of the EKF due to the bias in range $|\boldsymbol{\mu}_{t,3D,r}|$ (34) being reduced. $\beta_3 < 0.26$ and $\beta_4 < 0.27$ both result in $\Delta < 7\%$. $\beta_{3,4} < 0.2$ lead to $\Delta < 3\%$ and $\Delta < 4\%$, respectively. For $\beta_{3,4} < 0.1$, the difference is again almost negligible.

Finally, as a result of Figs. 5, 6, 9, and 10, it is difficult to verify the predicted difference in the critical bias significances $\beta_{crit,1,2,3}$ in (19), (39), and (40) although some evidence for it is present. Furthermore, restricting all bias significances β to values below 0.2 seems to be a good choice as this restricts the relative difference Δ to values below 4% in all simulated scenarios.

VIII. CONCLUSIONS

It has been shown that the second limit for the applicability of the classical linearized conversion in the 2D case, as postulated in [2], is very likely not to exist. Furthermore, the corresponding limits for the 3D case have been derived.

As explained, these limits are rather theoretical in nature. No conclusions can be drawn on the actual performance degradation of the EKF in real tracking applications. To this end, the performance of the EKF and the optimal BLUE filter has been compared in typical tracking scenarios for the 2D and 3D case. For a bias significance β smaller than 0.1, the performance degradation was found to be less than 1% in all simulated scenarios. For $\beta < 0.2$, it increased to up to 4%.

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Robust and Adaptive Actuator Failure Compensation Designs for a Rocket Fairing Structural-Acoustic Model

The actuator failure compensation problem is formulated for active vibration control of a rocket fairing structural-acoustic model with unknown actuator failures. Performance of a nominal optimal control scheme in the presence of actuator failures is studied to show the need of effective failure compensation. A robust control scheme and two adaptive control schemes are developed, which are able to ensure the closed-loop system signal boundedness in the presence of actuator failures whose failure pattern and values are unknown. The adaptive scheme for parameterizable failures ensures asymptotic stability despite failure uncertainties. Simulation results verified their failure compensation effectiveness.

I. INTRODUCTION

Microelectromechanical system (MEMS) technology makes it possible to use networks of effective and efficient actuators and sensors for many applications such as safe and low-cost rocket payload fairings with active vibration control. However, actuators (and sensors) may fail during system operation, and actuator failures may lead to performance deterioration or even instability of the rocket launch system. Actuator failures are often uncertain in failure patterns, failure time instants, and failure values, which introduce not only signal uncertainties but also structure uncertainties into the

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controlled system. For certain safety-critical systems such as the payload launch system, actuator failures, if not handled properly, may result in disasters. An accident which may be caused by actuator failures can be avoided if a control scheme can effectively make use of the remaining working actuators which have enough actuation capacity for ensuring stability. It is important to develop effective control systems, that is, to work robustly or take actions automatically whenever actuator failures occur, to generate control signals for the remaining actuators to ensure desired stability and tracking performance. Different approaches for dealing with actuator failures have been proposed [6, 2, 15, 3, 14].

This paper addresses the issues in developing stabilizing control designs for a rocket payload fairing structural-acoustic model with unknown actuator failures. In Section II we formulate the control problem, by presenting the system model, and an example to show the system performance with a nominal optimal controller in the presence of actuator failures. In Section III we develop a robust actuator failure compensation control design based on simultaneous stabilization of multi-model systems. In Section IV we develop two adaptive actuator failure compensation control designs: one for adaptive stabilization and one for both adaptive stabilization and regulation. Simulation results are presented to illustrate their effectiveness for actuator failure compensation.

II. PROBLEM STATEMENT

Active vibration control is an effective method for handling structural-acoustic vibration which may occur in a fairing system [7]. In the presence of actuator failures, a vibration control system, however, may lose its effectiveness.

A. Fairing System Model

A launch vehicle payload fairing is a protection-cover to protect a payload from wind pressure, high-heat and structure vibration, and to attenuate the fairing vibration, there are 300 MEMS sensor/actuator pairs mounted on its wall [4]. Our control objective is to suppress the launch vehicle fairing vibration even in the presence of dysfunctional actuators by constructing an effective reconfiguration strategy to compensate the detrimental effects caused by the failed actuators so as to guarantee the desired overall system performance.

The active vibration control approach is to use embedded, distributed, closed-loop sensor-controller-actuator systems to minimize the vibration. The structure typically has embedded or bonded transducers that continually transmit sensor measurements to an information processing system (a feedback controller). The processing system in turn determines appropriate action by sending signals back to the piezoelectric (PZT) actuators embedded in the structure, to control the fairing structural motion [13]. A variety of active vibration control approaches have been proposed to deal with the vibration damping problems such as direct model reference adaptive control [1], fuzzy logic [5].

For our work, we use the model-interaction approach for the fairing modeling [8, 11], which includes structural modeling and rigid-wall acoustic cavity modeling. The structural model for the fairing is formulated as

$$\begin{aligned} \dot{w}(t) &= A_s w(t) + B_s u(t) + H_s d(t) \\ y_s(t) &= C_s w(t) \end{aligned} \tag{1}$$

where $w(t) = [z^T(t), \dot{z}^T(t)]^T \in R^{2n_1}$ is the structural state vector with $z \in R^{n_1}$ and $\dot{z} \in R^{n_1}$ being structural displacement and velocity vectors for n_1 modes, respectively, $u(t) \in R^m$ is a vector of m control inputs at the structural nodes, that is, the structural actuator outputs whose components may fail during system operation, $y_s(t) \in R^{n_1}$ is a vector of structural outputs (displacements), $d(t) \in R^q$ is a disturbance vector, $A_s \in R^{2n_1 \times 2n_1}$, $B_s \in R^{2n_1 \times m}$, $C_s \in R^{n_1 \times 2n_1}$, and $H_s \in R^{2n_1 \times q}$ are structure-related matrices associated with each corresponding mode.

The model for the air cavity enclosed by the fairing structure can be expressed as

$$\begin{aligned} \dot{r}(t) &= A_a r(t) + B_a w(t) \\ y_a(t) &= C_a r(t) \end{aligned} \tag{2}$$

where $r(t) \in R^{2n_2}$, $y_a(t) \in R^l$ is the vector of pressures within the cavity, $A_a \in R^{2n_2 \times 2n_2}$, $B_a \in R^{2n_2 \times 2n_1}$, and $C_a \in R^{l \times 2n_2}$ are acoustic-related matrices for n_2 acoustic modes.

The overall structural-acoustic fairing model combines the structural and acoustic models as

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} A_s & B_{sa} \\ B_a & A_a \end{bmatrix} x(t) + \begin{bmatrix} B_s \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} H_s \\ 0 \end{bmatrix} d(t) \\ y_s(t) &= [C_s \quad 0] x(t) \end{aligned} \tag{3}$$

where $x(t) = [w^T(t), r^T(t)]^T \in R^n$, $n = 2n_1 + 2n_2$, is the state vector of the fully coupled fairing system, and $B_{sa} \in R^{2n_1 \times 2n_2}$ is the matrix associated with vibroacoustical-related pressure acting at the fairing structure. We assume here that the system disturbance $d(t)$ is zero, to address the actuator failure compensation problem only.

B. A Control System without Actuator Failures

To implement vibration suppression using a coupled dynamic model, a linear quadratic regulator (LQR) optimal control method was used in [8].

In the optimal control design [12] for a system

$$\dot{x} = Ax + Bu, \qquad x \in R^n, \qquad u \in R^m \tag{4}$$

the feedback control law is

$$u(t) = Kx(t) \tag{5}$$

where $K \in R^{m\times n}$ is a feedback gain matrix, which, based on the LQR theory, is chosen to minimize a quadratic performance index

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{6}$$

where $Q = Q^T \geq 0$ and $R = R^T > 0$ are the weighting matrices that serve as design parameters selected to provide suitable performance. Since $x(t)$ includes both the transmitted pressure states and the structure states, the control and system response energy can be directed at any combination of these quantities by different choices of Q and R. The feedback gain K is given as

$$K = -R^{-1}B^T P \tag{7}$$

where $P = P^T > 0$ is the matrix satisfying the Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \tag{8}$$

This LQR design ensures asymptotic stability of the closed-loop system $\dot{x}(t) = (A - BR^{-1}B^T P)x(t)$, while the performance index J in (6) is minimized.

C. Control System with Actuator Failures

In the above standard optimal control design, actuator failures was not considered. In this subsection, the performance of the optimal controller in the presence of actuator failures is studied.

One type of actuator failures that may occur in launch vehicle fairings is the jammed actuator fault, which can be modeled as [14]

$$u_i(t) = \bar{u}_i(t), \qquad t \geq t_i, \qquad i \in \{1,2,\ldots,m\} \tag{9}$$

where t_i is the unknown failure time instant and $\bar{u}_i(t)$ is the unknown failure signal. Supposing that at time t, there are $p < m$ actuator failures in the system, that is,

$$u_i(t) = \bar{u}_i(t), \qquad i = i_1, i_2, \ldots, i_p$$
$$\{i_1, i_2, \ldots, i_p\} \subset \{1,2,\ldots,m\} \tag{10}$$

we can rewrite the system (4) as

$$\dot{x} = Ax + Bu = Ax + \sum_{i\neq i_1,\ldots,i_p} b_i u_i + \sum_{i=i_1,\ldots,i_p} b_i \bar{u}_i(t). \tag{11}$$

In the presence of actuator failures, $u(t)$ can be expressed as

$$u = v(t) + \sigma(\bar{u} - v(t)) \tag{12}$$

where $v(t) \in R^m$ is the applied control input vector, $\bar{u} = [\bar{u}_1(t), \bar{u}_2(t), \ldots, \bar{u}_m(t)]^T$ is the failure vector, and σ represents the failure pattern defined as

$$\sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\} \tag{13}$$

$$\sigma_i = \begin{cases} 1 & \text{if the } i\text{th actuator has failed, i.e., } u_i = \bar{u}_i, \text{ since } t_i < t \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

It can be seen that when actuator failures take place, not only will the corresponding applied control inputs not be influenced, but also structural uncertainties will be brought into the system.

We first study the stability robustness of the optimal controller (5) in the presence of actuator failures. In this case, the applied control is $v(t) = Kx(t)$, and the actual input can be written as

$$u(t) = (I - \sigma)Kx(t) + \sigma\bar{u} \tag{15}$$

so that the resulting closed-loop system with the control law (5) becomes

$$\dot{x}(t) = (A + B(I - \sigma)K)x(t) + B\sigma\bar{u}. \tag{16}$$

To check the closed-loop system stability, the eigenvalues of $(A + B(I - \sigma)K)$ are examined.

For our study, we consider a single-mode fairing model (3) with

$$A = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415 \\ -0.1980 & -0.1150 & -0.0318 & 0.3 \\ -3.0500 & 1.1880 & -0.4650 & 0.9 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 1.55 & 0.75 \\ 0.975 & 0.8 & 0.85 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{17}$$

and $u = [u_1, u_2, u_3]^T$, that is, with three actuators u_1, u_2, u_3. We assume that one of these three actuators may fail during operation, so that there are four failure patterns: 1) $\sigma = 0$ (no failure), 2) $\sigma_1 = 1$, $\sigma_2 = 0$, $\sigma_3 = 0$ ($u_1 = \bar{u}_1$ fails), 3) $\sigma_1 = 0$, $\sigma_2 = 1$, $\sigma_3 = 0$ ($u_2 = \bar{u}_2$ fails), and 4) $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = 1$ ($u_3 = \bar{u}_3$ fails).

We calculate the feedback control gain K of (7) in the case of no failure, for $Q = I_4 \in R^{4\times 4}$, $R = I_3 \in R^{3\times 3}$. We then get the eigenvalues of $A + B(I - \sigma)K$ for different failure patterns of up to one failure. For the case when u_2 fails, the system (16) becomes unstable. Hence, a nominal optimal control design may not ensure stability in the presence of actuator failures. Effective control designs are needed to stabilize the system for all possible actuator failures.

Next, we develop such desirable control schemes to handle uncertain actuator failures.

III. ROBUST CONTROL DESIGN

In this section we treat the actuator failure compensation problem as a simultaneous stabilization problem for a set of systems resulted from different failure patterns. For the above system example, there are four systems, associated with four actuator failure patterns: 1) no failure, 2) actuator 1 fails, 3) actuator 2 fails, or 4) actuator 3 fails. Recent advances in linear matrix inequality based design techniques provide potential tools for simultaneous stabilization of more than two systems, which is applied to our actuator failure compensation problem.

A. Failure Compensation Scheme

A robust control design based on simultaneous stabilization of multi-model systems is applicable to our failure compensation problem with unknown actuator failures. Its goal is to design a single controller capable of simultaneously stabilizing a finite collection of systems corresponding to all possible failure patterns.

For a set of systems ($A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$), $i = 1,2,\ldots,N$, a simultaneous stabilizing gain K exists if there is a common $K \in R^{m\times n}$ and $P_i = P_i^T > 0$ for $i = 1,2,\ldots,N$, satisfying

$$P_i(A_i + B_iK) + (A_i^T + K^TB_i^T)P_i + K^TRK = -Q < 0, \quad R = R^T > 0. \tag{18}$$

This is an LQR based design which has certain optimality [10], in addition to the desired stabilization property which is employed for our failure compensation solution.

For actuator failure compensation, such a simultaneous stabilizing design $v(t) = Kx(t)$ exists if there is a common $K \in R^{m\times n}$ and a set of $P_\sigma = P_\sigma^T > 0$ for $\sigma \in \Sigma$, satisfying

$$P_\sigma(A + B(I-\sigma)K) + (A^T + K^T(I-\sigma)B^T)P_\sigma + Q + K^TRK = 0 \tag{19}$$

for all $\sigma \in \Sigma$, where Σ is the set of all failure patterns corresponding to all possible values of σ_i defined in (15), for up to certain number of actuator failures (for example, up to $m-q$ failures, $1 < q \le m$). For the above example, $m = 3$ and $q = 2$, and

$$\Sigma = \{\mathrm{diag}\{0,0,0\}, \mathrm{diag}\{1,0,0\}, \mathrm{diag}\{0,1,0\}, \mathrm{diag}\{0,0,1\}\} \tag{20}$$

that is, one K satisfies four Lyapunov equations in the form of (19) with four different P_σ corresponding to four failure cases in order to stabilize the system with four possible actuator failures.

A sufficient condition for the solvability of (19) is proposed in [10], that is, for any $X_\sigma > 0$, the following matrix inequality

$$\begin{aligned} P_\sigma A + A^TP_\sigma + Q - \Theta(P_\sigma, X_\sigma) & \\ + (R^{-1/2}(I-\sigma)B^TP_\sigma + R^{1/2}K)^T & \\ \times (R^{-1/2}(I-\sigma)B^TP_\sigma + R^{1/2}K) < 0 & \end{aligned} \tag{21}$$

is feasible, where

$$\begin{aligned} \Theta(P_\sigma, X_\sigma) = {} & X_\sigma B(I-\sigma)R^{-1}(I-\sigma)B^TP_\sigma \\ & + P_\sigma B(I-\sigma)R^{-1}(I-\sigma)B^TX_\sigma \\ & - X_\sigma B(I-\sigma)R^{-1}(I-\sigma)B^TX_\sigma. \end{aligned} \tag{22}$$

Using the Schur complement, the inequality (21) is equivalent to the matrix inequality

$$\begin{bmatrix} P_\sigma A + A^TP_\sigma + Q - \Theta(P_\sigma, X_\sigma) & (R^{-1/2}(I-\sigma)B^TP_\sigma + R^{1/2}K)^T \\ (R^{-1/2}(I-\sigma)B^TP_\sigma + R^{1/2}K) & -I \end{bmatrix} < 0. \tag{23}$$

An iterative algorithm is given in [10] to check the solvability of (23) and to obtain a desired solution K if it is solvable. The algorithm is initialized with an $X_\sigma = X_\sigma^T > 0$ satisfying (8) with P and B replaced by X_σ and $B(I-\sigma)$ for each $\sigma \in \Sigma$. The inequality (23) is then solved for a common $K = K_0$ and all $P_\sigma = P_{\sigma 0} = P_{\sigma 0}^T > 0$ for each $\sigma \in \Sigma$, by a linear matrix inequality (LMI) method. If such a set of solutions exist, then a set of solutions, K and $P_\sigma = P_\sigma^T > 0$, $\sigma \in \Sigma$, exist for (19). The iterative algorithm of [10] can then be continued with $X_\sigma = P_{\sigma 0}$ to solve the inequality (23) for a common $K = K_1$ and all $P_\sigma = P_{\sigma 1} = P_{\sigma 1}^T > 0$ for each $\sigma \in \Sigma$. This process can be carried out iteratively to calculate a sequence of K_i and $P_{\sigma i} = P_{\sigma i}^T > 0$, $\sigma \in \Sigma$. As $i \to \infty$, K_i and $P_{\sigma i}$ converge to the desired constant matrices K and P_σ. The iteration is stopped at step $i = i_f$ with $K = K_{i_f}$ when the $\|P_{\sigma i_f} - P_{\sigma i_f - 1}\|$ is small enough.

In summary, the actuator failure compensation problem is solvable with a robust control-based design, if the set of equations (19) are solvable, for which the sufficient condition is (23).

B. An Illustrative Example

For the system example (17), there are four different failure patterns given in (20) so that there are four matrix inequalities in the form of (23). We applied the algorithm of [10] to find out that those LMIs have feasible solutions and obtain the feedback gain matrix

$$K = \begin{bmatrix} 2.5956 & -0.2730 & -1.0617 & -0.8154 \\ 2.4081 & -0.4127 & -1.2002 & -0.9280 \\ 2.0027 & -0.0839 & -0.7777 & -0.5500 \end{bmatrix} \tag{24}$$

for which all four sets of eigenvalues of $(A + B(I-\sigma)K)$ are stable, that is, the four systems in (17) corresponding to four different actuator failure patterns are stabilized by a common feedback gain K. Our simulation results (not shown due to space limit) for the closed-loop system in the presence of the second actuator failure also indicated the desired stability (signal boundedness) property and small transient responses of the state and control input signals.

IV. ADAPTIVE CONTROL DESIGNS

Adaptive control of systems with actuator failures, aimed at compensating for actuator uncertainties with adaptive tuning of controller parameters based on system response errors, is studied in this section. Such a design is to adaptively stabilize the system whenever a new actuator failure pattern occurs. We first develop an adaptive scheme to stabilize the system by ensuring the closed-loop signal boundedness in the presence of actuator failures with bounded failure values. We then present a modified adaptive actuator failure compensation control scheme, for parameterizable actuator failures, to ensure asymptotic regulation of the system state variables to zero, in addition to signal boundedness, despite uncertain actuator failures. Desired system performance is proved analytically and illustrated by simulation results. In this study, we consider the system (11) with input expression (12) for an actuator failure pattern (10):

$$\dot{x} = Ax + Bu = Ax + \sum_{i \neq i_1,\ldots,i_p} b_i v_i + \sum_{i=i_1,\ldots,i_p} b_i \bar{u}_i(t) \tag{25}$$

$$u = v(t) + \sigma(\bar{u} - v(t)) \in R^m \tag{26}$$

supposing that at time t there are p actuator failures in the system, that is,

$$u_i = \bar{u}_i(t), \qquad i = i_1, i_2, \ldots, i_p$$
$$\{i_1, i_2, \ldots, i_p\} \subset \{1, 2, \ldots, m\} \tag{27}$$

where $\bar{u}$ and σ defined in (9) and (13), and $v(t) \in R^m$ is an applied control input.

A. Adaptive Design for Signal Boundedness

To design an adaptive control scheme capable of stabilizing the system (25) in the presence of unknown actuator failures, a basic requirement is that the system $(A, B(I - \sigma))$ is stabilizable for any actuator failure pattern σ under consideration, so that there is a gain matrix K_σ which depends on $(A, B(I - \sigma))$ for each failure pattern σ such that $v(t) = K_\sigma x(t)$ stabilizes the system (25). This requirement is necessary in the sense that if for a failure pattern σ there is no such a K_σ to stabilize the system (25) with the knowledge of actuator failures, then an adaptive solution also does not exit when actuator failures are unknown.

For uncertain actuator failures, an adaptive scheme is desired to stabilize the system with the presence of failures in any failure pattern $\sigma \in \Sigma$, where Σ is the set of all possible failure patterns. For the example (17), we have Σ given in (20).

For our adaptive control design, the following assumption is needed.

Assumption 1 (A, B) is stabilizable, and $\text{rank}[B(I - \sigma)] = \text{rank}[B]$, $\forall \sigma \in \Sigma$.

This assumption is a sufficient condition for the existence of a K_σ for each possible failure pattern σ to stabilize the system by the remaining actuators. Furthermore, the assumption also indicates that there is a common solution $P = P^T > 0$ to the Lyapunov equation $P[A + B(I - \sigma)K_\sigma] + [A^T + K_\sigma{}^T(I - \sigma)B^T]P = -Q$ all K_σ which correspond to all possible $\sigma \in \Sigma$. This assumption is satisfied by the example system (17) with Σ defined in (20).

In order to develop a stable adaptive scheme which is robust with respect to uncertainties caused by unknown actuator failures, we employ a robust adaptive design [9], which uses the knowledge of the upper bounds of the norm of the row vectors in $K_\sigma = [K_1, K_2, \ldots, K_m]^T$, that is, $\|K_i\|_2 \leq M_i$ where M_i is the known upper bound for $i = 1, 2, \ldots, m$.

We now present our first adaptive actuator failure compensation design and its properties.

THEOREM 1 *Under Assumption* 1, *the control law*

$$v(t) = \hat{K} x(t) \tag{28}$$

with $\hat{K} = [\hat{K}_1, \hat{K}_2, \ldots, \hat{K}_m]^T \in R^{m \times n}$ *updated by the adaptive laws*

$$\dot{\hat{K}}_i = -\Gamma_i x x^T P b_i - \Gamma_i \delta_i \hat{K}_i, \qquad i = 1, 2, \ldots, m \tag{29}$$

where $\Gamma_i = \Gamma_i^T > 0$, b_i *is the ith column of* B, *and*

$$\delta_i = \begin{cases} 0 & \text{if} \quad \|\hat{K}_i\|_2 < M_i \\ \delta_{0i}\left(\dfrac{\|\hat{K}_i\|_2}{M_i} - 1\right) & \text{if} \quad M_i \leq \|\hat{K}_i\|_2 < 2M_i \\ \delta_{0i} & \text{if} \quad \|\hat{K}_i\|_2 \geq 2M_i \end{cases} \tag{30}$$

ensure that all signals in the closed-loop system are bounded for any $\sigma \in \Sigma$.

PROOF Since (A, B) is stabilizable, there exists constant $K \in R^{m \times n}$ and $P \in R^{n \times n}$ such that

$$P(A + BK) + (A + BK)^T P = -Q < 0$$
$$P = P^T > 0, \quad Q = Q^T > 0. \tag{31}$$

The condition $\text{rank}[B(I - \sigma)] = \text{rank}[B]$ implies that a linear combination of columns in B can be expressed by a linear combination of those in $B(I - \sigma)$, that is, there exists a $K_\sigma \in R^{m \times n}$ such that

$$B(I - \sigma)K_\sigma = BK \tag{32}$$

for each $\sigma \in \Sigma$. Therefore, for each $\sigma \in \Sigma$, there is a K_σ satisfying

$$P[A + B(I-\sigma)K_\sigma] + [A^T + K_\sigma{}^T(I-\sigma)B^T]P = -Q < 0 \tag{33}$$

with the same $P = P^T > 0$ as that in (31).

Suppose that actuator failures happen at time instants t_k, with $t_k < t_{k+1}$, $k = 1,2,\ldots,N$. For the closed-loop system (27)–(30), we consider the Lyapunov function candidate

$$V = \tfrac{1}{2}x^T P x + \tfrac{1}{2}\sum_{i \neq i_1,i_2,\ldots,i_p} (\hat{K}_i - K_i)^T \Gamma_i^{-1}(\hat{K}_i - K_i) \tag{34}$$

for each time interval (t_k, t_{k+1}), $k = 0,1,\ldots,N$, with $t_0 = 0$ and $t_{N+1} = \infty$. The time-derivative of V in each time interval (t_k, t_{k+1}) associated with a certain failure pattern $\sigma \in \Sigma$ is

$$\dot{V} = -\tfrac{1}{2}(x - Q^{-1}P\sigma B\bar{u})^T Q (x - Q^{-1}P\sigma B\bar{u}) + \mu - \sum_{i \neq i_1,i_2,\ldots,i_p} \delta_i(\hat{K}_i - K_i)\hat{K}_i \tag{35}$$

where $\mu = \frac{1}{2}\bar{u}^T B^T \sigma P Q^{-1} P \sigma B \bar{u}$ is a nonnegative constant, and

$$\sum_{i \neq i_1,i_2,\ldots,i_p} \delta_i(\hat{K}_i - K_i)\hat{K}_i \geq 0. \tag{36}$$

Hence, the signals $x(t)$ and $\hat{K}_i(t)$, $i \neq i_1,i_2,\ldots,i_p$, are bounded in each time interval if the initial value of $V(t_k^+)$ with the corresponding time interval (t_k, t_{k+1}) is finite.

Note that the Lyapunov function V is not continuous at the time instants t_k, $k = 0,1,\ldots,N$. Each time when actuator fails, V has a jump with a finite value at that time instant. Here we consider the case that the actuator cannot work again once it fails. As a result, V is piecewise continuous with a finite number of discontinuous points. It can be obtained from (35) that $V(t_{k+1}^-)$ is bounded if $V(t_k^+)$ is finite, for the interval (t_k, t_{k+1}), which implies that $V(t_{k+1}^+)$ is bounded for the next interval (t_{k+1}, t_{k+2}), $k = 0,1,\ldots,N-1$, so that $V \in L^\infty$, $\forall t \geq 0$ with several jumps of finite values. Consequently, it is concluded that $x, \hat{K}_i \in L^\infty$.

For $\hat{K}_i(t)$, $i = i_1,i_2,\ldots,i_p$, considering the Lyapunov function candidate

$$V_i = \tfrac{1}{2}\hat{K}_i^T(t)\Gamma_i^{-1}\hat{K}_i(t), \qquad i = i_1,i_2,\ldots,i_p \tag{37}$$

we can show that there is a constant $\kappa_i > 0$ such that $\dot{V}_i < 0$ for $\|K_i\|_2 > \kappa_i$, that is, $\hat{K}_i(t)$ is bounded, $i \in \{i_1,i_2,\ldots,i_p\}$. Hence, all closed-loop signals are bounded.

Simulation results also indicated that the closed-loop system remains stable in the presence of the second actuator actuator failure, while the transient responses of the system state and input signals are small at the initial several seconds and then converge to constants.

B. Adaptive Design for Asymptotic State Regulation

When the actuator failure signals are parameterized by a set of unknown parameters and a set of known signals, a modified adaptive scheme can be derived, which is able to achieve asymptotic regulation of the closed-loop system states to zero in the presence of actuator failures.

A parameterizable failure is expressed as

$$u_i(t) = \bar{u}_i(t) = \sum_{j=1}^{s_i} \alpha_{ij} f_{ij}(t) = \alpha_i^T f_i(t), \qquad t \geq t_i, \qquad i \in \{1,2,\ldots,m\} \tag{38}$$

where $\alpha_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{is_i}]^T$ is a vector of some unknown failure parameters, and $f_i(t) = [f_{i1}(t), f_{i2}(t), \ldots, f_{is_i}(t)]^T$ is a vector of known signals.

We present the following adaptive failure compensation design and its properties for the system (25) with the unknown actuator failures given in (38).

THEOREM 2 *Under Assumption* 1, *the control law*

$$v(t) = \hat{K}x(t) + \sum_{j=1}^{m} \hat{\Theta}_j f_j(t) \tag{39}$$

with $\hat{K} = [\hat{K}_1, \hat{K}_2, \ldots, \hat{K}_m]^T \in R^{m\times n}$ *and* $\hat{\Theta}_j = [\hat{\Theta}_{j1}, \hat{\Theta}_{j2}, \ldots, \hat{\Theta}_{jm}]^T \in R^{m\times s_j}$ *updated by*

$$\dot{\hat{K}}_i = -\Gamma_i x x^T P b_i, \qquad i = 1,2,\ldots,m \tag{40}$$

$$\dot{\hat{\Theta}}_{ji} = -\Lambda_{ji} f_j(t) x^T P b_i, \qquad j = 1,2,\ldots,s_q, \quad i = 1,2,\ldots,m \tag{41}$$

where $\Gamma_i = \Gamma_i^T > 0$, $\Lambda_{ji} = \Lambda_{ji}^T > 0$, *and* b_i *is the ith column of B, ensures that all closed-loop system signals are bounded and* $\lim_{t\to\infty} x(t) = 0$, *for any* $\sigma \in \Sigma$.

PROOF In the proof of Theorem 1, we have shown that with Assumption 1, there exists a K_σ for each $\sigma \in \Sigma$ satisfying (33) with a common $P = P^T > 0$.

Suppose that at time t, there are $p < m$ actuator failures in the system, that is, $u_i(t) = \bar{u}_i(t)$, $i = i_1,i_2,\ldots,i_p$, $\{i_1,i_2,\ldots,i_p\} \subset \{1,2,\ldots,m\}$, and that actuator failures happen at time instants t_k, with $t_k < t_{k+1}$, $k = 1,2,\ldots,N$. For the system (25) with the adaptive controller (39)–(41), we consider the Lyapunov function candidate

$$V = \tfrac{1}{2}x^T P x + \tfrac{1}{2}\sum_{i \neq i_1,i_2,\ldots,i_p} (\hat{K}_i - K_i)^T \Gamma_i^{-1}(\hat{K}_i - K_i) + \tfrac{1}{2}\sum_{i \neq i_1,i_2,\ldots,i_p}\sum_{j=1}^{m} (\hat{\Theta}_{ji} - \Theta_{ji})^T \Lambda_{ji}^{-1}(\hat{\Theta}_{ji} - \Theta_{ji}) \tag{42}$$

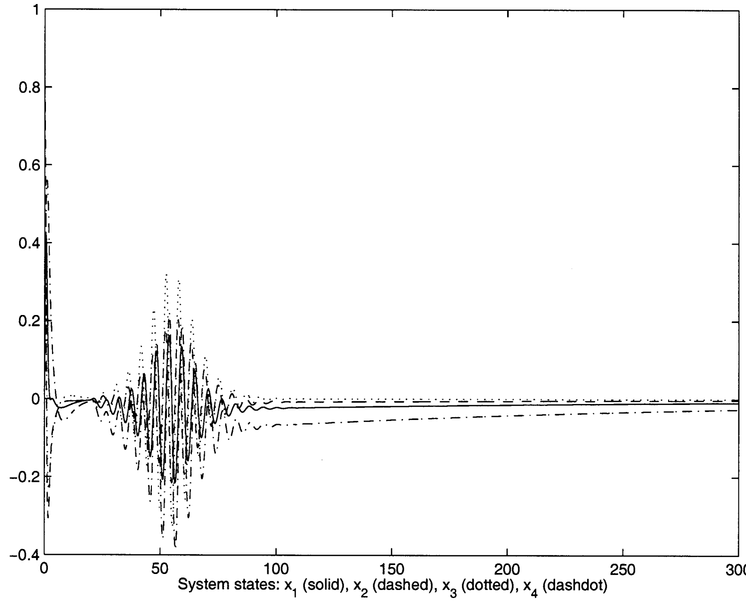

Fig. 1. System response of adaptive control design for asymptotic regulation.

for each time interval (t_k, t_{k+1}), $k = 0,1,\ldots,N$, with $t_0 = 0$ and $t_{N+1} = \infty$, where K_i is the ith row of K_σ, and Θ_{ji} is a solution of the following equation

$$\sum_{i \neq i_1, i_2, \ldots, i_p} b_i \Theta_{ji}^T = b_j \alpha_j^T, \qquad \text{for} \quad j = i_1, i_2, \ldots, i_p \tag{43}$$

and $\Theta_{ji} = 0$ otherwise. Notice that (46) is solvable because the equation (46) is equivalent to

$$B(I-\sigma)\Theta_j = b_j \alpha_j^T, \qquad \Theta_j = [\Theta_{j1}, \Theta_{j2}, \ldots, \Theta_{jm}]^T \in R^{m \times s_j} \tag{44}$$

which always has solutions due to the condition $\mathrm{rank}[B(I-\sigma)] = \mathrm{rank}[B]$.

The time-derivative of V in each (t_k, t_{k+1}) associated with a certain failure pattern $\sigma \in \Sigma$ is

$$\dot{V} = -\tfrac{1}{2} x^T Q x \leq 0. \tag{45}$$

Hence, $x \in L^2 \cap L^\infty$, and $\hat{K}_i \in L^\infty$ and $\hat{\Theta}_{ji} \in L^\infty$ for $i \neq i_1, i_2, \ldots, i_p$ and $j = 1,2,\ldots,m$, in each time interval (t_k, t_{k+1}) if the initial value $V(t_k^+)$ is finite. From (40), it follows that

$$[\Gamma_1^{-1}\dot{\hat{K}}_1, \Gamma_2^{-1}\dot{\hat{K}}_2, \ldots, \Gamma_m^{-1}\dot{\hat{K}}_m] = -xx^T PB. \tag{46}$$

With Assumption 1, we see that B can be represented by a linear combination b_i, $i \neq i_1, i_2, \ldots, i_p$, which implies that $\Gamma_i^{-1}\dot{\hat{K}}_i$, $i \in \{i_1, i_2, \ldots, i_p\}$, is a linear combination of $\Gamma_i^{-1}\dot{\hat{K}}_i$, $i \neq i_1, i_2, \ldots, i_p$. Therefore we also conclude that $\hat{K}_i \in L^\infty$ for $i = i_1, i_2, \ldots, i_p$; and similarly, we have $\hat{\Theta}_{ji} \in L^\infty$ for $i = i_1, i_2, \ldots, i_p$ and $j = 1,2,\ldots,m$, under the condition that $V(t_k^+)$ is finite.

The function V is not continuous at t_k, $k = 0,1,\ldots,N$, but only has finite value jumps at those time instants, that is, $V(t_k^+)$ is indeed finite. Thus, we conclude that $x \in L^2 \cap L^\infty$, $\hat{K} \in L^\infty$, and $\hat{\Theta}_j \in L^\infty$, $j = 1,2,\ldots,m$, $\forall t \geq 0$. In addition, since $v(t) \in L^\infty$ from (39) and $\dot{x} \in L^\infty$ from (25), given that $x(t) \in L^2$, we also have that $\lim_{t\to\infty} x(t) = 0$.

In summary, we have constructed, under Assumption 1, an adaptive actuator failure compensation control scheme which stabilizes the system $\dot{x} = Ax + Bu$ in the presence of uncertain actuator failures (38), and achieve asymptotic state regulation.

Simulation Results: We used the actuator failure $u_2(t) = \bar{u}_2 = (0.1 - u_2(20))(1 - e^{-0.1(t-20)}) + u_i(20)$, $t \geq 20$ s, and the matrices $Q = I_4$ and $R = I_3$. The initial conditions were chosen as $x(0) = [0,1,0.1,0.25]^T$, $\hat{K}_1(0) = [-2,-1,0.5,0]^T$, $\hat{K}_2(0) = [-3,-0.5,3,0.5]^T$, $\hat{K}_3(0) = [-1.5,-1,0,0.5]^T$, and $\hat{\theta}(0) = [0,0,0,0,0,0]^T$. Adaptive gains are $\Gamma_i = I$ and $\Lambda_i = \mathrm{diag}\{0.1, 1\}$ for $i = 1,2,3$. The simulation results for the system in the presence of the second actuator failing at 20th second are shown in Fig. 1, which indicate that the closed-loop system remains stable and the asymptotic state regulation is achieved in the presence of the actuator failure.

V. CONCLUSIONS

Actuator failures may cause performance deterioration or even instability in many safety-critical control systems. It is often unknown when an actuator fails and how much the failure is, while the remaining actuation can still be enough to accomplish a desired control task. The challenge is to develop a desirable feedback control scheme which is capable of utilizing the remaining actuation capacity in the presence of failure uncertainties. In this paper, we have demonstrated the undesirable effect of actuator failures and the desirable effectiveness of failure compensation, by developing three failure compensation control schemes. This work is illustrated in a framework of application to vibration control (stabilization and regulation) of a rocket payload fairing structural-acoustic model with unknown actuator failures. The robust control failure compensation scheme is based on an LMI method, under an LMI design condition. The two adaptive control failure compensation schemes are based on robust adaptive control and failure parameterization methods, under a matrix rank condition. While all three schemes ensure signal boundedness, the failure parameterization based design is able to achieve asymptotic state regulation, in spite of the failure uncertainties. Simulation results verified the effectiveness of the developed failure compensation schemes.

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Tracking with Distributed Sets of Proximity Sensors using Geometric Invariants

We propose a new approach to forming an estimate of a target track in a distributed sensor system using very limited sensor information. This approach uses a central fusion system that collects only the peak energy information from each sensor and assumes that the energy attenuates as a power law in range from the source. A geometrical invariance property of the proximity of the distributed sensors relative to a target track is used to generate potential target track paths. Numerical simulation examples are presented to illustrate the practicality of the technique.

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