Load-Reuse Analysis: Design and Evaluation

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Abstract

Load-reuse analysis finds instructions that repeatedly access the same memory location. This location can be promoted to a register, eliminating redundant loads by reusing the results of prior memory accesses. This paper develops a load-reuse analysis and designs a method for evaluating its precision.

In designing the analysis, we aspire for completeness—the goal of exposing all reuse that can be harvested by a subsequent program transformation. For register promotion, a suitable transformation is partial redundancy elimination (PRE). To approach the ideal goal of PRE-completeness, the load-reuse analysis is phrased as a data-flow problem on a program representation that is path-sensitive, as it detects reuse even when it originates in a different instruction along each control flow path. Furthermore, the analysis is comprehensive, as it treats scalar, array and pointer-based loads uniformly.

In evaluating the analysis, we compare it with an ideal analysis. By observing the run-time stream of memory references, we collect all PRE-exploitable reuse and treat it as the ideal analysis performance. To compare the (static) load-reuse analysis with the (dynamic) ideal reuse, we use an estimator algorithm that computes, given a data-flow solution and a program profile, the dynamic amount of reuse detected by the analysis. We developed a family of estimators that differ in how well they bound the profiling error inherent in the edge profile. By bounding the error, the estimators offer a precise and practical method for determining the run-time optimization benefit.

Our experiments show that about 55% of loads executed in Spec95 exhibit reuse. Of those, our analysis exposes about 80%.

Keywords: profile-guided optimizations, register promotion, program representations, data-flow analysis.

1 Introduction

Without comparison, caches are the best hardware defense against the von Neumann memory bottleneck. Capitalizing on data locality, caches win by reusing recent memory accesses. How can compilers benefit from these reuse opportunities? In the ideal case, the compiler promotes repeatedly accessed memory locations to registers. Register promotion is the best compiler solution for reducing the memory traffic. By removing redundant loads, it decreases the dynamic operation count and shortens instruction schedules. This paper focuses on compile-time detection of load reuse that is amenable to register promotion. We measure the amount of load reuse in programs, and design and evaluate an analysis for reuse detection.

Register promotion entails three subproblems. First, load-reuse analysis finds loads and stores that access the same address, together with the execution paths along which the reuse exists. In the example below, if $a_1$ always equals $a_4$ along path $p_1$, then load $a_4$ can benefit from reuse along $p_1$. Similarly for path $p_2$. Second, alias analysis verifies that the detected reuse is not disrupted by intervening stores. Below, if $a_0$ is never equal to $a_4$, then register promotion of $a_4$ is safe. Finally, a program transformation stores the prior memory access in a register and replaces the redundant load with a register reference. In the example, register promotion is not immediately applicable because load $a_4$ is not redundant on all paths. Such partial reuse can be compensated by hoisting a copy of the load along path $p_2$. Commonly, the hoisting is formulated as partial redundancy elimination (PRE) [26, 28, 35].

1 Load-reuse analysis:
   is $a_1 = a_4$ along path $p_1$?
   is $a_2 = a_4$ along path $p_2$?

2 Alias analysis:
   is $a_0 \neq a_4$?

3 Program transformation:
   hoist load $a_4$ along path $p_3$

Detecting reuse is profitable even when register promotion is prevented (due to aliasing or lack of registers). In such a case, the PRE transformation step can employ alternative, albeit less effective, reuse mechanisms. When promotion is unsafe due to interfering stores, the redundant load can be replaced with a data-speculative load, which works as a register reference when the kill did not occur, but as a load when it did [6, 20, 23, 38]. When registers are not available, load reuse can be exploited using software cache control [20, 23, 33]. By directing which loaded values remain in the cache and which bypass it, the compiler can improve the suboptimal hardware cache replacement strategy.

This paper focuses on the first component of register promotion, load-reuse analysis. Because an optimization is only as powerful as its analysis, improving the precision of the analysis is of high
The second component, alias analysis, has a different aim: while load-reuse analysis detects memory references that must go to the same location, alias analysis finds those that may, thus identifying killing stores. Recent research indicates that, for register promotion, a simple alias analysis may be sufficient [18, 27]. The third component, PRE transformation, was explored in [10], where we describe how to effect a complete removal of all detected reuse. In this paper we concentrate on increasing the amount of detected reuse.

**Design.** The design of the load-reuse analysis emphasizes scalability and completeness. Scalability is achieved by developing a sparse, SSA-based program representation, which grows moderately with the program size.

The analysis is PRE-complete if it detects all reuse that the PRE transformation can exploit. Aiming for PRE-completeness is not a narrow goal, as PRE covers most scalar transformations based on data-flow analysis. It generalizes common-subexpression elimination, loop-invariant code motion, and constant propagation. Beyond the power of the PRE class are, however, loop optimizations, such as loop fusion and interchange. These array-oriented transformations can be used as a preprocessing, locality-improving phase, after which PRE can harvest the scalar reuse opportunities [13].

To approach PRE-completeness, the load-reuse analysis is path-sensitive and comprehensive. Path-sensitivity has two flavors. First, we can expose reuse even when it exists only on a subset of paths coming to the load (in the example, path \( p_3 \) has no reuse). Second, we find the equivalence of address expressions even when it is path-specific (it is sufficient that \( a_1 \) equals \( a_2 \) along the path \( p_1 \) and not along all paths). The analysis is comprehensive in that memory references to scalars, arrays or pointer-based data structures are handled uniformly, without any high-level program information, such as type information.

Technically, our load-reuse analysis is formulated as a data-flow problem in order to directly guide the PRE transformation [10, 24]. In our analysis, data-flow problems are solved on the Value Name Graph (VNG) [5], a program representation that enhances data-flow analysis by exposing equivalence among address expressions. This paper extends the VNG representation in two directions. First, we improve its power by modeling indirect memory references. Second, because the original VNG [5] did not scale well, we develop a sparse VNG, based on the SSA form [17].

**Evaluation.** Typically, optimizations are evaluated by reporting the amount of computations removed. Unfortunately, such absolute measure says little about how much potential remains unexploited. Instead, our evaluation measures the level of PRE-completeness: how far is the analysis from an ideal one? Because detecting load reuse is in general undecidable, we can only hope to find an approximation of the ideal reuse amount. For that purpose, we perform a simulation-based limit study: by observing the dynamic stream of memory references, we find all reuse available under a given input and use it as an upper bound of the PRE-exploitable reuse in the program.

While the (static) load-reuse analysis identifies redundant loads and their reuse paths, the (dynamic) limit study yields the run-time number of redundantly executed loads. To compare these disparate quantities, we weight the static reuse using the program profile generated by the simulator. The result is the run-time amount of statically detected redundant loads. This amount can, besides measuring the precision of the analysis, guide the code-duplication trade-offs in code-restructuring optimizations [1, 7, 8, 29, 30], as described in [10]. Unfortunately, any method for computing the run-time amount of reuse from a profile is impaired by the profile's inherent inability to precisely reconstruct frequencies of execution paths on which reuse was detected by the analysis. This holds both for the commonly used edge profiles and for the more powerful path profiles, which record frequencies of not just CFG edges but also (some) finite paths [3]. While existing profile-directed optimizations disregard the profiling error [10, 32], our family of estimator algorithms computes its bounds. The estimators form a hierarchy: the more complex, the tighter the error bounds.

**Summary.** This paper culminates our efforts in developing a path-sensitive framework for value-flow optimizations [4–10]. Here, we develop a few missing pieces of the framework (sparse VNG, estimators) and also evaluate its effectiveness (on the load-reuse optimization, using the limit study).

In summary, the contributions of the paper are threefold:

1. **Load-reuse analysis:** we generalize the Value Name Graph representation [5] by supporting analysis of indirect memory accesses. We also develop a scalable, sparse version of the representation.
2. **Load-reuse limit study:** we develop a simulation-based method for detecting the amount and sources of load reuse in a program and use it to evaluate a static load-reuse analysis. The reuse available in significant benchmarks (Spec95) is reported.
3. **Profile-based estimators:** we develop algorithms that use edge profiles to assign a dynamic weight to an analysis-detected reuse. The estimators can be ordered by precision: even modest complexity is enough to use edge profiles and get sufficient precision.

Our entire experimental framework is summarized in Figure 1. Our experiments show that about 35% of loads executed in Spec95 could be removed through reuse. Of those, 80% are detected by our load-reuse analysis.

The rest of the paper is organized as shown in Figure 1. Section 2 describes the simulation-based reuse detection. Section 3 is devoted to the static load-reuse analysis. Section 4 presents the es-
timators and Section 5 evaluates the analysis. Finally, Section 6 concludes by discussing related work.

2 Dynamic Amount of Load Reuse

This section focuses on load reuse visible at runtime. We present a simulation-based limit study that has multiple uses: a) measuring the amount of reuse in programs (how large is the optimization potential of register promotion?), b) evaluating the load-reuse analysis by providing a reference point (how close is the analysis to its ideal performance?), and c) tuning the analysis (which are the redundantly executed load instructions?). In this section, we describe the design of our simulation and show that a large fraction (55%) of loads executed in Spec95 exhibits reuse opportunities.

The primary use of the limit study is to evaluate the precision of the load-reuse analysis. The precision is measured as the level of completeness. An analysis is \( T \)-complete if it detects all reuse that can be removed from the program with a program transformation \( T \). In this paper, \( T \) is the partial redundancy elimination (PRE)[10, 24, 28]. PRE is a code-motion transformation that can exploit reuse even when it exists only on a subset of execution paths incoming to the redundant load. Therefore, PRE has become the basis of modern register promotion techniques [6, 14, 26, 35].

Unfortunately, detecting load reuse is in general undecidable [31] and so no compile-time PRE-complete load-reuse analysis exists. Therefore, we use an empirical, run-time analysis that measures the reuse in the program as the program executes. In order to provide a close approximation of PRE-completeness, this simulation-based limit study should collect all reuse that PRE can remove, but no reuse that is beyond PRE's power. The simulation should thus mimic the character of the PRE transformation.

PRE removes redundancy by (conceptually) hoisting the partially redundant load against all control flow paths until it reaches a memory operation that generates the reuse. At this point, the contents of the promoted memory location is stored in a register that carries it to the original load. The reused value can be carried for a small number of loop iterations, using multiple registers [5, 14, 37]. In summary, the PRE operational restriction is that the redundant load can reuse a result of some other static instruction (or itself), where the result must be a small number of dynamic instances old.

The simulation algorithm reflects this PRE property. The run-time reuse is detected by remembering for each static memory instruction its access history: the dynamic stream of its recent addresses. A dynamic instance of a load is then redundant if a prior load or store accessed the same location without an intervening store. If an intervening store did occur, the load is still redundant; the intervening store becomes the reuse source.

As mentioned above in passing, the design of the simulation technique has two contradictory goals. On the one hand, the limit study should yield an upper bound: each reuse that can be removed with PRE must be detected. On the other hand, the bound should be tight: if a reuse for a given static load is intermittent (e.g., because it is sporadic or input dependent), it should be filtered out as noise. In the example below, the reuse between recurrent array accesses (i.e., between the store of \( A[i+2] \) and the load of \( A[i] \)) is PRE-exploitable by allocating two registers that will carry the value for two iterations [6, 12, 14]:

\[
\text{for } (i=0; i<N-2; i++) \{ \text{A[i+2] = A[i]; } \}
\]

On the other hand, the reuse below is noise. While some consécutive loads from the hash table may access the same location, the reuse is not guaranteed to occur each time the program takes the path across the loop backedge. Therefore, PRE cannot exploit this reuse.

\[
\text{while (c= read()) } \{ \text{.. = hashtab[hash(c)]; } \}
\]

To verify the PRE requirement that a path carries its reuse each time it is followed, the simulator would have to do extensive bookkeeping of followed paths. Consequently, we favor a noisier (less tight) upper bound on reuse over an expensive simulation. To reduce the noise, we limit the number of memory cells remembered in the access history of each static load and store. A small number \( h \) (1 to 4) of recent accesses is sufficient to capture most loop carried reuses, like the first example above [15].

As pointed out in Section 1, PRE is not capable of exploiting loop-level reuse, like the reuse between loads \( a \) and \( b \) below. Hoisting \( b \) does not work. Instead, the loops must be merged using loop fusion [13], after which PRE can harvest the reuse.

\[
\text{for } (i=0; i<N; i++) \{ \text{a: .. = A[i]; } \}
\]

\[
\text{for } (i=0; i<N; i++) \{ \text{b: .. = A[i]; } \}
\]

The simulation algorithm will (correctly) not identify the load \( b \) to be redundant (unless \( N \leq h \)) because the access history remembers only last \( h \) accesses made by load \( a \). Hence, the simulation is consistent with the power of PRE.

Reuse Level. Figure 7 plots the amount of simulation-observed load reuse. For each benchmark, the experiment was carried out at three points in the compilation: for the original program, after optimizations, and after register allocation. The compiler used was the latest public release of Impact [16]; the optimizations included the local, global, and loop invariant redundant load elimination, as well as superblock optimizations [22]. Note that while in the floating-point benchmarks the four on the right the removal of many loads was accompanied by the decrease of observable reuse, in the integer benchmarks the optimizer left many redundant loads unoptimized, which suggests that programs with complex control flow require more powerful, path-sensitive optimizations and/or better alias information. Also note the increase in observed reuse after register allocation, which is due to spill-code loads (the target processor was PA-7100).

We show the amount of reuse for the history depth 1 and 4. Increasing the history depth raises the observable reuse much more in integer programs than in the scientific ones, where more recurrent accesses would be expected. A manual examination of simulation results strongly suggests that the additional reuse collected at the deeper access history is mainly noise, similar to the intermittent reuse in the hash-table example above. Also shown in the graph is the fraction of reuse in which both the generator and the redundant load belong to the same procedure. These reuse patterns are not strictly intraprocedural, as the procedure might have returned and been called during the reuse. However, these “intraprocedural” reuse levels serve as a reference point for our intraprocedural load-reuse analysis (Section 5).

Input Variance. Profile-directed optimization and simulation-directed optimization design are valuable only if the program input exercises input-independent, pervasive program characteristics. How much does reuse vary across different inputs? We modified the inputs on several benchmarks and compared the observed reuse. The results are shown in Table 1. The input-based variation of the reuse level is within 18%, which may suggest that reuse is
largely input independent. The greatest difference is in m88ksim, in which each input directs the execution into different procedures. For the same reason, this benchmark has less reuse generated by stores in the test input (fractions add up to more than 100%; as a reuse instance may be generated by multiple instructions, a load and a store). We have manually examined compress and discovered that the lower reuse in the larger input is due to fewer noisy loads. Input variance may therefore be useful as a noise reduction mechanism; by taking intersection of reuse detected on different inputs, we may determine regular, statically detectable reuse.

Simulation Memory Requirements. While the simulation limit study is considerably more expensive than control flow profiling, it is used once (to design and tune the analysis) unlike the cheaper profiling which is repeated (to optimize each program). Still, the simulation speed was acceptable, at about 9.4 seconds per 1 million loads and stores executed (on PA-8000). The memory required varied greatly. The largest data structures were needed by swim (103MB + 32MB hash table) and the smallest by compress (4MB + the same hash table).

3 Load-Reuse Analysis

While the previous section described the approximate dynamic load-reuse analysis, this section presents the conservative static analysis.

Detecting load reuse reduces to finding path-sensitive must-alias information: we want to know which address expressions are always equivalent and along which control flow paths. Our analysis is formulated as a data-flow analysis, for two reasons. First, when detected reuse is expressed as a data-flow solution, it can directly guide the subsequent PRE transformation, which is driven by the data-flow problems of availability and anticipability [10, 24]. While the former problem exposes the reuse (by finding a prior load or store), the later verifies whether the PRE transformation is not harmful to the program (by determining whether the reuse can be consumed by a future load). The second reason is that data-flow analysis can leverage existing program representations [5, 6, 11, 19, 37] designed to expose reuse not accessible to the traditional data-flow analysis [24, 28].

The analysis in this paper is based on the Value Name Graph, a value-centric representation that enhances traditional data-flow analysis by appropriately naming the value that flows between equivalent computations [5]. A value flows between two (address) expressions if they compute the same value (address). In traditional data-flow analysis, each value is identified with its lexical name, e.g., its abstract syntax tree. When two names match, the addresses (may) compute identical values. But what name should be used when the value flows between equivalent addresses that have dif-

Recall that the use of may-alias information to disambiguate intervening stores is conceptually independent from detecting load reuse, as described in Section 1. This section also shows how may-aliasing is accounted for in our load-reuse analysis.
different names? The VNG overcomes the naming problem by synthesizing names that fully trace the flow of the analyzed value and by performing data-flow analysis on this synthesized name space. The synthesized names are created using symbolic substitutions along each control flow path; as a result, the VNG exposes equivalences among address expressions that become visible only after symbolic algebraic manipulations.

This paper addresses two deficiencies of the original VNG [5]. The first is the lack of expressiveness specific to detecting load reuse. Because the VNG only models value flow through arithmetic computations, it cannot trace flow of addresses through the memory, and hence cannot handle indirect addressing. The second deficiency is the high memory demands of the original VNG, a consequence of its rigorous reflection of algebraic characteristics of the value flow. In this paper, the VNG is made more effective by incorporating indirect addressing into the symbolic interpretation, and more efficient by developing a sparse VNG representation that is smaller and scalable.

Constructing the VNG. The VNG combines advantages of three orthogonal analysis approaches. Each of them overcomes different obstacles in equivalence detection: global value numbering finds equivalent expressions that have different names due to assignments to temporaries [34]; symbolic interpretation finds equivalences requiring algebraic simplification, such as recurrent array accesses [6, 12]; data-flow analysis connects expressions that may be equivalent only along some control flow paths [24, 28]. First, we sketch the construction of the original VNG enhanced to accommodate indirect addressing. The following subsection describes how to build the sparse VNG.

The construction has three steps, each corresponding to one of the underlying approaches. First, the symbolic interpretation creates names necessary to trace the value flow. Second, value numbering determines which names are synonymous references to the same value. The result of the first two steps is the VNG representation, on which the third step computes value-related data-flow problems, using any traditional data-flow analyzer.

Step 1: Create the symbolic names. The goal is to create sufficient names so that a value can be identified even when it flows outside the scope of the lexical name under which it was originally computed. Where the original name is not valid, we use an equivalent symbolic name. The symbolic names are created on demand by propagating backwards the address operand of each load. The propagation effectively creates a “symbolic” slice of the address operand, by substituting into the propagated address expression each relevant assignment and performing some algebraic simplification. While the original address operand represents the lexical name of the address value, the slice expression is the symbolic name. Below, we analyze the address of the load; its lexical name is $y + 4$. After this name is propagated through the preceding assignment, the name of the address changes to $2 \times x + 12$, which is a symbolic name. Note that the symbolic substitution was followed by algebraic simplification.

$$y := 2 \times x + 8 \quad \leftarrow \text{name} = 2 \times x + 12$$
$$z := \text{load} (y + 4) \quad \leftarrow \text{name} = y + 4$$

Due to loops, such a back-substitution process may not terminate. Therefore, we perform the substitution only for $w$ iterations of each loop, where $w$ is a small constant (1 to 4), analagetical to the access history $h$ used in the simulation in Section 2.

To accommodate indirect addressing, we enrich the symbolic language of value names with a pointer dereferencing operator $\ast$ and back substitution rules for loads and stores. Loads increase the direction level: when a name $t + 1$ is propagated backwards across $t := \text{load} L$, it will change to $\ast L + 1$. Stores may reduce the direction: across $\text{store} L, t$, the name $\ast L + 1$ will change to $t + 1$.

To obtain the performance reported in Section 5, it was sufficient to represent addresses with a symbolic name $E = c_0 + c_1 v_1 + \ldots + c_n v_n + \ast (E')$, where $c_i$ are literals, $v_i$ are program variables, and $E' = E \mid \epsilon$. The term $E'$ adds addressing direction. In the actual implementation, one may want to set a maximum number of direction levels, to limit the number of symbolic names created during back-substitution. In our experiments, we used level 0 (no $\ast$ operator in the address name) and level 1 (one $\ast$ operator in the address).

Figure 3(b) shows the VNG for the program in Figure 3(a). We illustrate the back-propagation using $p_4$, the address operand of the load in node 9. When propagating $p_4$ across the assignment $p_4 := p_3 + 1$ in node 7, the right-hand side $p_3 + 1$ is substituted into the current name $p_4$. We obtain $p_4 + 1$, which becomes another name for the analyzed address of the load (9). After crossing $p_4 := \text{load} L_p$ in node 6, $\ast L_p$ is substituted for $p_4$ and $\ast L_p + 1$ becomes yet another name for the address ($L_p$ is the address of the global variable $p$). The name will be further changed at nodes 4, 3, and 1. (Note that the Figure 3(b) is showing the VNG construction only along the then path.) The address operands of remaining memory operations will also undergo this back-propagation. The process of name creation is demand-driven, as only the necessary names are created.

When back-substitution is completed, the graph contains value threads that connect the different names of the analyzed value. Along the control flow path associated with a value thread, the threaded names refer to the same value. Therefore, all memory operations on a thread access the same memory location.

Step 2: Find synonymous names. The value threads are used by data-flow analysis to compute availability of prior memory accesses. For example, reuse exists between nodes 4 and 6, as they lie on the same thread. Unfortunately, threads alone cannot find reuse between the equivalent nodes 5 and 9, because they are not on the same thread. However, their address expressions ($t_1$ and $p_4$) are both symbolically reduced by the back-propagation step to the same name $p_4$, at the entry of node 2. This proves that both of these memory references must access the same memory location; the names from the two parallel threads are synonymous at each node, as expressed by the dashed edges.

We call the second step symbolic value numbering, as it extends the standard global value numbering [34] with the symbolic manipulation. It finds the synonymous names by “collapsing” the threads in a forward pass. Collapsing is performed by inserting store $\ast L_p + 1$ onto the thread connected with the dashed edge. The new store writes to the same location as its synonymous counterpart (store $t_1$) but is placed on the parallel thread, which enables detecting the reuse between node 5 and 9. The insertion of the store completes the VNG construction.

Step 3: Solve data-flow problems. Once the VNG is constructed, any conventional data-flow analysis can propagate facts along the threads augmented by the second step and answer the two questions posed by the FRA transformation: which memory addresses are equivalent, and along which control flow paths? In our example, the reuse between store (5) and load (9) will be revealed in the form of a memory access being available at node 9.

The Sparse VNG. Experiments with the original VNG (shown in Figure 3(b)) revealed three sources of inefficiencies preventing
practical deployment. First, the synonym relationships are maintained at each node, consuming much memory. Second, many symbolic names do not belong to any value thread on many nodes, wasting slots in data-flow bit-vectors. Such a construction runs out of 1GB virtual memory on some procedures that grew during inlining. Last, threads contain many switches between symbolic names, which reduces bit-vector parallelism, slowing down the analysis. We present here a sparse VNG representation. It reduces memory and time requirements, while maintaining the same power as the original formulation. The memory savings are more than 30-fold on some large procedures.

To obtain the sparse form, we skip the expensive Step 2 above and transform the VNG created in Step 1 into an SSA program with the following (local) transformation: first, for each symbolic name e we create a scalar variable, denoted \([e]\). Second, at CFG nodes where a name \(e_i\) is back-substituted into \(e_2\), we insert the assignment \([e_1] := [e_2]\). Figure 3(c) contains the result of such transformation. The memory references are correspondingly rewritten to refer to these new variables.

In this intermediate form, each \([e]\) variable will receive an SSA subscript after which the synonyms can be maintained globally using the global value numbering (GVN) [34], rather than on each node, which fixes our first deficiency. In our example, only \([L_p + 1]\) belongs to congruent class \(C_2\) and \([L_p]\) belongs to \(C_1\), each store to \(C_1\) must kill reuse in class \(C_2\) and vice versa. Therefore, in Figure 3(d), the store in node 8 would kill the reuse for the load in node 9. Depending on the optimizer, this kill may entirely destruct the reuse, preventing register promotion, or may mark only the reuse as unsafe, enabling its exploitation using a data-speculative load [20,23,33].

Killing stores. The VNG analysis detects reuse aggressively. Because the value threads extend uninterrupted across potentially killing stores, the VNG detects instructions that always read from the same location but it does not reflect that a store may change the contents of this location between these two reads. This exclusive focus on must-aliasing is an intentional design decision; by separating the killing effects, the VNG can detect a weaker form of reuse, one that may occasionally be interrupted, and exploit it with data-speculative loads, as mentioned in Section 1.

The killing information expressed as may-alias information can be accommodated in a natural way when data-flow analysis is computed on the sparse VNG. Using our running example, assume that \(p_4\) may equal \(L_p\). Because \([p_4]\) belongs to congruent class \(C_2\) and \([L_p]\) belongs to \(C_1\), each store to \(C_1\) must kill reuse in class \(C_2\) and vice versa. Therefore, in Figure 3(d), the store in node 8 would kill the reuse for the load in node 9. Depending on the optimizer, this kill may entirely destruct the reuse, preventing register promotion, or may mark only the reuse as unsafe, enabling its exploitation using a data-speculative load [20,23,33].
4 Estimators

The output of the load-reuse analysis is a data-flow solution that holds on paths along which reuse was detected. For any execution of the program, the total frequency of these reuse paths corresponds to the run-time number of loads that would be removed by a complete PRE transformation [10,36] and thus also to the dynamic amount of reuse detected by the analysis.

In this paper, an estimator is an algorithm that reconstructs the total frequency of reuse paths from a program profile. The estimator returns a profile-weighted reuse, which estimates the optimization benefit and thus can guide profile-directed optimizations [10,21]. In this paper, the weighted reuse serves as a measure of PRE-completeness: when the profile used by the estimator is generated by the limit-study simulator, the weighted reuse shows what fraction of the simulator-detected reuse was found by the analysis, and therefore indicates the precision of the analysis.

For pragmatic reasons, our estimator algorithms compute the optimization benefit from edge profiles, which are widely used and can be reused for various optimizations. Unfortunately, edge profiles contain an inherent profiling error. Because they do not capture branch correlation, they cannot reconstruct path frequencies faithfully to the actual execution, which prevents precise computation of weighted reuse. Existing estimators disregard the branch-correlation error. Built on the assumption that branches do not correlate, they are not concerned with how much the weighted reuse differs from the actual reuse [10,32]. To gain confidence in edge-profile-based estimates, we compute not a single reuse amount, but instead its lower and upper bounds, by assuming pessimistic and optimistic control flow scenarios.

This section presents five estimator algorithms that differ in their complexity and error-bounding precision. The practical reason for developing a hierarchy of increasingly better estimators (Figure 4) is that when a simpler (and faster) estimator yields loose bounds, the optimizer can run the next better (but slower) estimator and have a guarantee that the new bounds will not be worse. To further enhance practicality, the estimators use static analysis information that is also needed by the subsequent PRE transformation, which amortizes their cost.

While estimators cannot eliminate the inherent edge-profile error, by computing error bounds, they indicate the fundamental limitations of edge profiles. Our second best estimator was able to bound the error down to 5%, a 4-fold improvement over the simplest estimator. Therefore, with good algorithms, edge profiles seem to provide sufficient precision, at least for optimizations based on load reuse analysis. Other optimizations may still require correlated profiles, such as path profiles [2,3,39]. Unfortunately, even path profiles remedy the correlation problem only partially, as they measure the frequency of paths that may not fully overlap the detected reuse paths, thus capturing only part of the correlation needed to reconstruct the reuse weight. In fact, we are not aware of any profiling technique, short of a complete execution trace, that enables computing the weighted reuse with no profiling error. As a step towards this goal, the algorithmic abstraction behind our estimators formulates what profiling information enables error-free estimates.

The problem Statement: computing the weighted reuse. Figure 5(a) illustrates the problem of computing weighted reuse. Assume that the load-reuse analysis detected that the three loads refer always to the same memory location z. Also assume that the node D contains a (killing) store that may write to z, according to the alias analysis. Given the edge profile annotated on the CFG, what is the minimum and the maximum number of reuse opportunities on z permitted by that profile?2

The Estimators. The weighted reuse can be computed as the sum of frequencies over all reuse paths, i.e., all paths between the three loads in Figure 5(a), excluding the paths crossing the killing node D. Each time any of these paths is taken, exactly one load of z can be removed. Namely, the paths are [A, f, C], [C, j, i], [A, j, h, i, j, E], and [A, f, h, i, [k, l]+, E], where '+' denotes the usual non-zero repetition. Even if we could determine the frequency of each reuse path, summing their frequencies by enumerating them would not be feasible, as the loop generates infinitely many paths.

Rather than dealing with individual paths, our estimators find program points that efficiently summarize groups of paths with identical reuse properties. For the upper bound, we find a set of program points called generators, on which the reuse is available along all incoming paths. To compute the actual value of the upper bound, we determine how much reuse can flow between generators and the set of consumer points, on which a load consuming the reuse exists along each outgoing path. For the lower bound, we find the set of stealer points on which the reuse is available along no incoming paths. To arrive at the lower bound, we determine how much reuse-free flow can reach the consumers, stealing [3] the reuse flowing from the generators.

The estimators differ in how they compute these three sets and how precisely they account for the possible flow of reuse among them. Next, we present a brief overview of the individual estimators.

PRE is the simplest estimator. Mirroring closely the PRE transformation, generators are taken to be those instructions that generate the reuse; stealers are the points where a load is inserted by PRE to compensate partial redundancy; and consumers are the partially redundant loads. To determine which generators (or stealers) may provide (or steal) reuse for each consumer load, PRE uses control flow reachability.

The PRE estimator is imprecise because it includes in its worst-case assumptions also those reuse paths whose weight can be computed precisely even from the edge profile. Such paths can be excluded from the worst case by placing generator, stealer, and consumer points closer together, effectively reducing the number of paths among them. To find such a placement, the remaining estimators use the observation that all branch-correlation error harmful to reuse calculation can be contained into a special region, called a CMP region (short for code-motion-preventing region), originally developed to identify obstacles to code motion in a complete PRE transformation [10].

Footnote: The permitted minimum/maximum is a tight bound. Our estimators are not tight. Still, the more precise the estimator, the tighter the bounds it computes.
The CMP is the smallest multi-entry, multi-exit region in which the entries can be divided between generators and stealers, and the exits between consumers and (strict) non-consumers. Being the smallest such region, it finds the desired closest placement, considering in concert all reuse paths, not only those leading to a single load. The CMP contains all the error because, on each node in the CMP, the reuse is generated only along some incoming paths and can be consumed by a load only along some outgoing paths. Consequently, without the knowledge of branch correlation in the CMP, it is not possible to determine how much incoming reuse actually flowed to consumers in the profiled program execution. On the other hand, outside the CMP region, the reuse can be computed without an error. The CMP estimators thus focus on reducing the error contained in the CMP region, as follows:

\[ \text{CMP}^1 \] estimator conservatively assumes that there is a single CMP (hence the 1 in the name), in which all entries and exits are mutually reachable. This false reachability may connect consumers to spurious generators and stealers, producing loose bounds.

\[ \text{CMP}^2 \] attacks false reachability by partitioning the CMP region into connected CMP subregions, using control-flow reachability between CMP entries and exits. The individual connected CMPs are treated with the \[ \text{CMP}^1 \] estimator.

\[ \text{CMP}^\# \] exploits entry-exit reachability further. Compared to \[ \text{CMP}^2 \], it removes false reachability even within each connected CMP, by computing reuse as a network flow problem.

\[ \text{CMP}^{\text{Correlation Profiling}} \] exposes to the network flow computation all the CFG edges in the CMP, not just the summary entry-exit reachability information, thus exploiting a refined notion of reachability that accounts for how much reuse can flow between CMP entries and exits, and not just whether they are reachable.

**Objective**

To compute the upper bound on the reuse detected for load \( l \), we assume the most optimistic control flow scenario that all reuse generated in \( G(l) \) flows to \( l \). In other words, \( f(d) \) denotes the execution frequency for \( d \), where \( d \) is a CFG node or edge. Finally, \( L(P) \) denotes the set of loads in program \( P \).

\[ \text{PRE} (P) = \sum_{l \in L(P)} \min\{f(l), \sum_{g \in G(l)} f(g)\} \]

\[ L^{\text{PRE}} (P) = \sum_{l \in L(P)} \max\{0, f(l) - \sum_{s \in S(l)} f(s)\} \]

Let us apply the PRE estimator on the program in Figure 5(a). The bounds for loads \( A \) and \( C \) are trivial, as \( A \) is not redundant and \( C \) is fully redundant: \( L^{\text{PRE}} (A) = U^{\text{PRE}} (A) = 0 \) and
The profiling error affects only the partially redundant load $E$. Its generators and stealers are $G(E) = \{A, C\}$ and $S(E) = \{(g, h), (g, k), (D, E)\}$, yielding bounds $L^{UPRE}(E) = 45$ and $L^{UPRE}(E) = 135$. The total reuse for the program is $L^{PRE}(P) = 80$ and $U^{PRE}(P) = 170$, which is a $170/80 = 112.5\%$ error.

The large PRE’s error is not all due to the inherent deficiencies of the edge profile. The cause of the error is “overbooking” of a generator by multiple consumers. In Figure 5(a), load $A$ is a generator common to consumer loads $C$ and $E$, which together consume more reuse than $A$ can generate ($C$ counts 35 and $E$ counts 100). Technically, the cause of overbooking is that PRE charges the entire frequency contribution of a generator to multiple reuse paths that originate in the generator. Instead, the generator frequency should be divided among these paths. This can be done by moving the $A$ generator into the edges $(f, C)$ and $(f, h)$, which become the new generators, effectively dividing the contribution of $A$ among loads $C$ and $E$. The CMP region is an abstraction that divides the contribution of generators, stealers, and consumers. The CMP region for the running example is shown in Figure 5(b); it effectively excludes the reuse path $[A, f, C]$ from the worst-case considerations.

First, we present the definition of the CMP region. Formally, the CMP is a subgraph of the Sparse VNG. To simplify the presentation, we establish the restriction that the sparse VNG contains no $\phi$-nodes. Under this restriction of generality, each memory location has exactly one name. Without having to switch names, we can reason about estimators using the CFG, rather than the more general VNG. While the estimator extensions to handle an arbitrary VNG are small, their explanation is beyond the scope of this paper and can be found in [4].

Given the restriction, the CMP region is identified by solving the problems of anticipability and availability, which are defined as follows [10].

**Definition 1** Let $p$ be any path from the CFG start node to a node $n$. The contents of memory with address $x$ is available at $n$ along $p$ iff $x$ is loaded or stored on $p$ without a subsequent killing store.

Let $r$ be any path from $n$ to the CFG end node. The load of address $x$ is anticipated at $n$ along $r$ iff $x$ is loaded or stored on $r$ before any killing store or a store to $x$. The availability of $x$ at the entry of $n$ w.r.t. the incoming paths is defined as:

$$AVAIL_{n}[x] = \begin{cases} 
\text{Must} & \text{all} \\
\text{No} & \text{if } x \text{ is available along no paths.} \\
\text{May} & \text{some} 
\end{cases}$$

Anticipability (ANTIC) is defined analogously.

**Definition 2** The CMP region for address $x$, denoted $CMP[x]$, is a set of nodes $n$ where $AVAIL_{n}[x] = \text{May}$ and $ANTIC_{n}[x] = \text{May}.$

Figure 5(b) shows the CMP region for the address $x$. Each CMP region has a set of entry edges and exit edges. Each entry is either Must- or No-available; we denote them $N^M$ and $N^N$, respectively. The $M^M$ entries act as generators and the $N^N$ entries act as stealers. Similarly, exits are either Must- or No-anticipated, denoted $x^M$ and $x^N$, respectively. The $x^M$ act as consumer points. The non-consumer $x^N$ exits do not participate in the estimator algorithms. The CMP is the smallest region in which reuse is uncertain; generators cannot be moved closer to consumers, because they would enter the CMP regions, where reuse is not available along all incoming paths and thus they would no longer act as generators. Identical arguments prevent moving stealers and consumer points. The CMP thus maximizes the number of paths that can be excluded from the worst-case assumptions about branch correlations; outside the region, the reuse can be computed without any error, even from an edge profile. It can be shown that all reuse bypassing the CMP region can be measured by finding generator points on which the reuse is available along all incoming paths and will be consumed along all outgoing paths. Such generators have no branch-correlation uncertainty—they are definite. In Figure 5(b), the definite generator points are $m$ and $n$. Each of them provides 35 units of reuse that will be fully consumed (m’s by $C$ and n’s by $E$).

To formalize the above discussion, the CMP divides the reuse on an address $x$ into definite and uncertain. The definite reuse $R_d(x)$ has no error and equals the sum of frequencies of all definite generators $G_d(x)$. For the example in Figure 5, the definite reuse $R_d(x) = 70$. In the formulas below, $M(P)$ is the set of all address names mentioned in the program text. The definite generators $G_d(x)$ are placed as close to the consumers (the loads of $x$) as possible.

<table>
<thead>
<tr>
<th>all CMP estimators:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U^{CMP}(P) = \sum_{x \in M(P)} (R_d(x) + U_{u}^{CMP}(x))$</td>
</tr>
<tr>
<td>$L^{CMP}(P) = \sum_{x \in M(P)} (R_d(x) + L_{u}^{CMP}(x))$</td>
</tr>
<tr>
<td>$R_d(x) = \sum_{g \in G_d(x)} f(g)$</td>
</tr>
<tr>
<td>$G_d(x) = {(u, v) \mid AVAIL_{x,v}[u, x] = \text{Must} \land (AVAIL_{x,v}[v, x] = \text{May} \lor v = \text{load } x)}$</td>
</tr>
</tbody>
</table>

The CMP estimators differ in how they compute $U^{CMP}(P)$ and $L^{CMP}(P)$, which are the bounds of the uncertain component of the weighted reuse. Figure 6 compares the CMP estimators.

**CMP** is the simplest CMP-based estimator. It identifies CMP entries and exits and, to minimize its cost, assumes that each entry-exit pair is mutually reachable. The resulting optimistic scenario is that all $M^M$ entries are generators for all $x^M$ consumers. The upper bound is then the smaller of the total generator and the total consumer frequencies (Figure 6). The lower bound follows the same conservative assumption that the CMP region is fully connected. CMP is efficient; it computes only the ANTIC and AVAIL data-flow solutions. Entries and exits are identified by examining the two data-flow solutions locally at each node. Both the solutions and the entries are also needed by the PRE transformation [10]. For the running example in Figure 5(b), CMP yields $L^{CMP}(x) = 10$ and $U^{CMP}(x) = 60$. The total program bound is $L^{CMPP}(P) = 80$, $U^{CMPP}(P) = 130$, which improves PRE’s upper bound by removing overbooking of load $A$, reducing the error to $130/80 = 62.5\%$.

**CMP** improves precision by eliminating some false entry-exit reachability assumed by CMP. It identifies connected CMP subregions, thus partitioning generator, stealer, and consumer sets. The smaller sets result in less overestimation when considering the worst-case scenarios. The bounds are computed separately for each connected CMP and then summed. In practice, we observed that the partitioning of the CMP region produced the highest increase in
its lower bound: $J!z = \max\{0, (40 + 20) - 30\} = 30$, try (g, k), less reuse can be stolen than in CMP', which improves ure 5(c). Because CMP exit (i, j) is not reachable from CMP en-

LCMPf (P) = 100, vCMP' (z) = 130, which is a 130/100 = 30% error. 

shown in Figure 5(d). After the weak link is accounted for, the up-

der reuse, due to simple control flow. On the other hand, the reuse

can be (efficiently) performed prior to knowing the shapes of CMP

Figure 6: The CMP-based estimators: algorithms for computing the definite component of weighted reuse. $n^M$, $n^N$, and $x^M$ are the frequencies of the corresponding CMP entries and exits. $\text{maxflow}(u, v)$ denotes the maximum flow between vertices $u$ and $v$. CMP$\text{f}$ assumes all CMPs are one, i.e., that all entries and exits are mutually reachable. CMP$\text{e}$ separates connected CMPs, eliminating some false reachability. CMP$\text{p}$ exploits intra-CMP reachability, using a max-flow computation. CMP$\text{f}$ exposes to the max-flow all intra-CMP edges, including their actual profile weights.

<table>
<thead>
<tr>
<th>$U_u$</th>
<th>$\min{\sum_i n^M_i, \sum_j x^M_j}$</th>
<th>$\sum_i \min{\sum_j n^M_i, \sum_j x^M_j}$</th>
<th>maxflow$(N^M, X^M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_u$</td>
<td>$\max{0, \sum_i x^M - \sum_i n^M}$</td>
<td>$\sum_i \max{0, \sum_j x^M - \sum_j n^M}$</td>
<td>$\max{0, \sum_i x^M - \text{maxflow}(N^N, X^M)}$</td>
</tr>
</tbody>
</table>

Experiments: estimator precision. Figure 7 compares the precision of the estimators. For each benchmark, we plot the weighted reuse obtained by four estimators (we have not implemented CMP$\text{f}$). The reuse is broken up into four parts; the left two bars together represent the definite reuse component $R$, on which all benchmarks are normalized. The third and fourth bars are the lower and the upper bounds on the uncertain reuse. The floating-point benchmarks (the lower four) have nearly no uncertain reuse, due to simple control flow. On the other hand, the reuse in integer benchmarks has a significant uncertain component. We can observe that with good algorithms, the profiling error can be greatly reduced. Note that while, in theory, CMP$\text{p}$ is not strictly more precise than PRE (as the precision ordering shows), it performs much better in practice. In fact, CMP$\text{p}$ is appreciably better than CMP$\text{e}$ only on g020. Hence, due to its simplicity, CMP$\text{f}$ may be the estimator of choice. Overall, the average error was 15% for PRE and 5% for CMP$\text{f}$.

An important observation we made was that the estimator precision is strongly dependent on the pointer aliasing information. By interrupting some reuse paths, the killing stores induce more CMP regions, with more entries and exits, increasing the amount of uncertain reuse. For the comparison in Figure 7, we selected the configuration of load-reuse analysis that caused the largest estimator
5 Experiments

This section experimentally evaluates the load-store analysis from Section 3 in relation to the limit study from Section 2. Because our implementation of the analysis is intraprocedural, the reference point for comparison is the intraprocedurally observed reuse. To minimize noise in the baseline, we use the reuse collected at the access history $h = 1$. We analyzed the unoptimized source programs. In summary, for each benchmark, the baseline for comparison is the ‘X’ mark in the leftmost column in Figure 2. Figure 8 plots the amount of reuse discovered by the analysis. The plotted amount was computed as the mean average of the lower and upper bounds returned by the CMP estimator.

The load-reuse analysis was carried out under varying assumptions. The two highest bars in Figure 8 show the load detected at 1-level and O-level address indirection, respectively. Our implementation considered only indirect loads, not stores, which may explain the lack of indirect reuse in some benchmarks. To determine the reuse-detection power of the analysis, these two bars assumed perfect aliasing under which no stores along a reuse path would kill the detected reuse. While not all of this aggressively detected reuse can be promoted to registers, it can be exploited with alternative reuse mechanisms, such as data-speculative loads, as noted in Section 1.

Overall, the comparison with the limit study shows that our analysis is about 80% PRE-complete.

Aliasing. We also studied the killing effects of intervening stores and procedure calls. Because our compiler does not perform alias analysis, we considered three hypothetical levels of pointer aliasing precision, specified as follows: first, we assumed that only procedure calls killed the detected reuse; second, we added to the kill set all stores except for stores to global variables; third, all stores and procedure calls killed the reuse. Due to aggressive inlining, only a small amount of reuse was lost at procedure calls (the white bar segments). However, array and pointer stores remove almost one third of reuse (the dark, middle segments). While this pessimistic hypothetical aliasing gives disappointing results, other researchers showed that even a simple alias analysis may produce memory disambiguation that is near-optimal for purposes of register promotion [18, 27].

Register Pressure. Besides aliasing, a lack of registers is another reason why detected reuse may not lead to register promotion. The register pressure at a CFG node is the number memory locations whose reuse path crosses that node; each location needs one register. We averaged the register pressure over all nodes, weighting each node by its profile frequency. For the O-level perfect aliasing analysis configuration, the highest average register pressure was 34 registers for su2cor. Such an amount of registers will be soon available in general-purpose processors.

6 Related Work

Simulation-Based Analysis Evaluation. While in microarchitecture the use of upper-bound limit studies has become commonplace, in compiler optimization this trend is recent. In fact, [18] is the only simulation-based evaluation of an analysis known to us. Diwan et al use a simulator to derive an ideal performance of an algorithm for removing heap-based loads. The ideal performance is used to determine what alias analysis is near-optimal for the load removal, but still not too expensive. Our work differs in that we focus on load-reuse analysis, rather than on the may-alias analysis. Lams and Chandra developed a compiler auditor tool, which
analyzes the program trace to discover limitations and bugs in the compiler [25]. Reinman et al. developed a load-reuse profiler technique similar to our simulator limit-study, with the primary goal to give load-reuse hints to the processor [33].

Estimators. Frequency analysis is the only existing systematic method for profile-weighting a data-flow solution [32]. Unlike our estimators, it is based on edge profiles. Unlike the estimators, frequency analysis does not bound the profiling error. However, considering that the inherent edge-profile error is small, as suggested by our experiments, the maximum amount of error in the result of frequency analysis will be correspondingly small (the result always falls between our lower and upper bounds). Our estimators offer an alternative to frequency data-flow analysis. While frequency analysis requires an elimination-style data-flow solver, our estimators use reachability or network flow algorithms, which may be easier to implement. Due to the small size of the CMP region, estimators are expected to run faster than a frequency data-flow solver.

Load-Reuse Analysis. Traditionally, load removal is navigated by a lexical load-reuse analysis, in which only loads with identical names (scalars) or identical syntax-tree structure (record fields) can be detected as equivalent [18, 26, 26]. Techniques based on value numbering can match expressions that have different names, but their symbolic interpretation power is limited to handling copy assignments [34, 37]. Therefore, they cannot capture equivalences that require symbolic interpretation, such as the recurrent array accesses shown in Section 2 for which specialized techniques have been developed [6, 12, 14]. Our load-reuse analysis encapsulates both value numbering and symbolic capabilities. While it is less powerful that array dependence techniques [13], the experiments show that our analysis uncovers about 80% of opportunities exploitable by partial redundancy elimination, including array and pointer loads.

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References


