CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Practice exercise in class friday october 25, 2019

# Practice 07

You may answer any question with factorial, choose, and unresolved arithmetic notation, but may not use ellipses. For example, the following are all OK: $\boxed{120}$, $\boxed{5!}$, $\boxed{\frac{5\cdot4\cdot3\cdot2\cdot1}{(2\cdot1)(3\cdot2\cdot1)}}$, $\boxed{\binom{5}{3}}$; however, the following is *not* OK: $\boxed{10\cdot9\cdot8\cdots2\cdot1}$.

**problem 1** *Stand-alone problems*

1. $\binom{21}{8}$ ______ How many 8-element subsets of a 21-element set are there?

2. $2^5$ ______ How many strictly-increasing sequences of the numbers $\{1,2,3,4,5\}$ are there?

3. $5\times10^9-5\approx5\times10^9$ ______ My passphrase is a six-word extract taken randomly from the 5-billion-word string created by concatenating all Wikipedia articles. If no six-word string is repeated twice in that corpus, how many passwords can be created in this method?

4. $\frac{26!}{18!}\approx6.3\times10^{10}$ ______ My passphrase is an eight-character string made up of a random collection of lower-case letters (from the 26 letters a through z), without repeating any letter. How many passwords can be created in this method?

5. $26^8\approx2\times10^{11}$ ______ My passphrase is an eight-character string made up of a random collection of lower-case letters (from the 26 letters a through z), allowing letter repetitions. How many passwords can be created in this method?

6. $4\cdot6-3=21$ ______ I roll four fair six-sided dice and total the result. How many possible numbers could I roll?

7. $\frac{1}{6^4}=\frac{1}{1296}$ ______ I roll four fair six-sided dice and total the result. What is the chance the total will be 4?

8. $\frac{6^2+2(5^2+4^2+3^2+2^2+1^2)}{6^4}$ ______ I roll four fair six-sided dice and total the result. What is the chance the total will be 14?

PROBLEM 2 *Problems about Bogosort*

Bogosort sorts a list be shuffling it, checking to see if it is in order, and then shuffling again if not. We have two versions: version **R** shuffles randomly each time; version **U** other shuffles in a way that guarantees each shuffling will be unique (i.e., it never checks the same permuiation twice).

9. $\frac{1}{20!}$ If given a list of 20 distinct numbers, what is the chance **R** will get the sorted list after just one shuffle?

10. $1 \div \binom{20}{10}$ If given a list of 20 numbers consisting of ten 1s and ten 2s, what is the chance **R** will get the sorted list after just one shuffle?

11. $\frac{2}{20!}$ If given a list of 20 distinct numbers, 0 through 18 with 0 repeated in the list twice; what is the chance **U** will get the sorted list after just one shuffle?

12. $\frac{1}{20!} + \frac{20!-1}{20!^2} + \frac{(20!-1)^2}{20!^3}$ How likely **R** to get the right answer after no more than three tries given a list of 20 distinct numbers?

13. $\frac{3}{20!}$ How likely is **U** to get the right answer after no more than three tries given a list of 20 distinct numbers?

14. $n!$ If I know nothing about the contents of the list, but know it contains $n$ values, how many times could **U** shuffle the list in the worst case before it gets the list sorted?

15. all values distinct (continuing from the previous problem) Describe that worst-case list.