Lexical Analysis

Finite Automata

(Part 1 of 2)

HOCUS-POCUS, ABRACADABRA!

I COMMAND MY HOMEWORK TO DO ITSELF!
HOMEWORK, BE DONE!

FLIP FLIP FLIP

RATS.
Cunning Plan

• Informal Sketch of Lexical Analysis
  - LA identifies tokens from input string
  - lexer : (char list) → (token list)

• Issues in Lexical Analysis
  - Lookahead
  - Ambiguity

• Specifying Lexers
  - Regular Expressions
  - Examples
One-Slide Summary

• **Lexical analysis** turns a stream of characters into a stream of tokens.

• **Regular expressions** are a way to specify sets of strings. We use them to describe tokens.
Fold Batter Lightly ...

- \( \text{fold\_left } f \ a \ [1;\ldots;n] = f (\ldots (f (f a 1) 2)) n \)

\[
\text{fold\_left } (\text{fun } a \ e \ -> \ e :: a) \ [] \ [1;2;3] = [3;2;1]
\]
\[
\text{fold\_left } (\text{fun } a \ e \ -> \ a @ [e]) \ [] \ [1;2;3] = [1;2;3]
\]

- \( \text{fold\_right } f \ [1;\ldots;n] \ b = f 1 (f 2 (\ldots (f n b))) \)

\[
\text{fold\_right } (\text{fun } a \ e \ -> \ e :: a) \ [1;2;3] [] = [1;2;3]
\]
\[
\text{fold\_right } (\text{fun } e \ a \ -> \ a @ [e]) \ [1;2;3] [] = [3;2;1]
\]
Interpreter and Compiler Structure

Source → Lexical Analysis → List of Tokens → Parsing → Abstract Syntax Tree → Optimization

(Interpreter) → Run It!

(Compiler) → Code Generation → Assembly Code
Lexical Analysis

Regular Expressions

Lexical Analyzer Generator

Source

Lexical Analysis

Token List

Parsing

Syntax Tree

Context-Free Grammar

Parser Generator

PA2

PA3
Lexical Analysis

• What do we want to do? Example:
  
  ```
  if (i == j)
      z = 0;
  else
      z = 1;
  ```

• The input is just a sequence of characters:
  
  ```
  if (i == j)
      z = 0;
  else
      z = 1;
  ```

• Goal: partition input strings into substrings
  - And classify them according to their role
What's a Token?

- Output of lexical analysis is a list of tokens
- A token is a syntactic category
  - In English:
    - noun, verb, adjective, ...
  - In a programming language:
    - Identifier, Integer, Keyword, Whitespace, ...
- Parser relies on token distinctions:
  - e.g., identifiers are treated differently than keywords
Tokens

- **Tokens** correspond to sets of strings.

- **Identifier**: strings of letters or digits, starting with a letter

- **Integer**: a non-empty string of digits

- **Keyword**: “else” or “if” or “begin” or ...

- **Whitespace**: a non-empty sequence of blanks, newlines, and/or tabs

- **OpenPar**: a left-parenthesis
Lexical Analyzer: Build It!

- An implementation must do two things:
- Recognize substrings corresponding to tokens
- Return the value or **lexeme** of the token
  - The lexeme is the substring
Example

• Recall:
  ```c
  if (i == j) \n  \tz = 0; \n  \n  else \n  \tz = 1;
  ```

• Token-lexeme pairs returned by the lexer:
  - `<Keyword, “if”>`
  - `<Whitespace, “ ”>`
  - `<OpenPar, “(”>`
  - `<Identifier, “i”>`
  - `<Whitespace, “ ”>`
  - `<Relation, “==”>`
  - `<Whitespace, “ ”>`
  - `<…>`
Lexical Analyzer: Implementation

- The lexer usually *discards* “uninteresting” tokens that don't contribute to parsing.
- Examples: Whitespace, Comments
  - Exception: which language cares about whitespace?
- Question: What happens if we remove all whitespace and comments *prior* to lexing?
Lookahead

- The goal is to partition the string. That is implemented by reading left-to-right, recognizing one token at a time.
- **Lookahead** may be required to decide where one token ends and the next token begins
  - Even our simple example has lookahead issues
    - i vs. if
    - = vs. ==
Still Needed

- A way to describe the lexemes of each token
  - Recall: lexeme = “the substring corresponding to the token”

- A way to resolve ambiguities
  - Is if two variables i and f?
  - Is == two equal signs = =?
Languages

- **Definition.** Let $\Sigma$ be a set of characters. A **language over $\Sigma$** is a set of strings of characters drawn from $\Sigma$. $\Sigma$ is called the alphabet.
Examples of Languages

• Alphabet = English Characters
• Language = English Sentences
  - Note: *Not* every string on English characters is an English sentence.
  - Example: xayenb sbe'

• Alphabet = ASCII characters
• Language = C Programs
  - Note: ASCII character set is different from English character set.
Notation

- **Languages** are sets of strings

- We need some notation for specifying which sets we want
  - that is, which strings are in the set

- For lexical analysis we care about *regular languages*, which can be described using *regular expressions*. 
Regular Expressions

- Each **regular expression** is a notation for a regular language (a set of words)
  - You'll see the exact notation in minute!

- If **A** is a regular expression then we write **L(A)** to refer to the language denoted by **A**
Base Regular Expression

• Single character: 'c'
  - \( L('c') = \{ \text{“c”} \} \) (for any \( c \in \Sigma \))

• Concatenation: \( AB \)
  - A and B are other regular expressions
  - \( L(AB) = \{ ab \mid a \in L(A) \text{ and } b \in L(B) \} \)

• Example: \( L('i' 'f') = \{ \text{“if”} \} \)
  - We abbreviate 'i' 'f' as 'if'
Compound Regular Expressions

• Union

- \( L(A \mid B) = \{ s \mid s \in L(A) \text{ or } s \in L(B) \} \)

• Examples:

- \( L('if' \mid 'then' \mid 'else') = \{ "if", "then", "else" \} \)
- \( L('0'|'1'|'2'|'3'|'4'|'5'|'6'|'7'|'8'|'9') = \text{what?} \)

• Fun Example:

- \( L( ('0'|'1') ('0'|'1') ) = \{"00","01","10","11"\} \)
Starz!

- So far we have only finite languages
- Iteration: $A^*$
  - $L(A^*) = \{"\"\} \cup L(A) \cup L(AA) \cup L(AAA) \ldots$
- Examples:
  - $L('0^*) = \{"\", "0", "00", "000", "0000", \ldots \}$
  - $L('1''0^*) = \{"1", "10", "100", "1000", \ldots \}$
- Empty: $\varepsilon$
  - $L(\varepsilon) = \{ "\" \}$
The United States Forest Service's ursine mascot first appeared in 1944. Give his catchphrase safety message.
Natural Languages

- These languages, of which there are about 250, are often mutually intelligible and constitute a major branch of the Niger-Congo languages. They are spoken largely in central, east and southern Africa. Popular examples include Swahili, with 80 million speakers, Shona, with 11 million, and Zulu, with 10 million. They commonly use words such as *muntu* or *mutu* for “person”. Words such as bongos, chimpanzee, gumbo, jumbo, mambo, rumba and safari come from these languages.
In this 1958 Sheb Wooley song the pigeon-toed title character wears short shorts and wants to get a job in a rock'n'roll band playing the horn, but is perhaps best known for his skin tone and non-standard diet.
Example: Keyword

• Keyword: “else” or “if” or “begin” or ...

'else' | 'if' | 'begin' | ...

(Recall: 'else' abbreviates 'e' 'l' 's' 'e')
Example: Integers

- Integer: a non-empty string of digits

\[
\text{digit} = '0' \mid '1' \mid '2' \mid '3' \mid '4' \\
\quad \mid '5' \mid '6' \mid '7' \mid '8' \mid '9' \\
\text{number} = \text{digit} \ \text{digit}* \\
\]

Abbreviation: \( A+ = A \ A^* \)
Example: Identifier

- Identifier: string of letters or digits, starting with a letter

```
letter = 'A' | ... | 'Z' | 'a' | ... | 'z'
ident = letter ( letter | digit )* 
```

Is (letter* | digit*) the same?
Example: Whitespace

- Whitespace: a non-empty sequence of blanks, newlines, and tabs

\[
(\, | \t | \n) +
\]

or

\[
(\, | \t | \n | \r) +
\]
Example: Phone Numbers

- Regular expressions are everywhere!
- Consider: (434) 924-1021

\[ \Sigma = \{0, 1, 2, 3, \ldots, 9, (, ), -, \} \]

area = digit digit digit
exch = digit digit digit
phone = digit digit digit digit
dnumber = 
  \'(' area ')' exch '-' phone
Example: Email Addresses

- Consider weimer@cs.virginia.edu

\[ \Sigma = \{a, b, \ldots, z, ., @\} \]

name = letter+

address = name '@' name ('.' name)*
Regexp Summary

• Regular expressions describe many useful languages

• Next: Given a string s and a regexp R, is $s \in L(R)$

• But a yes/no answer is not enough!

• Instead: partition the input into lexemes

• We will adapt regular expression to this goal
Subsequent Outline

• Specifying lexical structure using regexps
• Finite Automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
• Implementation of Regular Expressions
  - Regexp -> NFA -> DFA -> Tables
  - The tables are the heart of the lexer, which is just a while loop that takes in the current input character and looks up the new state in the transition table.
Lexical Specification (1)

- Select a set of tokens
  - Number, Keyword, Identifier, …
- Write a regexp for the lexemes of each token
  - Number = digit+
  - Keyword = 'if' | 'else' | …
  - Identifier = letter ( letter | digit ) *
  - OpenPar = '('
  - …
Lexical Specification (2)

• Construct \( R \), matching all lexemes for all tokens:

\[
\begin{align*}
R &= \text{Keyword} \ | \ \text{Identifier} \ | \ \text{Number} \ | \ \ldots \\
R &= R_1 \ | \ R_2 \ | \ R_3 \ | \ \ldots
\end{align*}
\]

• Fact: if \( s \in L(R) \) then \( s \) is a lexeme
  - Furthermore, \( s \in L(R_j) \) for some \( j \)
  - This \( j \) determines the token that is reported
Lexical Specification (3)

- Let the input be $x_1 \ldots x_n$
  - Each $x_i$ is in the alphabet $\Sigma$

- For $1 \leq i \leq n$, check
  - $x_1 \ldots x_i \in L(R)$

- If so, it must be that
  - $x_1 \ldots x_i \in L(R_j)$ for some $j$

- Remove $x_1 \ldots x_i$ from the input and restart
Lexing Example

• R = Whitespace | Integer | Identifier | Plus

• Parse “f +3 +g”
  - “f” matches R, more precisely Identifier
  - “ “ matches R, more precisely Whitespace
  - “+” matches R, more precisely Plus
  - ...
  - The token-lexeme pairs are
    - <Identifier, “f”>
    - <Whitespace, “ “>
    - <Plus, “+”> ...

In the future, we'll just drop whitespace.
Ambiguities (1)

- There are ambiguities in the algorithm
- Example:
  - R = Whitespace | Integer | Identifier | Plus
- Parse “foo+3”
  - “f” matches R, more precisely Identifier
  - But also “fo” matches R, and “foo”, but not “foo+”
- How much input is used?
  - **Maximal Munch** rule: Pick the longest possible substring that matches R
Ambiguities (2)

- R = Whitespace | 'new' | Integer | Identifier
- Parse “new foo”
  - “new” matches R, more precisely 'new'
  - but also Identifier - which one do we pick?
- In general, use the rule listed first.
  - No, really.
- So we must list 'new' (and other keywords) before Identifier.
Error Handling

- \( R = \) Whitespace | Integer | Identifier | '+'
- Parse “=56”
  - No prefix matches \( R \): not “=”, nor “=5”, nor “=56”
- Problem: we can't just get stuck and die
- Solution:
  - Add a rule matching all “bad” strings
  - Put it last
- Lexer tools allow the writing of:
  - \( R = R1 \mid R2 \mid \ldots \mid Rn \mid Error \)
Summary

• Regular expressions provide a concise notation for **string patterns**
• Their use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
• Good algorithms known (next)
  - Requiring only a single pass over the input
  - And few operations per character (table lookup)