Lexical Analysis

Finite Automata

(Part 1 of 2)
Cunning Plan

- Informal Sketch of Lexical Analysis
  - LA identifies tokens from input string
  - lexer : (char list) → (token list)

- Issues in Lexical Analysis
  - Lookahead
  - Ambiguity

- Specifying Lexers
  - Regular Expressions
  - Examples
One-Slide Summary

• **Lexical analysis** turns a stream of characters into a stream of tokens.

• **Regular expressions** are a way to specify sets of strings. We use them to describe tokens.
Fold Batter Lightly ...

- \( \text{fold\_left } f \ a \ [1;\ldots;n] \equiv f \ (\ldots (f \ (f \ a \ 1) \ 2)) \ n \)
  
  \[
  \text{fold\_left } (\text{fun } a \ e \ -> \ e :: a) \ [] \ [1;2;3] \\
  = [3;2;1] \\
  \text{fold\_left } (\text{fun } a \ e \ -> \ a @ [e]) \ [] \ [1;2;3] \\
  = [1;2;3]
  \]

- \( \text{fold\_right } f \ [1;\ldots;n] \ b \equiv f \ 1 \ (f \ 2 \ (\ldots (f \ n \ b))) \)
  
  \[
  \text{fold\_right } (\text{fun } a \ e \ -> \ e :: a) \ [1;2;3] \ [] \\
  = [1;2;3] \\
  \text{fold\_right } (\text{fun } e \ a \ -> \ a @ [e]) \ [1;2;3] \ [] \\
  = [3;2;1]
  \]
Compiler Structure

- Regular Expressions
- Formal Grammar

- Lexer Generator
- Parser Generator

- Typing Rules
- Machine Description

- Lexical Analyzer
- Abstract Syntax Tree

- Parser
- Semantic Analysis

- Cool Source Code
- List of Tokens

- Code Generator
- Assembly Code

- Annotated Abstract Syntax Tree
Lexical Analysis

• What do we want to do? Example:
  
  ```
  if (i == j)
    z = 0;
  else
    z = 1;
  ```

• The input is just a sequence of characters:

  ```
  if (i == j)\n    z = 0;\n  else\n    z = 1;
  ```

• Goal: partition input strings into substrings
  - And classify them according to their role
What's a Token?

- Output of lexical analysis is a list of tokens
- A **token** is a syntactic category
  - In English:
    - noun, verb, adjective, ... 
  - In a programming language:
    - Identifier, Integer, Keyword, Whitespace, ...
- Parser relies on token distinctions:
  - e.g., identifiers are treated differently than keywords
Tokens

- **Tokens** correspond to **sets of strings**.

- **Identifier**: strings of letters or digits, starting with a letter

- **Integer**: a non-empty string of digits

- **Keyword**: “else” or “if” or “begin” or ...

- **Whitespace**: a non-empty sequence of blanks, newlines, and/or tabs

- **OpenPar**: a left-parenthesis
Lexical Analyzer: Build It!

- An implementation must do three things:
  - Recognize substrings corresponding to tokens
  - Return the value or lexeme of the token
    - The lexeme is the substring
  - Report errors intelligently!
Example

• Recall:

```c
if (i == j)
    z = 0;
else
    z = 1;
```

• Token-lexeme pairs returned by the lexer:

```plaintext
<Keyword, “if”>
<Whitespace, “ ”>
<OpenPar, “(”>
<Identifier, “i”>
<Whitespace, “ ”>
<Relation, “==”>
<Whitespace, “ ”>
...
Lexical Analyzer: Implementation

• The lexer usually *discards* “uninteresting” tokens that don't contribute to parsing.

• Examples: Whitespace, Comments
  - Exception: which languages care about whitespace?

• Question: What would happen if we removed all whitespace and comments *prior* to lexing?
Lookahead

- The goal is to partition the input string (source code). This is implemented by reading left-to-right, recognizing one token at a time.
- **Lookahead** may be required to decide where one token ends and the next token begins
  - Even our simple example has lookahead issues
    - \textit{i} vs. \textit{if}
    - \textit{=} vs. \textit{==}
Still Needed

• A way to describe the lexemes of each token
  - Recall: lexeme = “the substring corresponding to the token”

• A way to resolve ambiguities
  - Is if two variables i and f?
  - Is == two equal signs = =?
Languages

• Definition. Let $\Sigma$ be a set of characters. A language over $\Sigma$ is a set of strings of characters drawn from $\Sigma$. $\Sigma$ is called the alphabet.
Examples of Languages

• Alphabet = English Characters
• Language = English Sentences
  - Note: *Not* every string on English characters is an English sentence.
  - Example: xayenb sbe'

• Alphabet = ASCII characters
• Language = C Programs
  - Note: ASCII character set is different from English character set.
Notation

- **Languages** are sets of strings

- We need some notation for specifying which sets we want
  - that is, which strings are in the set

- For lexical analysis we care about *regular languages*, which can be described using *regular expressions*. 
Regular Expressions

• Each **regular expression** is a notation for a regular language (a set of words)
  - You'll see the exact notation in minute!

• If \( A \) is a regular expression then we write \( L(A) \) to refer to the language denoted by \( A \)
Base Regular Expression

• Single character: 'c'
  - \( L('c') = \{ \text{"c"} \} \)  (for any \( c \in \Sigma \))

• Concatenation: \( AB \)
  - \( A \) and \( B \) are other regular expressions
  - \( L(AB) = \{ ab \mid a \in L(A) \text{ and } b \in L(B) \} \)

• Example: \( L('i' 'f') = \{ \text{"if"} \} \)
  - We abbreviate 'i' 'f' as 'if'
Compound Regular Expressions

• Union
  - \( L(A \cup B) = \{ s \mid s \in L(A) \text{ or } s \in L(B) \} \)

• Examples:
  - \( L('if' \cup 'then' \cup 'else') = \{ "if", "then", "else" \} \)
  - \( L('0'|'1'|'2'|'3'|'4'|'5'|'6'|'7'|'8'|'9') = \text{what?} \)

• Fun Example:
  - \( L( ('0'|'1') ('0'|'1') ) = \{"00","01","10","11"\} \)
• So far we have only finite languages
• Iteration: $A^*$
  - $L(A^*) = \{"\"\} \cup L(A) \cup L(AA) \cup L(AAA) \ldots$
• Examples:
  - $L('0'^*) = \{"\"", "0", "00", "000", "0000", \ldots \}$
  - $L('1' '0'^*) = \{"1", "10", "100", "1000", \ldots \}$
• Empty: $\varepsilon$
  - $L(\varepsilon) = \{ \"\" \}$
Q: Events (603 / 842)

• This product was introduced on May 8, 1985, in one of the greatest consumer flops ever. It was effectively shelved on July 10 of the same year when its "red, white and you" predecessor was re-introduced and set as the default.
In this 1958 Sheb Wooley song the pigeon-toed title character wears short shorts and wants to get a job in a rock'n'roll band playing the horn, but is perhaps best known for his skin tone and non-standard diet.
6. "'Composition for College Writer's', seems like fun!"

19. The brunette thrusts out her carefully manicured hand. "Hi, my name is Kelly with one 'l'.

206. It was Chris, I was getting a real knack for hearing ....okay, ease-dropping. Like its not a common practice!

336. "Yeh, you were quite good, the way you jumped those, those ... thingy's," he said not able to think of the correct English word for the hurtles.
The United States Forest Service's ursine mascot first appeared in 1944. Give his catchphrase safety message.
Example: Keyword

- Keyword: “else” or “if” or “begin” or ...

'else' | 'if' | 'begin' | ...

(Recall: 'else' abbreviates 'e' 'l' 's' 'e')
Example: Integers

- **Integer**: a non-empty string of digits

\[
\text{digit} = '0' \mid '1' \mid '2' \mid '3' \mid '4' \mid '5' \mid '6' \mid '7' \mid '8' \mid '9'
\]

\[
\text{number} = \text{digit} \ \text{digit}^*
\]

**Abbreviation**: \( A^+ = A \ A^* \)
Example: Identifier

- Identifier: string of letters or digits, starting with a letter

\[
\text{letter} = 'A' \mid \ldots \mid 'Z' \mid 'a' \mid \ldots \mid 'z' \\
\text{ident} = \text{letter} \ ( \text{letter} \mid \text{digit} \ )^* \\
\]

Is \( (\text{letter}^* \mid \text{digit}^*) \) the same?
Example: Whitespace

- Whitespace: a non-empty sequence of blanks, newlines, and tabs

\[(\ ' \ | \ \t \ | \ \n \ ) +\]

or

\[(\ ' \ | \ \t \ | \ \n \ | \ \r \ ) +\]
Example: Phone Numbers

- Regular expressions are everywhere!
- Consider: (434) 924-1021

$$\Sigma = \{0, 1, 2, 3, \ldots, 9, (, ), -\}$$

- area = digit digit digit
- exch = digit digit digit
- phone = digit digit digit digit
- number = '()' area ')' exch '-' phone
Example: Email Addresses

- Consider weimer@cs.virginia.edu

\[ \Sigma = \{a, b, \ldots, z, ., @\} \]

\[ \text{name} = \text{letter}^+ \]

\[ \text{address} = \text{name} '@' \text{name} ('..' \text{name})^* \]
Regexp Summary

• Regular expressions describe many useful languages
• Next: Given a string $s$ and a regexp $R$, is $s \in L(R)$
• But a yes/no answer is not enough!
  - Recall: we also need the substring (lexeme)
• Instead: partition the input into lexemes
• We will adapt regular expression to this goal
Subsequent Outline

• Specifying lexical structure using regexps
• Finite Automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
• Implementation of Regular Expressions
  - Regexp -> NFA -> DFA -> tables
Lexical Specification (1)

• Select a set of tokens
  - Number, Keyword, Identifier, …

• Write a regexp for the lexemes of each token
  - Number = digit+
  - Keyword = 'if' | 'else' | …
  - Identifier = letter ( letter | digit ) *
  - OpenPar = '('
  - …
Lexical Specification (2)

• Construct $R$, matching all lexemes for all tokens:

\[
R = \text{Keyword} \mid \text{Identifier} \mid \text{Number} \mid \ldots
\]

\[
R = R_1 \mid R_2 \mid R_3 \mid \ldots
\]

• Fact: if $s \in L(R)$ then $s$ is a lexeme
  - Furthermore, $s \in L(R_j)$ for some $j$
  - This $j$ determines the token that is reported
Lexical Specification (3)

- Let the input be $x_1 \ldots x_n$
  - Each $x_i$ is in the alphabet $\Sigma$
- For $1 \leq i \leq n$, check
  - $x_1 \ldots x_i \in L(R)$
- If so, it must be that
  - $x_1 \ldots x_i \in L(R_j)$ for some $j$
- Remove $x_1 \ldots x_i$ from the input and restart
Lexing Example

- \( R = \) Whitespace | Integer | Identifier | Plus
- Parse “f +3 +g”
  - “f” matches \( R \), more precisely Identifier
  - “ “ matches \( R \), more precisely Whitespace
  - “+” matches \( R \), more precisely Plus
  - ...
  - The token-lexeme pairs are
    - <Identifier, “f”>
    - <Whitespace, “ “>
    - <Plus, “+”> ...

In the future, we'll just drop whitespace.
Ambiguities (1)

• There are ambiguities in the algorithm
• Example:
  - $R = \text{Whitespace} \mid \text{Integer} \mid \text{Identifier} \mid \text{Plus}$
• Parse “foo+3”
  - “f” matches $R$, more precisely Identifier
  - But also “fo” matches $R$, and “foo”, but not “foo+”
• How much input is used?
  - **Maximal Munch** rule: Pick the longest possible substring that matches $R$
Ambiguities (2)

• $R = \text{Whitespace} \mid 'new' \mid \text{Integer} \mid \text{Identifier}$

• Parse “new foo”
  - “new” matches $R$, more precisely 'new'
  - but also Identifier - which one do we pick?

• In general, use the rule listed first
  - No, really.

• So we must list 'new' (and other keywords) before Identifier.
  - That is, you must list it first when writing PA2.
Error Handling

• \( R = \text{Whitespace} \mid \text{Integer} \mid \text{Identifier} \mid '+' \)

• Parse “=56”
  - No prefix matches \( R \): not “=”, nor “=5”, nor “=56”

• Problem: we can't just get stuck and die

• Solution:
  - Add a rule matching all “bad” strings
  - Put it last

• Lexer tools allow the writing of:
  - \( R = R_1 \mid R_2 \mid \ldots \mid R_n \mid \text{Error} \)
Summary

- Regular expressions provide a concise notation for string patterns
- Their use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Requiring only a single pass over the input
  - And few operations per character (table lookup)