UVa - cs302: Theory of Computation Spring 2008

Problem Set 4 - Turing Machines

Due: Tuesday, 18 March (2:02pm)

This problem set covers material from Classes 12-13 (through 13 March) and Sipser's book through the end of Chapter 3. For full credit, answers must be concise, clear, and convincing, not just correct. Please **staple** your answer sheets before class.

Honor Policy. For this assignment, we will use the "Tila Tequila" collaboration policy as described on Problem Set 2.

Problem 1: Describing a Turing Machine. Design a deterministic Turing Machine that decides the language $\{0^n1^n2^n \mid n \geq 0\}$. You should provide descriptions of your Turing Machine at the three levels of detail described in Section 3.3:

a. Provide a *high-level description* of your machine.

b. Provide an *implementation description* of your machine.

c. Provide a full *formal description* of your machine (this is tedious, but everyone should do it once!).

Problem 2: Deciding $0^n1^{n^2}$. Provide an *implementation description* of a Turing Machine that decides the language $0^n1^{n^2}$ for $n \geq 0$.

Problem 3: Equivalence of 2-DPDA+ϵ and Turing Machine. In Class 12, we informally argued that a 2-stack deterministic pushdown automaton with forced ϵ-transitions is equivalent to a Turing Machine. For this problem, prove one direction of the equivalence: that a 2-DPDA+ϵ can simulate any Turing Machine. (Hint: your proof should show how to construct a 2-DPDA+ϵ from a given Turing Machine. You will need to introduce a formal notation for describing a 2-DPDA+ϵ.)

Problem 4: Robustness of Turing's Model. (Sipser problem 3.11) A *Turing machine with doubly infinite tape* is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that the class of languages recognized by a Turing machine with doubly infinite tape is equivalent to the class of languages recognized by a regular Turing machine.

Problem 5: Deciders and Recognizers. In Class 12, we argued that DFAs, DPDAs, and NFAs always terminate, hence the sets of languages that can be recognized by these machines are equivalent to the sets of languages that can be decided by them. By contrast, a Turing Machine may run forever on some inputs, so there may be languages which can be recognized but not decided by a Turing Machine (we will see examples of such languages next week). What about nondeterministic pushdown automata?

a. Draw a Venn diagram including these sets:

   NDPDA-Recognizable: languages that can be recognized by a NDPDA.
   NDPDA-Decidable: languages that can be decided by a NDPDA.
   TM-Decidable: languages that can be decided by a Turing Machine.
   TM-Recognizable: languages that can be recognized by a Turing Machine.

b. (bonus) Prove that your diagram correctly depicts the relationship between *NDPDA-Recognizable* and *NDPDA-Decidable*. (For example, if your diagram showed them as the same circle, prove that the sets are equivalent. If your diagram shows one set inside the other, prove that all languages in one set are in the other set but that there is some language in the outer set that is outside the inner set.)

c. (bonus) Prove that your diagram correctly depicts the relationship between *NDPDA-Recognizable* and *TM-Decidable*.