Homework

Divide and Conquer Exercise 1 due September 7
Divide and Conquer Exercise 2 due September 11
  • Written (use LaTeX!)
  • Asymptotic notation
  • Recurrences
  • Divide and conquer
Recurrence Solving Techniques

- Tree
- Guess/Check
- “Cookbook”
- Substitution
Tree Method

\[ T(n) = 2T\left(\frac{n}{2}\right) + n \]
Tree Method

\[ T(n) = 2T \left( \frac{n}{2} \right) + n \]

Number of subproblems

1
Tree Method

\[ T(n) = 2T \left( \frac{n}{2} \right) + n \]

Number of subproblems

Cost to combine

1

n

2

n/2

4

n/4

2^k

\[ \frac{n}{2^k} = 1 \]
Tree Method

3. Use asymptotic notation to simplify

\[ T(n) = 2T\left(\frac{n}{2}\right) + n \]

How many levels?

Problem size at \( k \)th level: \( \frac{n}{2^k} \)

Base case: \( n = 1 \)

At level \( k \), it should be the case that \( \frac{n}{2^k} = 1 \)

\[ n = 2^k \Rightarrow k = \log_2 n \]

<table>
<thead>
<tr>
<th>Number of subproblems</th>
<th>Cost to combine</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( n )</td>
</tr>
<tr>
<td>2</td>
<td>( n/2 )</td>
</tr>
<tr>
<td>4</td>
<td>( n/4 )</td>
</tr>
<tr>
<td>( 2^k )</td>
<td>( \frac{n}{2^k} = 1 )</td>
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</tbody>
</table>
Tree Method

3. Use asymptotic notation to simplify

\[ T(n) = 2T(n/2) + n \]

\[ k = \log_2 n \]

What is the cost?

Cost at level \( i \):

\[ 2^i \cdot \frac{n}{2^i} = n \]

Total cost:

\[ T(n) = \sum_{i=0}^{\log_2 n} n = n \sum_{i=0}^{\log_2 n} 1 = n \log_2 n \]

\[ = \Theta(n \log n) \]
Tree Method

\[ T(n) = 3T \left( \frac{n}{2} \right) + n \]
Tree Method

\[ T(n) = 3T \left( \frac{n}{4} \right) + 10 \]
Tree Method

\[ T(n) = T(n - 2) + n^2 \]
Tree Method

\[ T(n) = T\left(\frac{2n}{3}\right) + T\left(\frac{n}{3}\right) + n \]