CS4102 - ALGORITHMS - S'16 — ABHI SHELAT
DUE WED APR 13, 2016 AT 5P VIA COURSE WEBSITE

H6.

You may collaborate with other students on the homework but you must submit your own individually written solution, identify your collaborators, and acknowledge any external sources that you consult. Do no submit a solution that you cannot explain to me.

PROBLEM 1 *Another all pairs shortest path*

Set $\textsc{bshort}_{i,j,k}$ to be the shortest path from i to j that uses only k hops. Note this is different from the ASHORT variable that we used in class in that we do not restrict the intermediate nodes to be $1 \dots k$ in this formulation. State a recursive formula for BSHORT. Devise an algorithm that uses this recurrence. What is the running time of this algorithm?

PROBLEM 2 *Negative Cycles*

In lecture, we argued that the Bellman-Ford algorithm can be made to detect a negative cycle by running for one more iteration and seeing if any of the verticies have a shorter path. Prove this by showing that if graph $G = (V, E, w)$ has a negative cycle, then there exists a vertex $v \in V$ for which $\textsc{short}_{V-1,v} > \textsc{short}_{V,v}$.

PROBLEM 3 *Number of shortest paths*

Given a graph $G = (V, E)$, and a starting node s, let $\delta(s, v)$ be the length of the shortest path when each edge has unit cost between s and v. Design an algorithm that computes the number of distinct paths from s to v that have length $\delta(s, v)$.

PROBLEM 4 *Why does BFS work?*

$\textsc{bfs}(G = (V, E), s)$

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d_v ← ∞ for all v ∈ V
d_s ← 0
Q ← ∅
ENQUEUE(Q, s)
while Q ≠ ∅
      do u ← DEQUEUE(Q)
         for each v ∈ Adj(u)
             do if d_v = ∞
                   d_v ← d_u + 1
                   ENQUEUE(Q, v)
```

Theorem 1 *Let $G = (V, E)$ be a graph and suppose that BFS is run on G from vertex $s \in V$. Then, when BFS terminates, $d_v = \delta(s, v)$ for all $v \in V$. Here $\delta(s, v)$ is the length of the shortest path (in terms of edges) from s to v in G and $\delta(s, v) = \infty$ if there is no path from s to v in G.*

Prove the theorem.