Enhancing Feedback Control Scheduling Performance by On-line Quantification and Suppression of Measurement Disturbance

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***Abstract*— It has been shown that feedback control is effective to support the specified performance of dynamic systems that are both resource insufficient and exhibit unpredictable workloads. In the control of continuous and physical systems, the controlled system is sampled as fast as possible to capture the system dynamics. In general, this property cannot be applied to the control of computer systems, as the measured variables are of statistical nature, e.g., deadline miss ratio. In this paper we quantize the disturbance present in the measured variable as a function of the sampling period and we propose a measurement disturbance suppressive control structure. The experiments we have carried out show that a controller using the proposed control structure outperforms a traditional control structure, with regard to performance reliability and adaptation.**

Keywords: QoS management, feedback control, disturbance modeling, disturbance suppression.

I. Introduction

In recent years a new class of soft real-time systems operating in open environments has emerged, e.g., web applications, e-commerce, and data-intensive applications. These applications typically operate in open and unpredictable environments, in which arrival patterns and the resource requirements of tasks are in general unknown. This implies that tasks cannot be subject to exact schedulability analysis given the lack of a priori knowledge of the workload, making transient overloads inevitable. Furthermore, these systems are becoming larger and more complex, and at the same time they are being used in applications where performance guarantees are needed. Feedback control scheduling has been introduced as a promising foundation for performance control of complex real-time systems [2], [3], [6–9], [11], where the system designer/operator explicitly specifies the desired performance of the system in terms of a performance reference. It has been shown that feedback control is highly effective to support the specified performance of dynamic systems that are both resource insufficient and that exhibit unpredictable workloads.

When controlling physical and continuous systems, the sampling period selection is of paramount importance. The sampling period must be chosen such that the dynamics of the controlled system is captured and in general the sampling rate is set to the maximum that the controller and the AD/DA converters can manage [4]. However, when controlling computer systems, one cannot sample the controlled system arbitrarily fast. Usually, the measured variables are of a statistical and averaging nature, e.g., utilization or deadline miss ratio. To form these metrics requires an underlying data set, which must be large enough to give an acceptable accuracy of the behavior of the controlled system. To obtain a large data set we have to set the sampling period to a large value, meaning that we gather data over a larger time window. Doing so, however, results in an unresponsive system as the controller is rarely invoked and, hence, does not react fast enough to failures or changes in workload. Therefore, ideally we want to choose a low sampling period, to react to changes in the controlled system, still being able to base the control actions on a valid and accurate representation of the controlled system. This enables controllers to be more efficient in keeping the actual performance at the reference performance. This in turn increases the reliability of the system and implies a more controlled worst-case performance of the closed-loop system and faster convergence toward the desired performance. For example, consider a service provider streaming video to a set of clients. The service provider sets the reference quality of the streams to a certain level, and also gives requirements on worst-case quality and how fast the quality should converge toward the reference quality in the case of a transient overload or a disturbance in the system (e.g., a server goes down). Now, using a more accurate measurement of the controlled variable, enables the service provider to provide streams with more reliable quality and faster performance adaptation, as compared to using less accurate measurements. Hence, the quality is regulated much more efficiently even in the case of transient system overloads.

The contributions of this paper are as follows:

- a model of the controlled system incorporating the uncertainty in measurements, induced by the data set size,
- a method for quantizing the uncertainty of a measurement, given the data set size,
- a feedback control structure that is insensitive to the uncertainty of the measured variable.

To the best of our knowledge, this is the first paper proposing a model of the measurement disturbance and describing a

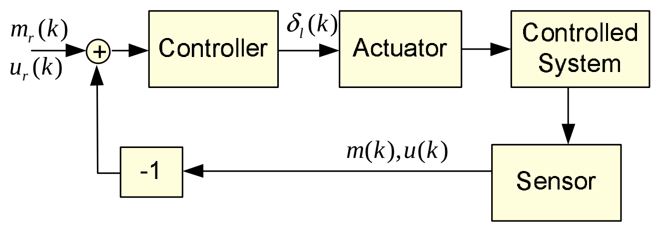


Fig. 1. Feedback loop structure.

feedback control structure that suppresses the measurement disturbance. We show through experiments that this approach results in a faster QoS adaptation and reliability in the face of changes to the controlled system, compared to using a traditional feedback structure where no disturbance suppression is done.

The remainder of this paper is organized as follows. The problem formulation is given in Section II. In Section IV we present our approach, and in Section V, we evaluate the performance of the disturbance suppressive feedback structure. In Section VI we give an overview on related work, followed by Section VII, where conclusions and future work are discussed.

II. Problem Formulation

We adopt the following notation of describing discrete variables in the time domain. A sampled variable $a(k)$ refers to the value of the variable a at time kT, where T is the sampling period and k is the sampling instant. For the rest of this paper, we sometimes drop k where the notion of time is not of primary interest. A typical structure of a feedback control system is given in Figure 1. Input to the controller is the difference between the reference $y_r(k)$, representing the desired performance of the controlled system, and the actual system performance given by the measured variable $y(k)$. Based on the performance error $y_r(k) - y(k)$ the controller computes a change $\delta_l(k)$ to the estimated admitted workload $l(k)$. The control problem is how to compute the manipulated variable $\delta_l(k)$ such that the difference between the desired performance and the actual performance is minimized, i.e., we want to minimize $(y_r(k) - y(k))^2$.

Typically, one is interested in controlling the performance of real-time systems using the metrics utilization or deadline miss ratio. We define the utilization as,

$$u(k) = \frac{t_B(k)}{t_T}, \quad (1)$$

where $t_B(k)$ is the number of time units that the system is busy computing, and t_T is the total number of monitored time units. We say that a task is terminated when it has completed or missed its deadline. Let the deadline miss ratio,

$$m(k) = \frac{n_M(k)}{n_T(k)}, \quad (2)$$

be the ratio of tasks that have missed their deadline, where $n_M(k)$ is the number of tasks that have missed their deadline and $n_T(k)$ is the number of terminated admitted tasks in the time interval $[(k-1)T, kT]$. Considering the goal of feedback control scheduling, i.e., minimizing $(y_r(k) - y(k))^2$, we note that it is impossible to keep $y(k)$ at $y_r(k)$. Before justifying this claim, let $\bar{y}$ denote the average value of $y(k)$ and let the variance of $y(k)$ be,

$$R_y = E\{(y(k) - \bar{y})^2\}, \quad (3)$$

the expected value of the squared deviation from $\bar{y}$.

Intuitively, the variance in utilization, i.e. $R_u = E\{(u(k) - \bar{u})^2\}$, and the variance in deadline miss ratio, i.e. $R_m = E\{(m(k) - \bar{m})^2\}$, are caused by transient increases or decreases in load, due to inaccurate execution time estimation errors, resource conflicts causing blocking, restart, and abortion of tasks. We refer to unpredictable changes in load as the system disturbance, causing variance in $u(k)$ and $m(k)$. However, there is also a second component contributing to variations in $u(k)$ and $m(k)$, namely, the disturbance arising from the averaging operation. For example, consider that we are measuring deadline miss ratio over a set of tasks for a given load. We note that R_m increases as $n_T(k)$ decreases. In the extreme case when $n_T(k)$ is one, $m(k)$ is equal to one or zero. Since $n_T(k)$ is small we obtain a poor presentation of the actual system performance, which in turn implies a negative effect on the feedback control scheduling performance, i.e., we observe large deviations between $y(k)$ and $y_r(k)$. We view the type of variance that arises from the averaging operation as a measurement disturbance that deteriorates the actual image of the system. Now, we know that $n_T(k)$ increases as T increases and, thus, by increasing the sampling period we lower R_u and R_m. However, this typically degrades the responsiveness of the controller [4], resulting in a slower reaction to changes in $u(k)$ and $m(k)$. Ideally, we want to have a low sampling period, to respond promptly to changes in the controlled variable, while experiencing a low measurement disturbance. The problems we address in this work are the following:

- How can we formalize and model system and measurement disturbances?
- How can we quantify the measurement disturbance?
- Given a sampling period T, how can we efficiently suppress the measurement disturbance to obtain a more accurate image of the behavior of the controlled system?
- What is the gain in performance control, with respect to minimizing $(y_r(k) - y(k))^2$, when suppressing the measurement disturbance?

In summary, our findings provide an insight on how the computation of utilization and deadline miss ratio give rise to measurement disturbance and also how we can use this knowledge to lower the measurement disturbance to achieve better performance control.

III. Task Model

We consider a real-time system as the controlled system, where there is one CPU as the main processing element. A task τ_i is classified as either a periodic or an aperiodic task. See Table I for a complete task model. A task τ_i has a set of

Attribute	Periodic Tasks	Aperiodic Tasks
d_i	$d_i = p_i$	$d_i = i_{A,i}$
$l_{E,i}[s_j]$	$l_{E,i}[s_j] = x_{E,i}[s_j]/p_i$	$l_{E,i}[s_j] = x_{E,i}[s_j]/r_{E,i}$
$l_{A,i}[s_j]$	$l_{A,i}[s_j] = x_{A,i}[s_j]/p_i$	$l_{A,i}[s_j] = x_{A,i}[s_j]/r_{A,i}$

TABLE I

THE ASSUMED TASK MODEL.

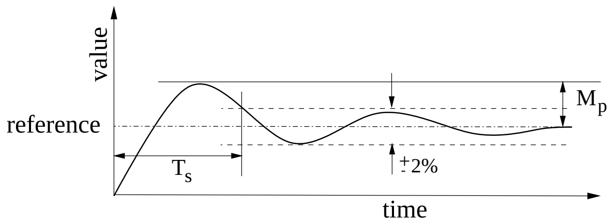


Fig. 2. Definition of settling time (T_s) and overshoot (M_p).

QoS levels $\{s_1, \ldots, s_j, \ldots, s_N\}$, where $N \geq 2$. A task has the following characteristics:

- period p_i (periodic tasks),
- estimated mean inter-arrival time $r_{E,i}$ (aperiodic tasks),
- actual mean inter-arrival time $r_{A,i}$ (aperiodic tasks),
- relative deadline d_i,
- estimated execution time $x_{E,i}[s_j]$,
- actual execution time $x_{A,i}[s_j]$,
- estimated load, $l_{E,i}[s_j]$,
- actual load, $l_{A,i}[s_j]$, and
- quality of result $q_i[s_j]$ produced when terminated, where $0 \leq q_i[s_j] \leq 1$.

The quality $q_i[s_j]$ of τ_i increases as j increases. Similarly, $x_{E,i}[s_j]$ and $x_{A,i}[s_j]$ increase as j increases. In the simplest case when $N = 2$, $q_i[s_1] = 0$, $q_i[s_2] = 1$, $x_{E,i}[s_1] = x_{A,i}[s_1] = 0$, $x_{E,i}[s_2] > 0$, $x_{A,i}[s_2] > 0$, represents a task model where tasks are either rejected or admitted for execution. Upon arrival, a task presents its estimated average load $l_{E,i}[s_j]$ and its relative deadline d_i to the system. The actual load of the task $l_{A,i}[s_j]$ is not known in advance due to variations in execution time.

IV. APPROACH

A. An Overview of the Approach

Generally, the performance of a controller improves with decreasing sampling period as the closed-loop system becomes more responsive to the difference between the controlled variable and its reference [4]. The reference may vary over time as a result of changes to the desired performance, or the controlled variable may vary due to changes in workload characteristics, e.g., a sudden change in execution time estimation error. The desired transient-state performance is usually expressed in terms of the maximum overshoot and the settling time [4], [6], [8], as shown in Figure 2. The maximum overshoot M_p is the worst-case system performance in the transient system state. The settling time T_s is the time for the transient overshoot to decay and reach the steady state performance and, hence, it is a measure of system adaptability.

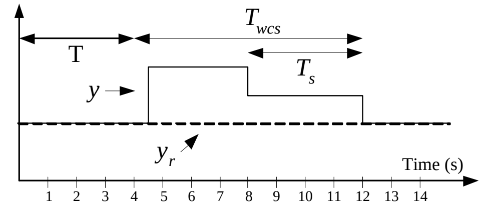


Fig. 3. The worst-case settling time depends on the sampling period.

Now assume that the sampling period is T. At time kT the feedback loop is closed, meaning that the controlled system is sampled, the manipulated variable is recomputed, and the actuator is invoked. Between each sampling no feedback information is sent to the controller and the system is running in open-loop and unresponsive to changes in the controlled variable. By definition, if the reference or the controlled variable suddenly changes at kT, then it takes T_s seconds for the controlled variable to reach the steady-state. However, consider the case when the controlled variable suddenly increases immediately after kT, e.g., the deadline miss ratio increases due to an execution time estimation error. Then, in the worst-case it takes $T_{wcs} = T + T_s$ for the controlled variable to reach the steady-state, as the change in the controlled variables is not detected until the next sampling instant at time $(k+1)T$. This is shown in Figure 3, where at time $4.5s$ the controlled variable suddenly increases. This increase is not detected until time $8s$ when the controlled variable is sampled. As T_s is $4s$, it takes a total of $7.5s$ for the controlled variable to reach steady-state. Hence, T must be chosen such that T_{wcs} satisfies the desired response time.

Further T_s may be influenced by T depending on the methodology used when tuning the controllers. Assume that we want to control $m(k)$ by using a P-controller [4] that is tuned by following the method presented in [8]. We apply a step to the reference to simulate the effect of a change in reference or a transient overload [8]. The settling time of $m(k)$ is $9s$ when $T = 1s$, whereas the settling time of $m(k)$ is $90s$ when $T = 10s$. Hence, the settling time increases as the sampling period increases, meaning that the controller is less responsive to changes in the reference or transient overloads.

We advocate an approach where the sampling period is chosen such that the transient-state specification based on the worst-case settling time, e.g. $T_{wcs} \leq 10s$ and $M_p \leq 10\%$, is satisfied and where the effects of the measurement disturbance due to the chosen sampling period is suppressed using estimators. For example, if T_{wcs} must be less than $10s$, and T_s is simulated to be $9s$, then T must be less or equal to $1s$. In the following sections we provide a model of the controlled system and we show that the measurement disturbance due to the averaging operation increases significantly as the sampling period decreases. We propose a feedback control structure that suppresses the measurement disturbance, resulting in an enhancement in the performance control.

B. A State-Space Model of the Controlled System

We adopt the model presented by Lu et al. [8], describing the utilization and the deadline miss ratio in the face of changes to estimated requested workload. We choose this model due to its simplicity and sufficiently precise description of the dynamics of a real-time system. We extend the model to capture the system disturbance and the measurement disturbance.

Starting from the control input, the workload of admitted tasks $l(k+1)$ in the next sampling period is changed through the manipulated variable $\delta_l(k)$ and the system disturbance δ_{wl}, given by

$$l(k+1) = l(k) + \delta_l(k) + \delta_{wl}(k). \tag{4}$$

As mentioned previously in Section II, the system disturbance $\delta_{wl}(k)$ arises from incomplete knowledge about the controlled system, e.g. unknown execution times and resource conflicts.

We say that an output signal is saturated when it remains unchanged even though the input signal is altered. The relationship between the admitted workload $l(k)$ and the utilization $u(k)$ is non-linear due to saturation. When $l(k)$ is less or equal to 100%, i.e. the CPU is underutilized, then $u(k)$ is not saturated and is equal to $l(k)$. However, when $u(k)$ is saturated, i.e., $l(k)$ is greater than 100%, then $u(k)$ remains at 100%, despite changes to $l(k)$. When the CPU is underutilized, we add a utilization measurement disturbance $\delta_{wu}(k)$ to $l(k)$ when forming $u(k)$, i.e.,

$$u(k) = \begin{cases} l(k) + \delta_{wu}(k), & l(k) + \delta_{wu}(k) \leq 100\% \\ 100\%, & l(k) + \delta_{wu}(k) > 100\%. \end{cases} \tag{5}$$

Continuing with $m(k)$, the relationship between the actual workload $l(k)$ and $m(k)$ is non-linear due to saturation. Let $l_{Th}(k)$ be the workload threshold of tasks in the k^{th} period for which admitted tasks are schedulable. $m(k)$ is saturated when $l(k) \leq l_{Th}(k)$, and remains zero despite changes to $\delta_l(k)$, i.e.,

$$m(k) = 0, \qquad l(k) \leq l_{Th}(k). \tag{6}$$

When not saturated, $m(k)$ increases non-linearly with $l(k)$. Since feedback control relies on linear systems, we linearize the relationship between $l(k)$ and $m(k)$ by forming the derivative between $l(k)$ and $m(k)$ at the vicinity of the performance reference m_r, i.e., $g_M = \frac{dm(k)}{dl(k)}$. To capture the deadline miss ratio measurement disturbance $\delta_{wm}(k)$, we model $m(k)$ as,

$$m(k) = g_M l(k) + \delta_{wm}(k). \tag{7}$$

Finally, under the conditions $l(k) \leq 100\%$ and $l_{Th} < l(k)$, we obtain the following state-space model from (4)-(7),

$$\begin{aligned} l(k+1) &= l(k) + \delta_l(k) + \delta_{wl}(k) \\ u(k) &= l(k) + \delta_{wu}(k) \\ m(k) &= g_M l(k) + \delta_{wm}(k), \end{aligned} \tag{8}$$

where $l(k)$ is the state of the system and $u(k)$ and $m(k)$ are the measured outputs of the system. Above, we have developed the state-space model (8) representing the dynamics of $u(k)$ and $m(k)$ given the input $\delta_l(k)$, the load disturbance $\delta_{wl}(k)$, and utilization and deadline miss ratio measurement disturbances $\delta_{wu}(k)$ and $\delta_{wm}(k)$, respectively. In the following sections we use this model to design control structures that suppress the measurement disturbance.

C. Suppressing the Measurement Disturbance

In the remaining of this section we focus on suppression of the deadline miss ratio disturbance $\delta_{wm}(k)$, as the suppression of the utilization measurement disturbance is analogous. Define $\hat{y}(k)$ as the estimation of a true signal $y(k)$. An initial step to suppressing the measurement disturbance is to use the open-loop estimator [5] given by,

$$\begin{aligned} \hat{l}(k+1) &= \hat{l}(k) + \delta_l(k) \\ \hat{m}(k) &= g_M \hat{l}(k). \end{aligned} \tag{9}$$

Basically, this estimator is running open-loop and not utilizing any incoming measurements. Eventually the estimations would diverge from the true system state due to incomplete knowledge of the controlled system, e.g., inaccurate execution time estimations. However, if we form the difference between the measured output and the estimated output and constantly correct the model with this error, the divergence is minimized. The idea is to construct a feedback system around the open-loop estimator (9) with the estimation error as feedback. Let $\hat{y}(k|b)$ be the estimated value of $y(k)$, predicted at time bT. For example, $\hat{y}(k|k-1)$ refers to the estimated value of $y(k)$, predicted at time $(k-1)T$. The new estimator is given by,

$$\begin{aligned} \hat{l}(k+1|k) &= \hat{l}(k|k) + \delta_l(k) \\ \hat{l}(k|k) &= \hat{l}(k|k-1) + K_m(m(k) - g_M \hat{l}(k|k-1)) \\ \hat{m}(k|k) &= \hat{m}(k) = g_M \hat{l}(k|k) \end{aligned} \tag{10}$$

where K_m is the deadline miss ratio feedback gain. Assume that the current time is kT. The next estimated load $\hat{l}(k+1|k)$ is the sum of the current estimated load $\hat{l}(k|k)$ and $\delta_l(k)$. The current estimated load $\hat{l}(k|k)$ is the previously predicted current estimated load $\hat{l}(k|k-1)$, which is adjusted with respect to $m(k)$. Remember that $m(k)$ is related to $l(k)$ and, hence, $m(k)$ is an indirect measure of the current load. By setting K_m to a large value, the estimation follows the true system state to a larger extent, as the impact of the difference in measurement and estimation, i.e., $m(k) - \hat{m}(k)$, is large. However, a large K_m implies that the measurement disturbance has a large influence on the state estimation. Hence, if the measurement disturbance is small and the system disturbance is large, then we should choose a large K_m. In contrast, if the measurement disturbance is large and the system disturbance is small, then we set K_m to a small value to suppress the measurement disturbance. Applying this principle to the control of deadline miss ratio, we show in Section IV-D that a large $n_T(k)$ implies a small measurement disturbance and, hence, K_m should be set to a large value. However, if $n_T(k)$ is small, meaning that the measurement disturbance is large, then K_m should be set to a small value to eliminate the disturbance due to the averaging operation.

To compute a suitable value for K_m, we need to model and quantify the magnitude of the system and measurement disturbances. We assume that $\delta_{wm}(k)$ does not depend on its previous nor future values, i.e., there is no correlation between $\delta_{wm}(k)$ and $\delta_{wm}(k+b)$, where $k \neq b$. Similarly we assume that $\delta_{wu}(k)$ is uncorrelated with $\delta_{wu}(k+b)$, and that $\delta_{wl}(k)$ is uncorrelated with $\delta_{wl}(k+b)$. Hence, we model the system and the measurement disturbances as white noise [10], with zero average values, i.e.,

$$E\{\delta_{wu}(k)\} = E\{\delta_{wm}(k)\} = E\{\delta_{wl}(k)\} = 0. \tag{11}$$

and we define the variance of the system and the measurement disturbances as,

$$R_{\delta wu} = E\{\delta_{wu}^2(k)\}, R_{\delta wm} = E\{\delta_{wm}^2(k)\}, \tag{12}$$
$$R_{\delta wl} = E\{\delta_{wl}^2(k)\}. \tag{13}$$

As $R_{\delta wu}(k)$ and $R_{\delta wm}(k)$ increase, the magnitude of the measurement disturbances increases. Having modeled the disturbances, the next step becomes to compute K_m, which is given by the following theorem.

Theorem 1: The K_m that minimizes the difference between the true system state and the estimated system state, i.e. $|l(k) - \hat{l}(k)|$, is

$$K_m = \frac{g_M H_m}{g_M^2 H_m + R_{\delta wm}} \tag{14}$$

where

$$H_m = \frac{R_{\delta wl}}{2} + \sqrt{\frac{R_{\delta wl}^2}{4} + \frac{R_{\delta wl} R_{\delta wm}}{g_M}}. \tag{15}$$

Proof: Given the model (8), where $\delta_{wl}(k)$ and $\delta_{wm}(k)$ are white noise, and the estimator (10), the optimal choice of K_m, follows directly from the Kalman filter, see e.g. [4], [5]. Equation (15) is the solution to the corresponding riccati equation [4], [5]. ■

In this regard, the estimator (10) where K_m is set according to Theorem 1 is an optimal estimator, meaning that it produces estimations that are closest to the true system state among all estimators. The corresponding estimator for the utilization is,

$$\begin{aligned} \hat{l}(k+1|k) &= \hat{l}(k|k) + \delta_l(k) \\ \hat{l}(k|k) &= \hat{l}(k|k-1) + K_u(u(k) - \hat{l}(k|k-1)) \\ \hat{u}(k|k) &= \hat{u}(k) = \hat{l}(k|k) \end{aligned} \tag{16}$$

where,

$$K_u = \frac{H_u}{H_u + R_{\delta wu}} \tag{17}$$
$$H_u = \frac{R_{\delta wl}}{2} + \sqrt{\frac{R_{\delta wl}^2}{4} + R_{\delta wl} R_{\delta wu}}.$$

Figure 4 plots K_m as a function of $R_{\delta wl}$ and $R_{\delta wm}$ according to (14) and (15), where g_M is one. There are some interesting issues to consider here. We notice that K_m decreases as the measurement disturbance $R_{\delta wm}(k)$ increases, meaning the measurement values have less impact on the estimation. We recall that the system disturbance represents variations in load and model inaccuracies, and as the system disturbance increases the estimation should rely more on the measurement measurements to achieve better tracking of the true state of the system. Hence, as $R_{\delta wl}(k)$ increases K_m should increase as is shown in Figure 4.

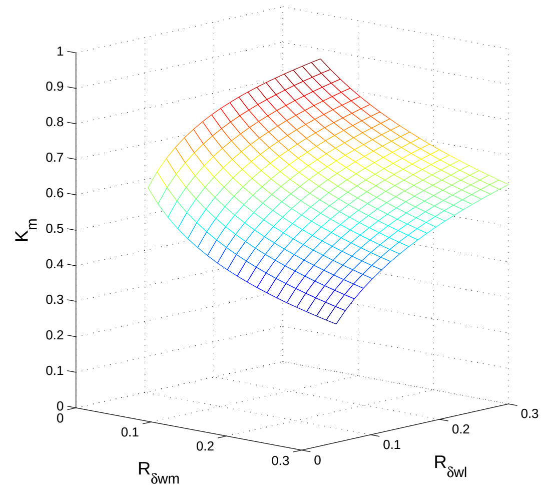


Fig. 4. The optimal choice of K_m given $R_{\delta wl}$ and $R_{\delta wm}$.

Above we have derived an state estimator that given the magnitude of the measurement and system disturbances produces optimal estimations of the controlled system state.

D. Quantifying the Measurement Disturbance Variance

In this section we provide the theory for computing $R_{\delta wm}$ and $R_{\delta wu}$. As in the previous section we only show the derivation for $R_{\delta wm}$ as computing $R_{\delta wu}$ is analogous. Consider the model given by (8). Using (11), we notice that,

$$E\{m(k)\} = E\{g_M l(k)\} = g_M E\{l(k)\} = g_M \bar{l} = \bar{m}, \tag{18}$$

i.e., the average deadline miss ratio is equal to the average of $g_M l(k)$, which we denote with $\bar{m}$. To compute $R_{\delta wm}$ we consider the variations in $m(k)$ to originate from $\delta_{wm}(k)$ only, i.e., we consider $g_M l(k)$ to be constant and equal to $\bar{m}$. Under this assumption the following theorem shows how $R_{\delta wm}$ is computed.

Theorem 2: Let $n_T \geq 1$ denote the number terminated tasks, n_M denote the number of tasks that have missed their deadlines, and $\bar{m}$ denote the average deadline miss ratio. Then the variance in $\delta_{wm}(k)$ is,

$$R_{\delta wm} = \sum_{n_M=0}^{n_T} \left(\frac{n_M}{n_T}\right)^2 \frac{n_T!}{n_M!(n_T - n_M)!} \bar{m}^{n_M} (1-\bar{m})^{n_T - n_M} - \bar{m}^2. \tag{19}$$

Proof: Utilizing the assumption $\bar{m} = g_M l(k)$, we get from (8) and (12) that,

$$R_{\delta wm} = E\{(m(k) - \bar{m})^2\} = E\{m^2(k)\} - \bar{m}^2. \tag{20}$$

By definition,

$$E\{m^2(k)\} = \sum_{\text{all } i} m_i^2(k) P_m(n_M(k), n_T(k)) \quad (21)$$

where $m_i(k)$ denotes the possible values that $m(k)$ can take and P_m is the discrete probability density function giving the probability of $m_i(k)$ assuming the value $\frac{n_M(k)}{n_T(k)}$. For example, if $n_T(k) = 1$, then $m_1(k) = 0$ and $m_2(k) = 1$, and similarly if $n_T(k) = 2$, then $m_1(k) = 0$, $m_2(k) = 0.5$, and $m_3(k) = 1$. In general, $m_i(k)$ can assume the values,

$$m_i(k) = \frac{n_M(k)}{n_T(k)}, 0 \leq n_M(k) \leq n_T(k). \quad (22)$$

To compute P_m, consider a set of tasks $\{\tau_1, \ldots, \tau_i, \ldots, \tau_{n_T}\}$ with the deadline miss indicators $\{d_1, \ldots, d_i, \ldots, d_{n_T}\}$, where $d_i = 1$ means that τ_i has missed its deadline and $d_i = 0$ means that τ_i did not miss its deadline. The probability of $d_i = 1$ is $\bar{m}$, whereas the probability of $d_i = 0$ is $1 - \bar{m}$, i.e.,

$$P_d(d_i) = \begin{cases} \bar{m}, & d_i = 1 \\ 1 - \bar{m}, & d_i = 0. \end{cases} \quad (23)$$

The probability of n_M out of n_T task missing their deadlines is given by the binomial distribution and, hence, the probability of m_i assuming the value $\frac{n_M}{n_T}$ is,

$$P_m(n_M, n_T) = \frac{n_T!}{n_M!(n_T - n_M)!} \bar{m}^{n_M} (1 - \bar{m})^{n_T - n_M}. \quad (24)$$

Substituting m_i and P_m in (21) by (22) and (24), gives,

$$E\{m^2(k)\} = \sum_{n_M=0}^{n_T} \left(\frac{n_M}{n_T}\right)^2 \frac{n_T!}{n_M!(n_T - n_M)!} \bar{m}^{n_M} (1 - \bar{m})^{n_T - n_M} \quad (25)$$

and by inserting (25) in (20) we finally obtain,

$$R_{\delta wm} = \sum_{n_M=0}^{n_T} \left(\frac{n_M}{n_T}\right)^2 \frac{n_T!}{n_M!(n_T - n_M)!} \bar{m}^{n_M} (1 - \bar{m})^{n_T - n_M} - \bar{m}^2.$$

■

The derivation of $R_{deltawu}$ is analogous to the derivation of $R_{\delta wm}$, hence,

$$R_{\delta wu} = \sum_{t_B=0}^{t_T} \left(\frac{t_B}{t_T}\right)^2 \frac{t_T!}{t_B!(t_T - t_B)!} \bar{u}^{t_B} (1 - \bar{u})^{t_T - t_B} - \bar{u}^2, \quad (26)$$

where $\bar{u} = E\{l(k)\}$.

Figure 5 shows how $R_{\delta wm}$ varies as a function of $\bar{m}$ and n_T. As expected, $R_{\delta wm}$ decreases as n_T increases meaning that the measurement disturbance originating from the averaging operation decreases in intensity. For a certain n_T, the intensity of the measurement disturbance peaks when the average deadline miss ratio $\bar{m}$ is 0.5. The intensity $R_{\delta wm}$ is zero when $\bar{m}$ is zero or one, which is expected as we have no variation in $m(k)$ at zero or one deadline miss ratio.

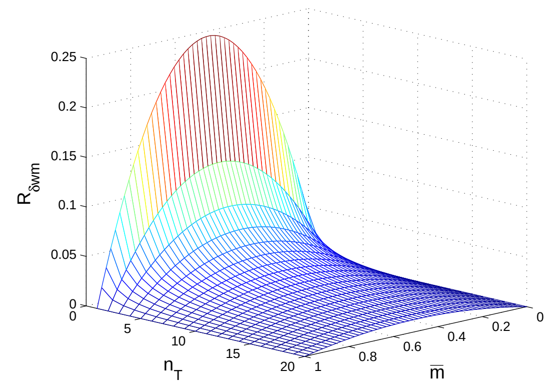


Fig. 5. $R_{\delta wm}$ as a function of $\bar{m}$ and n_T.

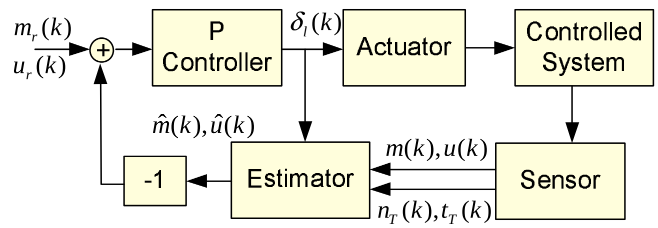


Fig. 6. The Feedback Structure for Measurement Disturbance Suppression.

E. A Measurement Disturbance Suppressive Feedback Control Structure

The feedback control structure that suppresses the measurement disturbance consists of the classical feedback loop and the additional estimator, as shown in Figure 6. We use a P-controller [4], [5], where $\delta_l(k) = K_P(y_r(k) - y(k))$. The estimators (10) and (16) are used to estimate the deadline miss ratio and the utilization. According to the separation principle [4], [5], the design of the controller and the estimator is disjoint, meaning that the tuning of one does not affect the other one. The controller is designed using profiling data and a tuning method, e.g. [8], and the estimator is designed using the profiling data and (10). Hence, the separation principle reduces significantly the design complexity.

At time kT, $m(k)$ and $u(k)$ are formed by the sensor and returned to the estimator along with $n_T(k)$ and $t_T(k)$. $R_{\delta wm}(k)$ and $R_{\delta wu}(k)$ are then formed using (19) and (26). Having obtained $R_{\delta wm}(k)$ and $R_{\delta wu}(k)$, the estimator feedback gains are computed according to (14) and (17). Note that updating the feedback gains requires a time-variant estimator [4], [5]. However, in this work we approximate the time-variant estimator with a time-invariant estimator as an initial approach. In our future work we extend the estimator to an time-variant estimator.

Once, the feedback gains are updated, the estimators (10) and (16) are used to compute $\hat{m}(k)$ and $\hat{u}(k)$. The P-controller then computes $\delta_l(k)$ using the estimates $\hat{m}(k)$ and $\hat{u}(k)$. The effect of using an estimator is the following. As $n_T(k)$ or

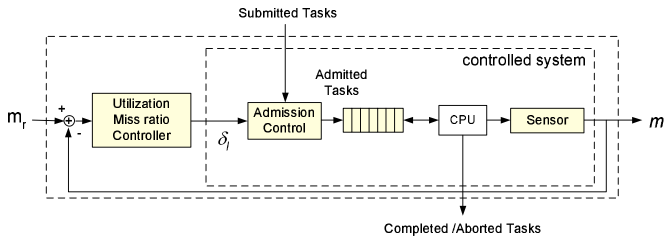


Fig. 7. The Simulated System Architecture.

$t_T(k)$ increases, then we should trust $m(k)$ and $u(k)$ more as the deadline miss ratio and utilization are based on a larger data set. An increase in $n_T(k)$ or $t_T(k)$ corresponds to a decrease in $R_{\delta wm}(k)$ and $R_{\delta wu}(k)$ which in turn results in an increase in K_m and K_u (see Figure 4 and 5). An increase in K_m and K_u corresponds to the estimators relying more on the measured data and basing the estimation less on predictions. Similarly, as $n_T(k)$ or $t_T(k)$ decrease, estimations are based more on predictions and less on the measurements. The result of this is an efficient way of suppressing disturbances present in $m(k)$ and $u(k)$ that arise from the averaging operation. This means that we present a more accurate image of the system state to the P-controller which is able to enhance the performance management in terms of lowering $(m_r^2(k) - m(k))^2$ and $(u_r^2(k) - u(k))^2$. The latter implies a more reliable performance and system adaptability as the actual system performance is closer to the desired system performance.

V. Performance Evaluation

The main objective of the performance evaluation is to determine the suitability of the proposed approach, namely, using an estimator to suppress the measurement disturbance. In this regard we perform an experiment where the performance of a feedback loop with an estimator is compared to the performance of a traditional feedback loop without an estimator. The following describes the simulator used to carry out the performance evaluation, followed by definition of performance metrics and the result of the experiments.

A. Simulator Setup

The simulated workload consists of aperiodic tasks, as an aperiodic task set implies an increased unpredictability in workload, hence, a greater challenge on the control of performance. The general outline of the feedback control scheduling architecture is given in Figure 7. We assume a workload model where each task has two QoS levels, i.e. $N = 2$, $q_i[s_1] = 0$, $q_i[s_2] = 1$, $x_{E,i}[s_1] = x_{A,i}[s_1] = 0$, $x_{E,i}[s_2] > 0$, and $x_{A,i}[s_2] > 0$ (see Section III), i.e., a task is either admitted for execution or rejected. Input to the controlled system is the set of arriving submitted task and the change to the admitted estimated workload $\delta_l(k)$. Output from the controlled system is the set of terminated tasks and $m(k)$. The goal is to minimize $(m_r - m(k))^2$. Based on $\delta_l(k)$, the admission controller enforces the workload adjustment. A task is admitted if its load added to the admitted load is less than $l(k)$. The workload model of the tasks is described as follows. The estimated execution time $x_{E,i}[s_2]$ of a task τ_i is uniformly distributed according to $U : (50ms, 300ms)$. Upon generation of a task an actual execution time given by the normal distribution $N : (x_{E,i}[s_2], \sqrt{x_{E,i}[s_2]})$ is associated with τ_i. The deadline is set to $a_i + x_{E,i}[s_2] \times slackfactor$, where a_i denotes the arrival time of τ_i. The slack factor is uniformly distributed according to $U : (10, 30)$. The inter-arrival time is exponentially distributed with the mean inter-arrival time set to $x_{E,i}[s_2] \times slackfactor$.

In our experiments, one simulation run lasts for 10 minutes of simulated time. For all the performance data, we have taken the average of 10 simulation runs and derived 95% confidence intervals.

1) Performance Metrics: In Section II we argued that the goal of feedback control is to minimize the difference between the actual system performance and the desired system performance. Therefore, we distinguish the performance of controllers by how well they force $m(k)$ to follow m_r, despite presence of system and measurement disturbances. In our simulations we evaluate the controllers with respect to,

$$J_a = \frac{1}{S}\sum_{k=1}^{S} |m_r - m(k)|$$

$$J_s = \frac{1}{S}\sum_{k=1}^{S} (m_r - m(k))^2$$

where S is the number of samples taken. By J_s and J_a we can establish how well the controllers are able to keep $m(k)$ near m_r. The lower J_s and J_a are, the better a controller is able to keep $m(k)$ near m_r, and also the faster $m(k)$ converges toward m_r.

B. Evaluation of Controller Performance

We know that the control performance is directly related to $R_{\delta wm}$, which in turn depends on n_T. Therefore it is interesting to observe the performance of a feedback loop as n_T and, hence, T varies. We show that using the deadline miss ratio estimator (10) significantly reduces J_a and J_s for low sampling periods. This implies that the measurement disturbance is suppressed, resulting in a more efficient control of the deadline miss ratio. In this experiment we vary m_r according to $0.05, 0.10, \ldots, 0.30$, and vary T according to $0.50, 1.00, \ldots, 5.00s$. The results are shown in Figures 8-12.

Starting with Figure 8 we see that n_T increases as T increases and, hence, for larger T we expect a lower magnitude of the measurement disturbance. Further, n_T also increases as the deadline miss ratio reference m_r increases. An increase in T and, consequently, an increase in n_T results in a lower measurement disturbance, meaning that we can rely on the measured deadline miss ratio to a greater extent, which corresponds to an increase in K_m. This is shown in Figure 9, where K_m increases as T increases.

The measured J_a and J_s when an estimator is used (figure b), respectively not used (figure a), are presented in Figures 10 and 11. The case when $m_r = 0.10$ is given in Figure

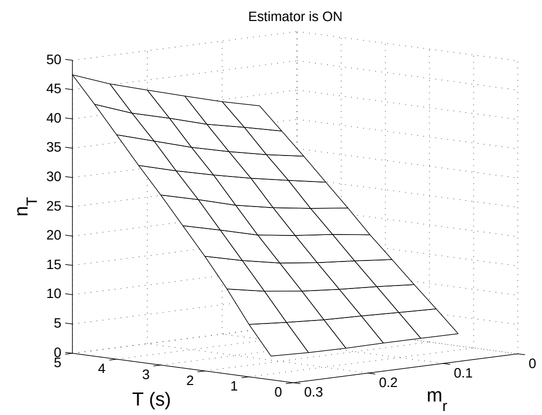


Fig. 8. Measured n_T

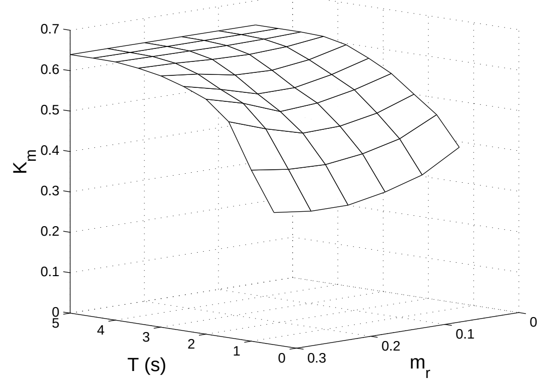
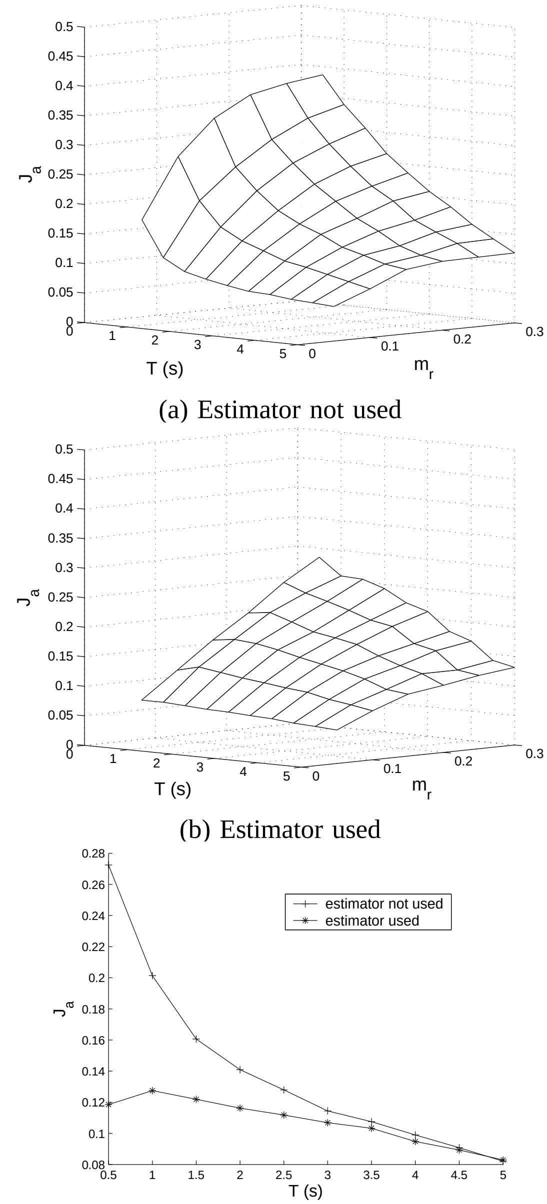


Fig. 9. Measured K_m



(a) Estimator not used

(b) Estimator used

(c) $m_r = 0.10$

Fig. 10. Measured J_a

10(c). Remember that as J_a and J_s decrease, the actual system performance is closer to the desired system performance. The difference in J_a decreases as T increases due to the decreasing magnitude of the measurement disturbance. For low sampling periods, a significant decrease in J_a is achieved when an estimator is used as compared to the case when an estimator is not used. Considering Figure 10(c) at $T = 0.5s$, J_a is 0.12 when an estimator is used, compared to 0.28 when an estimator is not used, i.e., we have a difference of about 0.16. Similarly, the difference in J_a is 0.07 for the case $T = 1s$, showing that even at the sampling period of one second we have a great difference in J_a. Hence, the actual deadline miss ratio is closer to its reference when an estimator is used.

We also study $m(k)$ in the time domain to obtain a better understanding of how a certain J_a corresponds to variations in $m(k)$. Figure 12 shows the deadline miss ratio in the time domain for the experiment corresponding to $T = 1s$ and $m_r = 0.10$. As we can see $m(k)$ oscillates heavily around the reference in the absence of an estimator, see Figure 12(a). However, the deviations are significantly reduced when using an estimator, as shown in Figure 12(b). In the absence of the estimator, large deviations in $m(k)$ due to the measurement disturbance are not filtered. As a consequence, the controller tries to compensate for the changes in $m(k)$ by changing the requested workload, which results in an over compensation and, hence, $m(k)$ deviates even more from m_r. However, an estimator is able to suppress variations in $m(k)$ due to the measurement disturbance or equivalently the averaging operation. Consequently, a less noisy measurement is presented to the controller which in turn enhances the control performance.

In summary we have shown that a lower sampling period increases the disturbance in the measurements. The performance is improved when using estimators for suppressing the measurement disturbance. We have observed that $m(k)$ is closer to the reference m_r, implying that we achieve improved performance reliability and adaptation. Hence, when using an

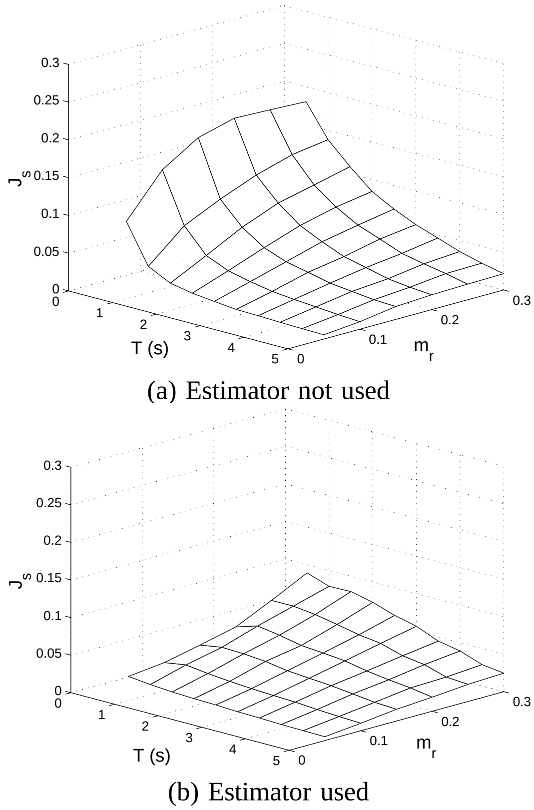


Fig. 11. Measured J_s, estimator off

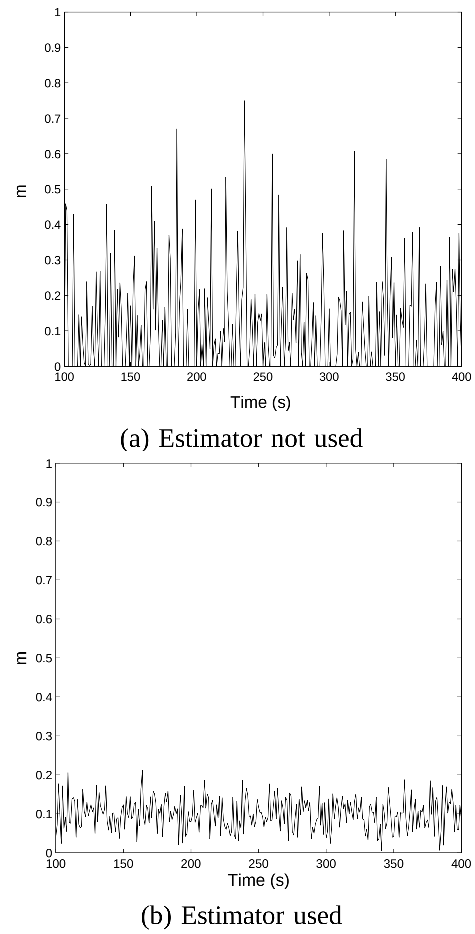


Fig. 12. $m(k)$ in the time domain.

estimator the actual system performance is closer to the desired system performance than the case when the estimator is not used.

VI. Related work

Lu et al. introduced a feedback control scheduling framework for controlling utilization and deadline miss ratio [8]. In our previous work we controlled the performance of real-time databases using feedback control [2]. Parekh et al. use feedback control scheduling to control the length of a queue of remote procedure calls (RPCs) arriving at a server [11]. Li et al. [7] proposed a task model for QoS adaptations. In their model, there are dependencies among the tasks, characterized by input quality and output quality. The goal is to design controllers that force the target task to maintain the same output quality at a desired QoS reference. Abdelzaher et al. presented control algorithms for managing service delay and queue length of requests arriving at web servers [1]. None of the approaches above have considered the effects of the sampling period on the measured variable. Lu et al. [9] introduced an architecture for differentiated caching services. The desired relation between the hit ratios of different content classes is enforced using per-class feedback control loops. The authors note that the variance of the hit ratio for a class may be large for small sampling periods. They use a low pass filter to smoothen out large deviations in hit ratio. However, they do not model and quantify the measurement disturbance and a method for tuning the low pass filter is not given. Further, a low pass filter does not produce optimal estimations of the controlled variable. In contrast, we have modeled and quantified the measurement disturbance and derived an optimal estimator for the controlled variable.

VII. Conclusions

The emergence of real-time systems operating in open and unpredictable environments has resulted in a paradigm shift in techniques for managing system resources. Using feedback control has shown to be effective for a large class of real-time systems with unpredictable workload characteristics. Although there is a great body of knowledge in the control community dealing with the control of dynamic systems, not all techniques and results can be directly mapped into the domain of computer performance control. The measured variables typically used to describe the performance of computer systems are of statistical nature, as they are formed over a data set. In this paper we have shown how the sampling period selection influences the characteristics of the measurements and, hence, the control performance. The disturbance in the measurement increases as the sampling period decreases, due to the decreasing size of the data set that is used to compute the measured variable. Still a large sampling period is not desired as the control would become less responsive to changes in the controlled variable. To solve the problem of the sampling period selection we have proposed an approach consisting of choosing a suitable sampling period to capture the system dynamics, and an estimator that produces estimations of the controlled variable. Experimental results show that this approach results in improved control performance as the actual performance is closer to the desired level. This increases the reliability of the system and implies a more controlled worst-case performance and faster convergence toward the desired performance.

In this work we considered the metrics deadline miss ratio and utilization. In our future work, we will extend our approach by considering the quality of the results that the tasks produce to be the metrics. Also we will consider time-variant estimators, where the estimation parameters are varied according to changes in the controlled system.

ACKNOWLEDGMENT

This work was funded, in part by CUGS (the National Graduate School in Computer Science, Sweden), CENIIT (Center for Industrial Information Technology) under contract 01.07, ISIS (Information Systems for Industrial Control and Supervision), and NSF grants CCR-0098269 and IIS-0208758.

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APPENDIX

In this section we provide the theory for computing $A_{\delta wm} = E\{|\delta_{wm}(k)|\}$ and $A_{\delta wu} = E\{|\delta_{wm}(k)|\}$. We only show the derivation for $A_{\delta wm}$ as computing $A_{\delta wu}$ is analogous. To compute $A_{\delta wm}$ we consider the variations in $m(k)$ to originate from $\delta_{wm}(k)$ only, i.e., we consider $g_M l(k)$ to be constant and equal to $\bar{m}$. Hence, we have that,

$$A_{\delta wm} = E\{|\delta_{wm}(k)|\} = E\{|m(k) - \bar{m}|\}. \tag{27}$$

Knowing $A_{\delta wm}$ is interesting as it gives how much in average $m(k)$ deviates from $\bar{m}$, as given by the following theorem.

Theorem 3: Let $n_T \geq 1$ denote the number terminated tasks, n_M denote the number of tasks that have missed their deadlines, and $\bar{m}$ denote the average deadline miss ratio. Then the average deviation in $\delta_{wm}(k)$ is,

$$A_{\delta wm} = \sum_{n_M=0}^{n_T} \left|\left(\frac{n_M}{n_T}\right) - \bar{m}\right| \frac{n_T!}{n_M!(n_T - n_M)!} \bar{m}^{n_M}(1-\bar{m})^{n_T - n_M}. \tag{28}$$

Proof: Let, $a(k) = |m(k) - \bar{m}|$. Then by definition,

$$A_{\delta wm} = E\{a(k)\} = \sum_{\text{all } i} a_i P_a(a_i) \tag{29}$$

where P_a gives the probability of $a(k)$ being equal to $a_i = |\frac{n_M}{n_T} - \bar{m}|$. We note that there are at most two solutions to the equation $a_i = \epsilon$, given by,

$$n_{M1} = n_T(\bar{m} + \epsilon), \tag{30}$$

$$n_{M2} = n_T(\bar{m} - \epsilon), \tag{31}$$

such that $0 \leq n_{M1} \leq n_T$ and $0 \leq n_{M2} \leq n_T$. If $a_i = \epsilon$ only has the solution (30), then $P_a(\epsilon) = P_m(n_{M1}, n_T)$. Similarly, if $a_i = \epsilon$ only has the solution (31), then $P_a(\epsilon) = P_m(n_{M2}, n_T)$. If $a_i = \epsilon$ has two solutions, then $P_a(\epsilon) = P_m(n_{M1}, n_T) + P_m(n_{M2}, n_T)$. The latter implies that,

$$\epsilon P_a(\epsilon) = |\frac{n_{M1}}{n_T} - \bar{m}|P_m(n_{M1}, n_T) + |\frac{n_{M2}}{n_T} - \bar{m}|P_m(n_{M2}, n_T) \tag{32}$$

By replacing P_a with P_m according to above, we finally obtain (28) by using (24). ■

For example, choosing $n_T = 25$ and $\bar{m} = 0.20$ gives $A_{\delta wm} = 0.0627$, i.e., in average $m(k)$ deviates from $\bar{m}$ by 0.0627. This corresponds for example to the sequence $m(k), m(k+1), m(k+2) = 0.1373, 0.2627, 0.1373$. As we can see, the deviation is quite large even for a great n_T.

Above we have showed how the average deviation between $m(k)$ and $\bar{m}$, as a function of n_T and $\bar{m}$, is computed. This can be used to calculate the expected variation of $m(k)$ around $\bar{m}$.