CS4501: Introduction to Computer Vision
Projective Geometry and Light

Various slides from previous courses by:
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What is a camera?
Who invented cameras?
Photography – Life of a Photograph - from Light to Pixels
Shutter Speed / Aperture Size / ISO sensitivity
Today’s Class

• Recap from Last Class on Shutter Speed / Aperture / ISO sensitivity
• Camera Parameters
• Brief Introduction to Projective Geometry (Computer Graphics)
• Light Models (Diffuse Light vs Specular Light)
Life of a Photograph

Camera Irradiance → Optics → Aperture → Shutter → Camera Body

Sensor (CCD/CMOS) → Gain (ISO) → A/D → RAW

Demosaic (Sharpen) → White Balance → Gamma/curve → Compress → JPEG

Slide by Steve Seitz
How to Shoot Photos in Manual?

- Shutter speed
- Aperture size
  - Focus / Auto-focus (Yes, you can shoot in manual and also probably should focus in manual)
- ISO sensitivity
Fast Shutter Speed

- Doesn’t allow much light
- Large Aperture Size

Slow Shutter Speed

- Allows a lot of light
- Small Aperture Size

ISO sensitivity – Should be small ideally

https://www.exposureguide.com/iso-sensitivity/
Projection: world coordinates $\rightarrow$ image coordinates

If $X = 2$, $Y = 3$, $Z = 5$, and $f = 2$

What are $U$ and $V$?
Projection: world coordinates $\rightarrow$ image coordinates

Camera Center $(0, 0, 0)$

\[
\begin{align*}
U &= -X \cdot \frac{f}{Z} \\
V &= -Y \cdot \frac{f}{Z}
\end{align*}
\]

\[
P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]

\[
\begin{align*}
U &= -2 \cdot \frac{2}{5} \\
V &= -3 \cdot \frac{2}{5}
\end{align*}
\]
Projection: world coordinates $\rightarrow$ image coordinates

$$p = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$
Homogeneous coordinates vs Cartesian coordinates

Conversion

Converting to *homogeneous* coordinates

\[(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\]  
homogeneous image coordinates

\[(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\]  
homogeneous scene coordinates

Converting *from* homogeneous coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \\
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)
\]
Homogeneous coordinates vs Cartesian coordinates

**Invariant to scaling**

\[
\begin{bmatrix}
    kx \\
    ky \\
    kw
\end{bmatrix} = k \begin{bmatrix}
    x \\
    y \\
    w
\end{bmatrix} \Rightarrow \begin{bmatrix}
    kx/kw \\
    ky/kw \\
    1
\end{bmatrix} = \begin{bmatrix}
    x/w \\
    y/w \\
    1
\end{bmatrix}
\]

Homogeneous Coordinates \quad Cartesian Coordinates

Point in Cartesian is ray in Homogeneous
Projection: world coordinates $\rightarrow$ image coordinates

$$P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$${\bf p} = \begin{bmatrix} U \\ V \end{bmatrix}$$

$$U = -X \frac{f}{Z} \quad U = -2 \frac{2}{5}$$

$$V = -Y \frac{f}{Z} \quad V = -3 \frac{2}{5}$$

Camera Center
(0, 0, 0)
Projection matrix: from World to Image Coordinates

![Diagram showing projection matrix from World to Image Coordinates]

**Intrinsic Assumptions**
- Unit aspect ratio
- Optical center at (0,0)
- No skew

**Extrinsic Assumptions**
- No rotation
- Camera at (0,0,0)

\[
x = K[I \ 0] X
\]

\[
\begin{bmatrix}
u \\
v \\
w \\
1
\end{bmatrix} = \begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

Slide Credit: Savarese
Remove assumption: known optical center

Intrinsic Assumptions
• Unit aspect ratio
• No skew

Extrinsic Assumptions
• No rotation
• Camera at (0,0,0)

\[
x = K[I \ 0]X
\]

\[
w = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}
\]

\[
\begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]
Remove assumption: square pixels

Intrinsic Assumptions
• No skew

Extrinsic Assumptions
• No rotation
• Camera at (0,0,0)

\[ x = K[I \ 0]X \]
Remove assumption: non-skewed pixels

Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

\[ x = K[I \ 0]X \]

\[
\begin{bmatrix}
 u \\
 v \\
 1
\end{bmatrix} =
\begin{bmatrix}
 \alpha & s & u_0 & 0 \\
 0 & \beta & v_0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

Note: different books use different notation for parameters
Oriented and Translated Camera
Allow camera translation

**Intrinsic Assumptions**

**Extrinsic Assumptions**
- No rotation

\[ x = K[I \ t]X \]

\[ w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]
3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:

\[
R_x (\alpha) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{bmatrix}
\]

\[
R_y (\beta) = \begin{bmatrix}
\cos \beta & 0 & \sin \beta \\
0 & 1 & 0 \\
-\sin \beta & 0 & \cos \beta
\end{bmatrix}
\]

\[
R_z (\gamma) = \begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
Allow camera rotation

\[ x = K[R \ t]X \]

\[
\begin{bmatrix}
  u \\
  v \\
  1
\end{bmatrix} =
\begin{bmatrix}
  \alpha & s & u_0 \\
  0 & \beta & v_0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  r_{11} & r_{12} & r_{13} & t_x \\
  r_{21} & r_{22} & r_{23} & t_y \\
  r_{31} & r_{32} & r_{33} & t_z \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
  1
\end{bmatrix}
\]
Projection matrix (Word Coordinates to Image Coordinates)

\[
x = K[R \quad t]X
\]

**Intrinsic Camera Properties:** $K$

**Extrinsic Camera Properties:** $[R \ t]$
Degrees of freedom

\[ x = K[R \quad t]X \]

\[
\begin{bmatrix}
u \\
v \\
1
\end{bmatrix} = \begin{bmatrix}
\alpha & s & u_0 \\
0 & \beta & v_0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
r_{11} & r_{12} & r_{13} & t_x \\
r_{21} & r_{22} & r_{23} & t_y \\
r_{31} & r_{32} & r_{33} & t_z
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]
Assignment – World Coordinates to Image Coordinates

What the camera can “see” assuming this is a wireframe object

(0,0,0) (0,0,3) (1,1,4)
Things to Remember for Quiz

- Pinhole camera model
- Focal length in the pinhole camera model
- Shutter Time / Aperture / ISO
- Homogeneous Coordinates
- Extrinsic Camera Properties and Intrinsic Camera Properties
- Describe mathematically (and intuitively) the conversion process from World Coordinates to Image Coordinates
Light

- What determines the color of a pixel?
BRDF (Bidirectional reflectance distribution function)

\[ E_{\text{surface}}(\theta_i, \phi_i) \sim \cos \theta_i \ L_{\text{surface}}(\theta_i, \phi_i) \]
BRDF (Bidirectional reflectance distribution function)

\[
E_{\text{surface}}(\theta_i, \phi_i) \quad \text{Irradiance at Surface in direction } (\theta_i, \phi_i)
\]

\[
L_{\text{surface}}(\theta_r, \phi_r) \quad \text{Radiance of Surface in direction } (\theta_r, \phi_r)
\]

\[
\text{BRDF} : f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L_{\text{surface}}(\theta_r, \phi_r)}{E_{\text{surface}}(\theta_i, \phi_i)}
\]

Slide by Aaron Bobick
Reflection

• Body Reflection:
  - Diffuse Reflection
  - Matte Appearance
  - Non-Homogeneous Medium
  - Clay, paper, etc

• Surface Reflection:
  - Specular Reflection
  - Glossy Appearance
  - Highlights
  - Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection
Phong Reflection Model

- The BRDF of many surfaces can be approximated by
  The Lambertian + Specular Model
Phong Reflection Model

\( \hat{L}_m \), which is the direction vector from the point on the surface toward each light source (\( m \) specifies the light source),
\( \hat{N} \), which is the normal at this point on the surface,
\( \hat{R}_m \), which is the direction that a perfectly reflected ray of light would take from this point on the surface, and
\( \hat{V} \), which is the direction pointing towards the viewer (such as a virtual camera).

\[
I_p = k_a i_a + \sum_{m \in \text{lights}} (k_d (\hat{L}_m \cdot \hat{N}) i_{m,d} + k_s (\hat{R}_m \cdot \hat{V}) \alpha i_{m,s}).
\]
Phong Reflection Model - Recap

\[ I_p = k_a i_a + \sum_{m \in \text{lights}} (k_d (\hat{L}_m \cdot \hat{N}) i_{m,d} + k_s (\hat{R}_m \cdot \hat{V})^\alpha i_{m,s}). \]
Phong Reflection Model - Recap

\[ I_p = k_a i_a + \sum_{m \in \text{lights}} (k_d (\hat{L}_m \cdot \hat{N}) i_{m,d} + k_s (\hat{R}_m \cdot \hat{V})^\alpha i_{m,s}). \]

Phong Reflection Model - Recap

\[ I_p = k_a i_a + \sum_{m \in \text{lights}} (k_d (\hat{L}_m \cdot \hat{N}) i_{m,d}) + k_s (\hat{R}_m \cdot \hat{V})^\alpha i_{m,s} \].

[Reference](https://en.wikipedia.org/wiki/Phong_reflection_model)
Phong Reflection Model - Recap

\[ I_p = k_a i_a + \sum_{m \in \text{lights}} (k_d (\hat{L}_m \cdot \hat{N})i_{m,d} + k_s (\hat{R}_m \cdot \hat{V})^\alpha i_{m,s}). \]

Phong’s Shading / Illumination Model

- Originally from Vietnam / PhD from Utah, Professor at Utah, and later Stanford.
- Died at age 32 from leukemia
- Phong’s professor Ivan Sutherland went on to win the Turing Award (Nobel Prize in CS) for lifelong contributions to Computer Graphics
Same ideas used in Computer Graphics

• Ray Tracing
• Radiosity
• Photon Mapping
Reflection

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc

Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:
Diffuse Reflection – Lambertian Surface / BRDF

• Only body reflection, and no specular reflection
• BRDF is independent of outgoing direction
• BRDF depends on indent direction (foreshortening)

• Light intensity does not depend on the outgoing direction. Only incoming.

• It is independent of where the viewer stands.

• Smooth surface, not glossy. Can think of any examples?

Slide by Aaron Bobick
CAN’T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?
The other extreme – Only Specular Reflection

How about a mirror?

Reflection **ONLY** at mirror angle
Problem in Computer Vision: Intrinsic Image Decomposition

Given this

Extract this

Images by Marc Serra
Problem in Computer Vision: Shape from Shading

Given this

Extract this

Images by Aaron Bobick
A photon’s life choices

• Absorption
• Diffusion
• Reflection
• Transparency
• Refraction
• Fluorescence
• Subsurface scattering
• Phosphorescence
• Interreflection
A photon’s life choices

- **Absorption**
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection
A photon’s life choices

- Absorption
- **Diffuse Reflection**
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection
A photon’s life choices

• Absorption
• Diffusion
• **Specular Reflection**
• Transparency
• Refraction
• Fluorescence
• Subsurface scattering
• Phosphorescence
• Interreflection
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- **Transparency**
- Refraction
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A photon’s life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- **Refraction**
  - Fluorescence
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A photon’s life choices

• Absorption
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• Refraction
• **Fluorescence**
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A photon’s life choices

- Absorption
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- Fluorescence
- **Subsurface scattering**
- Phosphorescence
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A photon’s life choices

- Absorption
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- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- **Phosphorescence**
- Interreflection
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- **Interreflection**
The human eye is a camera! (not quite)

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
- What’s the “film”?
  - photoreceptor cells (rods and cones) in the **retina**
Next Class: Image Processing and Image Filters
Questions?