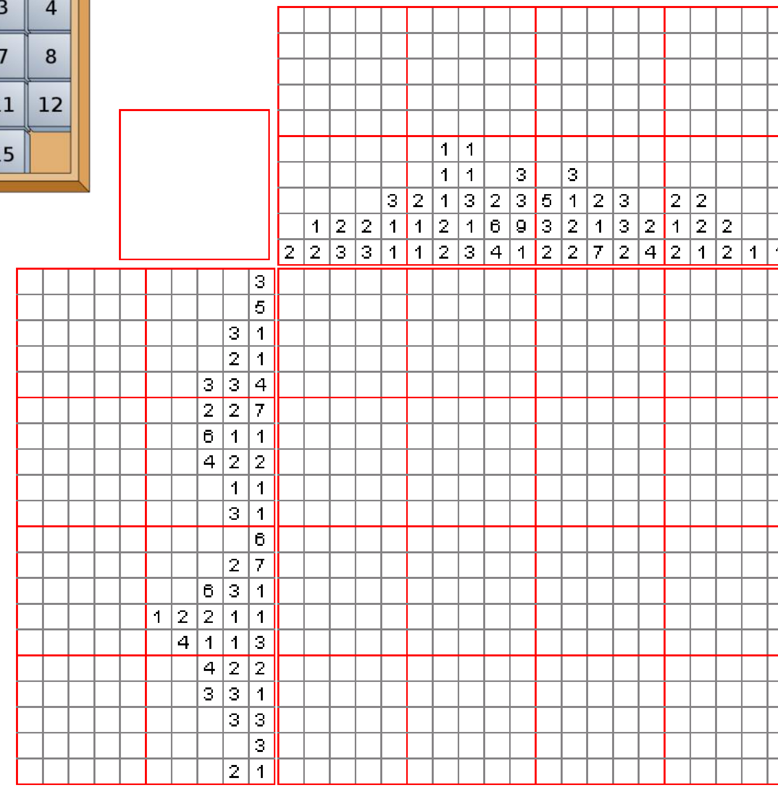
Q: Why "thesis" and not "theorem"?

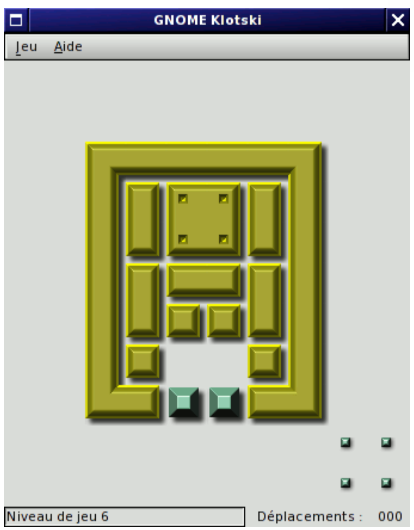


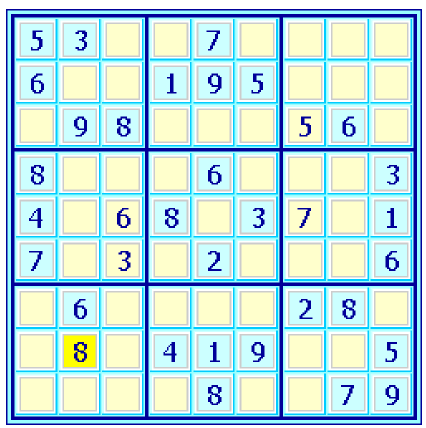
Undefined / informal tasks: produce (or even identify) good music, art, poetry, humor, aesthetics, justice, truth, etc.

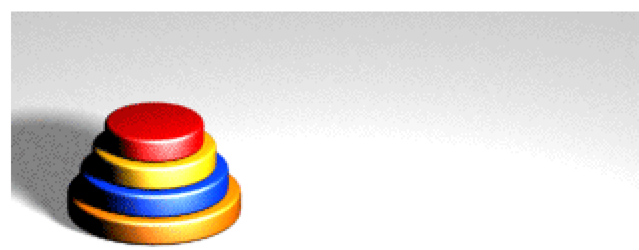
Things We Can Compute

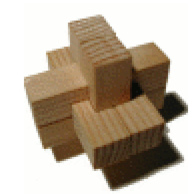
Puzzles:





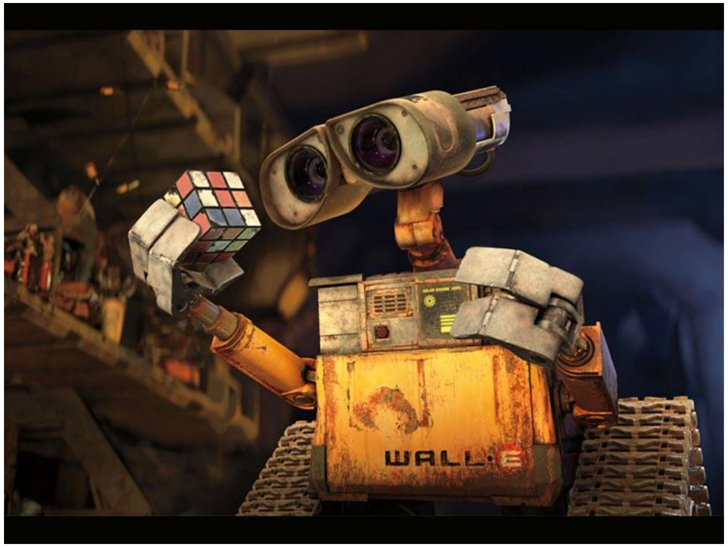


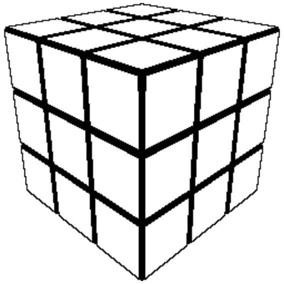


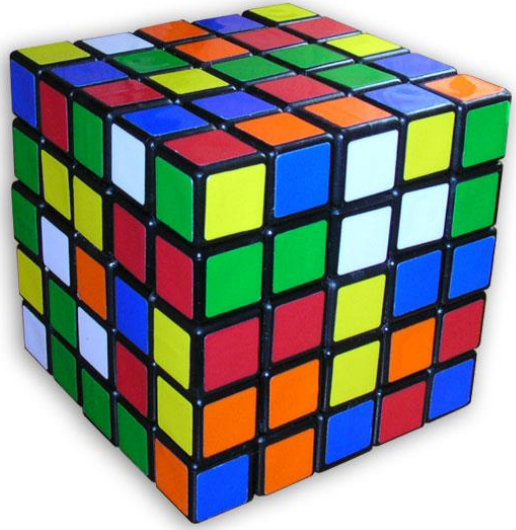


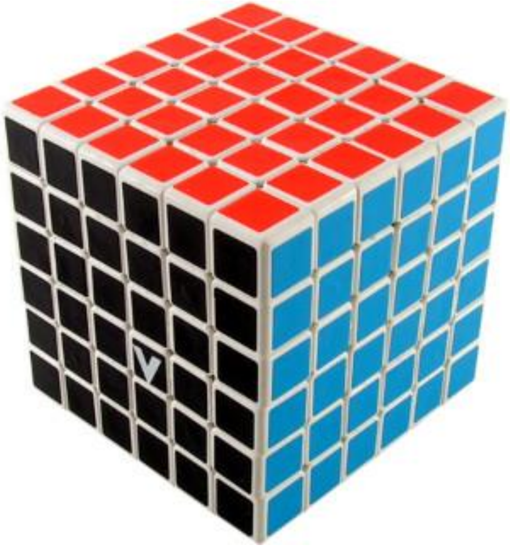
Well-defined (albeit large) discrete solution spaces

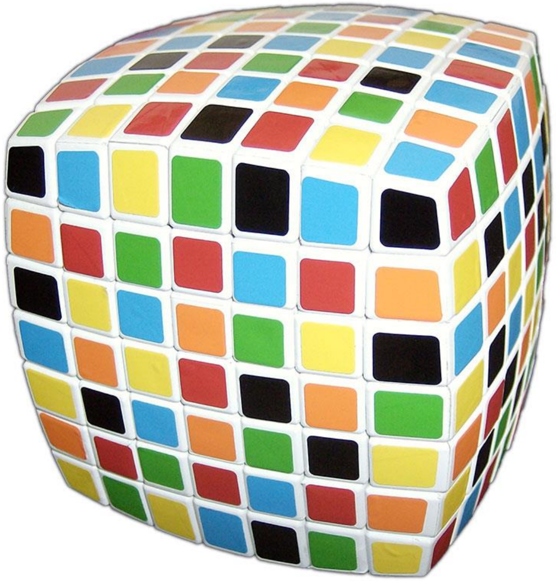
Things We Can Compute





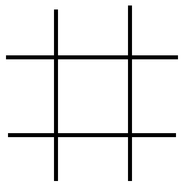


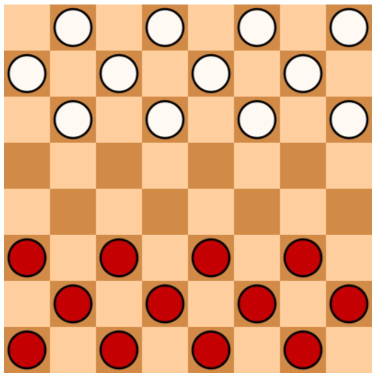


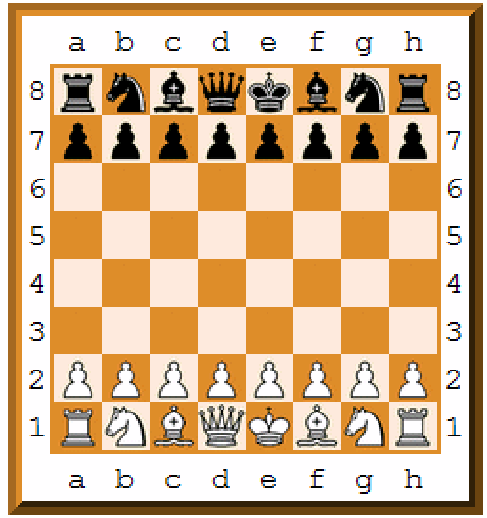


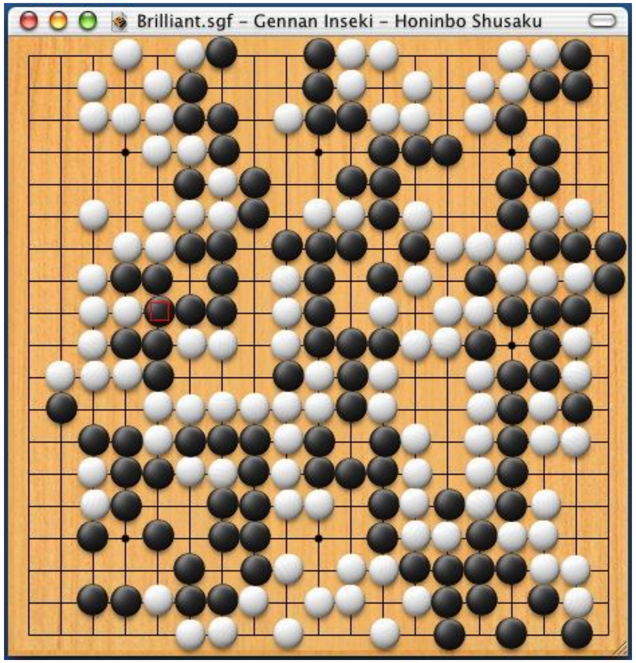
Things We Can Compute

Games:









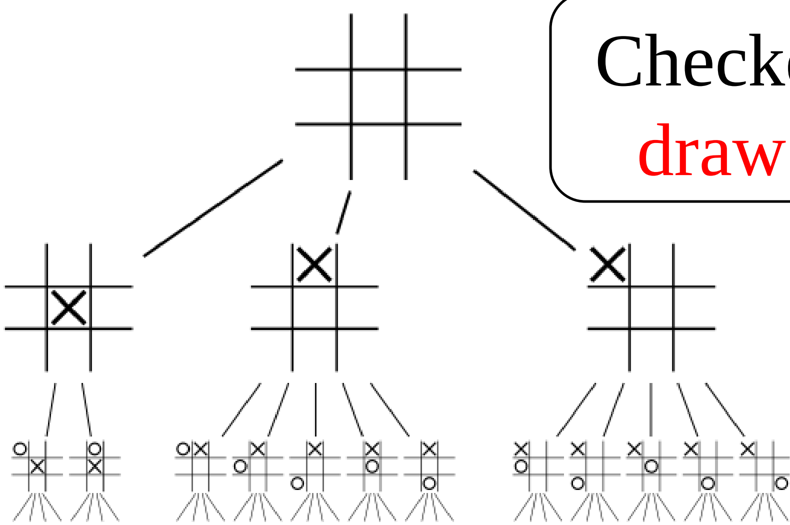
Tree size:

765

10^{20}

10^{50}

10^{171}

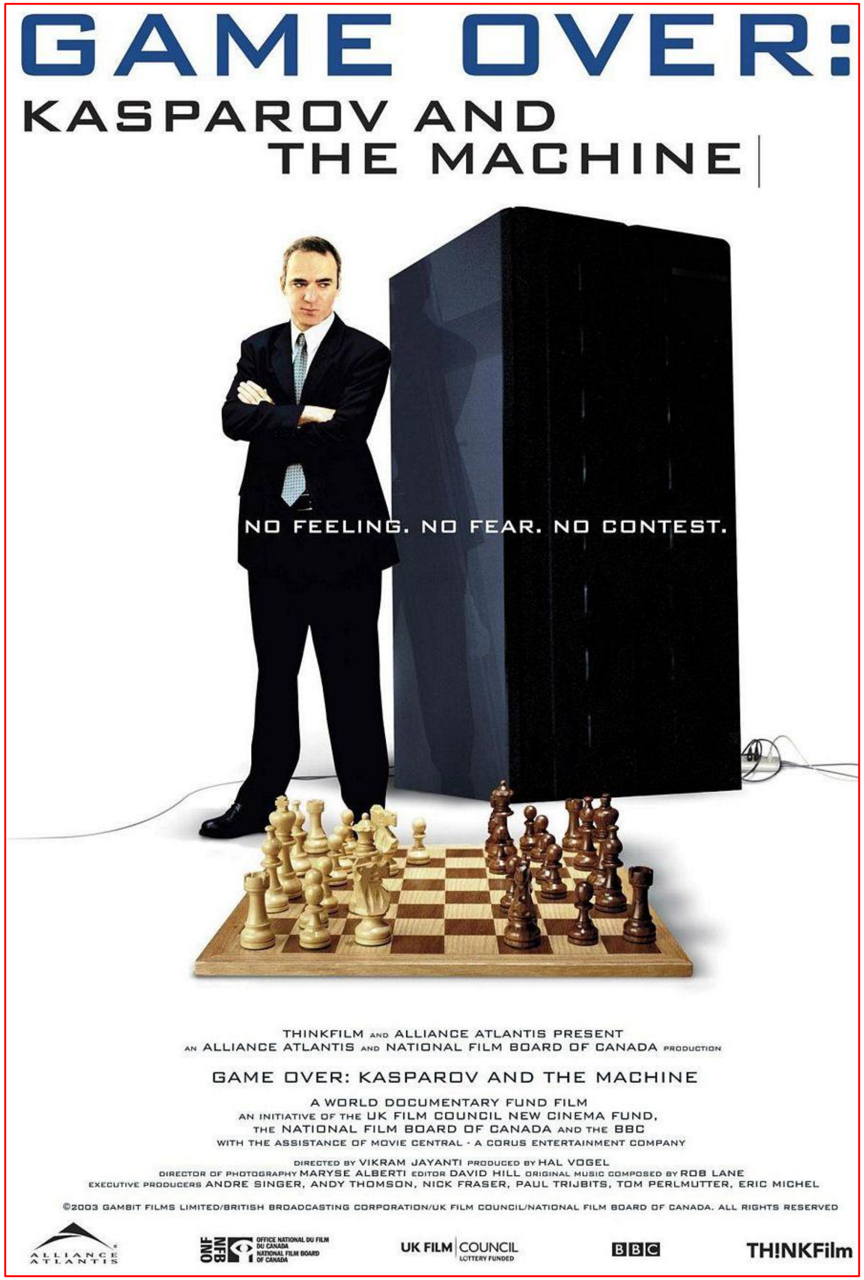


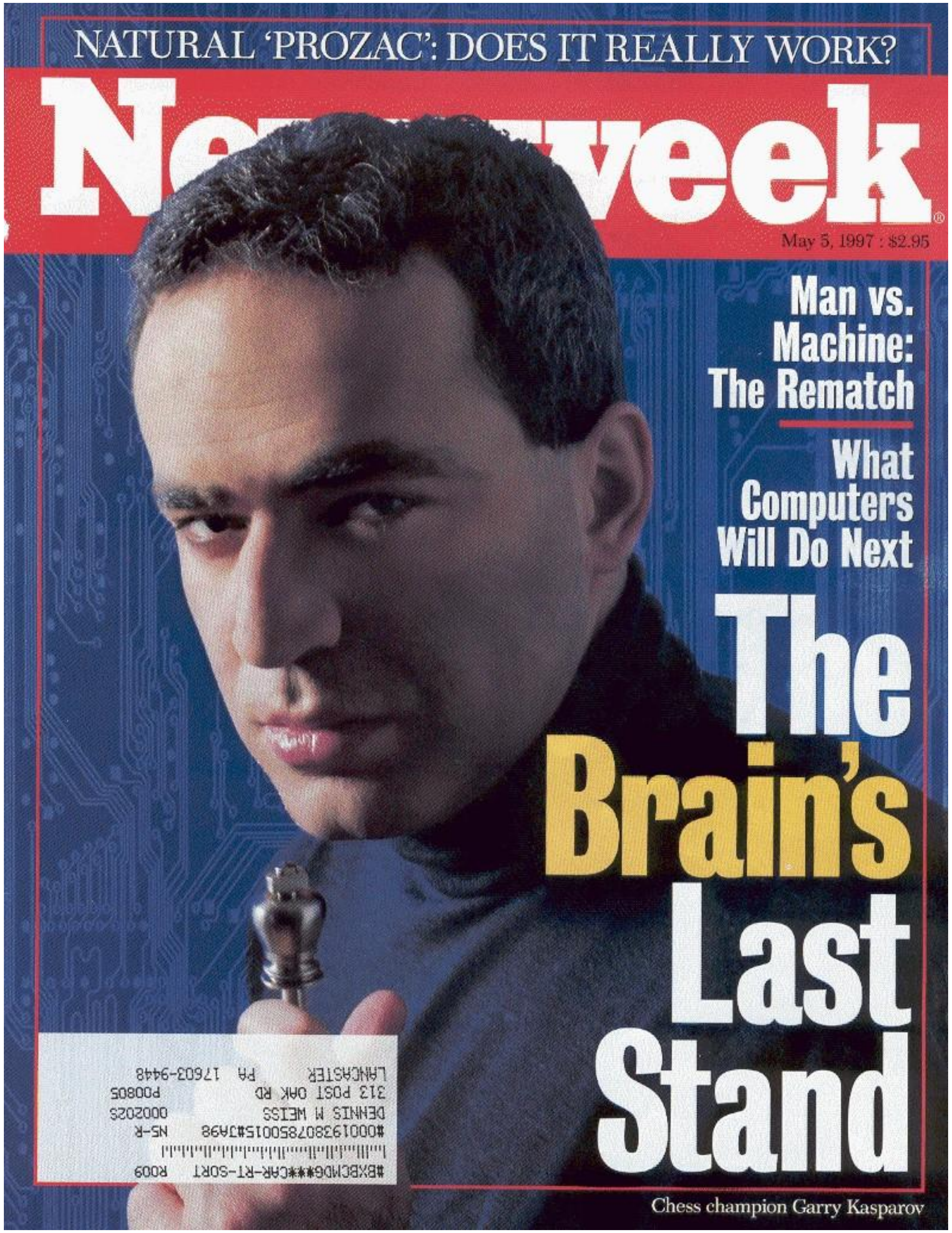
Checkers proven draw in 2007!

Things We Can Compute

IBM's "Deep Blue" becomes Chess world champion in 1997

Things We Can Compute





Things We Can Compute

"Watson" AI becomes Jeopardy world champion in 2011

Things We Can Compute

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Technology

Google achieves AI 'breakthrough' by beating Go champion

27 January 2016 | Technology

Google's DeepMind division has achieved a landmark in AI

A Google artificial intelligence program has beaten the European champion of the board game Go.

The Chinese game is viewed as a much tougher challenge than chess for computers because there are many more ways a Go match can play out.

The tech company's DeepMind division said **its software had beaten its human rival** five games to nil.

Google AI in landmark victory over Go grandmaster

Fan Hui, three-time champion of the east Asian board game, lost to DeepMind's program AlphaGo in five straight games

Fan Hui makes a move against AlphaGo in DeepMind's HQ in King's Cross. Photograph: Google DeepMind

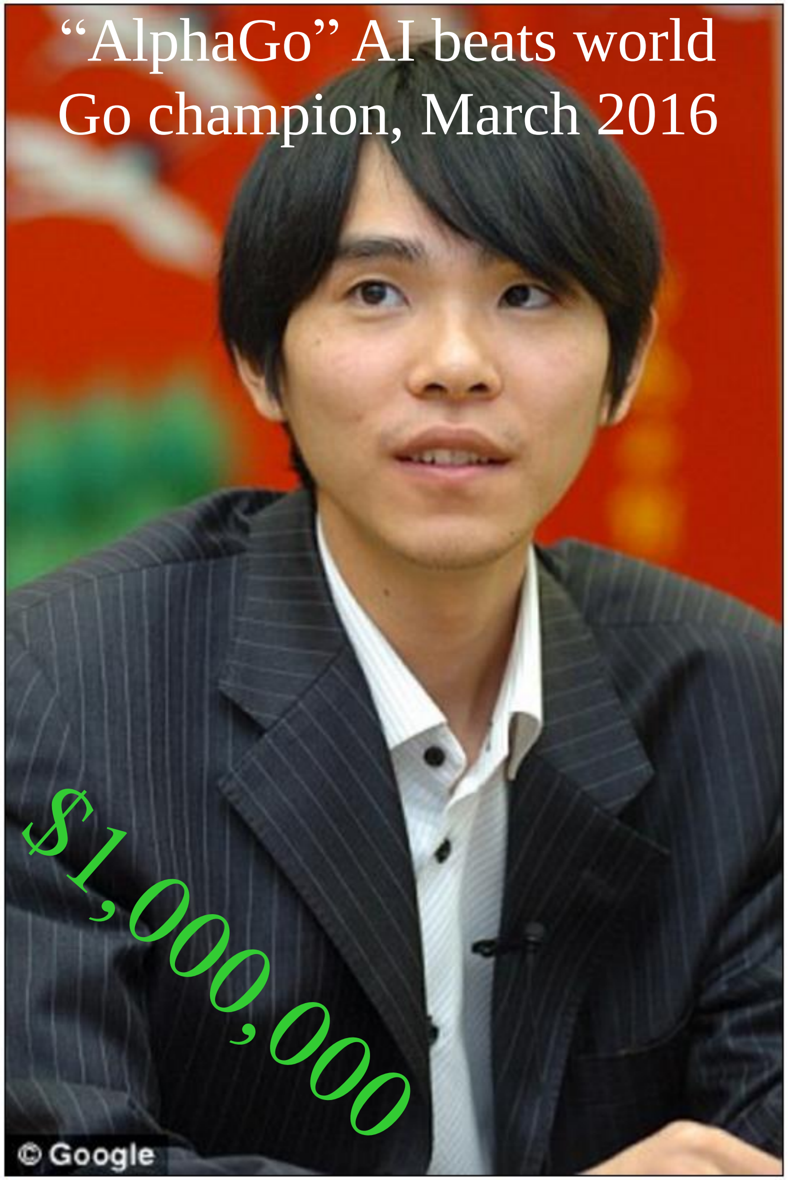
When Gary Kasparov lost to chess computer Deep Blue in 1997, IBM marked a milestone in the history of artificial intelligence. On Wednesday, in a research paper released in Nature, Google earned its own position in the history books, with the announcement that its subsidiary DeepMind has built a system capable of beating the best human players in the world at the east Asian board game Go.

Go, a game that involves placing black or white tiles on a 19x19 board and trying to remove your opponents', is far more difficult for a computer to master than a game such as chess.

DeepMind's software, AlphaGo, successfully beat the three-time European Go champion Fan Hui 5-0 in a series of games at the company's headquarters in King's Cross last October. Dr Tanguy Chouard, a senior editor at Nature who attended the matches as part of the review process, described the victory as "really chilling to watch".

"It was one of the most exciting moments of my career," he added. "But with the usual mixed feelings ... in the quiet room downstairs, one couldn't help but root for the poor human being beaten."

Things We Can Compute



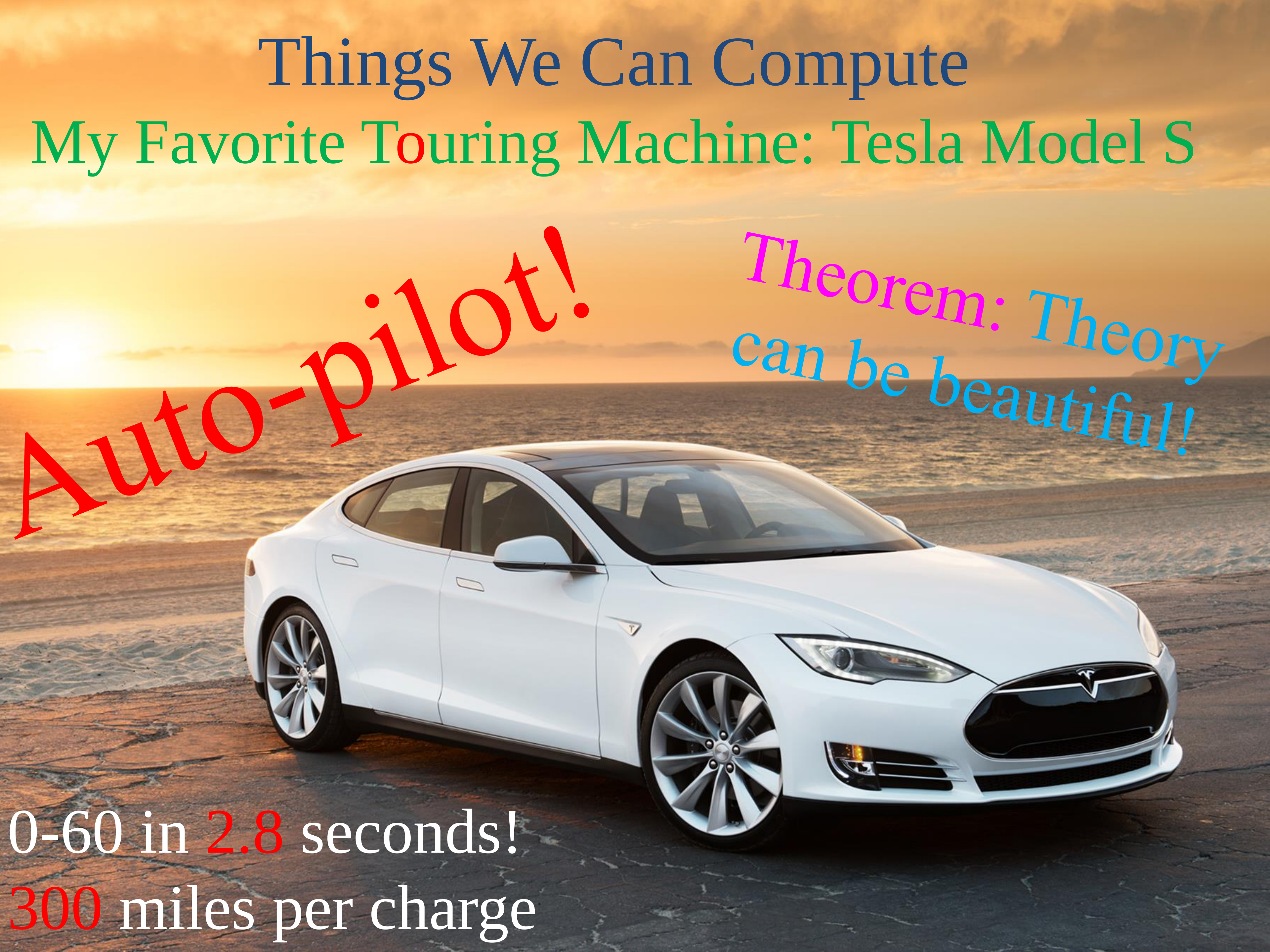
Now the machine has beaten Fan Hui (pictured left) it will face the top human player - Lee Sedol (right) of South Korea – at a meeting in Seoul in March, with the winner to be awarded $1 million (£701,607)

Things We Can Compute

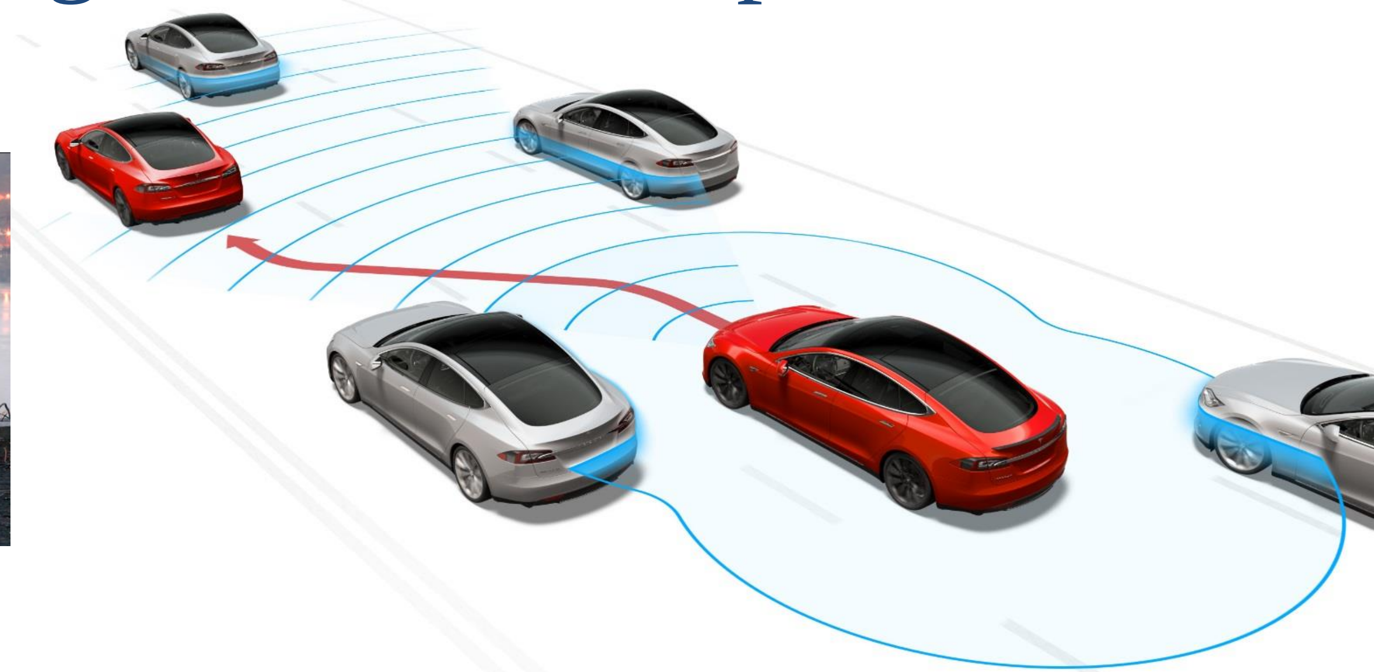
"AlphaGo" AI beats world Go champion March 2016

Things We Can Compute

My Favorite Touring Machine: Tesla Model S



Things We Can Compute



Things We Can Compute

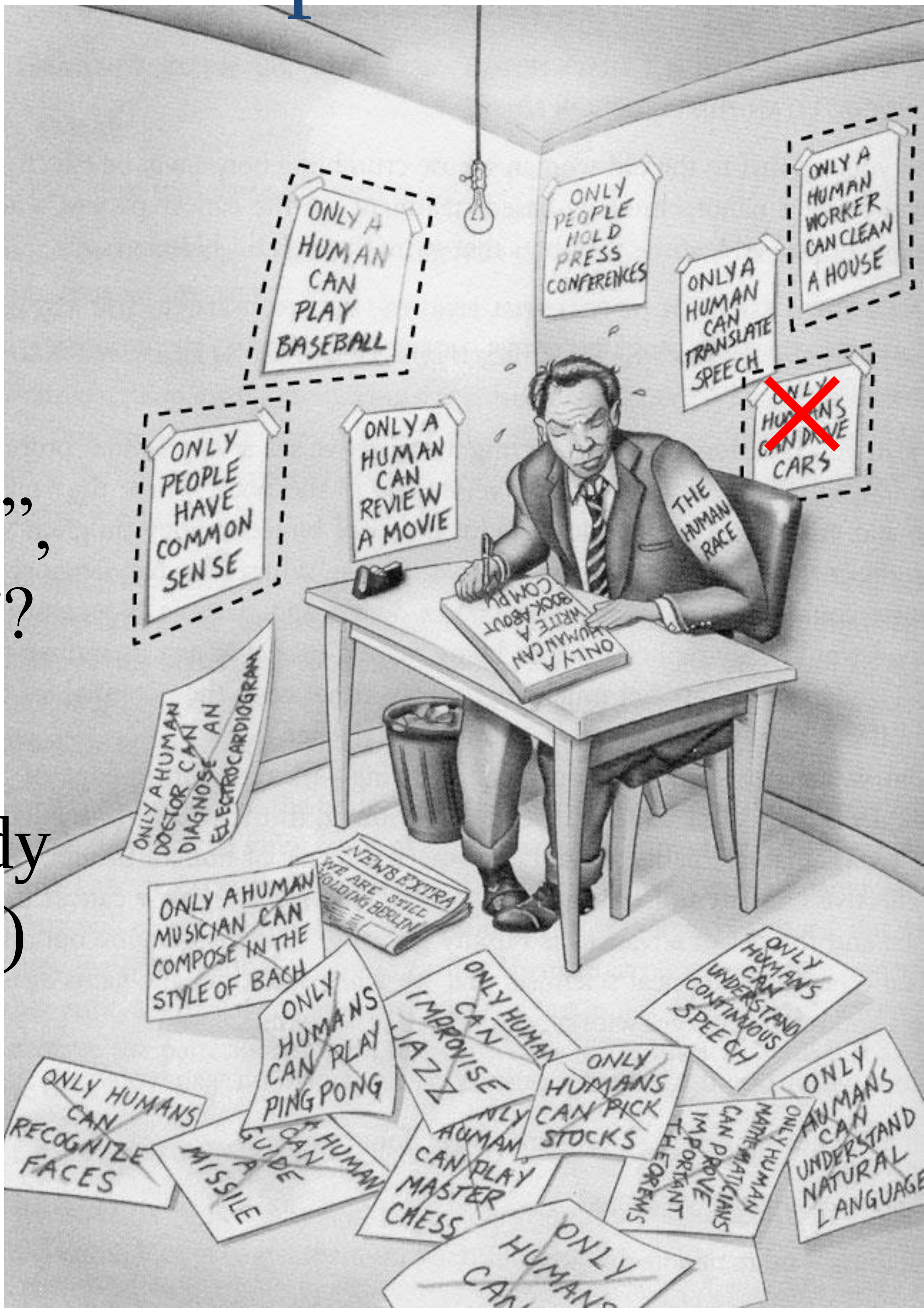
Fact: gap is narrowing between natural and artificial intelligence

Q: Will this gap ever close?
A: Probably yes.

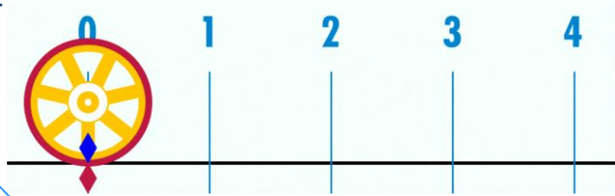
Q: What is "intelligence", "mind", "consciousness", "sentience"?
A: We still don't know.

- In many areas machines already exceeded humans (e.g., games)
- These trends are accelerating!

Q: Where is technology going?
A: We still don't know.



Things We Can Compute



Digits of Pi:

$$\sum_{n=0}^{\infty} \frac{4(-1)^n}{2n+1} = \pi$$

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} \cdots.$$

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdots$$

$$\pi = \sqrt{12}\left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \cdots\right)$$

$$\frac{\pi}{4} = 4 \arctan\frac{1}{5} - \arctan\frac{1}{239}$$

$$\pi = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \ddots}}}}$$

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$$

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

$$\pi = \sum_{k=0}^{\infty} \frac{1}{16^k}\left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right),$$

$$\frac{426880\sqrt{10005}}{\pi} = \sum_{k=0}^{\infty} \frac{(6k)!(13591409+545140134k)}{(3k)!(k!)^3(-640320)^{3k}}$$

$$\pi = \frac{1}{2^6} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{10n}} \left(-\frac{2^5}{4n+1} - \frac{1}{4n+3} + \frac{2^8}{10n+1} - \frac{2^6}{10n+3} - \frac{2^2}{10n+5} - \frac{2^2}{10n+7} + \frac{1}{10n+9}\right)$$

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230
664709384460955058223172535940812848111745028410270193852110555964462294895493038196442881097566593344612847564823378
678316527120190914564856692346034861045432664821339360726024914127372458700660631558817488152092096282925409171536436
789259036001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274956
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506040092770167113900984882401285836160356370766010471018194295559619894676783744944825537977472684710404753464620804
66842590694912 ...

Things We Can Compute

Prime numbers:

Theorems:

∃ an infinity of primes

∃ # primes ≤ n → $n / \log_e n$

∃ arbitrarily large prime gaps

Prime numbers

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Open problems:

∃ an infinity of prime pairs? (i.e., p & p+2) ?

Goldbach's conjecture (verified for all $n<10^{18}$):

every even integer >2 is the sum of two primes ?

Largest known prime: $2^{43,112,609}-1$ (12,978,189 digits)


53
223
547
1033
1677
HOLD ON THERE, MR. WEBSTER. 1677 ISN'T PRIME— IT'S DIVISIBLE BY 43.

Things We Can Compute

More prime numbers theorems:

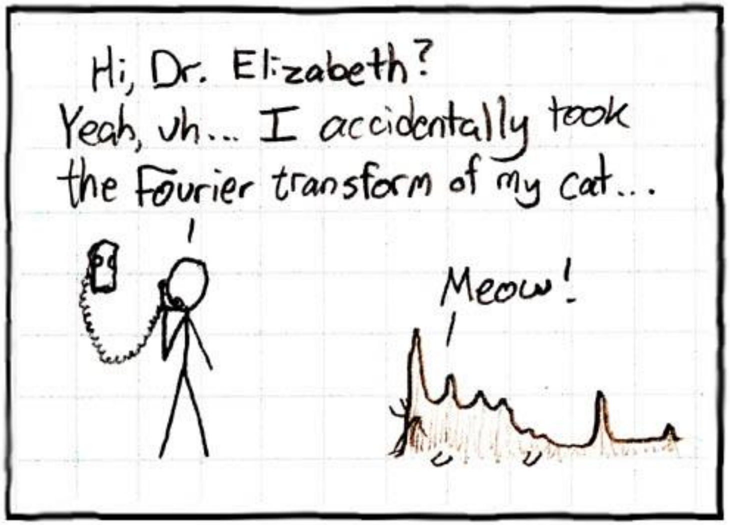
No polynomial yields only primes.

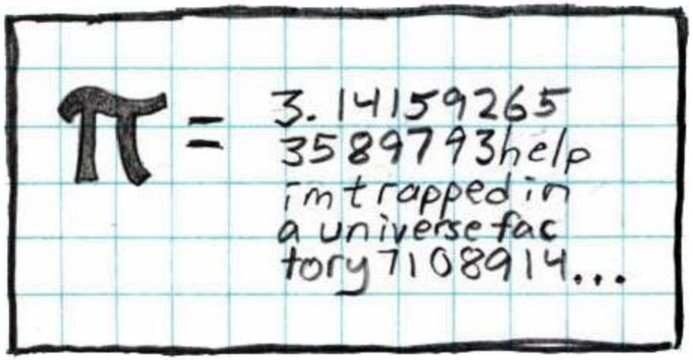
N^2+n+41 yields 40 consecutive primes for $0 \le n \le 39$.

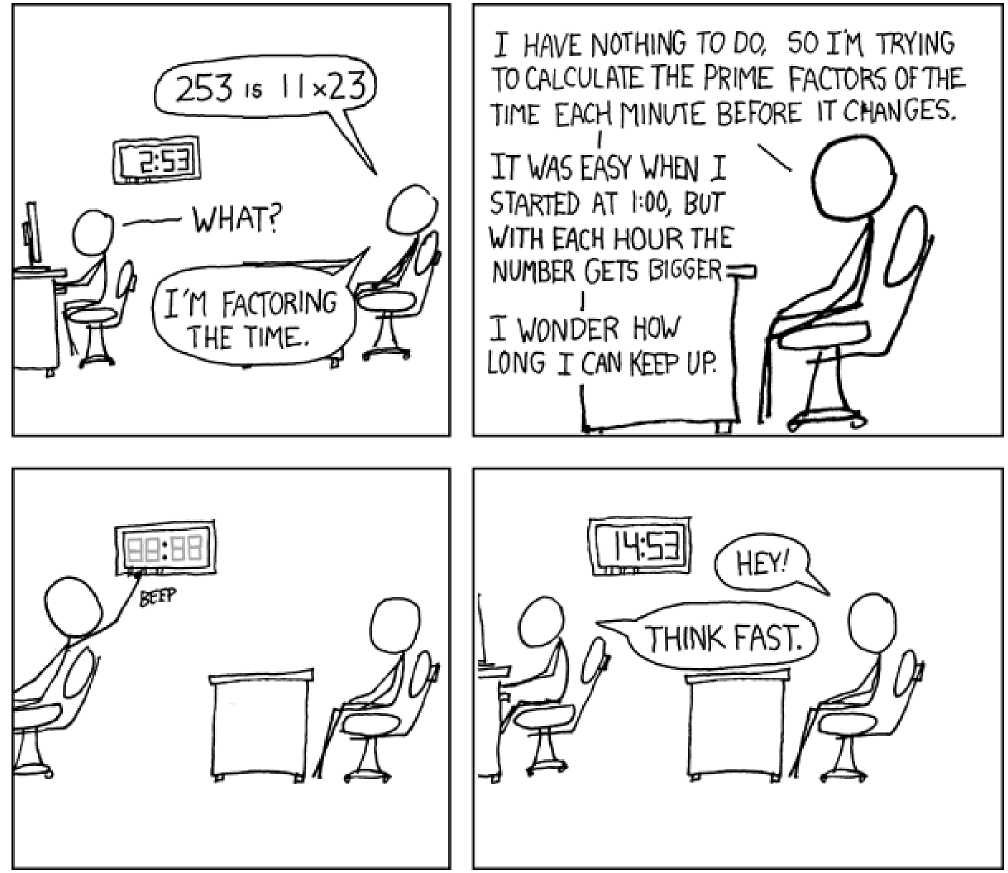
The set of primes coincides exactly with the positive values of the following 26-variable polynomial:

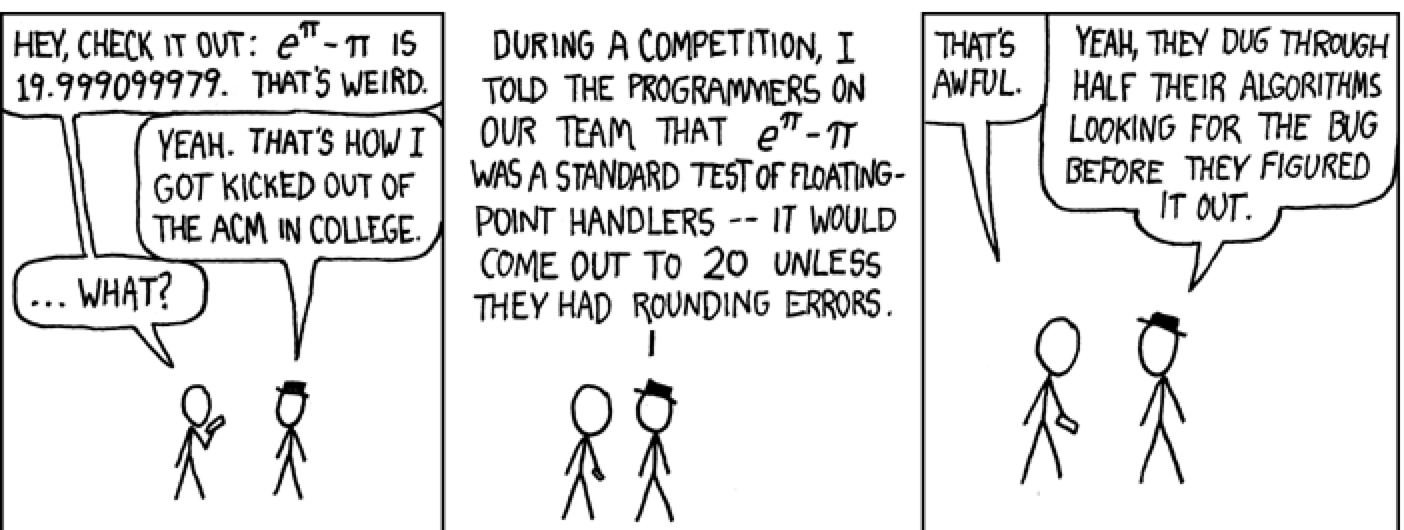
$$(k + 2)(1 - [wz + h + j - q]^2 - [(gk + 2g + k + 1)(h + j) + h - z]^2 - [16(k + 1)^3(k + 2)(n + 1)^2 + 1 - f^2]^2 - [2n + p + q + z - e]^2 - [e^3(e + 2)(a + 1)^2 + 1 - o^2]^2 - [(a^2 - 1)y^2 + 1 - x^2]^2 - [16r^2y^4(a^2 - 1) + 1 - u^2]^2 - [n + l + v - y]^2 - [(a^2 - 1)l^2 + 1 - m^2]^2 - [ai + k + 1 - l - i]^2 - [((a + u^2(u^2 - a))^2 - 1)(n + 4dy)^2 + 1 - (x + cu)^2]^2 - [p + l(a - n - 1) + b(2an + 2a - n^2 - 2n - 2) - m]^2 - [q + y(a - p - 1) + s(2ap + 2a - p^2 - 2p - 2) - x]^2 - [z + pl(a - p) + t(2ap - p^2 - 1) - pm]^2)$$

as a, b, c, … , z range over the nonnegative integers!


Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took the Fourier transform of my cat...
Meow!

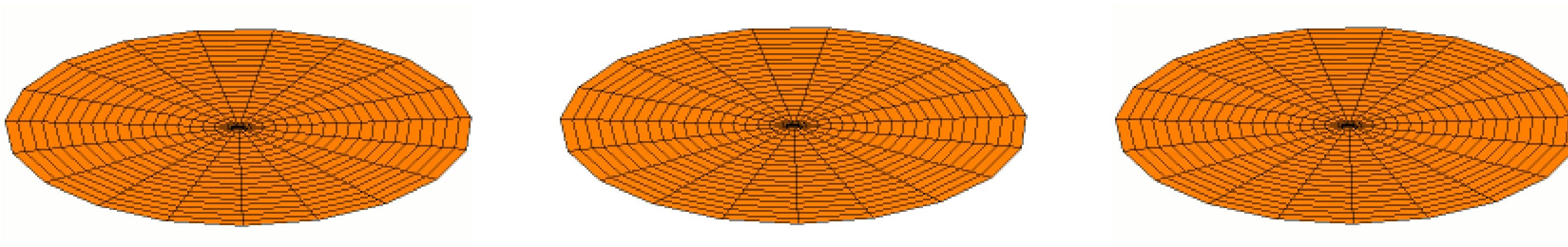

π = 3.14159265 358979 3help imtrappedin a universefac tory7108914...


2:53
253 is 11×23
WHAT?
I'M FACTORING THE TIME.
I HAVE NOTHING TO DO, SO I'M TRYING TO CALCULATE THE PRIME FACTORS OF THE TIME EACH MINUTE BEFORE IT CHANGES.
IT WAS EASY WHEN I STARTED AT 1:00, BUT WITH EACH HOUR THE NUMBER GETS BIGGER
I WONDER HOW LONG I CAN KEEP UP.
88:88
BEEP
14:53
HEY!
THINK FAST.

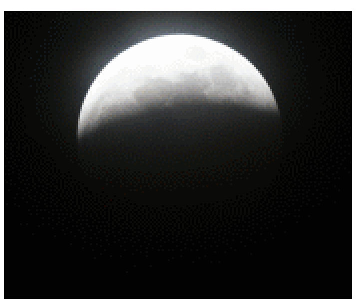

HEY, CHECK IT OUT: $e^{\pi} - \pi$ IS 19.999099979. THAT'S WEIRD.
YEAH. THAT'S HOW I GOT KICKED OUT OF THE ACM IN COLLEGE.
... WHAT?
DURING A COMPETITION, I TOLD THE PROGRAMMERS ON OUR TEAM THAT $e^{\pi} - \pi$ WAS A STANDARD TEST OF FLOATING-POINT HANDLERS -- IT WOULD COME OUT TO 20 UNLESS THEY HAD ROUNDING ERRORS.
THAT'S AWFUL.
YEAH, THEY DUG THROUGH HALF THEIR ALGORITHMS LOOKING FOR THE BUG BEFORE THEY FIGURED IT OUT.

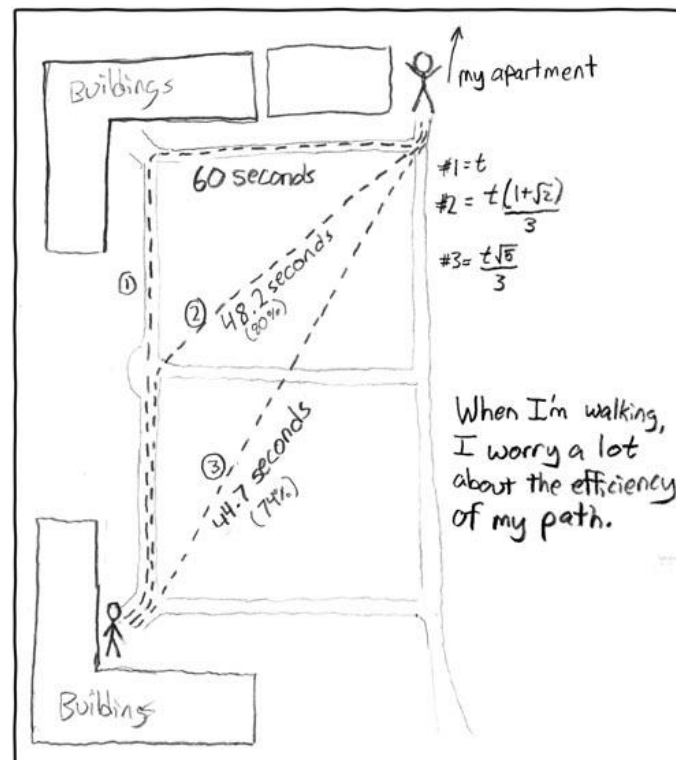
Things We Can Compute

Harmonics:



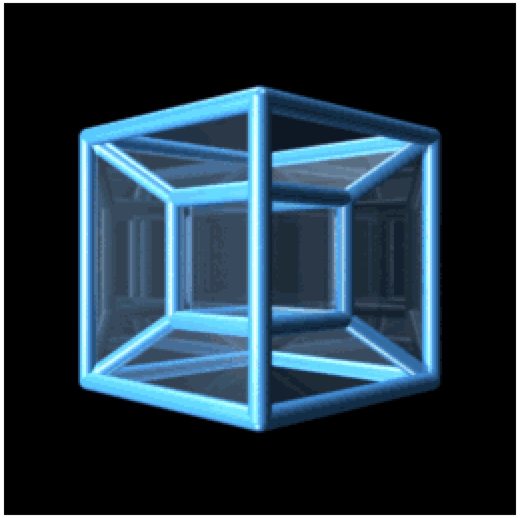
Eclipses:



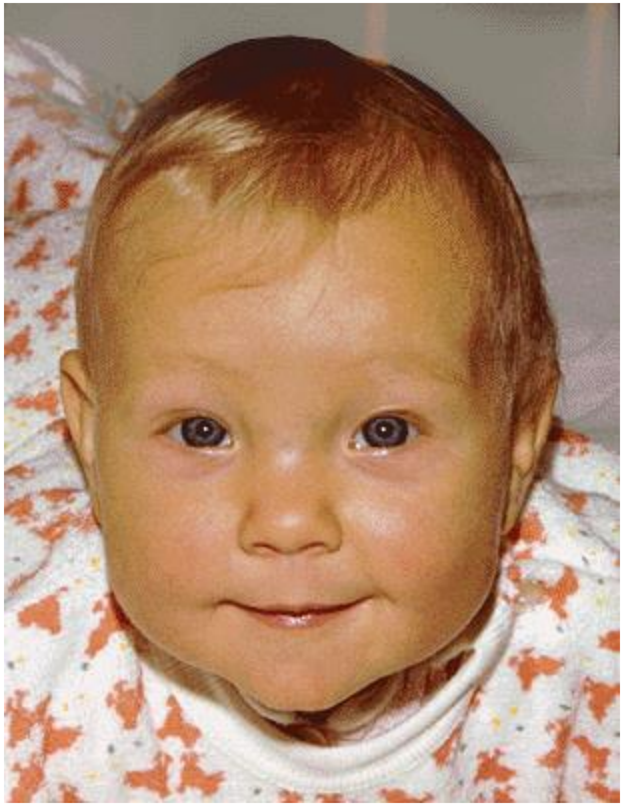


Things We Can Compute

Visualization:

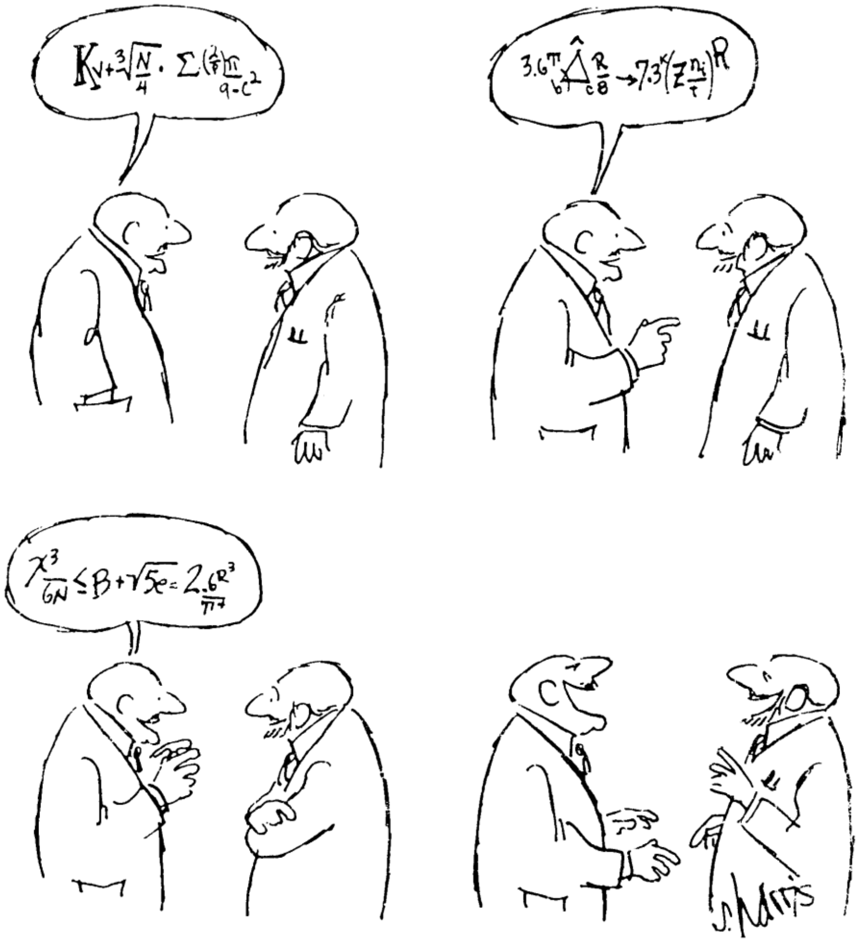


Morphing:



What Can't We Compute and Why?

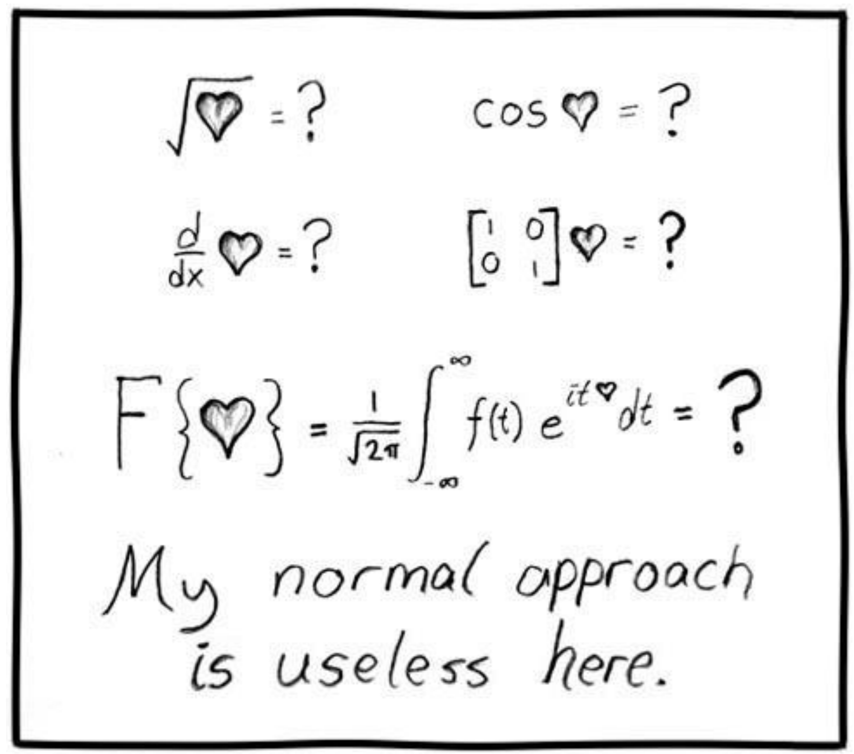
Humor:



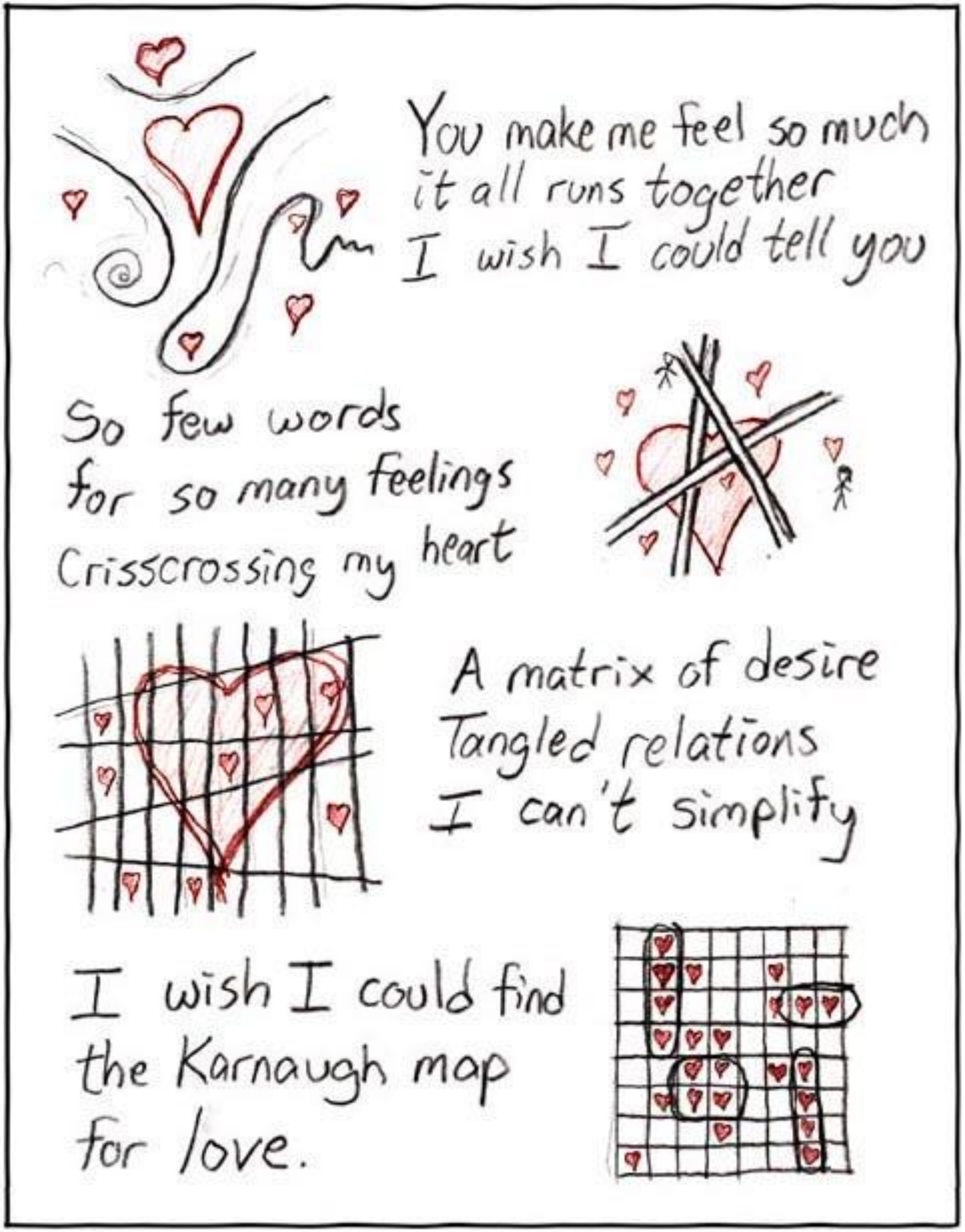
Issues: not well-defined, subjective, ambiguous

What Can’t We Compute and Why?

Emotions:

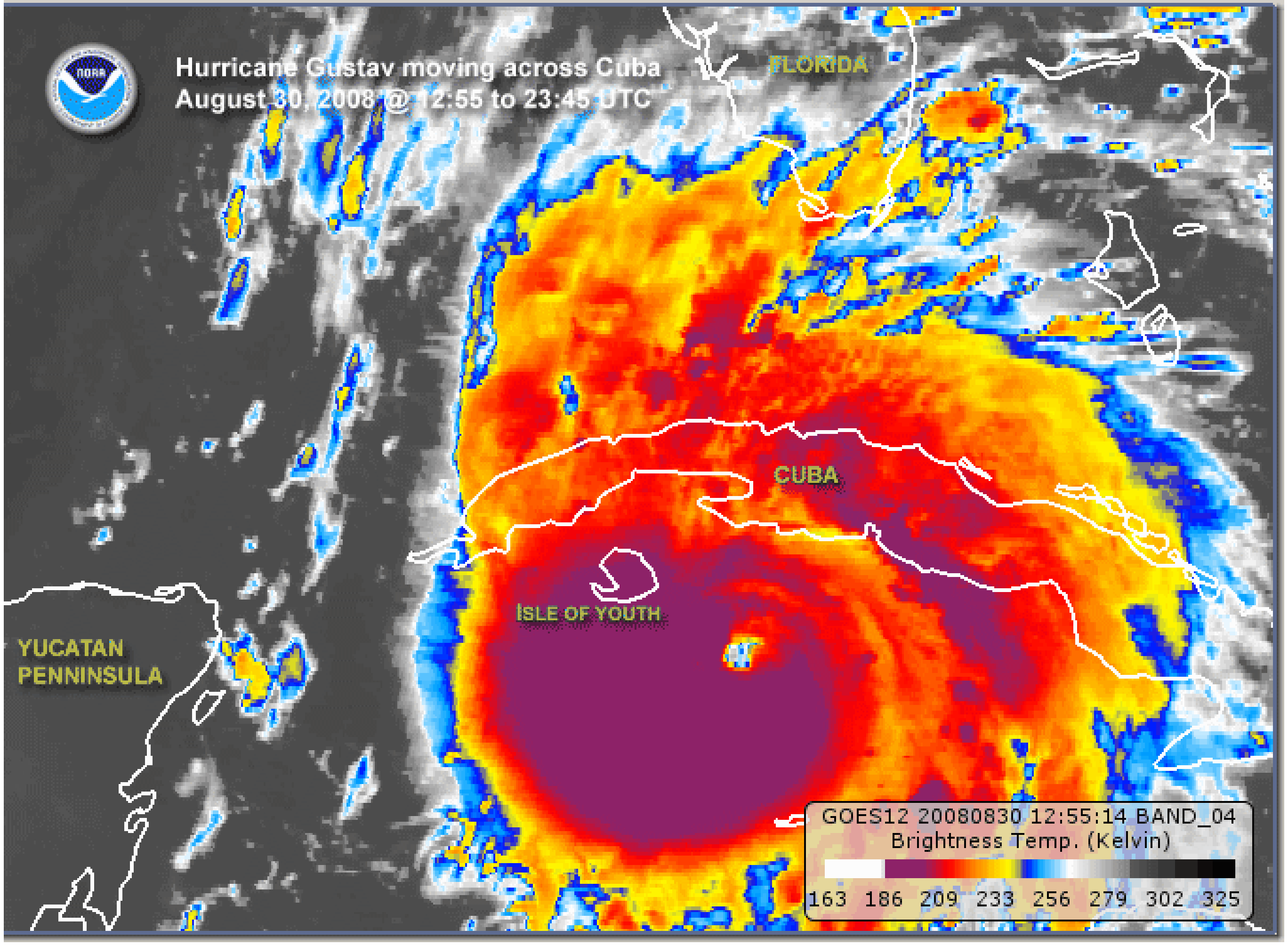


Issues: not well-defined, subjective, ambiguous


You make me feel so much
it all runs together
I wish I could tell you
So few words
for so many feelings
Crisscrossing my heart
A matrix of desire
Tangled relations
I can't simplify
I wish I could find
the Karnaugh map
for love.

What Can't We Compute and Why?

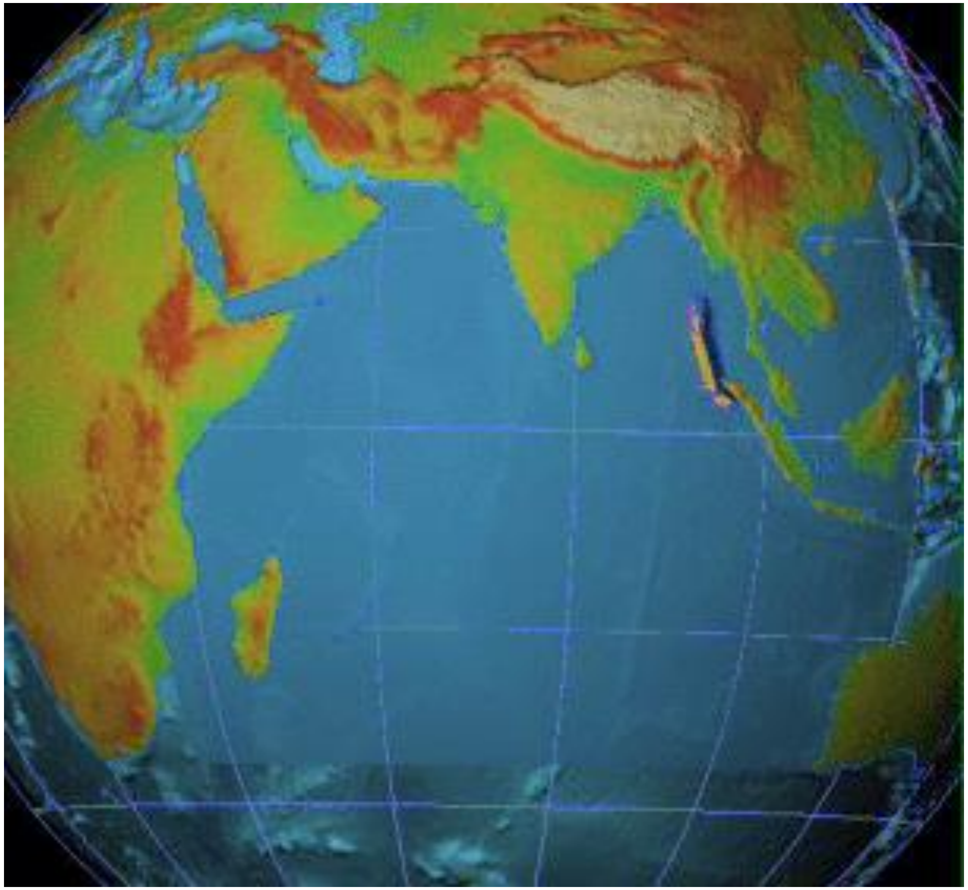
Weather:



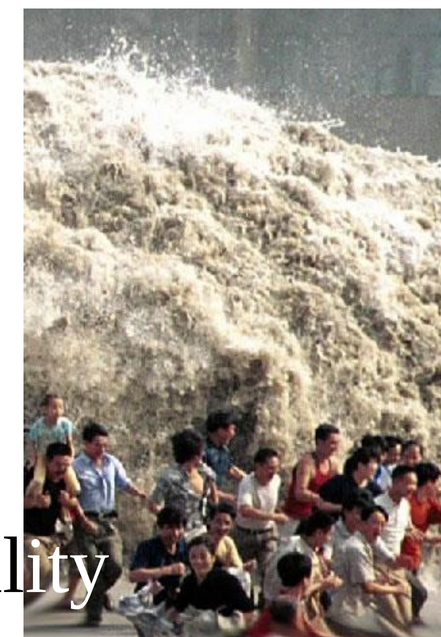
Issues: chaos, insufficient data, undecidability

What Can't We Compute and Why?

Tsunamis:



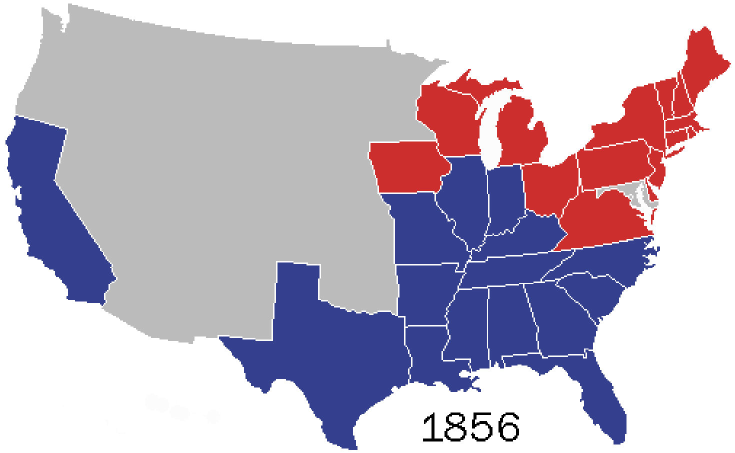
Dec 2004 trunami, 225,000 dead
Energy: 9.5 teratons, 100-ft waves



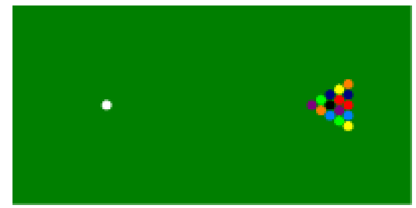
Issues: chaos, insufficient data, undecidability

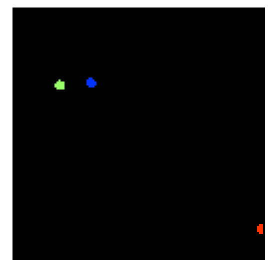
What Can't We Compute and Why?

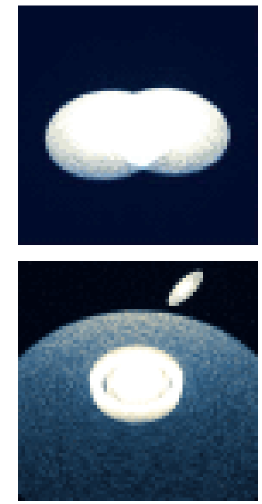
Elections:



N-Body systems:

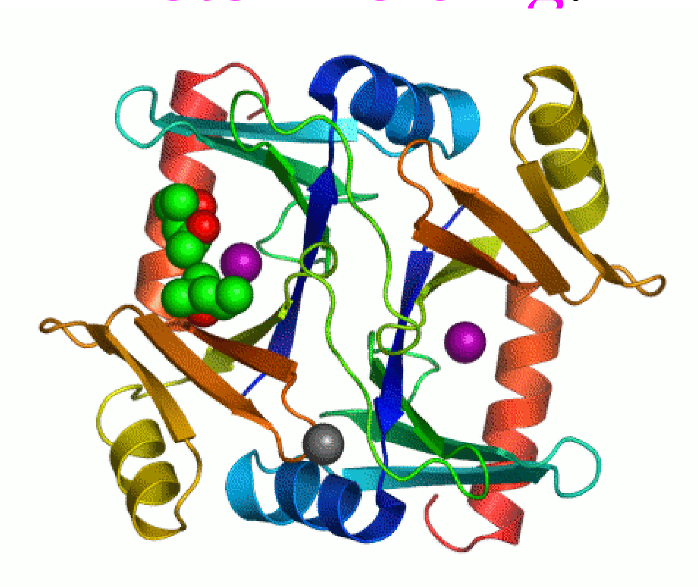




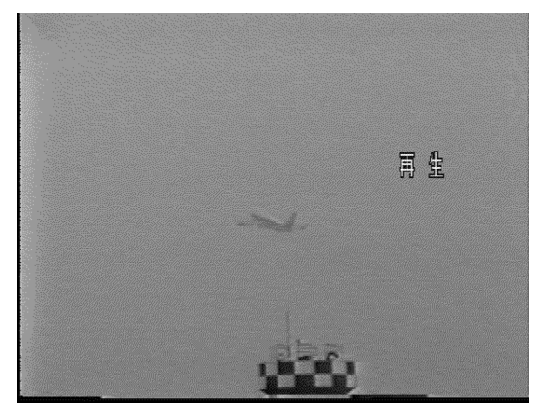


Galactic collisions

Protein folding:

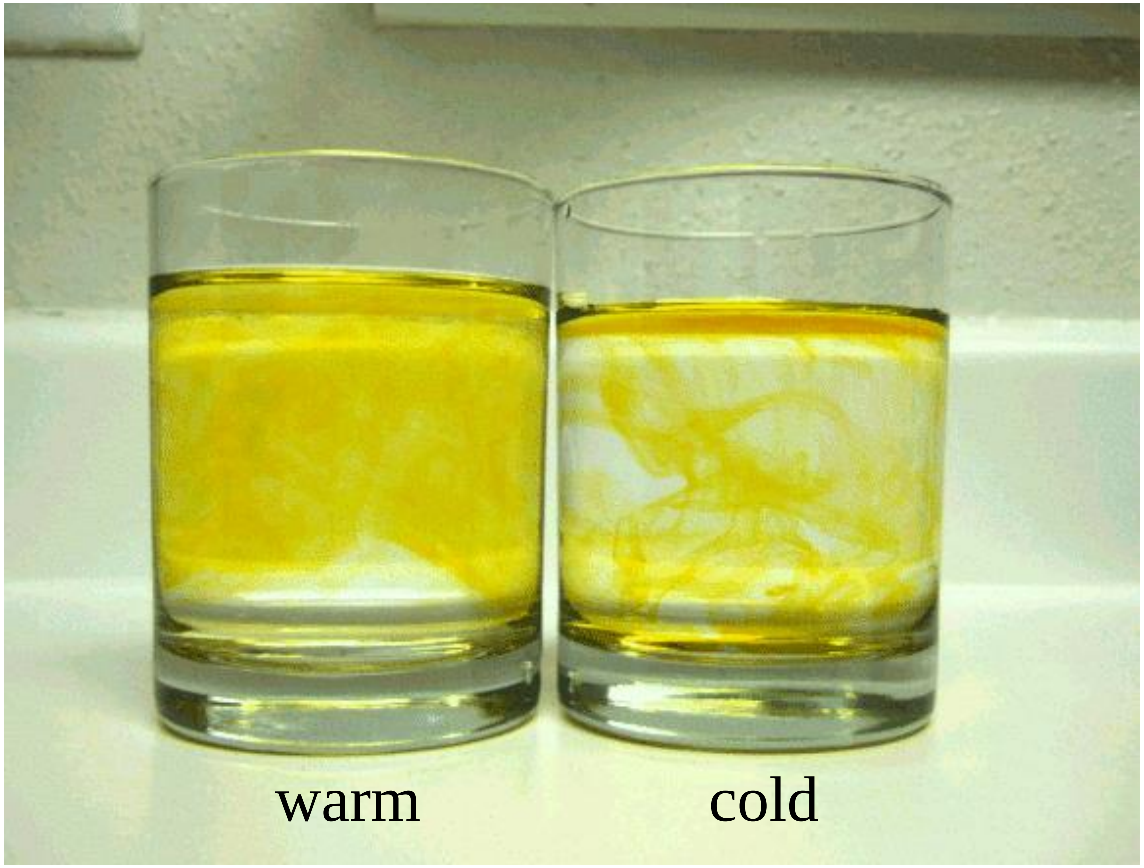


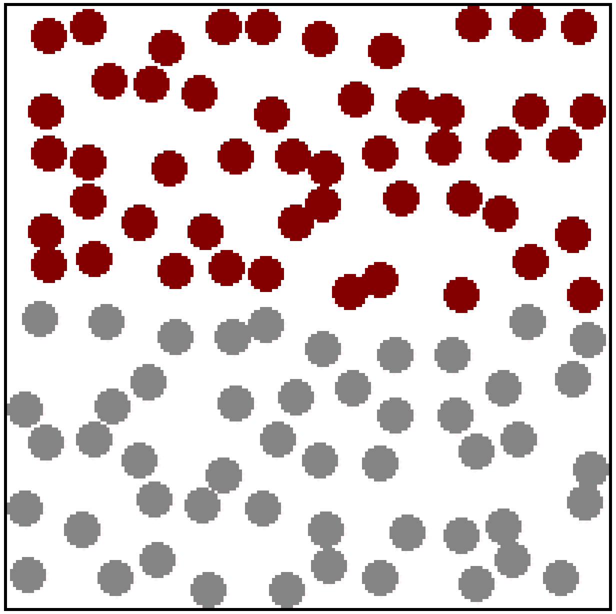
Lightning:



What Can't We Compute and Why?

Diffussion:

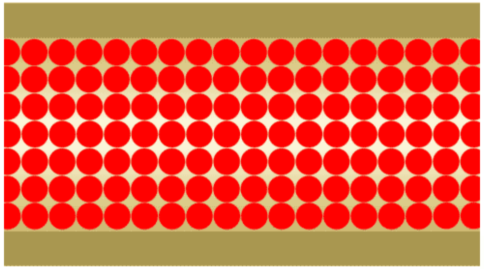


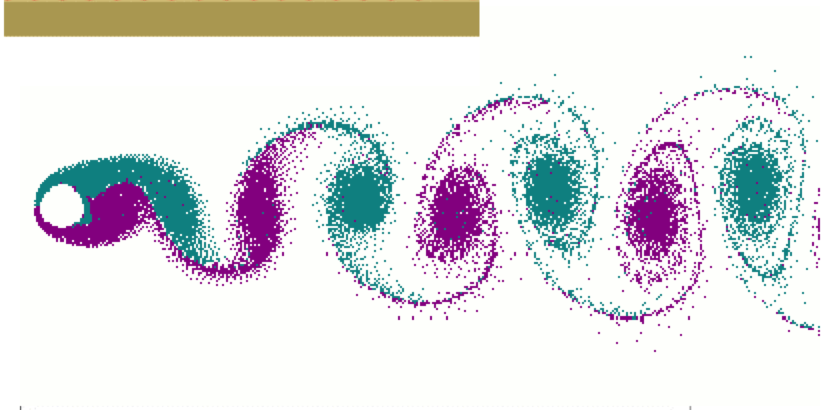


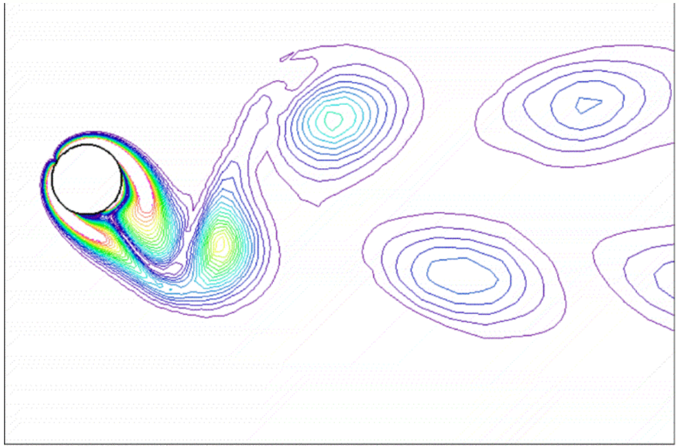
Issues: chaos, insufficient data, undecidability

What Can't We Compute and Why?

Turbulance:



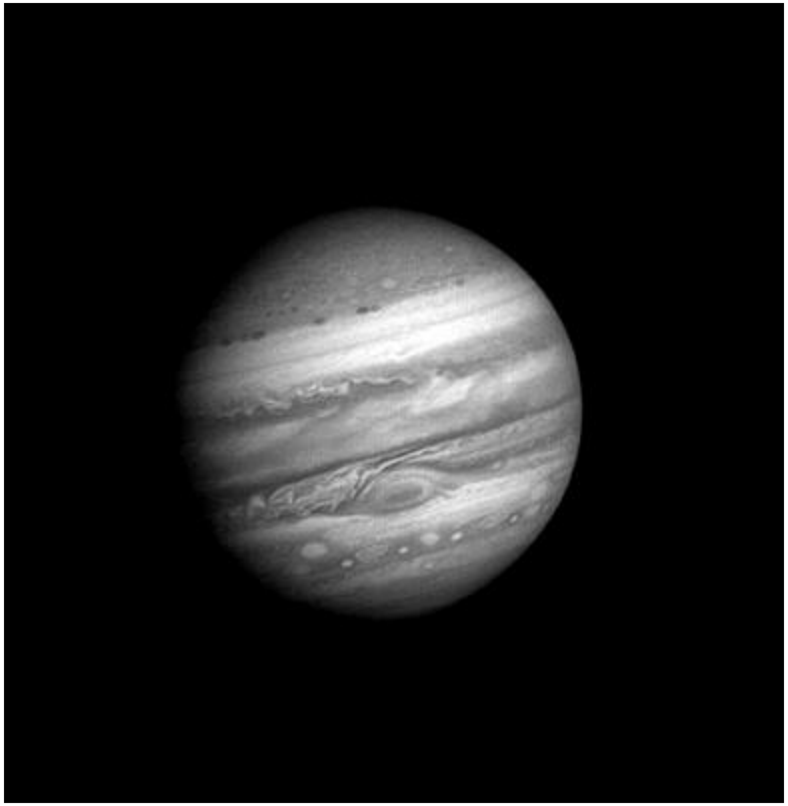




Issues: chaos, insufficient data, undecidability

What Can't We Compute and Why?

Turbulance:



Issues: chaos, insufficient data, undecidability

The Euler and Navier–Stokes equations describe the motion of a fluid in $\mathbb{R}^n$ ($n = 2$ or 3). These equations are to be solved for an unknown velocity vector $u(x,t) = (u_i(x,t))_{1\le i\le n} \in \mathbb{R}^n$ and pressure $p(x,t) \in \mathbb{R}$, defined for position $x \in \mathbb{R}^n$ and time $t \ge 0$. We restrict attention here to incompressible fluids filling all of $\mathbb{R}^n$. The *Navier–Stokes* equations are then given by

$$\frac{\partial}{\partial t}u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t) \qquad (x \in \mathbb{R}^n, t \ge 0), \tag{1}$$

$$\operatorname{div} u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \qquad (x \in \mathbb{R}^n, t \ge 0) \tag{2}$$

with initial conditions

$$u(x,0) = u^\circ(x) \qquad (x \in \mathbb{R}^n). \tag{3}$$

Here, $u^\circ(x)$ is a given, C^∞ divergence-free vector field on $\mathbb{R}^n$, $f_i(x,t)$ are the components of a given, externally applied force (e.g. gravity), ν is a positive coefficient (the viscosity), and $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ is the Laplacian in the space variables. The *Euler equations* are equations (1), (2), (3) with ν set equal to zero.

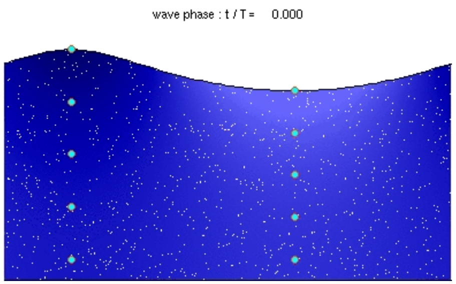
Equation (1) is just Newton's law $f = ma$ for a fluid element subject to the external force $f = (f_i(x,t))_{1\le i\le n}$ and to the forces arising from pressure and friction. Equation (2) just says that the fluid is incompressible. For physically reasonable

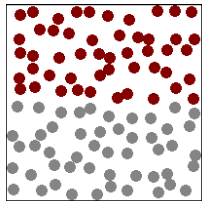
Theory vs. Reality Chasms

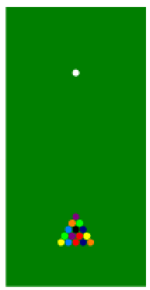
Navier–Stokes equations:

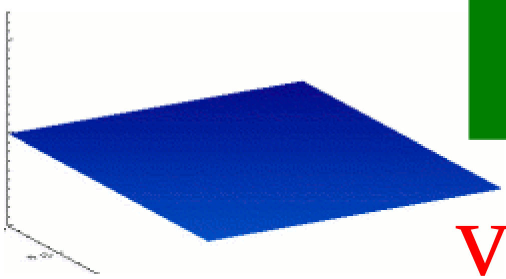
$$\frac{\partial}{\partial t}u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t)$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \nabla \cdot \mathbb{T} + \mathbf{f} \qquad \operatorname{div} u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0$$









vs.

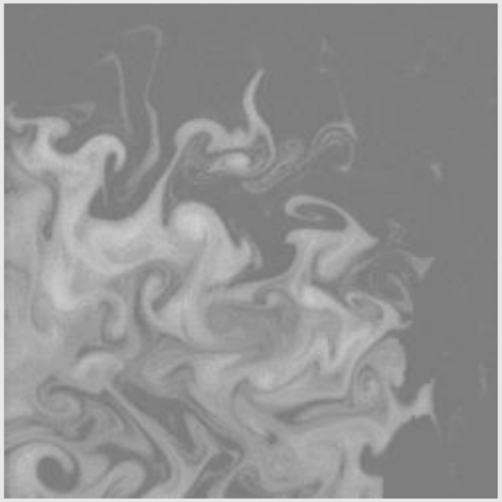
Clay Mathematics Institute

Dedicated to increasing and disseminating mathematical knowledge

Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

- The Millennium Problems
- Official Problem Description — Charles Fefferman
- Lecture by Luis Cafarelli (video)



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Millennium Problems

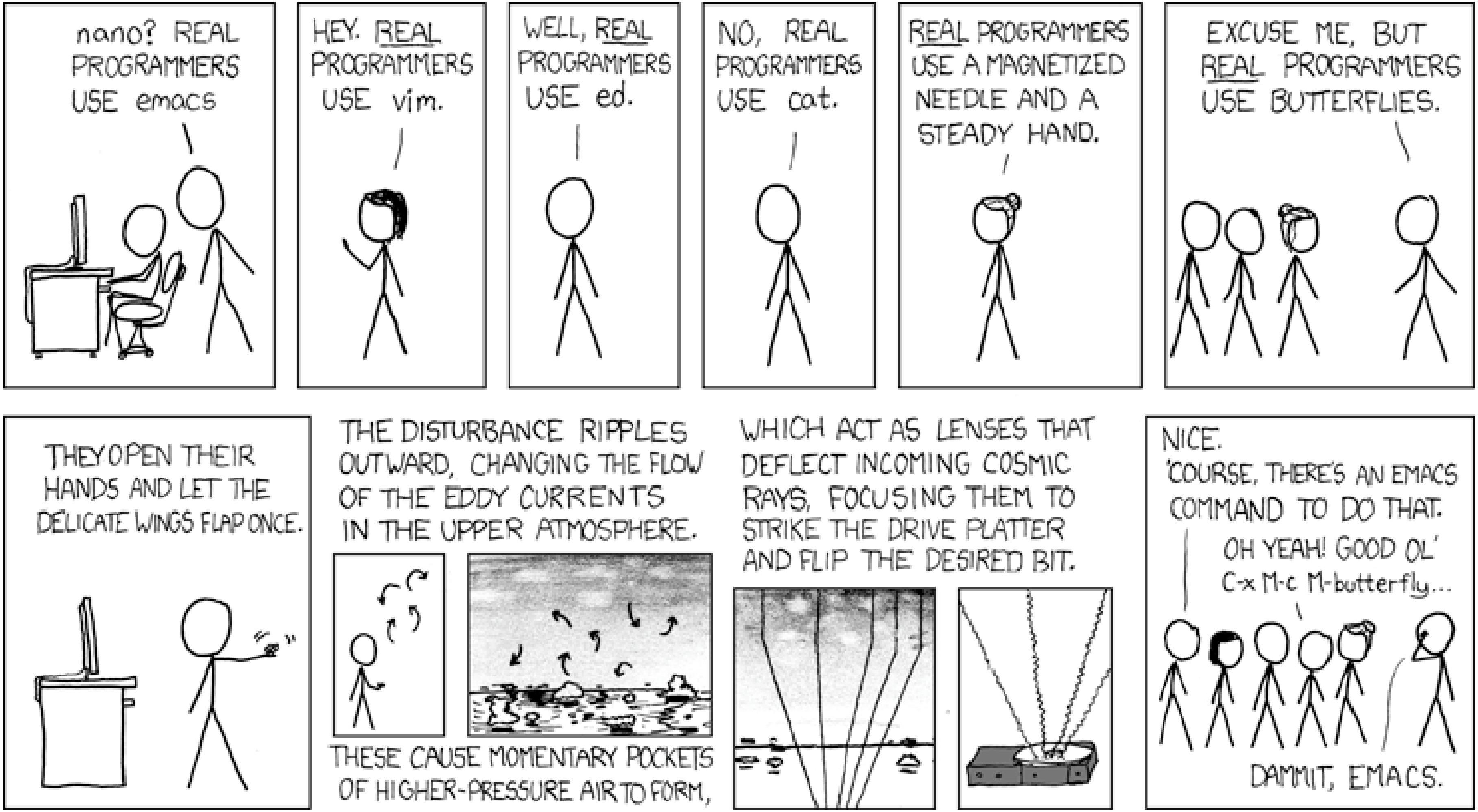
In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven *Prize Problems*. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each. During the Millennium Meeting held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled *The Importance of Mathematics*, aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

One hundred years earlier, on August 8, 1900, David Hilbert delivered his famous lecture about open mathematical problems at the second International Congress of Mathematicians in Paris. This influenced our decision to announce the millennium problems as the central theme of a Paris meeting.

The rules for the award of the prize have the endorsement of the CMI Scientific Advisory Board and the approval of the Directors. The members of these boards have the responsibility to preserve the nature, the integrity, and the spirit of this prize.

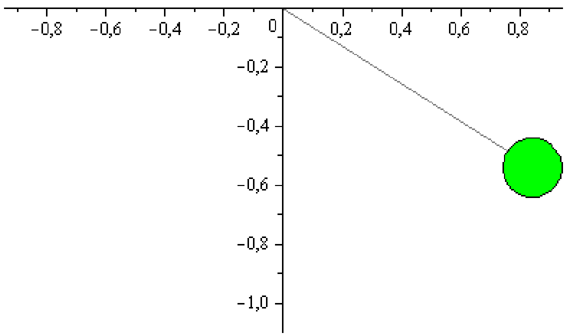
- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP ??
- ~~Poincaré Conjecture~~ Perelman 2006
- Riemann Hypothesis
- Yang-Mills Theory

- Rules
- Millennium Meeting Videos


nano? REAL PROGRAMMERS USE emacs
HEY. REAL PROGRAMMERS USE vim.
WELL, REAL PROGRAMMERS USE ed.
NO, REAL PROGRAMMERS USE cat.
REAL PROGRAMMERS USE A MAGNETIZED NEEDLE AND A STEADY HAND.
EXCUSE ME, BUT REAL PROGRAMMERS USE BUTTERFLIES.
THEY OPEN THEIR HANDS AND LET THE DELICATE WINGS FLAP ONCE.
THE DISTURBANCE RIPPLES OUTWARD, CHANGING THE FLOW OF THE EDDY CURRENTS IN THE UPPER ATMOSPHERE.
THESE CAUSE MOMENTARY POCKETS OF HIGHER-PRESSURE AIR TO FORM,
WHICH ACT AS LENSES THAT DEFLECT INCOMING COSMIC RAYS, FOCUSING THEM TO STRIKE THE DRIVE PLATTER AND FLIP THE DESIRED BIT.
NICE.
'COURSE, THERE'S AN EMACS COMMAND TO DO THAT.
OH YEAH! GOOD OL' C-x M-c M-butterfly...
DAMMIT, EMACS.

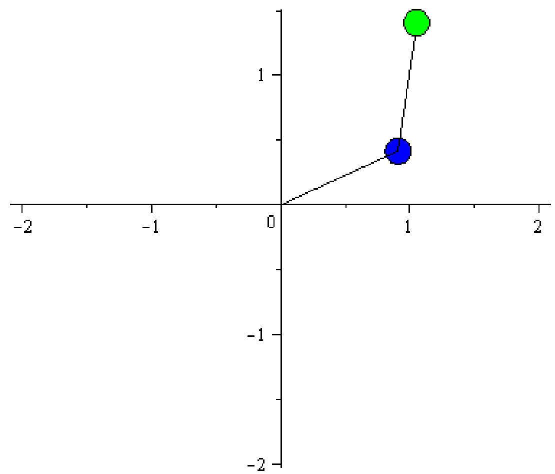
Simple Chaotic Systems

Compound pendulums:



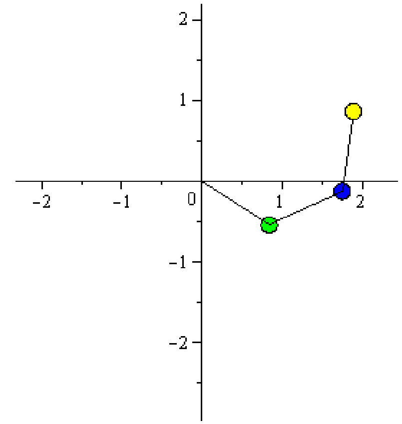
$\frac{d^2}{dt^2}\phi(t)+\frac{g}{l}\phi(t)=0 \Rightarrow \phi(t)=\phi(0)\cos\left(\sqrt{\frac{g}{l}}\, t\right)$

$l=1\text{ m};\ \phi(0)=1\text{ rad};\ \frac{d}{dt}\phi(0)=0;\ T=2\pi\sqrt{\frac{1\text{ m}}{g}}\sim 2\text{ s}$



$l_1=l_2=1\text{ m};\ m_1=m_2=1\text{ kg};$

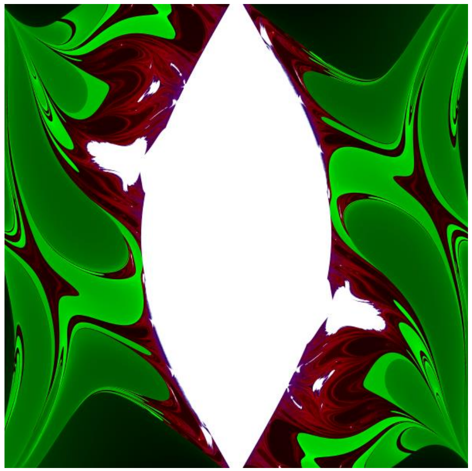
$\phi_1(0)=2\text{ rad};\ \phi_2(0)=3\text{ rad};\ \frac{d}{dt}\phi_1(0)=\frac{d}{dt}\phi_2(0)=0$



$l_1=l_2=l_3=1\text{ m};\ m_1=m_2=m_3=1\text{ kg};$

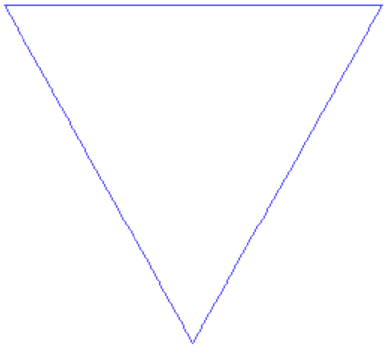
$\phi_1(0)=1\text{ rad};\ \phi_2(0)=2\text{ rad};\ \phi_3(0)=3\text{ rad};$

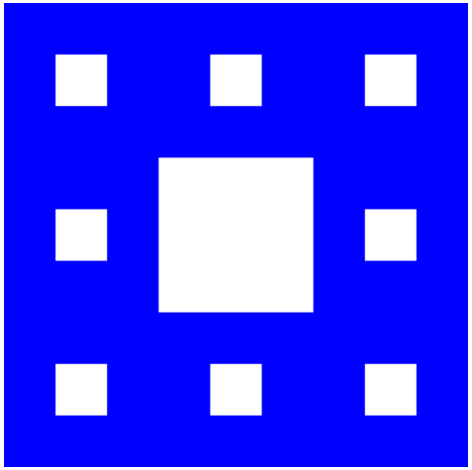
$\frac{d}{dt}\phi_1(0)=\frac{d}{dt}\phi_2(0)=\frac{d}{dt}\phi_3(0)=0$

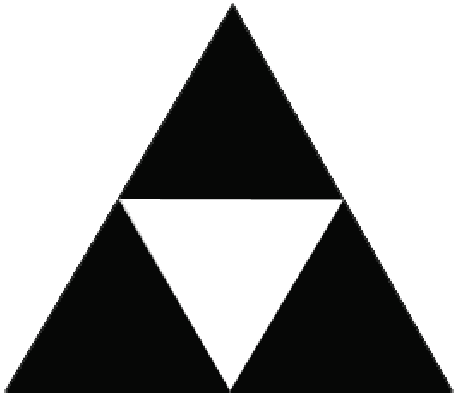


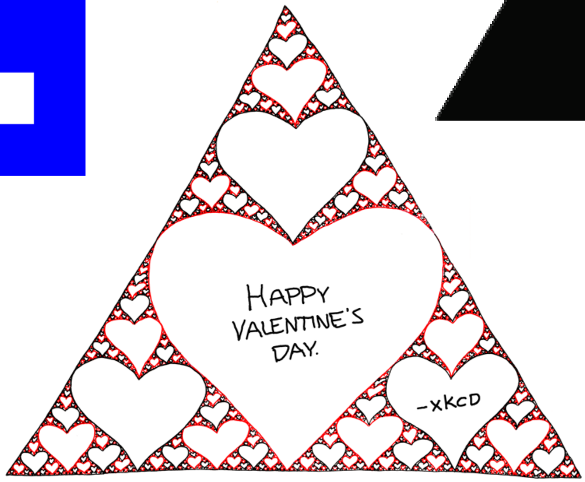
Issues: chaos, undecidability

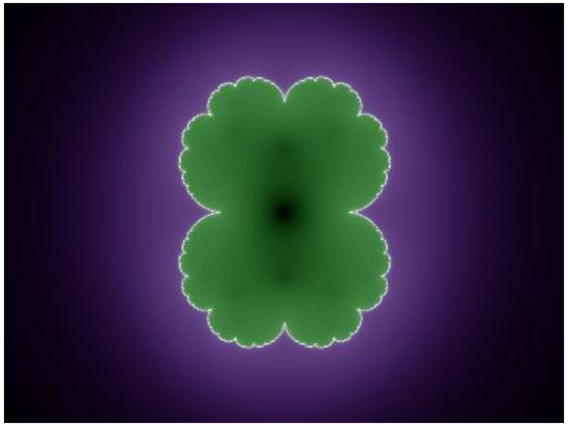
Simple Chaotic Systems: Fractals

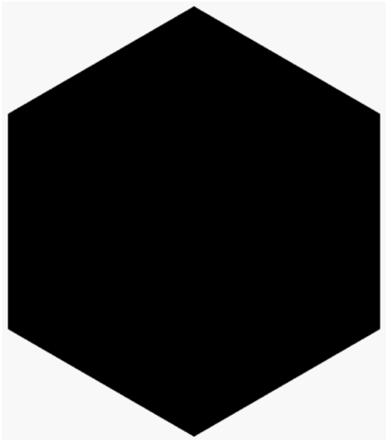


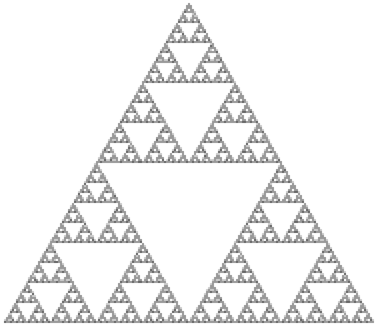


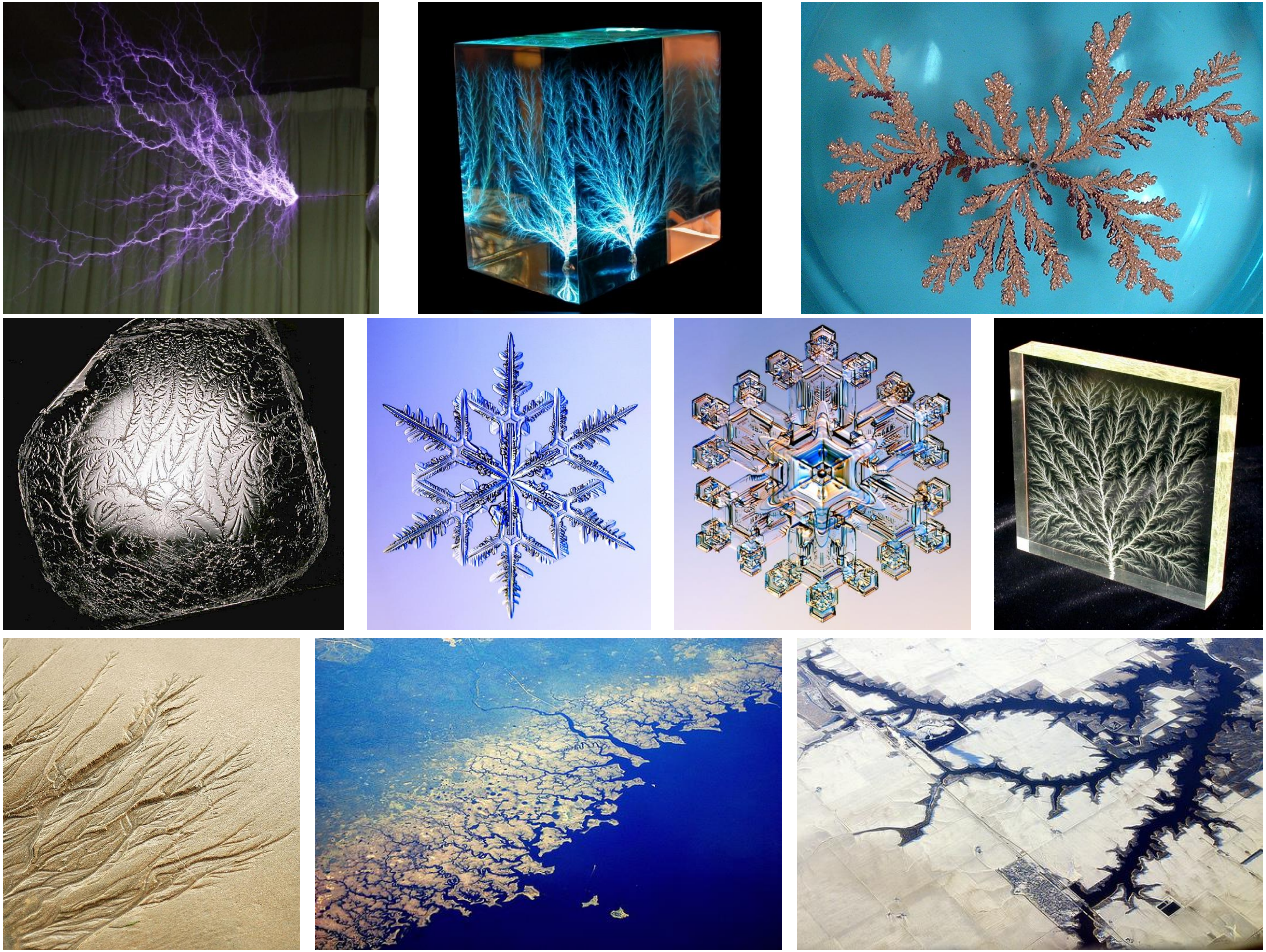






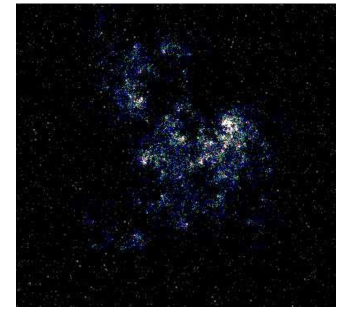


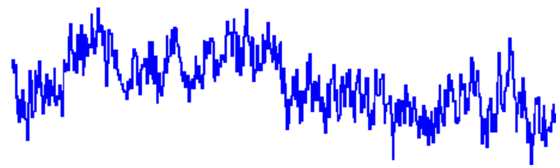


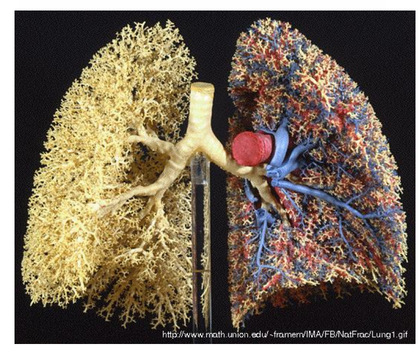


Applications of Fractals

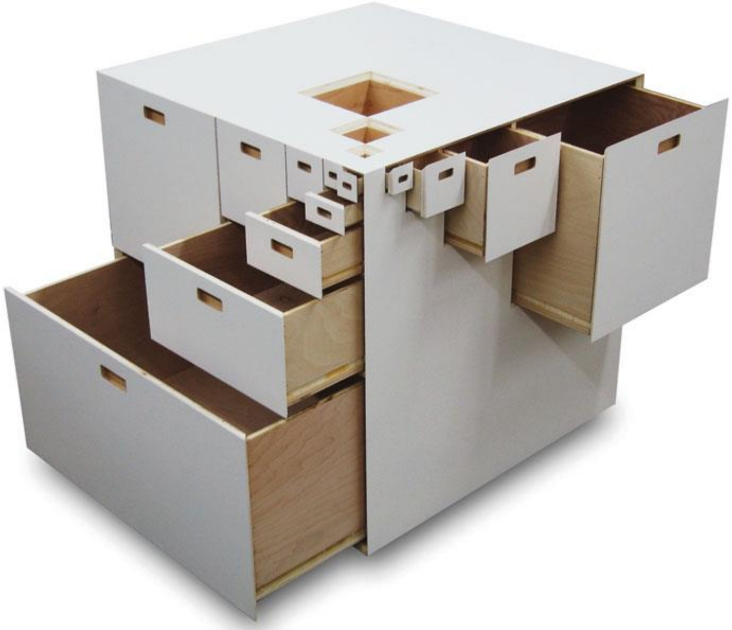
- Compressing images
- Simulating galaxies
- Analyzing markets
- Generating music
- Modeling weather
- Movie special effects
- Designing video games
- Describing crystal growth
- Understanding anatomy
- Explaining plant forms
- Tracking populations
- Fashion design

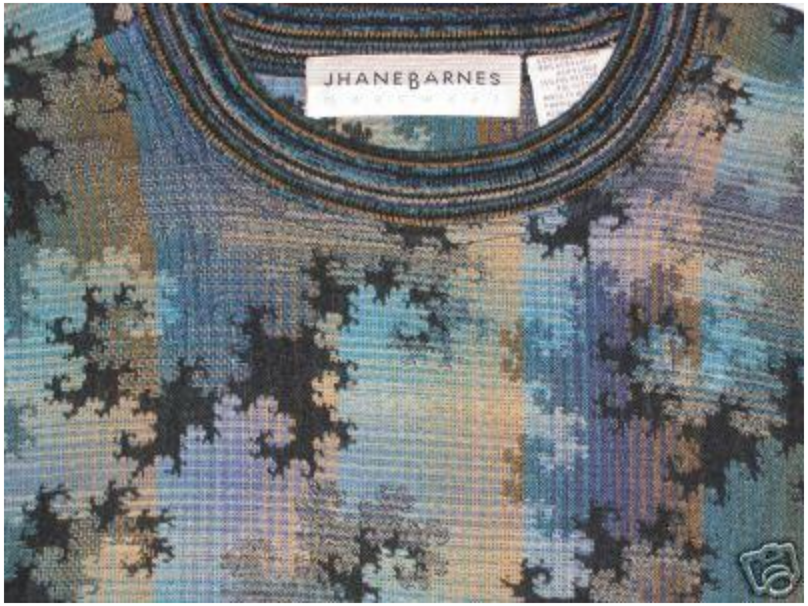






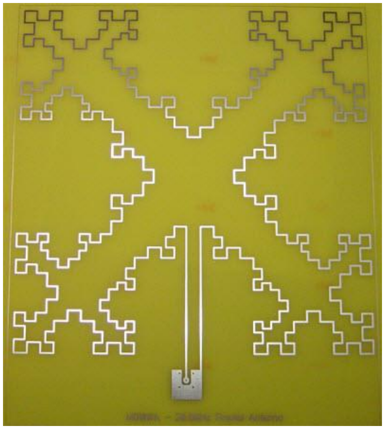
Applications of Fractals

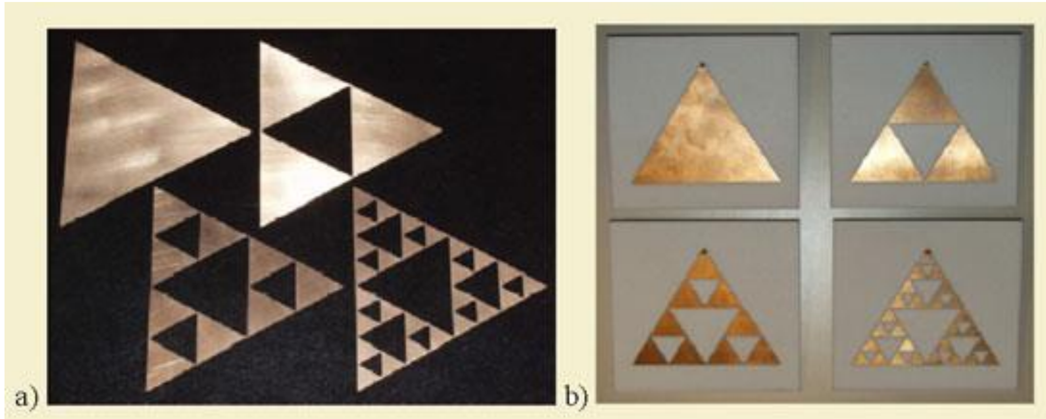


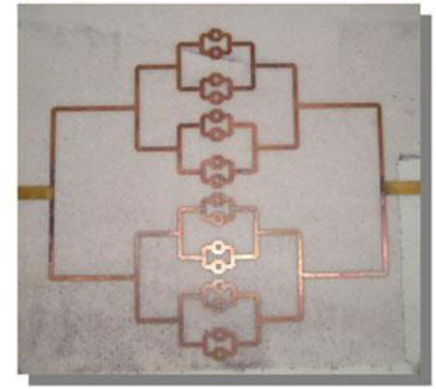


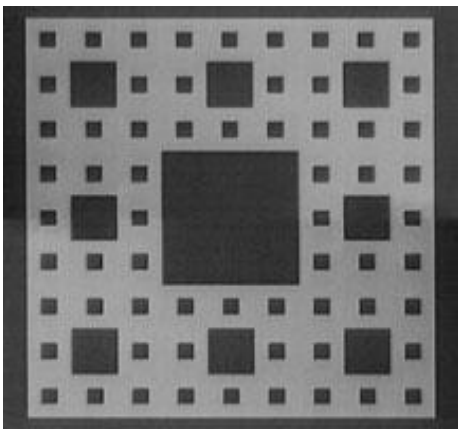
"WE DID THE WHOLE ROOM OVER IN FRACTALS."

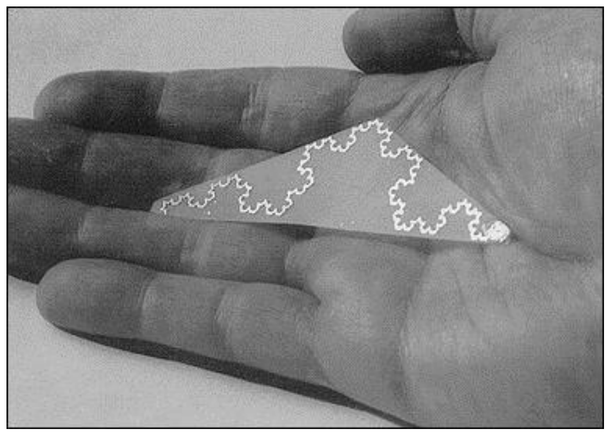
Fractal Antennas

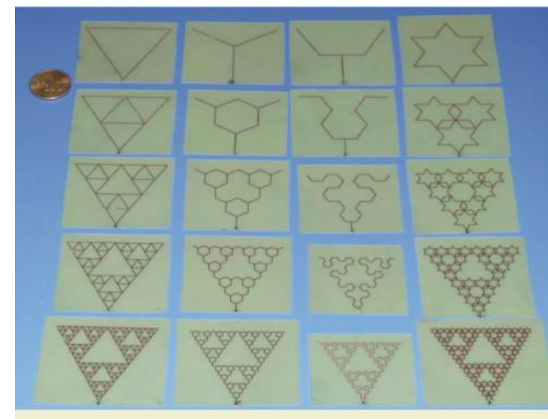


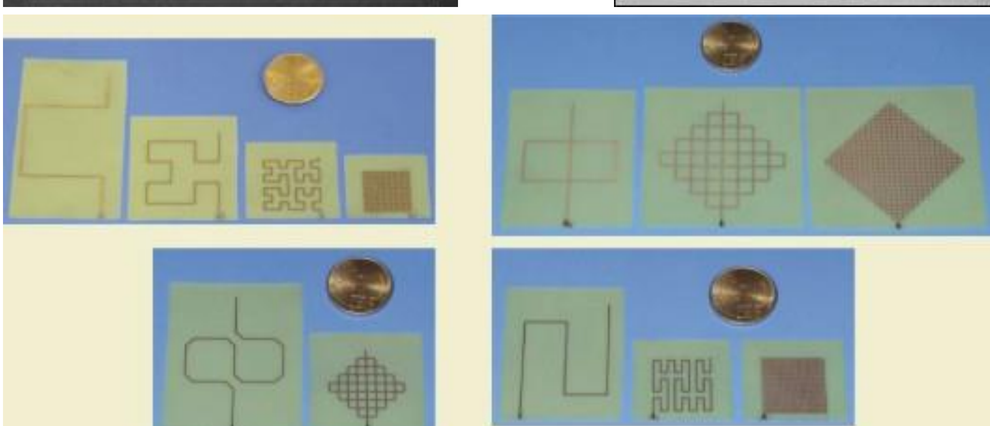


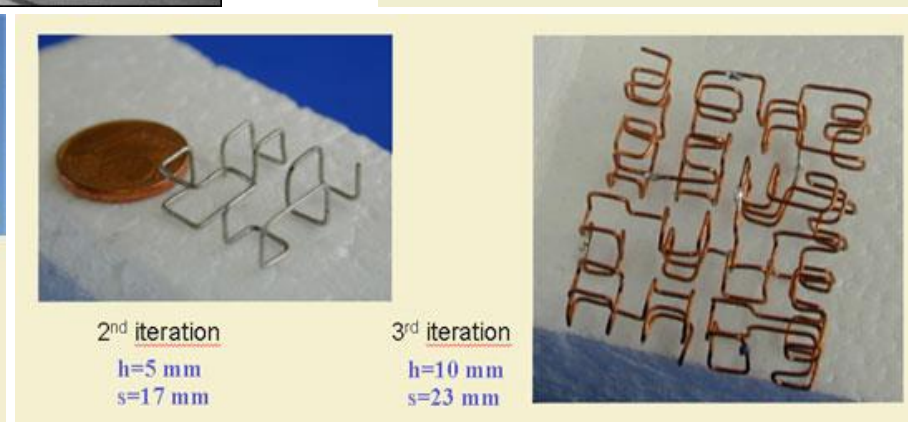


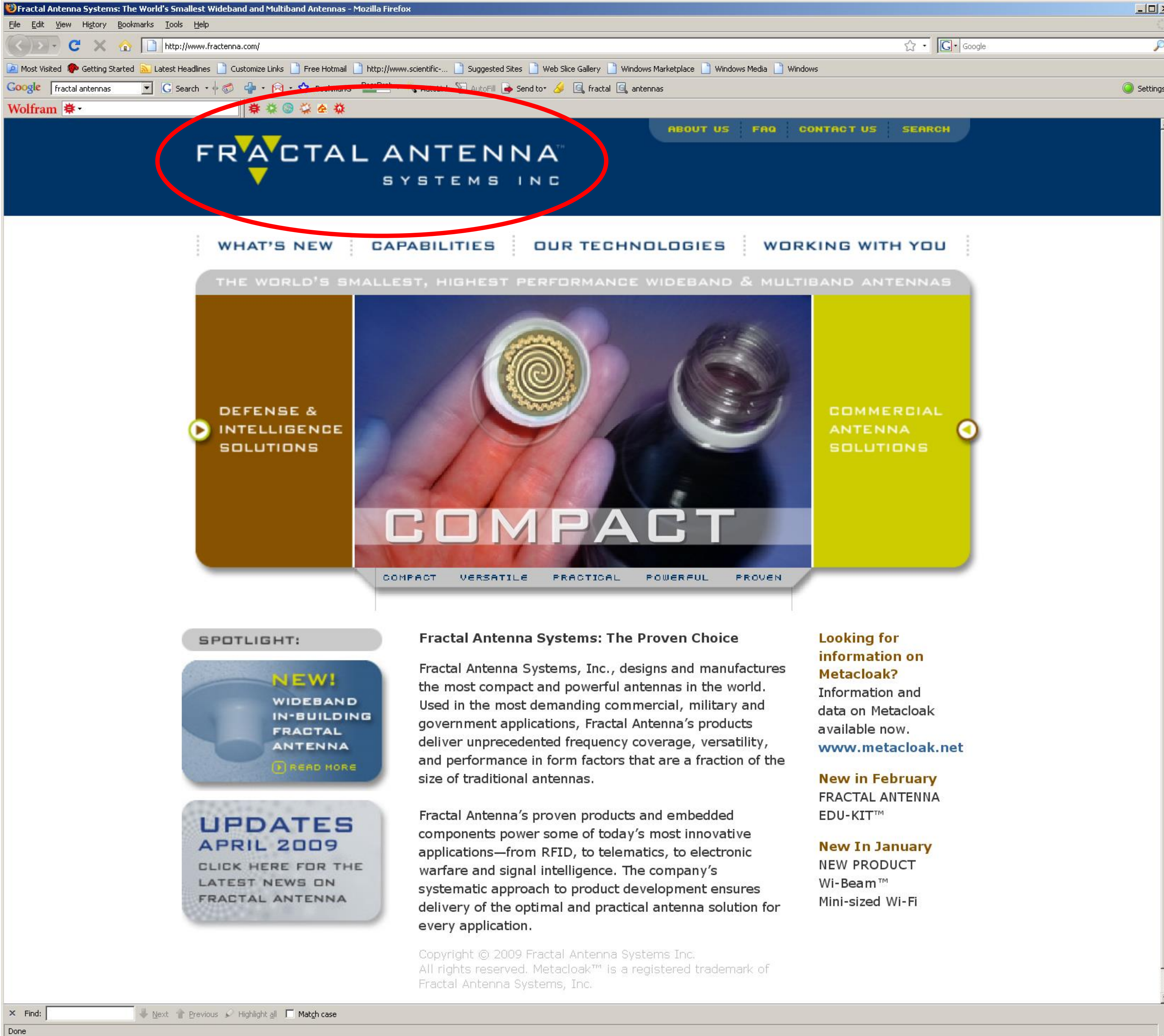








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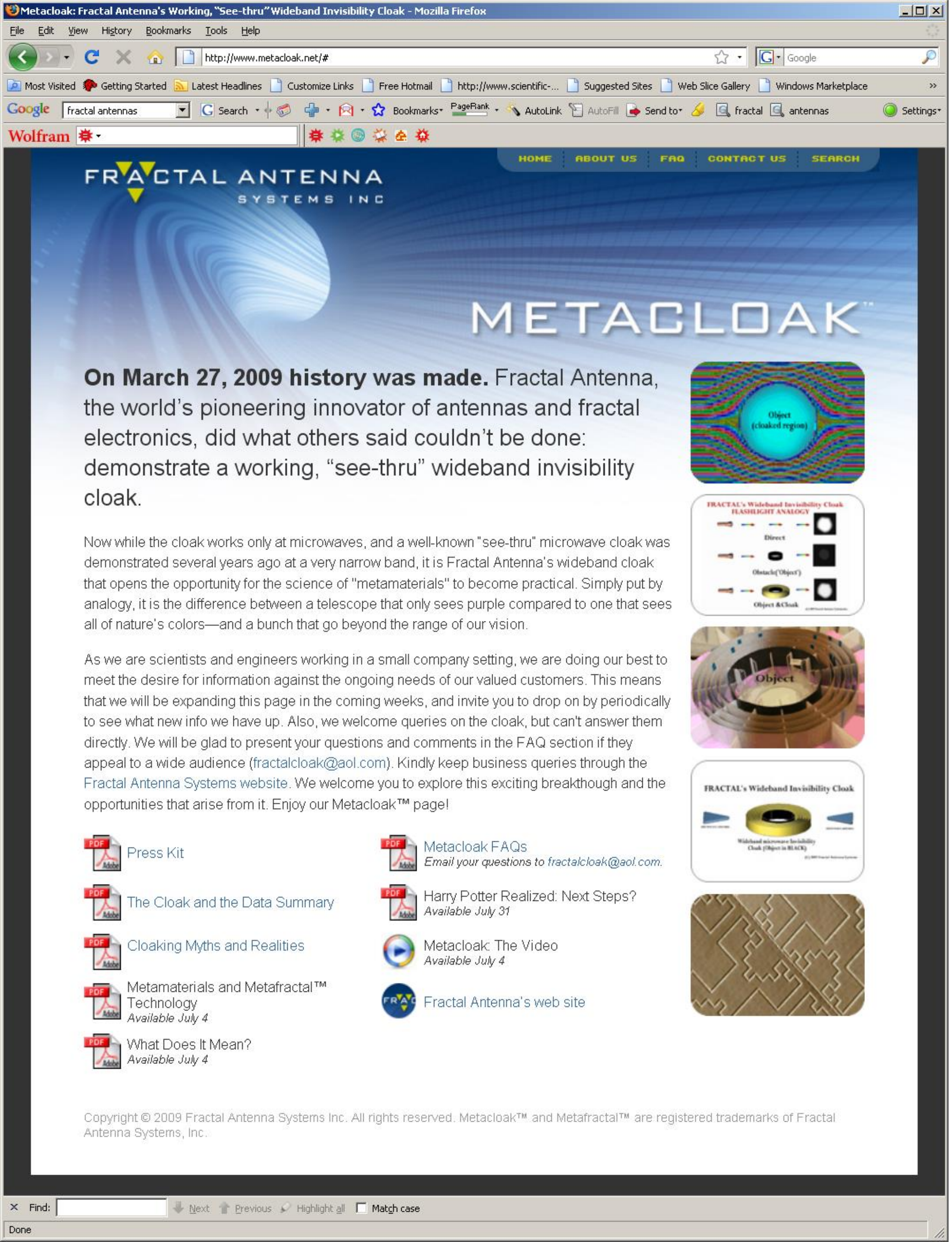
Metacloak: Fractal Antenna's Working, "See-thru" Wideband Invisibility Cloak - Mozilla Firefox

http://www.metacloak.net/#

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FRACTAL ANTENNA SYSTEMS INC

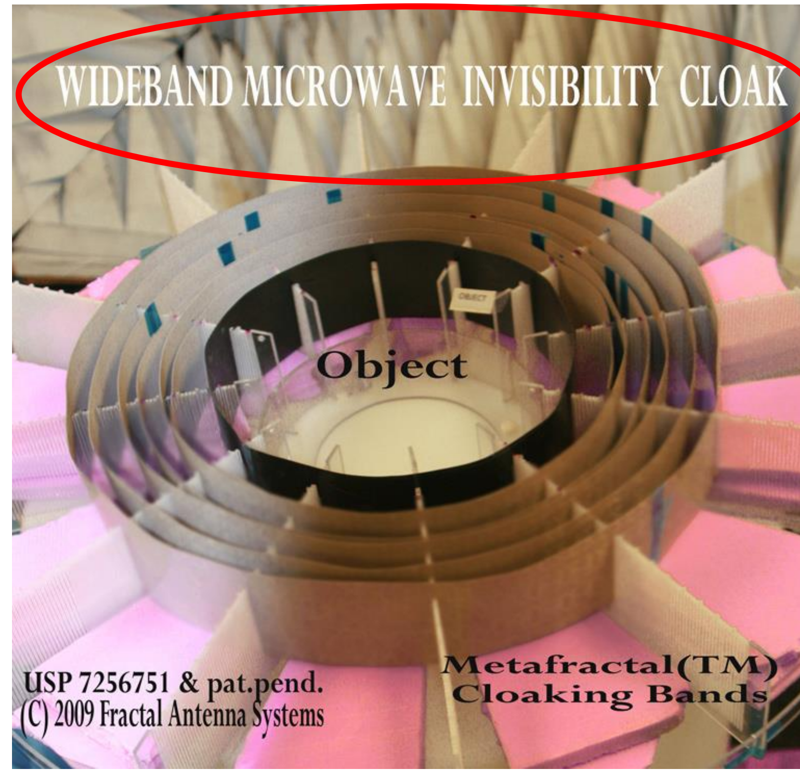
METACLOAK™

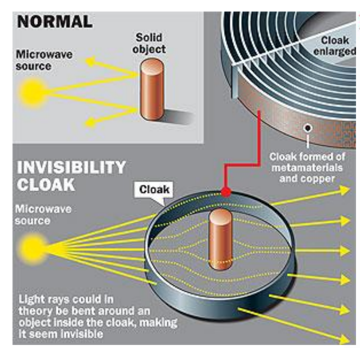
On March 27, 2009 history was made. Fractal Antenna, the world's pioneering innovator of antennas and fractal electronics, did what others said couldn't be done: demonstrate a working, "see-thru" wideband invisibility cloak.

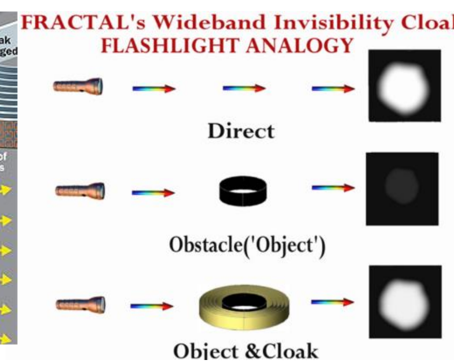
Now while the cloak works only at microwaves, and a well-known "see-thru" microwave cloak was demonstrated several years ago at a very narrow band, it is Fractal Antenna's wideband cloak that opens the opportunity for the science of "metamaterials" to become practical. Simply put by analogy, it is the difference between a telescope that only sees purple compared to one that sees all of nature's colors—and a bunch that go beyond the range of our vision.

As we are scientists and engineers working in a small company setting, we are doing our best to meet the desire for information against the ongoing needs of our valued customers. This means that we will be expanding this page in the coming weeks, and invite you to drop on by periodically to see what new info we have up. Also, we welcome queries on the cloak, but can't answer them directly. We will be glad to present your questions and comments in the FAQ section if they appeal to a wide audience (fractalcloak@aol.com). Kindly keep business queries through the Fractal Antenna Systems website. We welcome you to explore this exciting breakthough and the opportunities that arise from it. Enjoy our Metacloak™ page!

- Press Kit
- The Cloak and the Data Summary
- Cloaking Myths and Realities
- Metamaterials and Metafractal™ Technology *Available July 4*
- What Does It Mean? *Available July 4*
- Metacloak FAQs *Email your questions to fractalcloak@aol.com.*
- Harry Potter Realized: Next Steps? *Available July 31*
- Metacloak: The Video *Available July 4*
- Fractal Antenna's web site



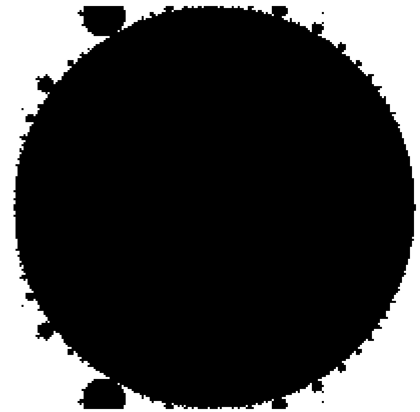




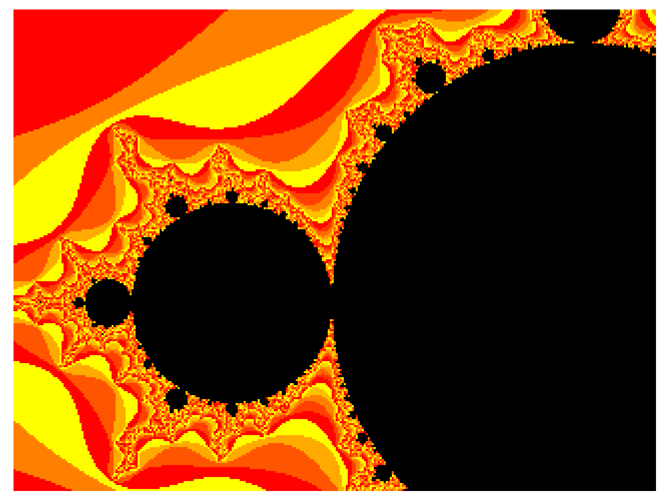
Simple Chaotic Systems: Fractals

$P_c(z)=z^2+c$ $\quad$ $P_c:\mathbb{C}\rightarrow\mathbb{C}$

$Q(c,n)= P_c(Q(c,n-1))$ $\quad$ $Q(c,0)=0$

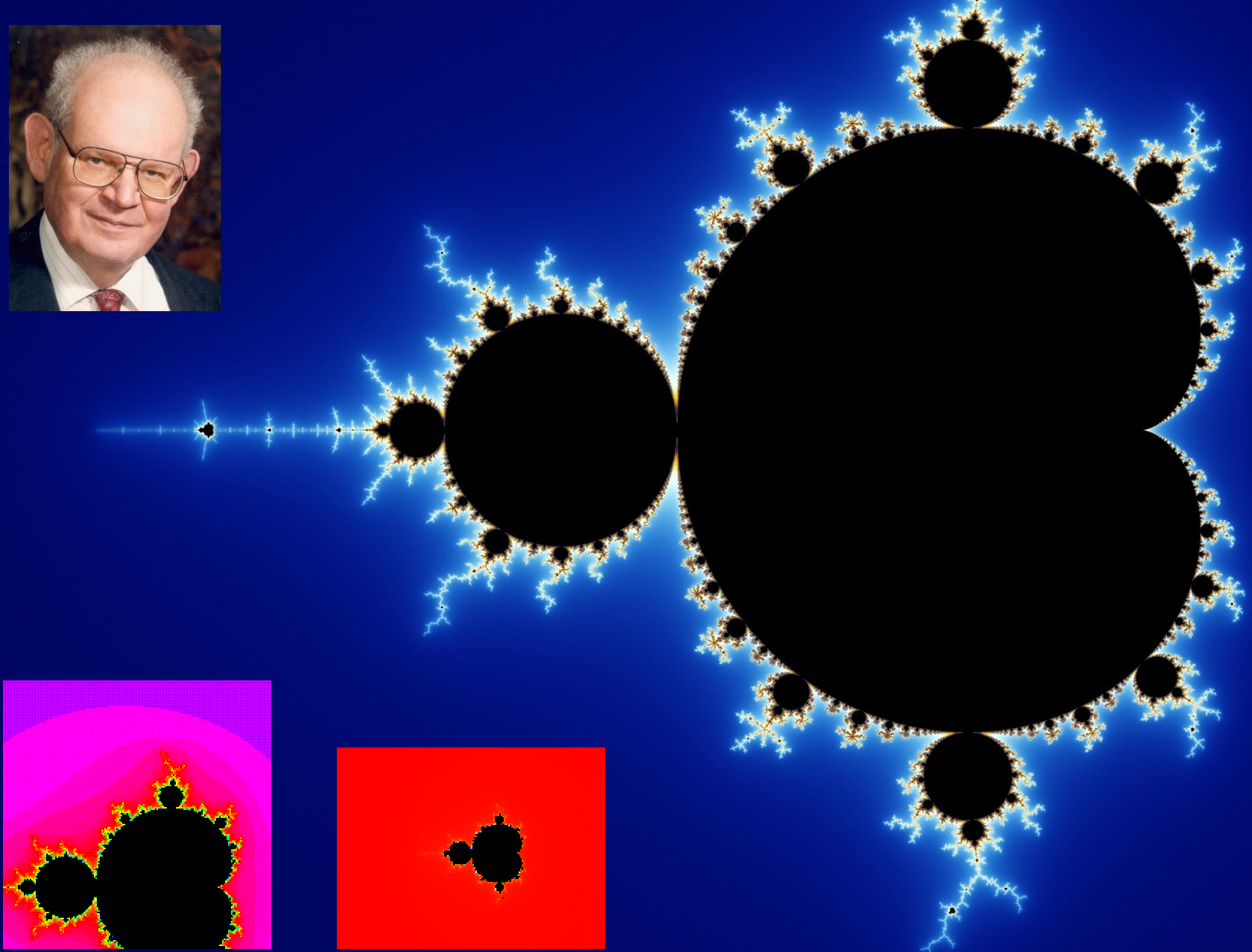


Mandelbrot set $= \{c\in\mathbb{C} \mid Q(c,n)<2\ \forall n\}$

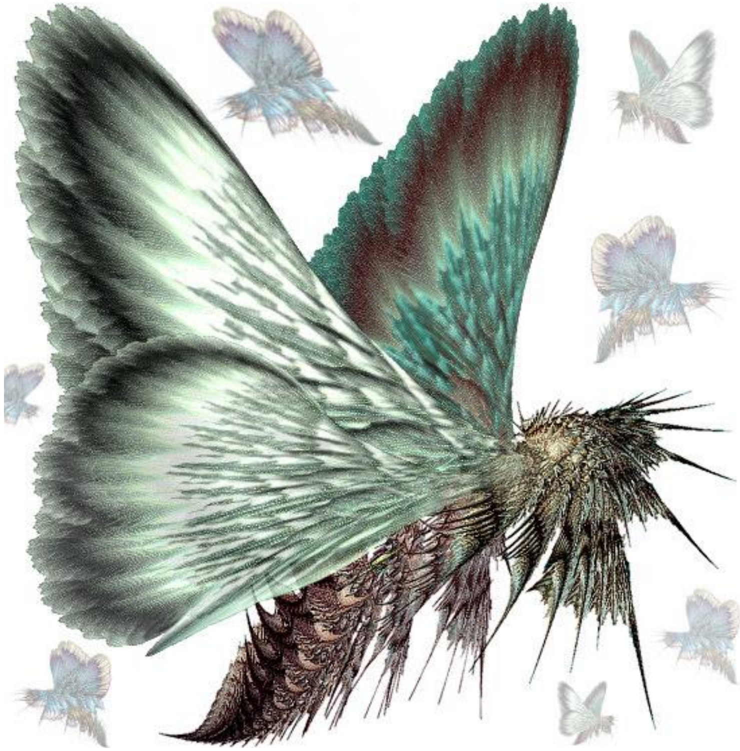


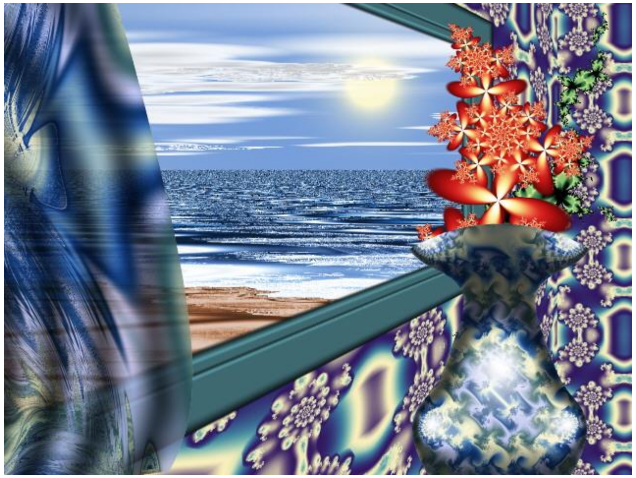
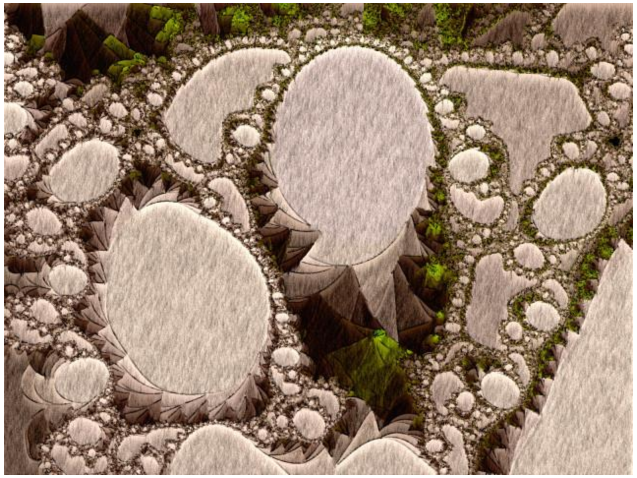
Issues: chaos, undecidability, incompleteness

The Mandelbrot Set



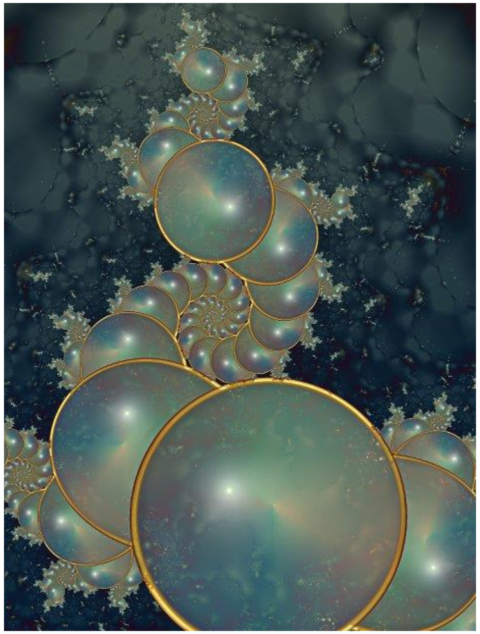
Fractal Art

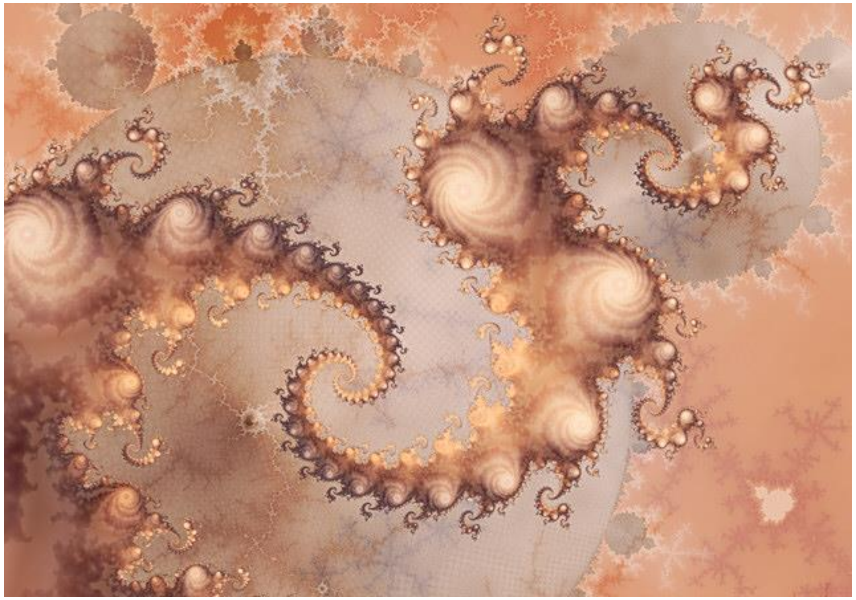
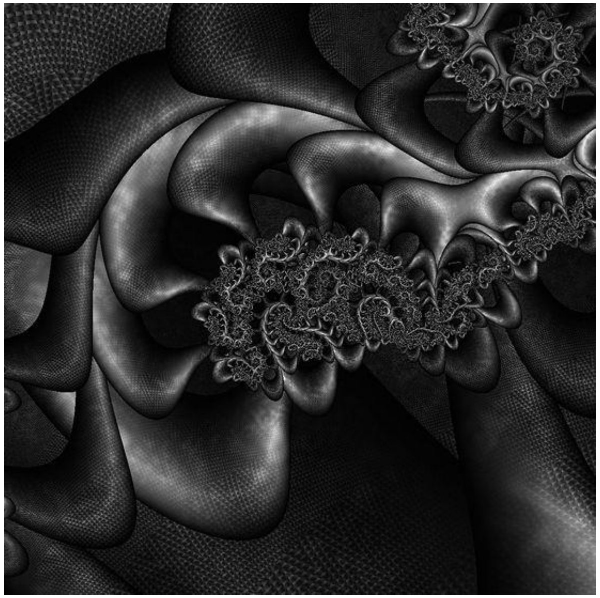
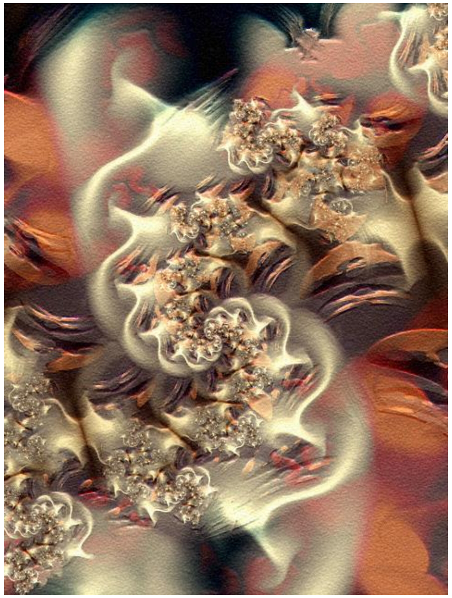


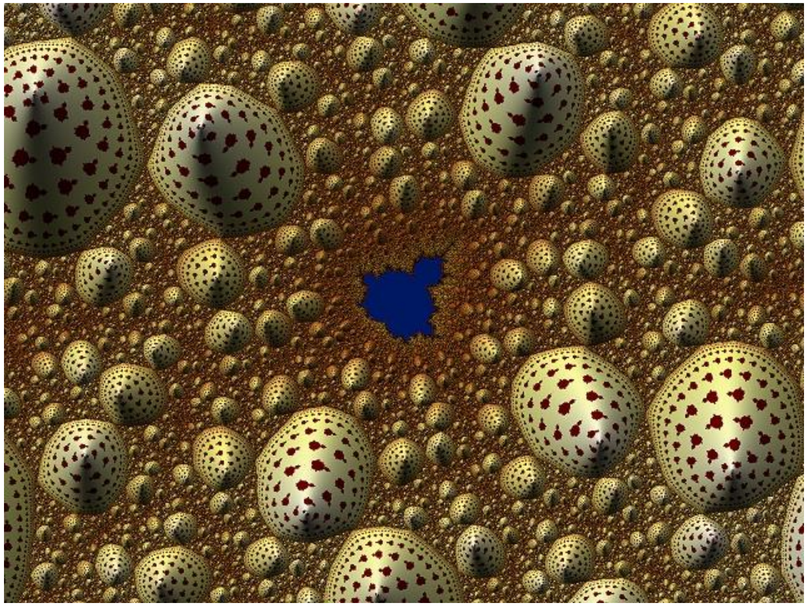

Fractal Art

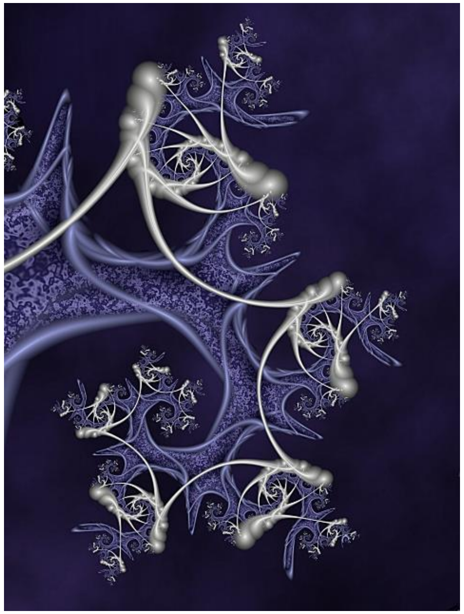


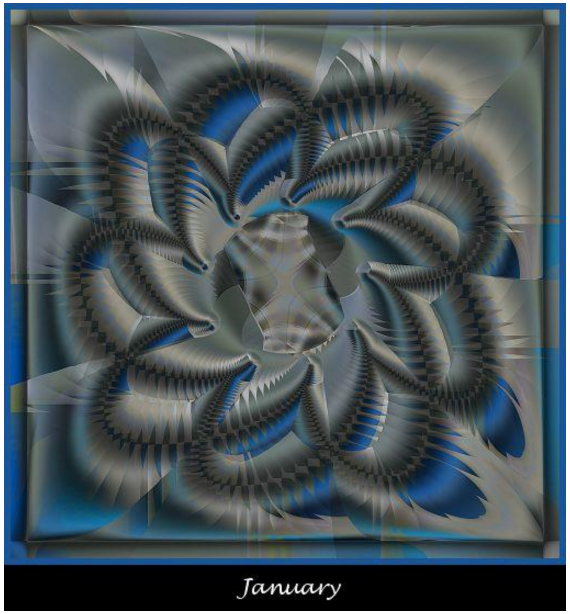


Fractal Art







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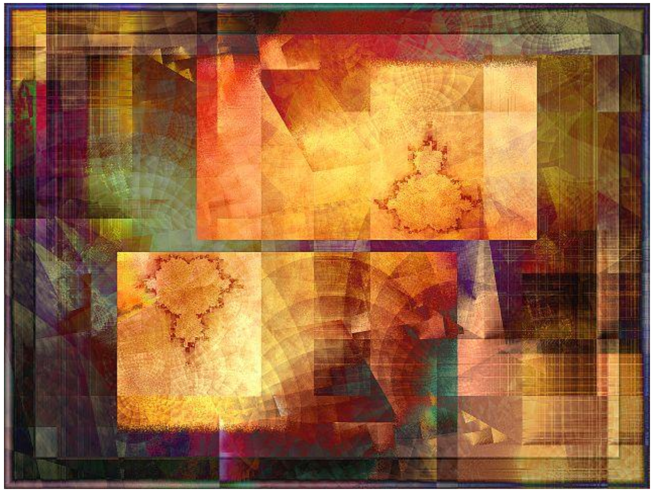

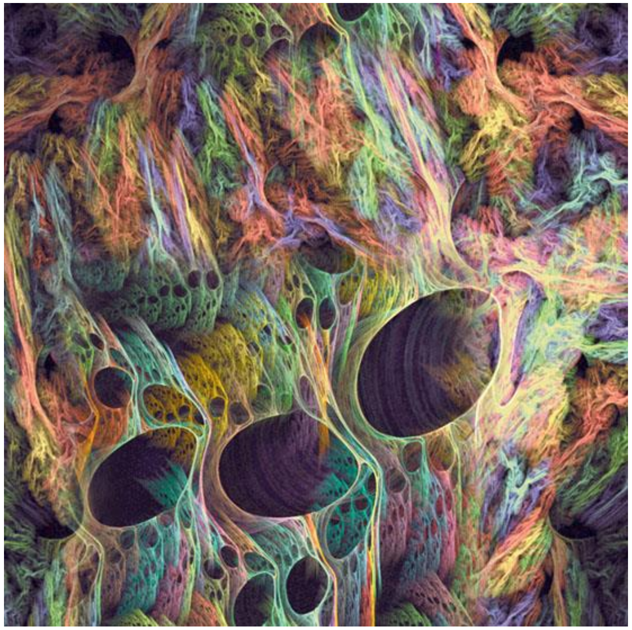

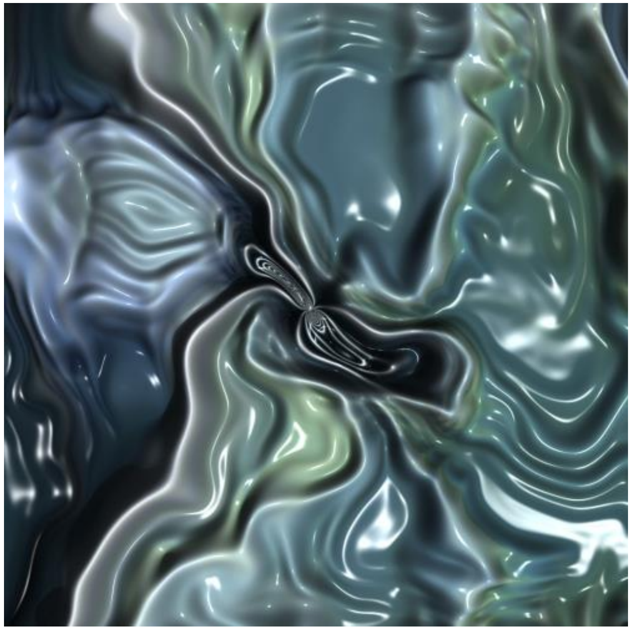

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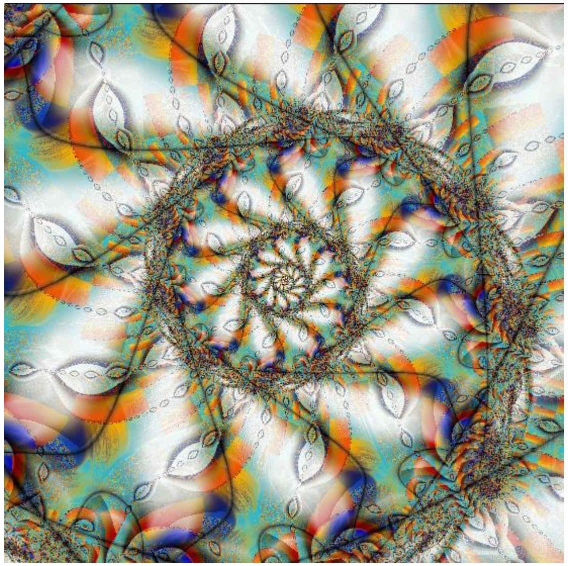
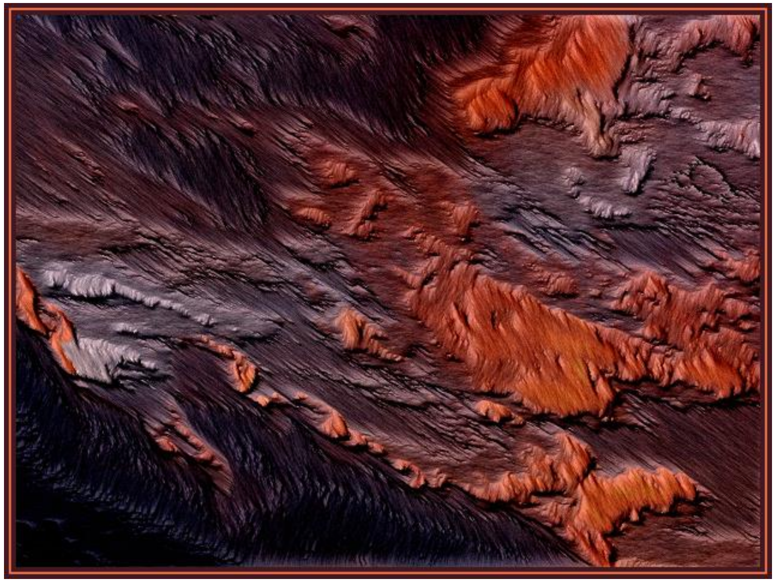
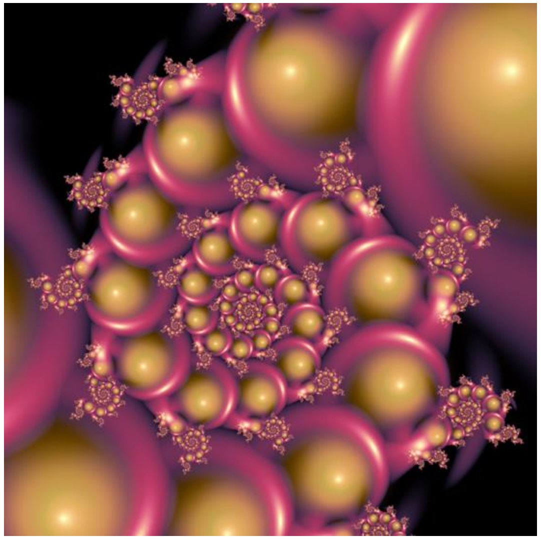


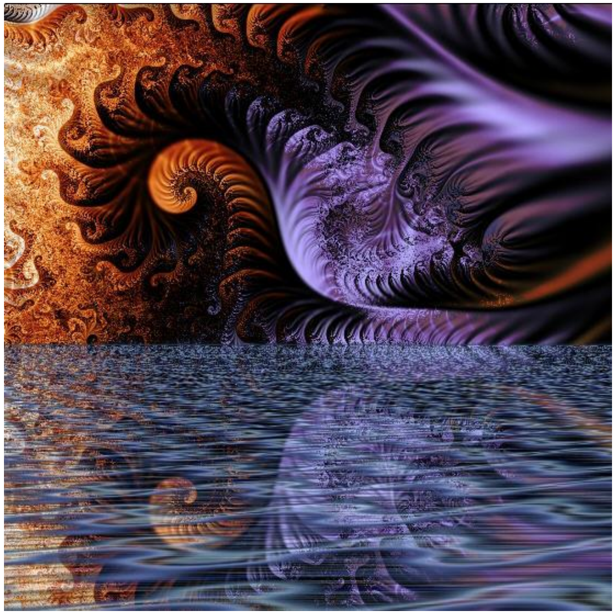

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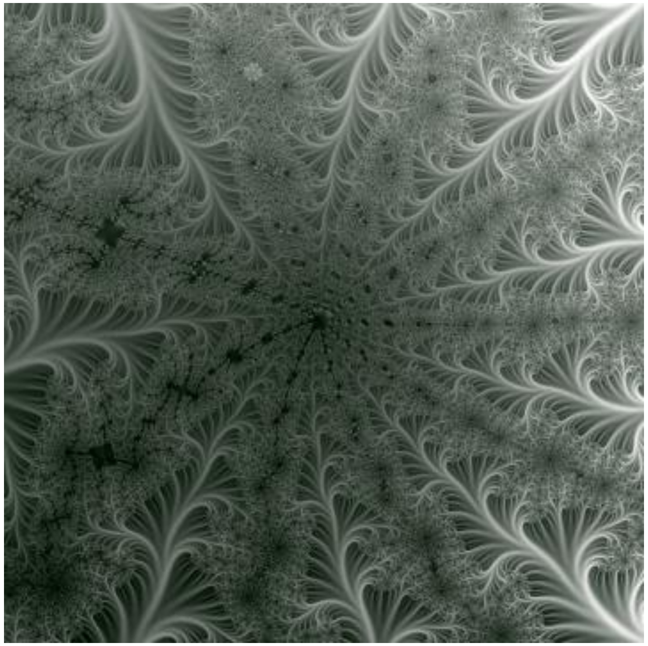

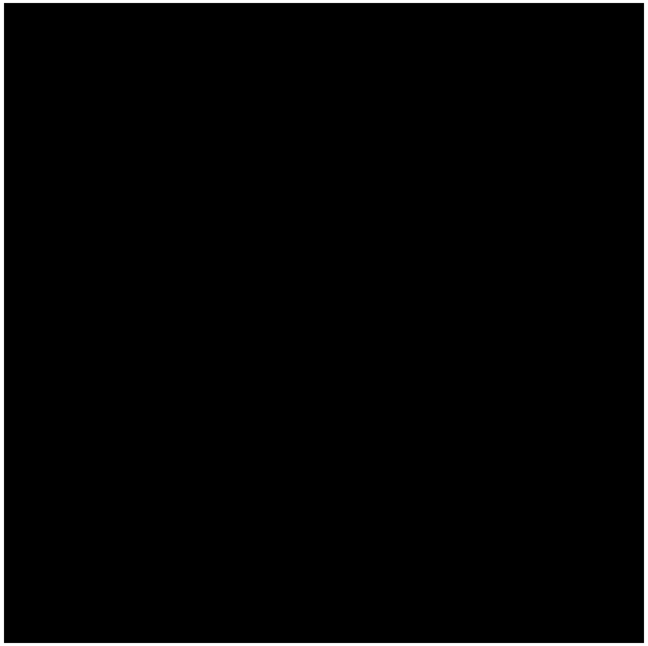

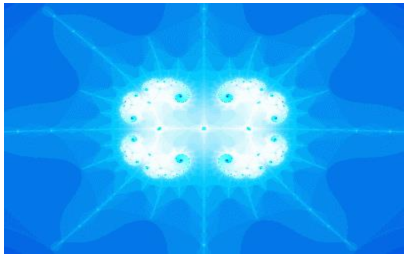

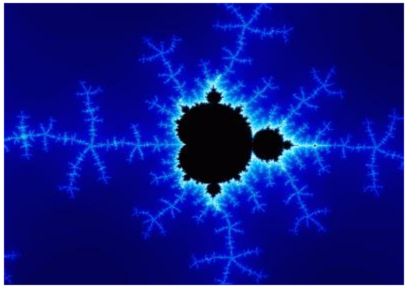

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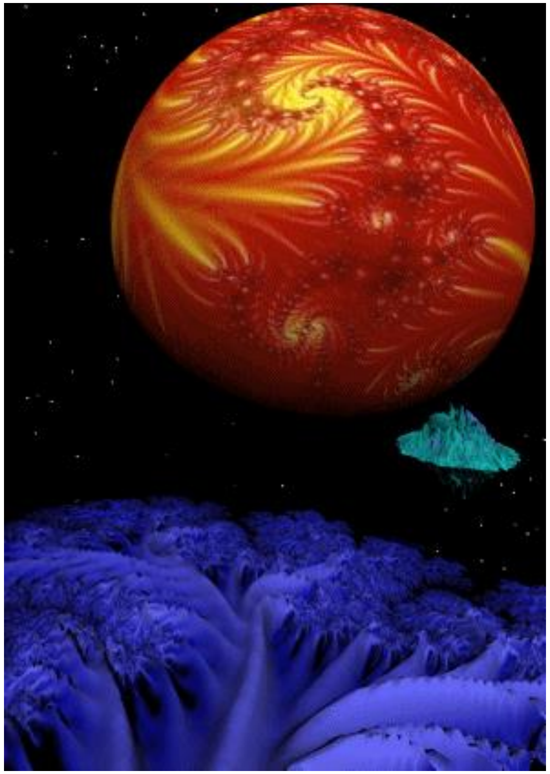
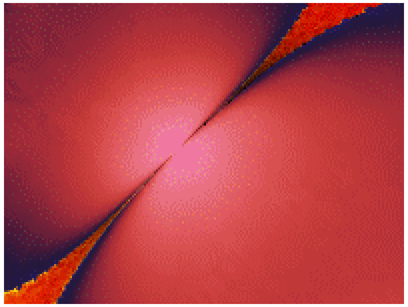
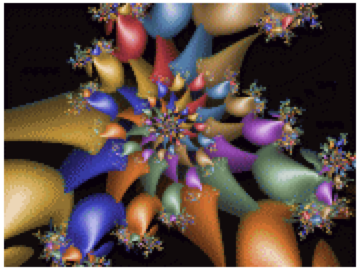
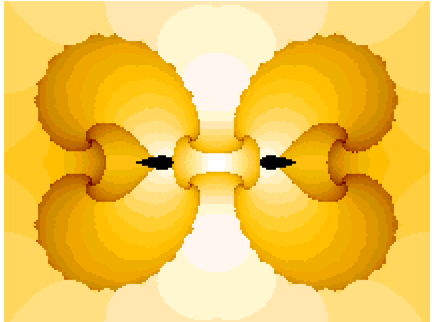
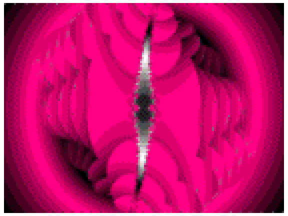
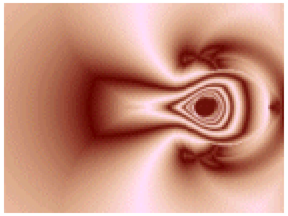
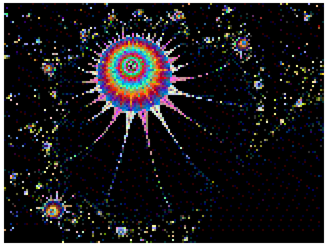
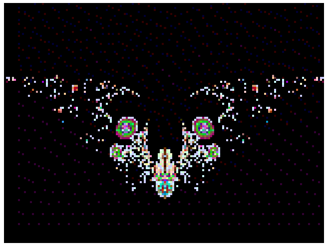
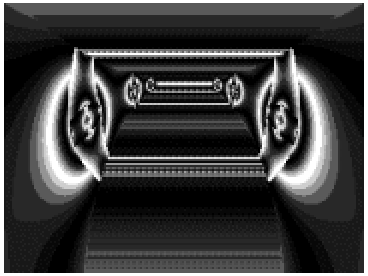


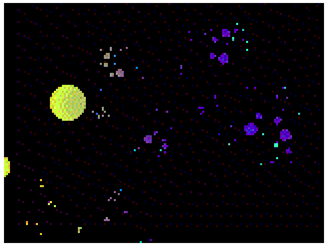


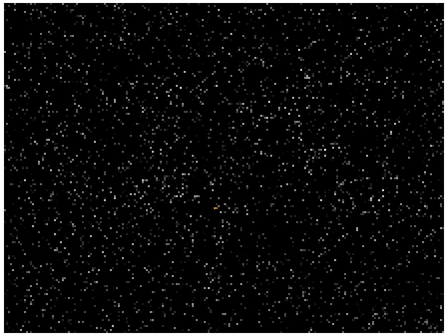


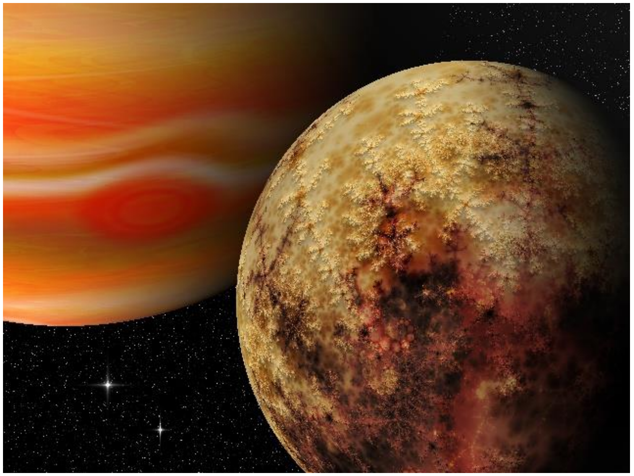
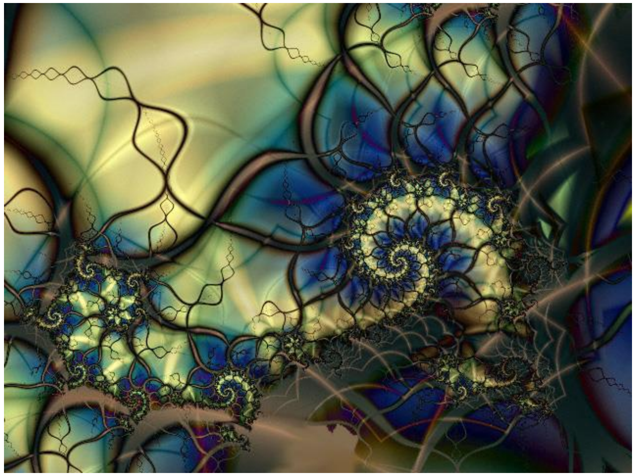







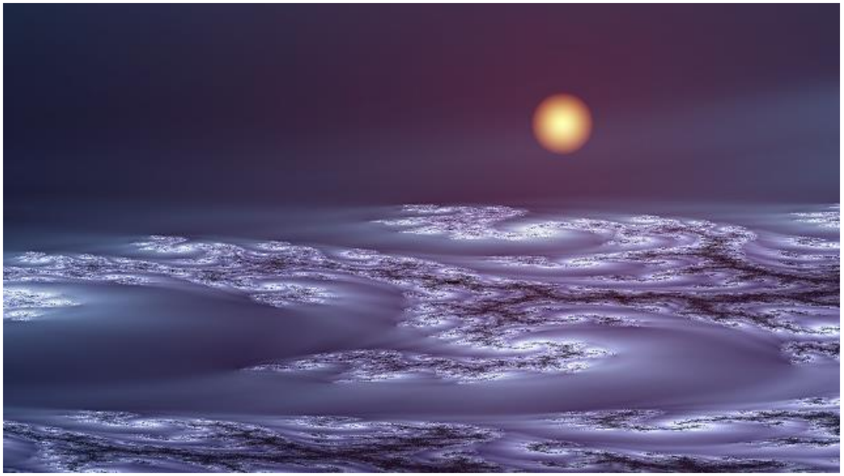
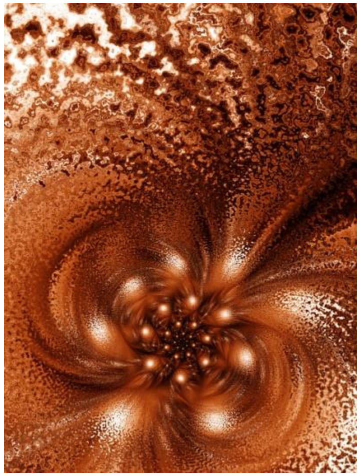
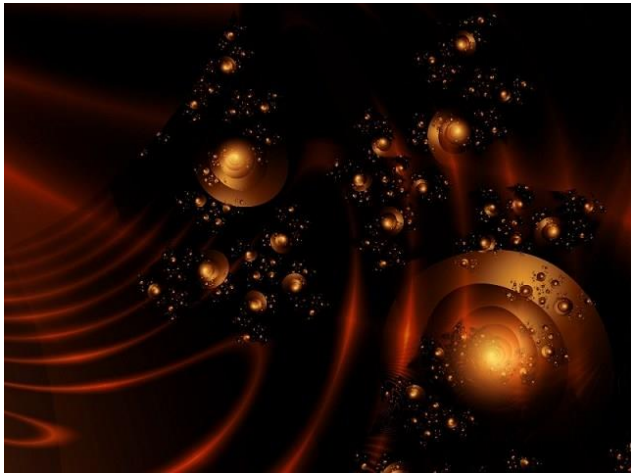


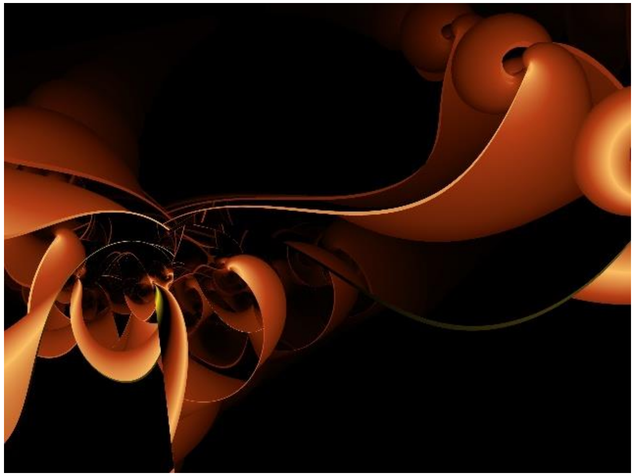
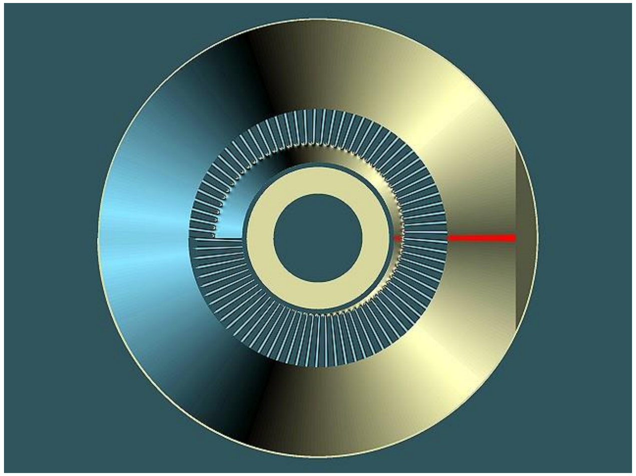
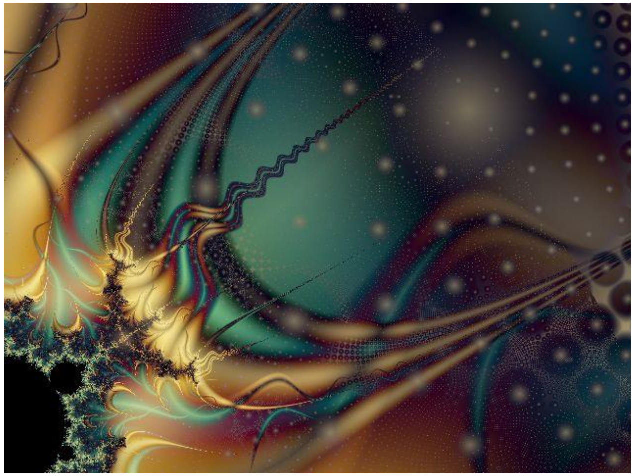
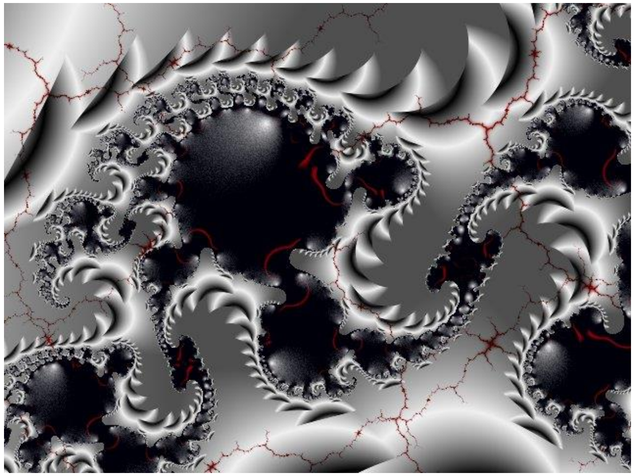
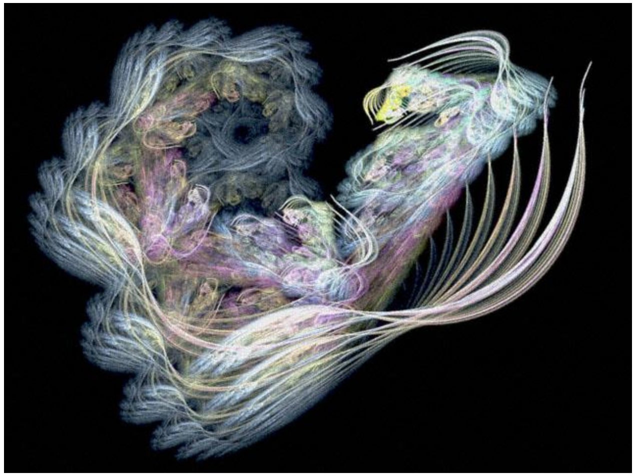
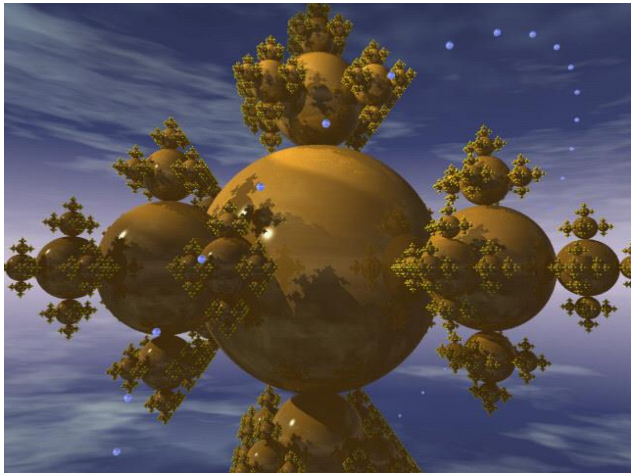
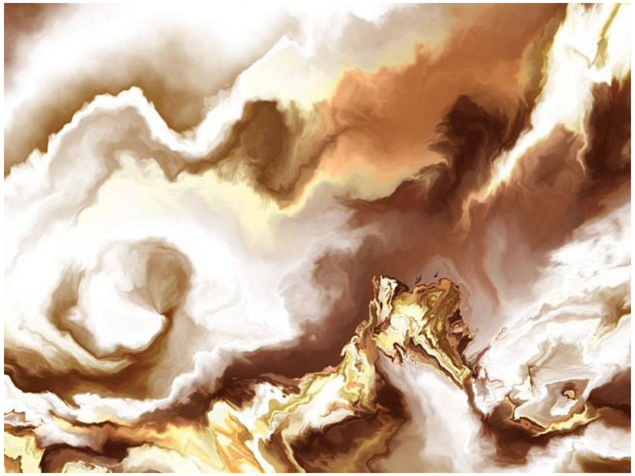
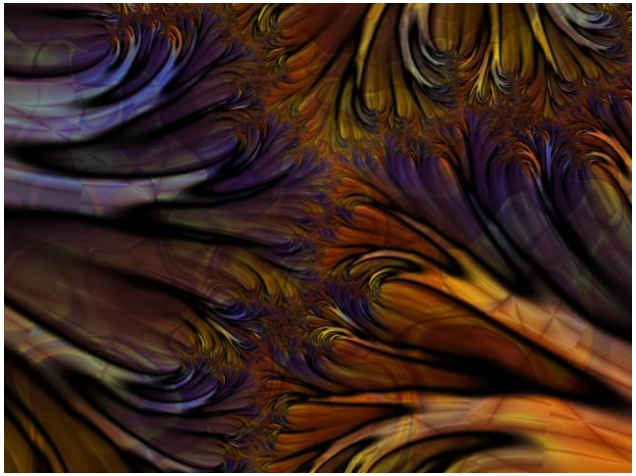
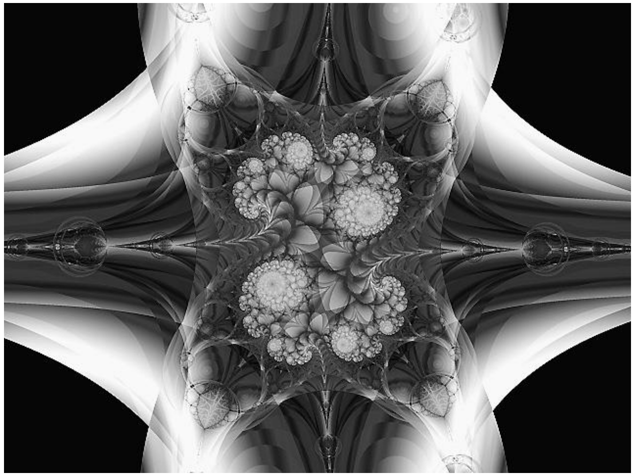


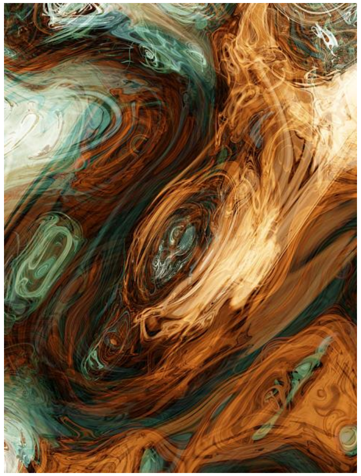
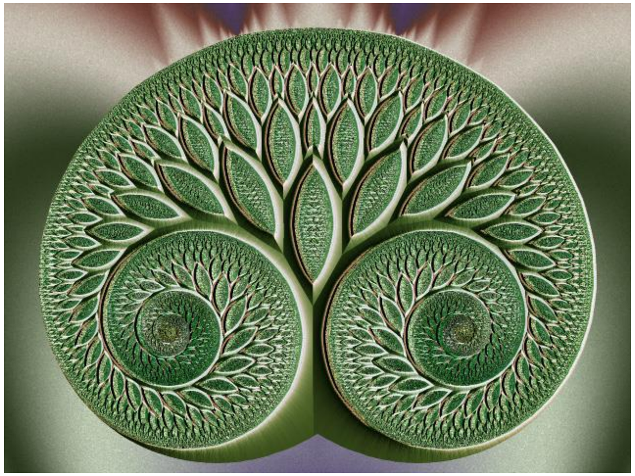
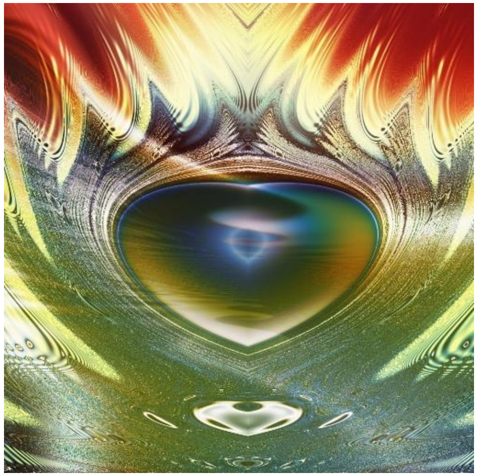


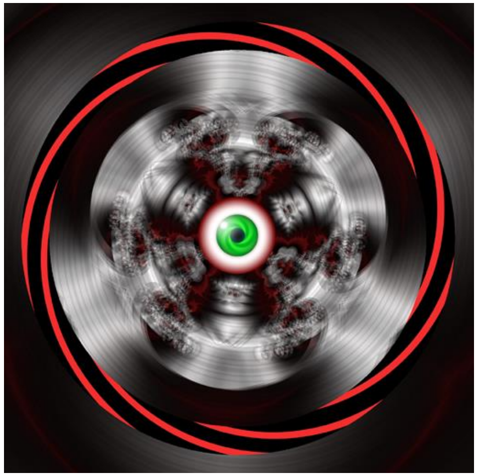



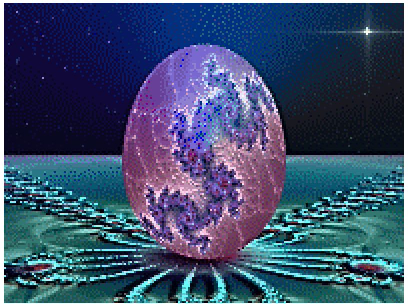
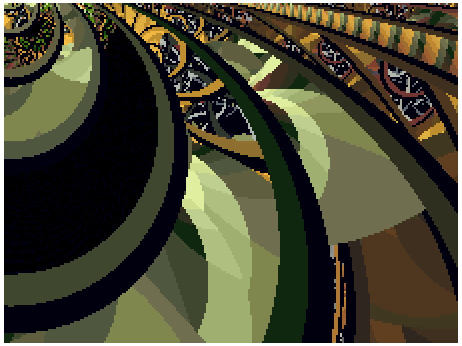
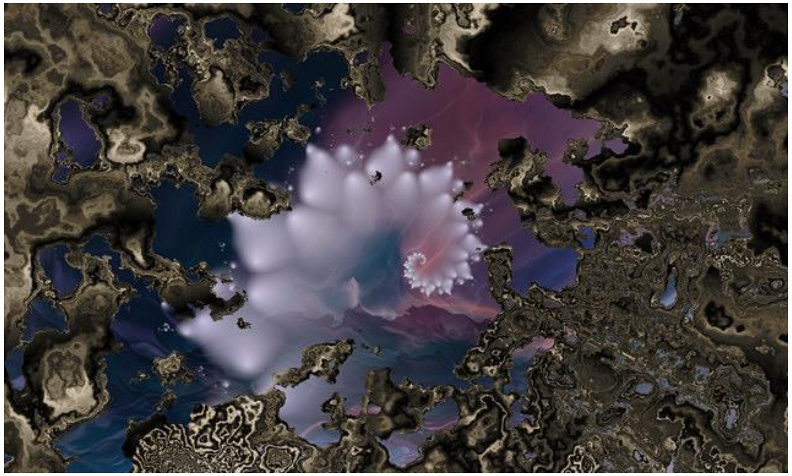


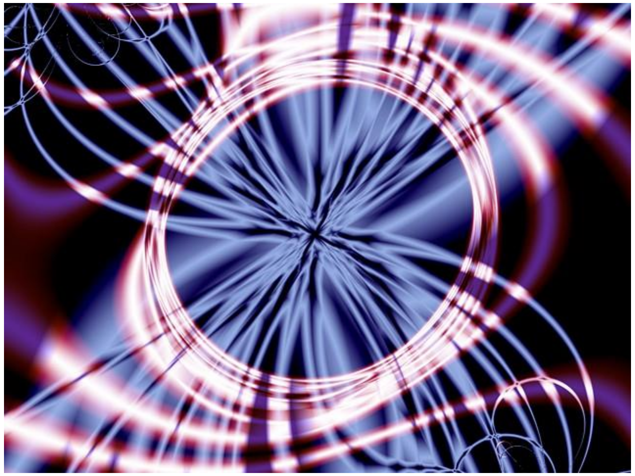
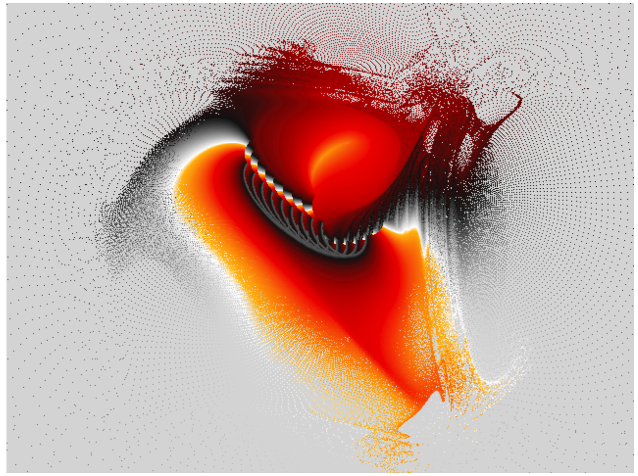
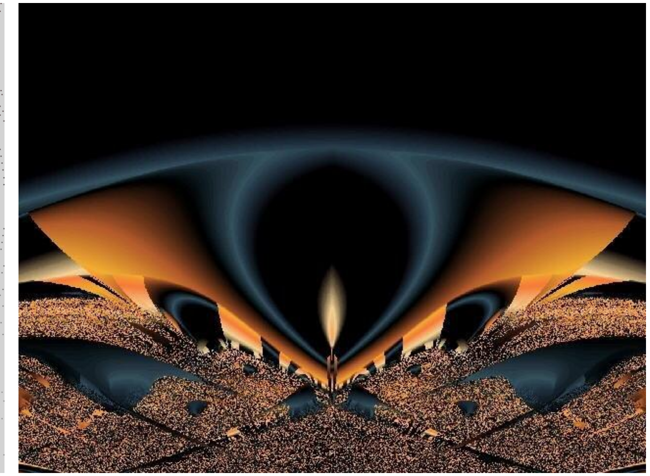
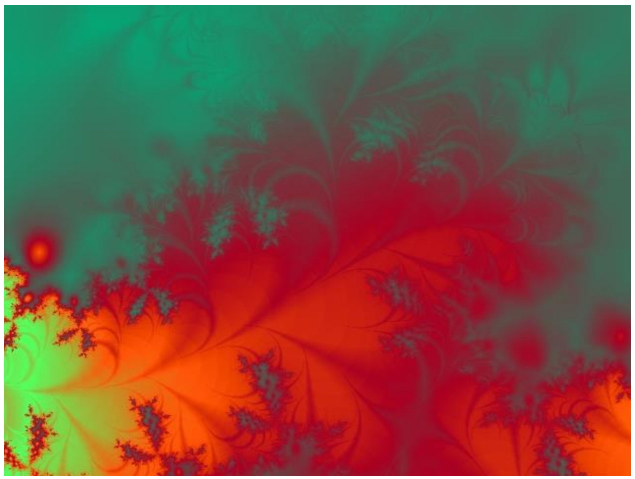
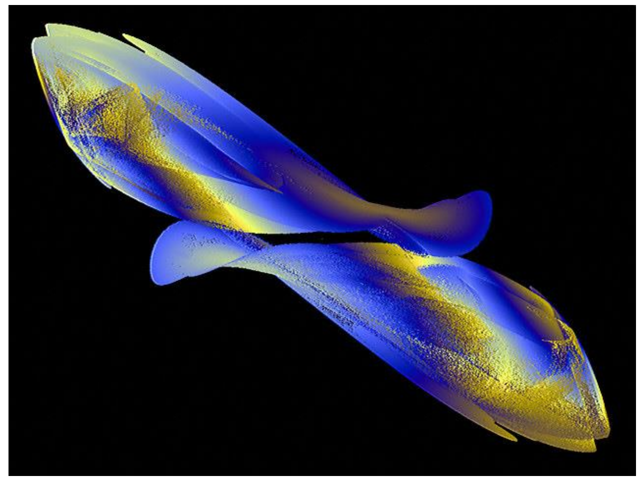





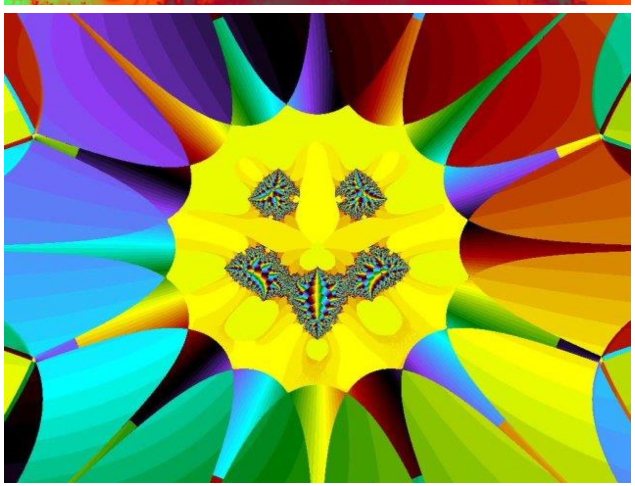
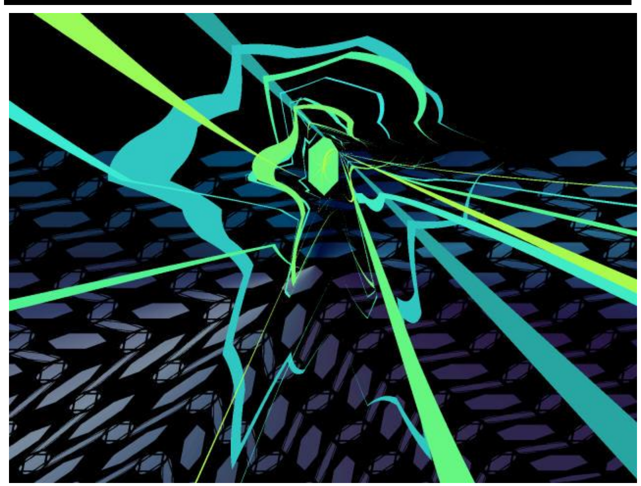
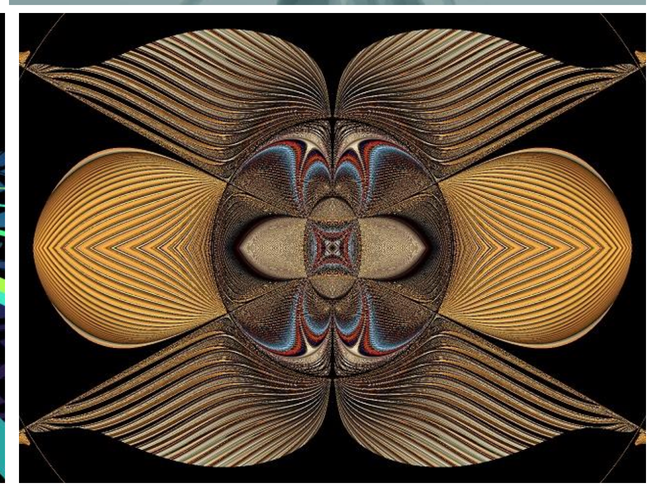










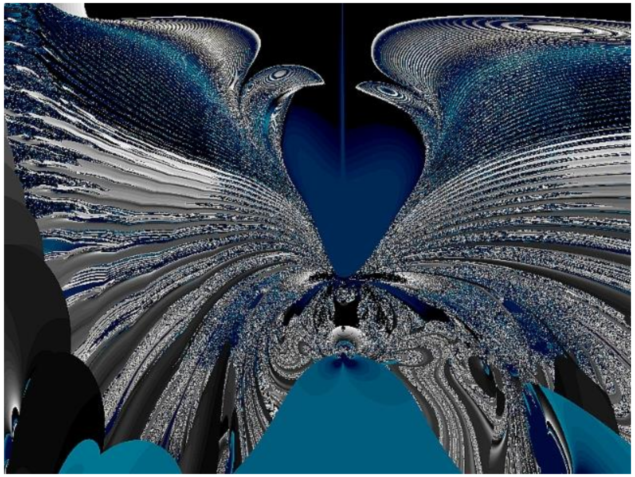
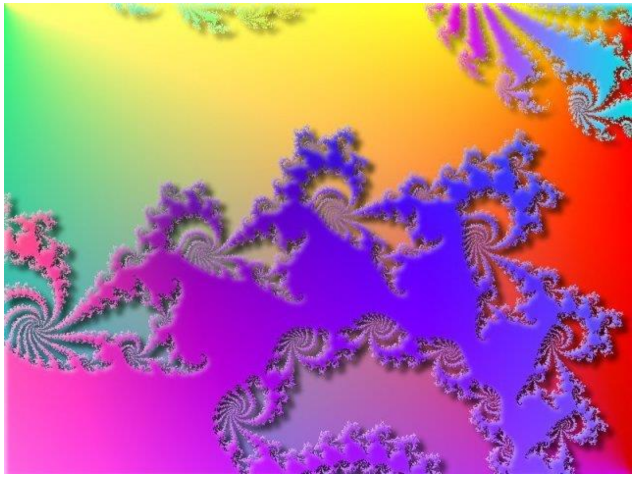





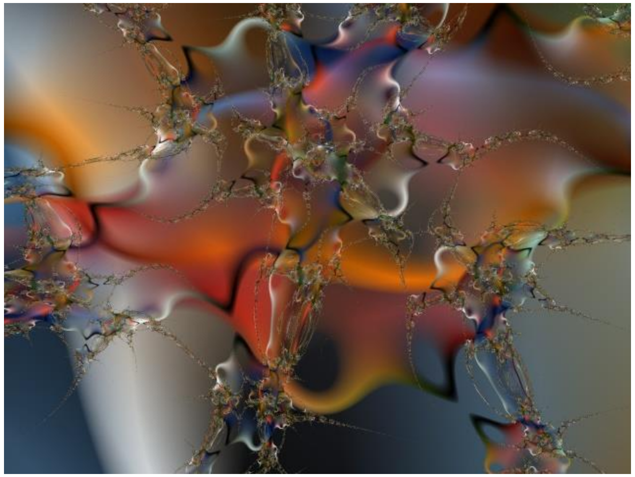
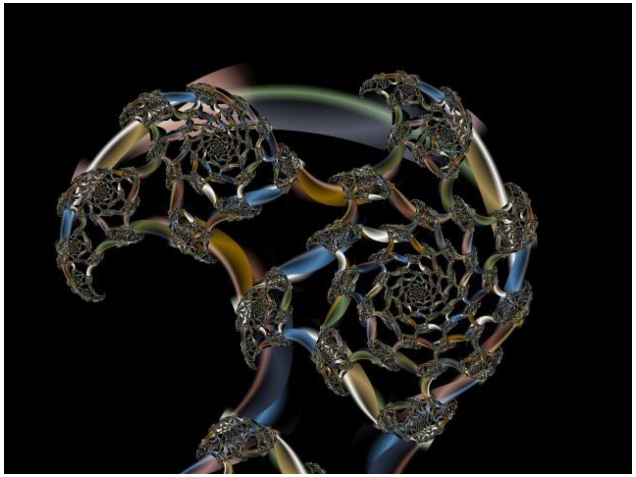
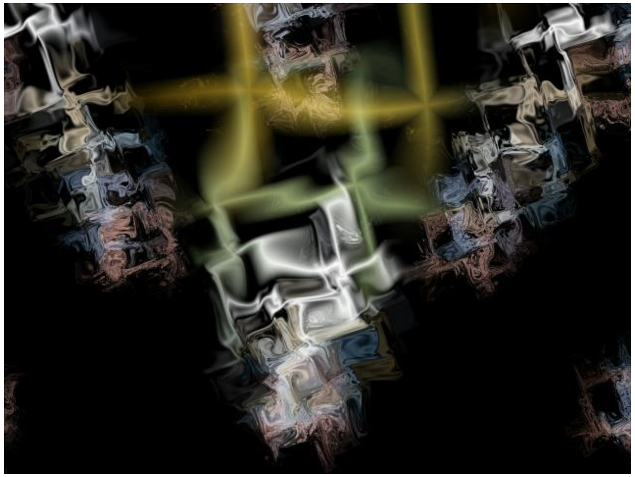
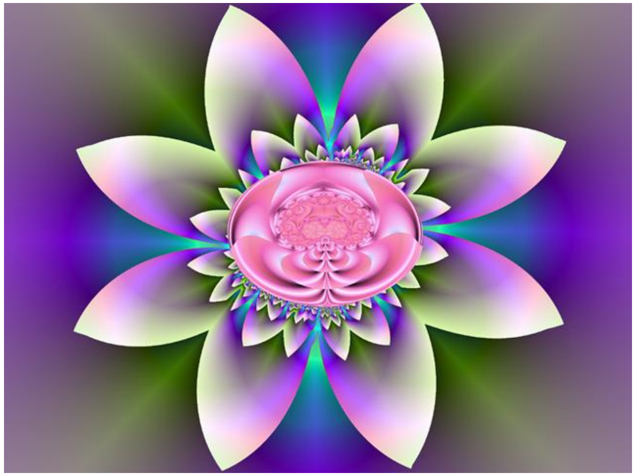


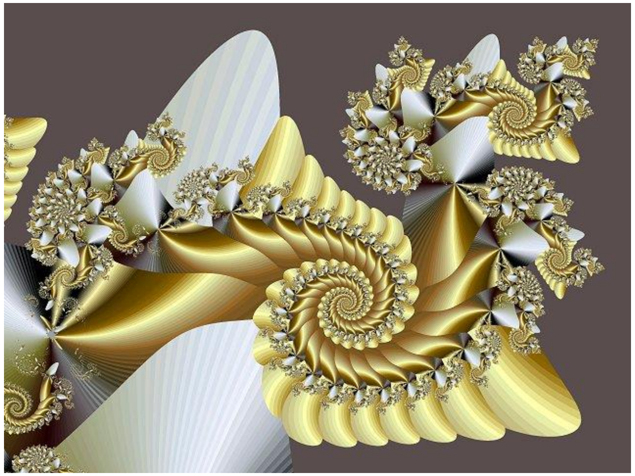




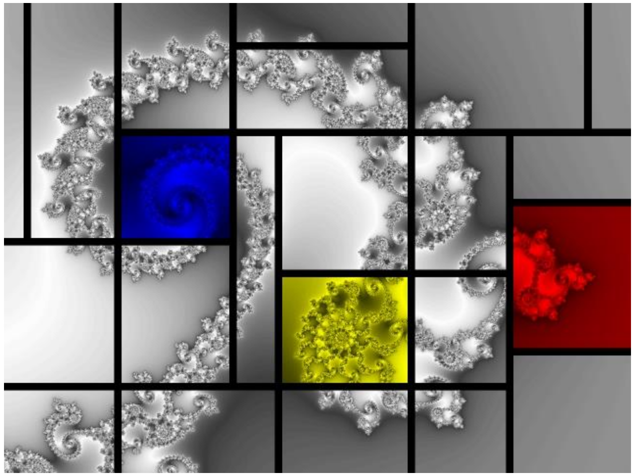
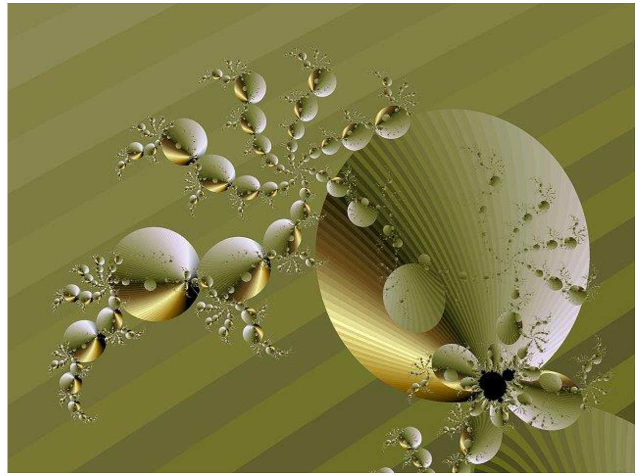






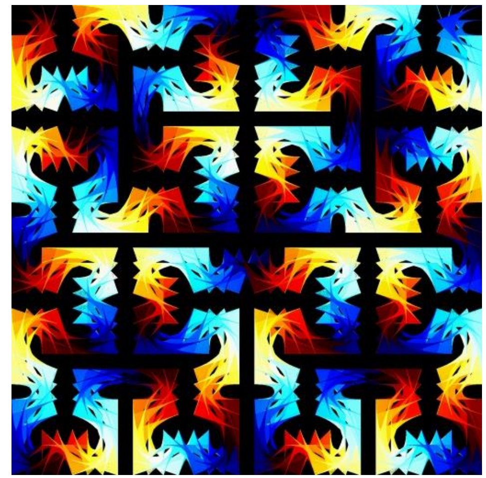
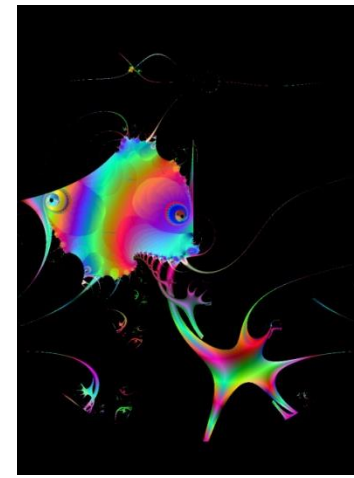



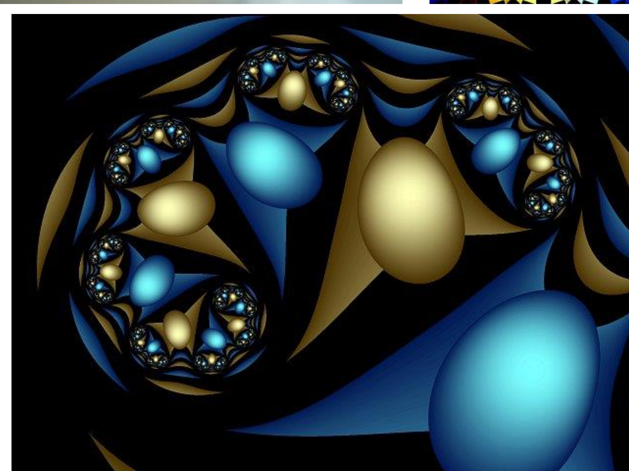
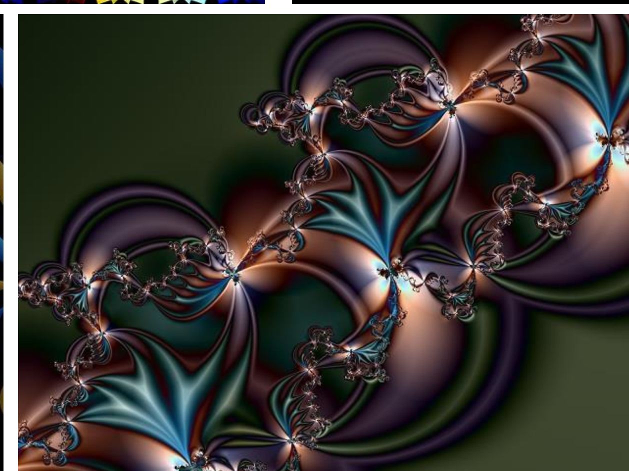





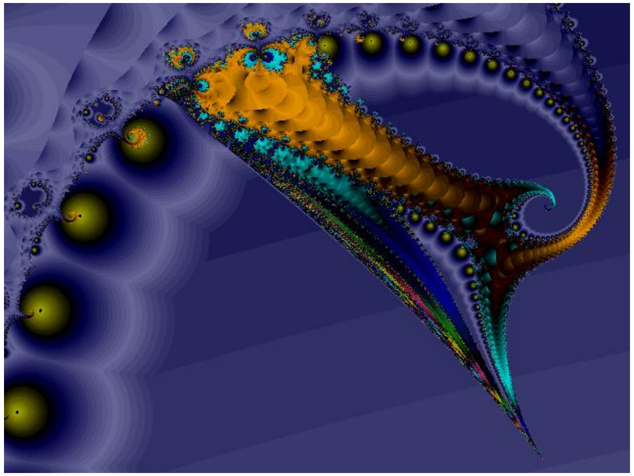
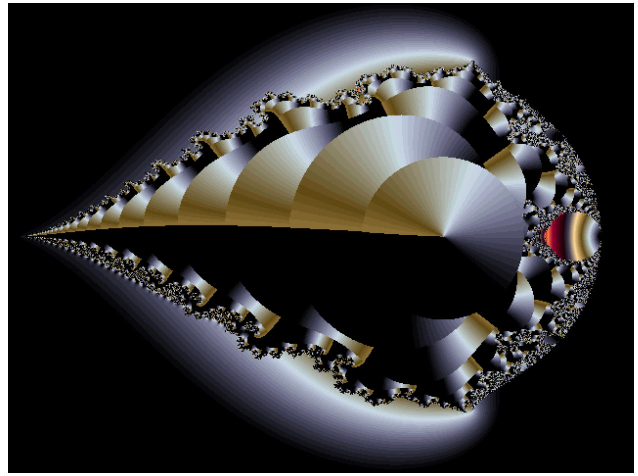
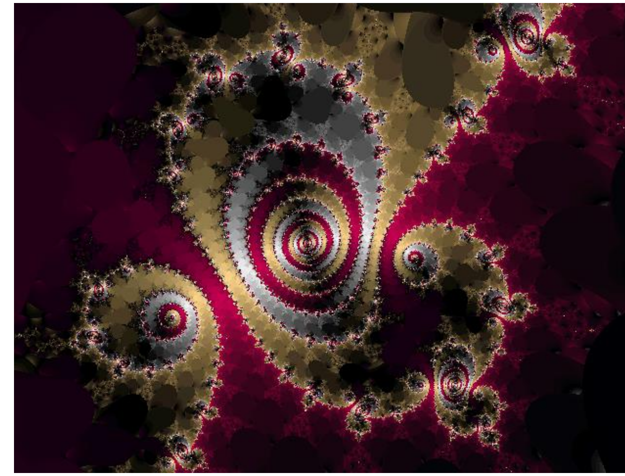
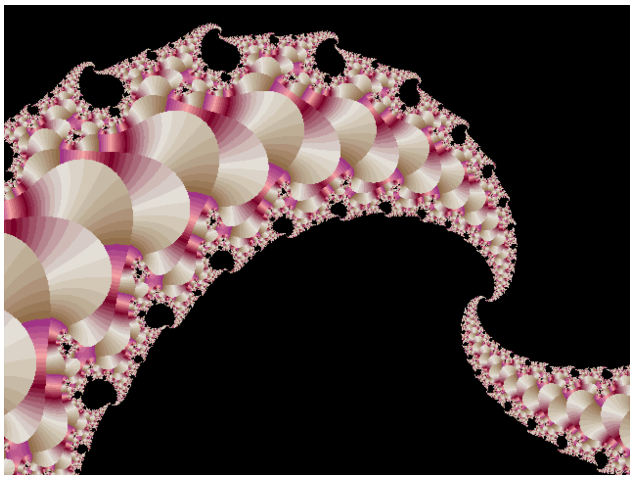
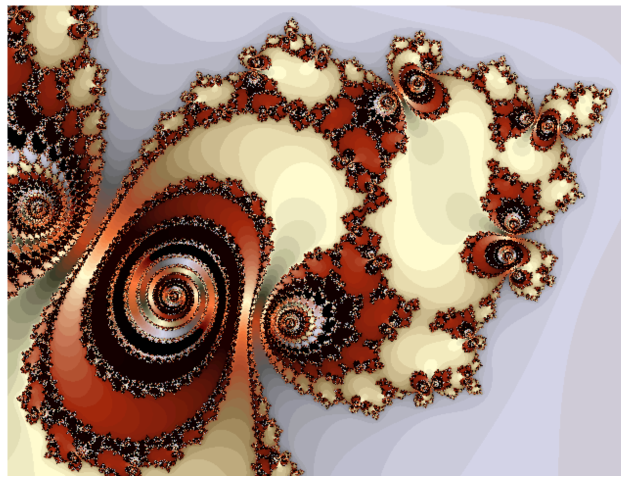
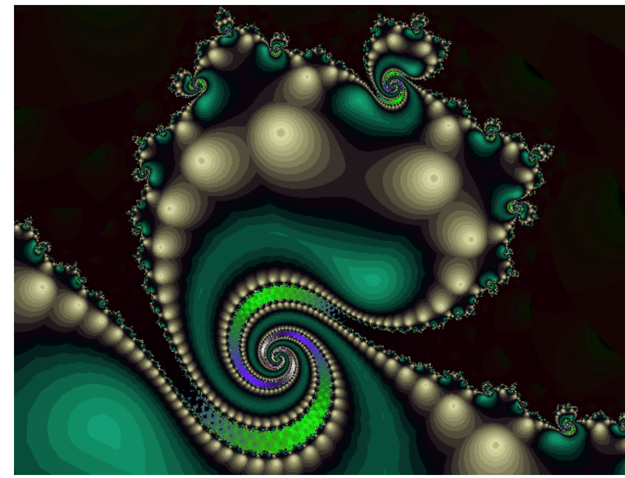


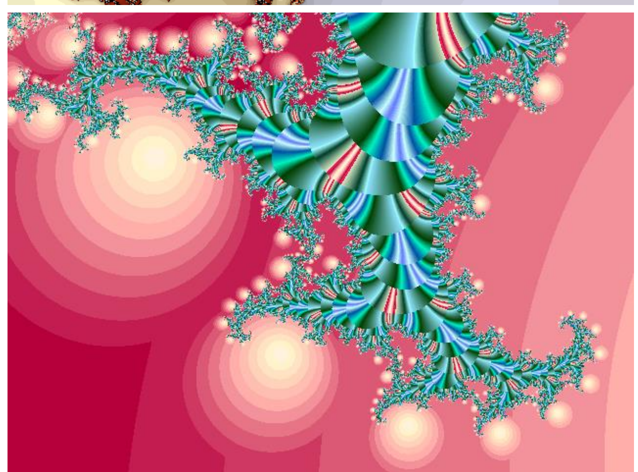
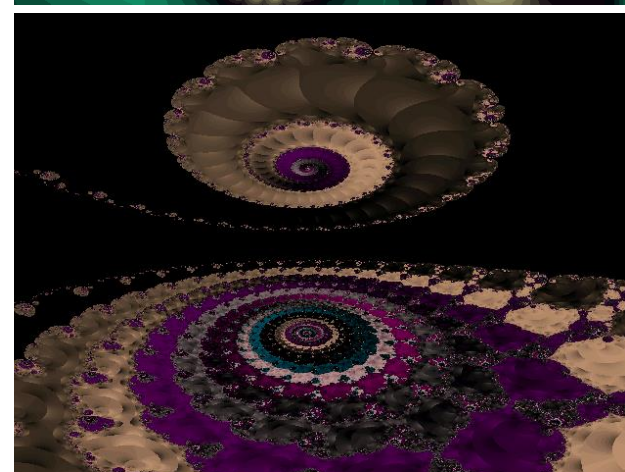



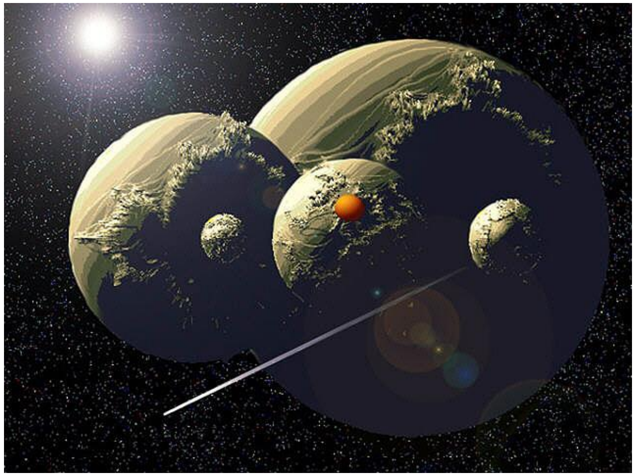
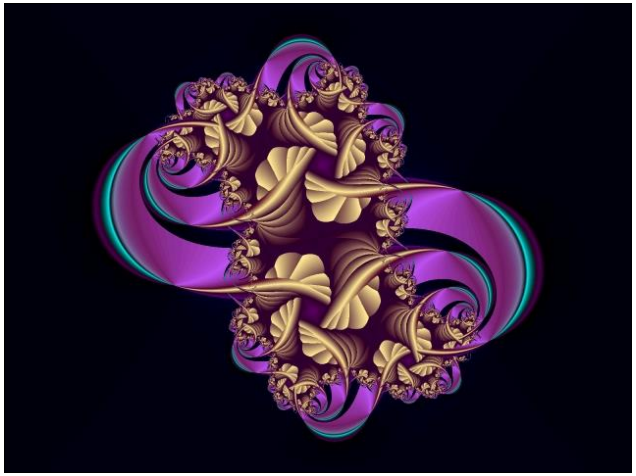
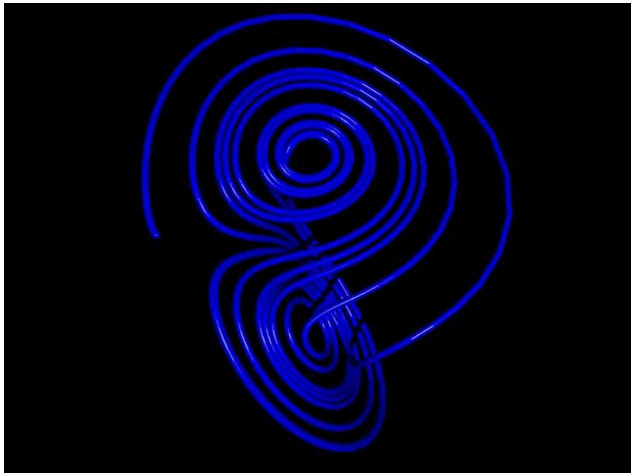
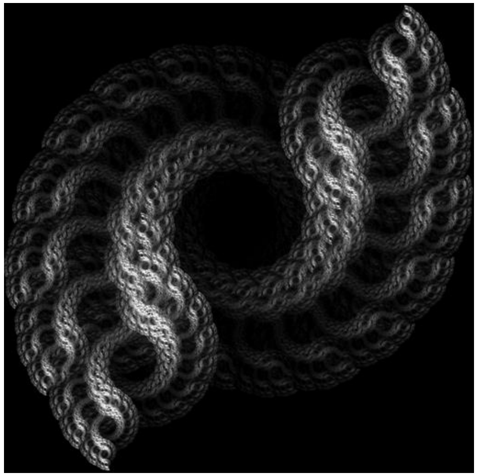
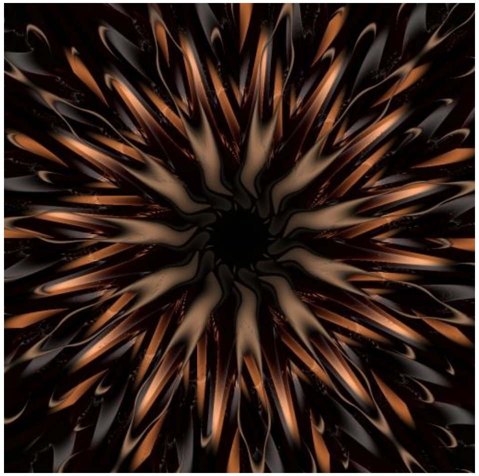


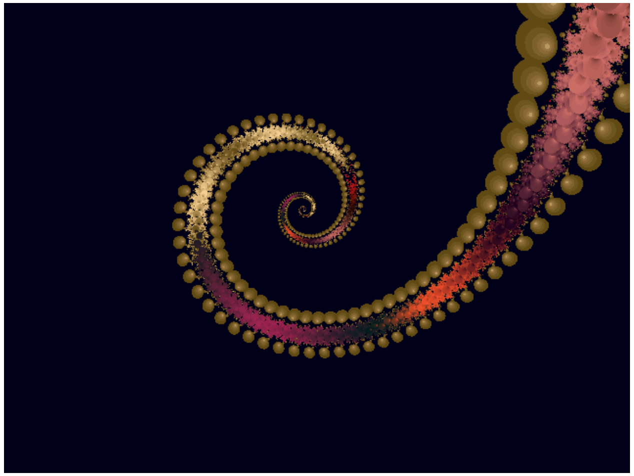



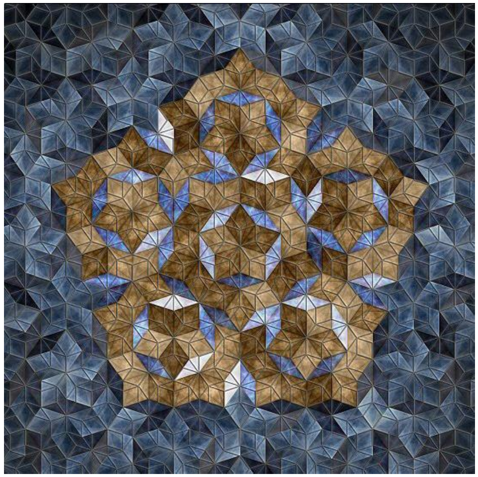









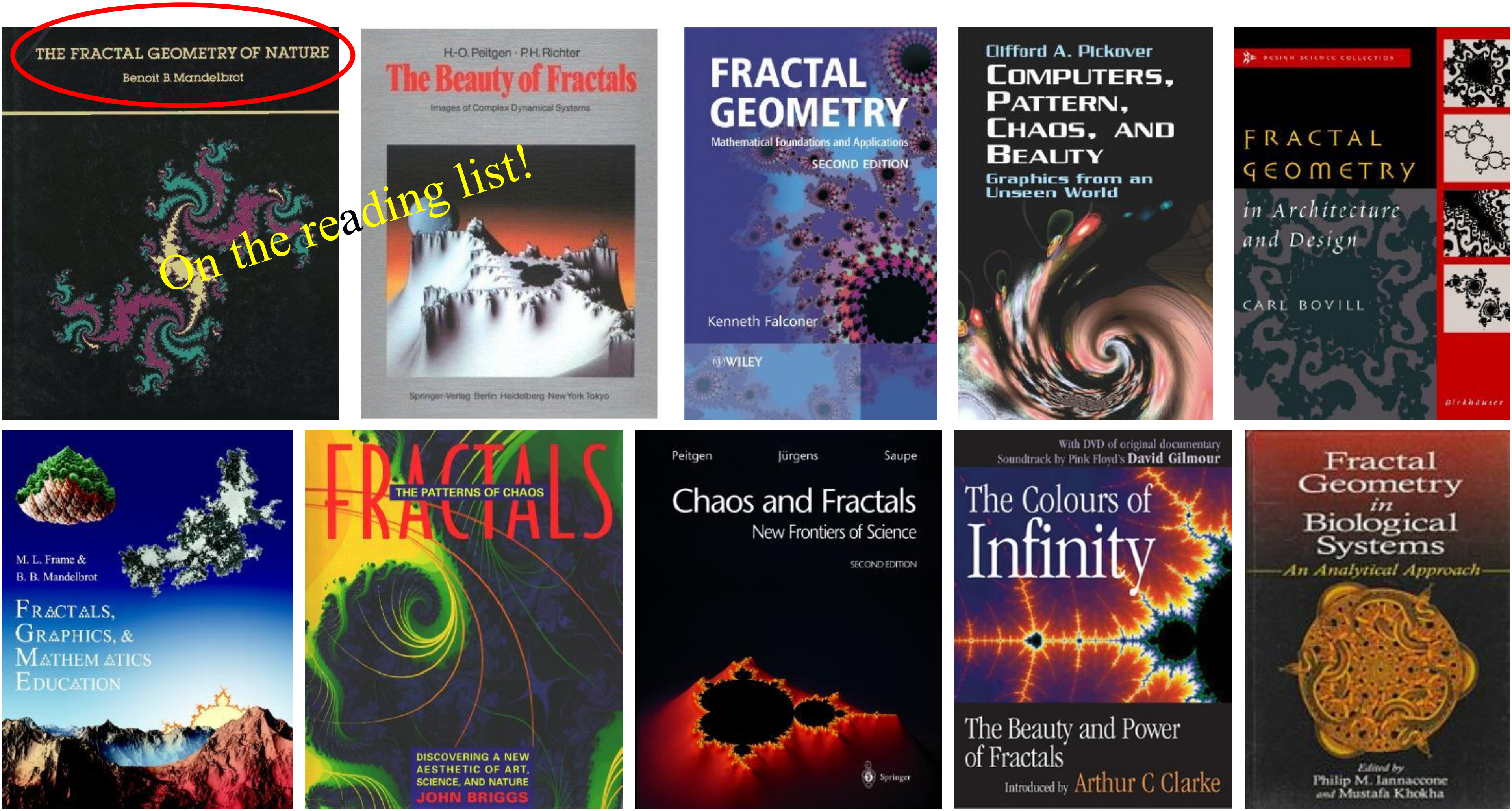




More on Fractals

Fractal Art Contests: www.fractalartcontests.com

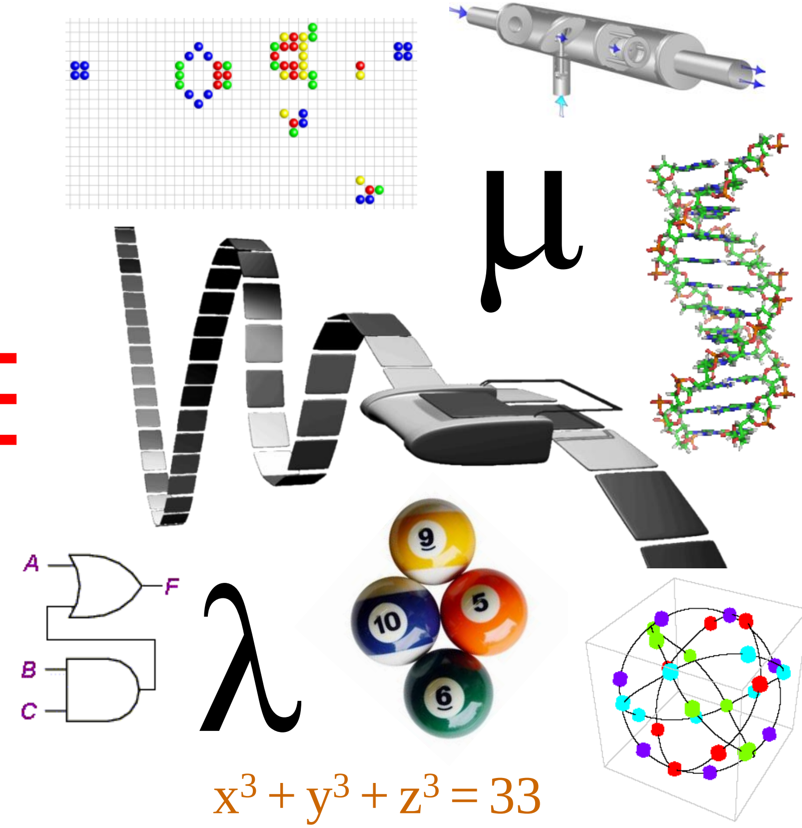
www.wikipedia.org/wiki/Mandelbrot_set



The Church-Turing Thesis

Q: What does it mean “to be computable”?

≡



$x^3 + y^3 + z^3 = 33$

The Church-Turing Thesis: Anything that is “intuitively computable” is also Turing-machine computable.

“Le Penseur”
Auguste Rodin, 1902
Musée Rodin, Paris