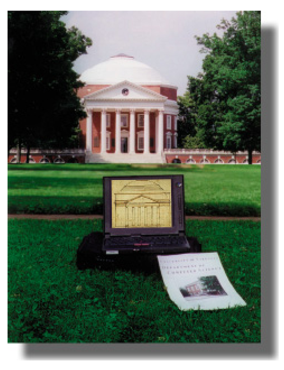
Moving-Target TSP and Related Problems



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Introduction

- **Classical TSP:** sites to be visited are stationary
- **Related work:**
  - Time-dependent TSP (cost to travel between stationary sites changes with time)
- **Our contribution:**
  - Moving-Target TSP formulation
  - Algorithms for variants of Moving-Target TSP
- **Motivation:**
  - Supply ships resupply patrolling boats
  - Planes intercept mobile ground units

Problem Formulation

Given:

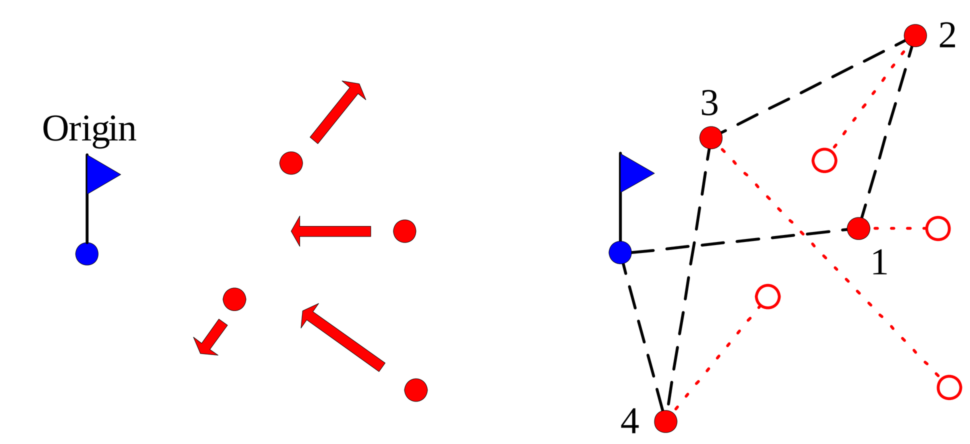
- A set $S = \{s_1, s_2,..., s_n\}$ of ***targets***
- Each target s_i has constant velocity $\mathbf{v_i}$
- Each target s_i starts moving from a position $\mathbf{p_i} \in \Re^n$
- A ***pursuer*** starting at origin with maximum speed $v > |\mathbf{v_i}|$

Find:

- A fastest tour which intercepts all targets

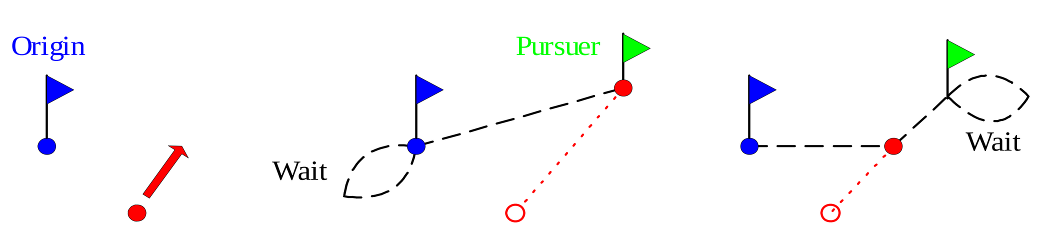
Variations (Moving-Target Vehicle Routing Problem):

- Multiple pursuers j = 1.. k
- Pursuers have a given capacity to fill target demand



Basic Observation

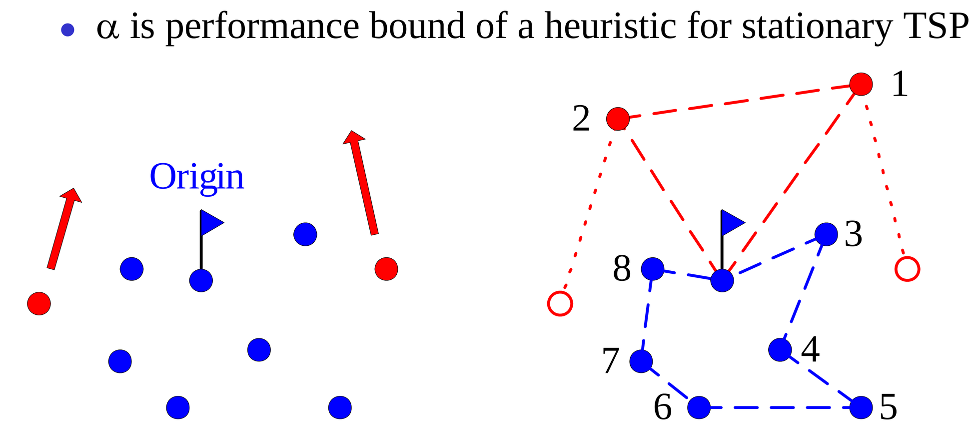
- **Lemma:** Optimal tours have no waiting periods



- **Corollary:** Pursuer always moves at max speed

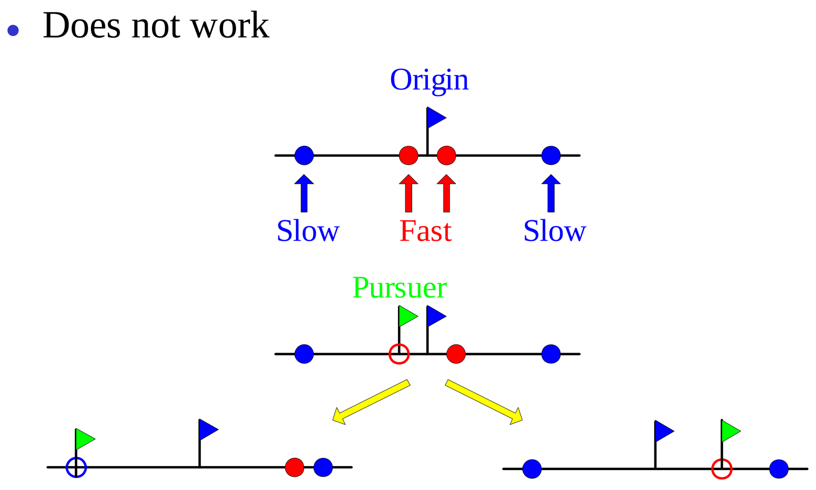
Few Moving Targets

- **Few (O(log n/log log n)) moving targets ⇒**
  - Efficient $(1+\alpha)$-approximation algorithm
  - α is performance bound of a heuristic for stationary TSP



One-Dimensional Variant

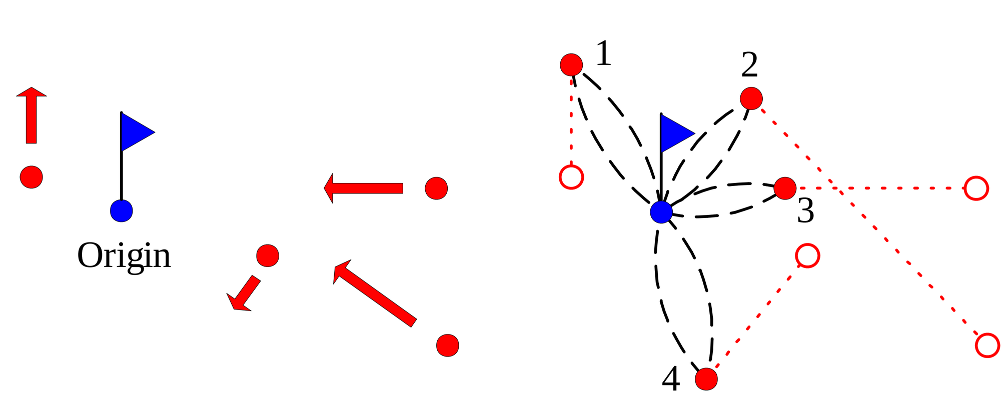
- **Trivial Algorithm:**
  - Intercept all targets on one side of the origin first
  - Then intercept targets on the other side
  - Does not work



- **Lemma:** Pursuer can change direction only after intercepting the fastest target
- Dynamic programming solution
- $O(n^2)$ algorithm
- Implemented and verified

Moving-Target TSP with Resupply

- Pursuer can intercept only one target before requiring resupply at the origin
  - Targets move directly away (or towards) the origin
- Corresponds to Moving-Target VRP except:
  - Single pursuer
  - Pursuer supply = target demand
- **Solution** (when targets move away from origin):
  - Intercept targets in order of increasing d_i/v_i
  - Algebraic proof for two targets
  - For n targets, swapping targets improves tour



- Targets intercept origin ⇒ Implicit change in target velocity
- **Valid tour**: No target passes through the origin
- **Theorem**: If the tour in which pursuer intercepts
  - Targets moving away in ascending order of d_i/v_i
  - Approaching targets in descending order of d_i/v_i

  is valid, then it is optimal
- **Problem:** Find the fastest *valid* tour
- **Theorem:** The tour where the pursuer intercepts targets in descending order of d_i/v_i is always valid and the slowest
- **Theorem:** Slowest tour ≤ **2**×(optimal valid tour)

Multi-Pursuer Moving-Target TSP with Resupply

- Moving-Target Vehicle Routing Problem with
  - Multiple pursuers
  - Pursuer supply = target demand
  - All targets are moving away
- Minimization Objectives:
  - **Makespan** = when the last pursuer at the base
    - Standard for multiprocessor scheduling
    - NP-hard even for stationary targets
  - **Total time** = total time while pursuers are moving
    - Standard for classical vehicle routing problem
    - Trivial for stationary case
- **Theorem**: Total time objective is NP-hard

- If all targets have the same d_i/v_i
  - Total time objective is still NP-hard
  - Approximation algorithm with

    Performance ratio = $\max_i \{(1 + v_i) / (1 - v_i)\}$
- If all targets have the same speed v_i
  - Nontrivial result
  - The following strategy is optimal:

    send next available pursuer after the closest target

Summary

- Formulation of Moving-Target TSP
- Exact algorithm for one-dimensional version
- Heuristic when few targets are moving
- Approximate and exact heuristics for selected variants of Moving-Target TSP with Resupply

Future Work

- 2-Dimensional Moving-Target TSP
- Moving-Target TSP with Resupply
  - When targets can pass through origin or when finding the fastest valid tour:
    - Prove/Disprove NP-Hardness
    - Find efficient algorithms/heuristics
- Multi-Pursuer Moving-Target TSP