A+K+Q Game Rules

- 3 card deck: Ace > King > Queen
- 2 Players, each player gets one card face-up
- Higher card wins

Without secrecy, stakes, betting, it's not poker!

A+K+Q Game Rules

- 3 card deck: Ace > King > Queen
- 2 Players, each player gets one card face-down
- Higher card wins
- **Betting:** (half+street game)
  - **Ante:** 1 chip
  - **Player 1:** bet 1, or check
  - **Player 2:** call or fold
- **Stakes:** scheduling signup order by chip count

Loosely based on Bill Chen and Jerrod Ankenman, *The Mathematics of Poker.*
A+K+Q Analysis

Better to be player 1 or player 2?

Easy Decisions:

Hard Decisions:

Game Payoffs (Player 1, Player 2)

Zero-Sum Game

\[ \sum_{p \in \text{Players}} \text{Gain}(p) = 0 \]

Payoffs for Player 1

Strategic Domination

Strategy A **dominates** Strategy B if Strategy A always produces a better outcome than Strategy B regardless of the other player's action.
Eliminating Dominated Strategies

<table>
<thead>
<tr>
<th>Player 1:</th>
<th>Ace</th>
<th>King</th>
<th>Queen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 2:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ace</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>King</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Queen</td>
<td></td>
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</tbody>
</table>

Simplified Payoff Matrix

<table>
<thead>
<tr>
<th>Player 1:</th>
<th>Ace</th>
<th>King</th>
<th>Queen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 2:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ace</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>King</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Queen</td>
<td></td>
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</tbody>
</table>

The Tough Decisions

What if Player 1 never bluffs?

Expected Value

\[ EV = \sum_{e \in \text{Events}} Pr(e) \text{Value}(e) \]

Never Bluff Strategy

\[ EV_1 = \frac{1}{3}(1) + \frac{1}{3}(-\frac{1}{2} + \frac{1}{2}) + \frac{1}{3}(-1) = 0 \]

The Tough Decisions

What if Player 1 always bluffs?
Always Bluff Strategy

Player 1: A K Q

<table>
<thead>
<tr>
<th>Player 2</th>
<th>A Call</th>
<th>K Call</th>
<th>Q Fold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bet</td>
<td>-1</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>Check</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Call</td>
<td>+2</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>Call</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fold</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>Fold</td>
<td></td>
<td></td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ EV_{1/\text{Call}K} = \frac{1}{3}(\frac{1}{2}(+2) + \frac{1}{2}(+1)) + \frac{1}{3}(\frac{1}{2} - \frac{1}{2}) + \frac{1}{3}(\frac{1}{2}(-2)) = -\frac{1}{6} \]

\[ EV_{1/\text{Fold}K} = \frac{1}{3}(1) + \frac{1}{3}(+2) + \frac{1}{3}(\frac{1}{2}(-2) + \frac{1}{2}(1)) = \frac{1}{3} \]

Recap

If player 1 never bluffs: \( EV_1 = 0 \)

If player 1 always bluffs: \( EV_1 = -\frac{1}{6} \)

Is this a break-even game for Player 1?

Course Overview

- **Topics**
  - Game Theory
  - Machine Learning
  - Anything else relevant to building a poker bot
- **Format**: most classes will be student-led
  - Present a topic and/or research paper

Class Leader Expectations

- **At least two weeks*** before your scheduled class:
  - Let me know what you are planning on doing (talk to me after class or email)
- **At least one week** before your scheduled class:
  - Post on the course blog a description of the class topic and links to any reading/preparation materials
- **At the class**: lead an interesting class, bring any needed materials
- **Later that day**: post class materials on the course blog
- **Follow-up**: respond to any comments on the course blog

* If you signed up for Feb 1, you’re already late!

Course Project

Build a poker bot capable of competing in the Sixth Annual Computer Poker Competition
http://www.computerpokercompetition.org/

Work in small (2-4) person teams
A few preliminary projects earlier
Combine ideas/code/results from best teams

Note: overlaps with USENIX Security, August 9-12 (also in San Francisco)

My (Lack of) Qualifications

- I do research in computer security
- I have very limited knowledge and experience in game theory, machine learning, etc.
- I am (probably) a fairly lousy poker player

This course will be a shared learning experience, and will only work well if everyone contributes to make it interesting and worthwhile.
Things to Do

• Submit course survey
• Print and sign course contract: bring to Tuesday’s class
• Reading for Tuesday: Chapters 1 and 2 of Darse Billings’ dissertation

Recap Recap

If player 1 never bluffs: \( EV_1 = 0 \)
If player 1 always bluffs: \( EV_1 = -\frac{1}{6} \)

Looks like a break-even game for Player 1: is there a better strategy?

Mixed Strategy

<table>
<thead>
<tr>
<th>Player 1</th>
<th>A</th>
<th>K</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bet</strong></td>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td><strong>Check</strong></td>
<td>-2</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

Never Bluff \( EV_1 = 0 \)
Always Bluff \( EV_1 = -\frac{1}{6} \)

Pure strategy: always do the same action for a given input state.
Mixed strategy: probabilistically select from a set of pure strategies.

Strategies

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bluff with Queen</td>
<td>Call with King</td>
</tr>
<tr>
<td>Check with Queen</td>
<td>Fold with King</td>
</tr>
</tbody>
</table>

\[
EV_1(<S_{Bluff},T_{Call}>) = -\frac{1}{6} \quad EV_1(<S_{Bluff},T_{Fold}>) = \frac{1}{6}
\]
\[
EV_1(<S_{Check},T_{Call}>) = \frac{1}{6} \quad EV_1(<S_{Check},T_{Fold}>) = 0
\]

Finding the best strategy for Player 1: assume Player 2 plays optimally.

Nash Equilibrium

• Player 1 is making the best decision she can, taking into account Player 2’s decisions.
• Player 2 is making the best decision he can, taking into about Player 1’s decisions.
• Neither player can improve its expected value by deviating from the strategy.

Hence, to find the best strategy for Player 1, we need to find a strategy that makes Player 2 indifferent between his options.

John Nash (born 1928)

Equilibrium Points in N-Person Games, 1950
Winning the AKQ Game

\[ EV_1(<S_{Bluff}, T_{Call}>) = -\frac{1}{6} \]
\[ EV_1(<S_{Bluff}, T_{Fold}>) = \frac{1}{6} \]
\[ EV_1(<S_{Check}, T_{Call}>) = \frac{1}{6} \]
\[ EV_1(<S_{Check}, T_{Fold}>) = 0 \]

<table>
<thead>
<tr>
<th></th>
<th>Bluff</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>Fold</td>
<td>+1</td>
<td>0</td>
</tr>
</tbody>
</table>

Player 1 wants to make Player 2 indifferent between \( T_{Call} \) and \( T_{Fold} \)

\[
EV(<S, T_{Call}>) = EV(<S, T_{Fold}>) \frac{S \text{ it Prob }}{2} \\
EV(<S, T_{Call}>) = -2(Pr[AJ]) - 2(Pr[QJ]) \\
EV(<S, T_{Fold}>) = -1
\]

Charge

- Submit course survey
- Print and sign course contract: bring to Tuesday’s class
- Reading for Tuesday: Chapters 1 and 2 of Darse Billings’ dissertation

Readings posted now. Everything else will be posted on the course site (by tomorrow!):

http://www.cs.virginia.edu/evans/poker

If you are signed up for February 1, by tomorrow: contact me about plans for class.