

You may answer any question with factorial, choose, and unresolved arithmetic notation, but may not use ellipses. For example, the following are all OK:  $\boxed{120}$ ,  $\boxed{5!}$ ,  $\boxed{\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)}}$ ,  $\boxed{\binom{5}{3}}$ ; however, the following is *not* OK:  $\boxed{10 \cdot 9 \cdot 8 \cdots 2 \cdot 1}$ .

## PROBLEM 1 *Stand-alone problems*

1. \_\_\_\_\_ How many 8-element subsets of a 21-element set are there?
2. \_\_\_\_\_ How many strictly-increasing sequences of the numbers  $\{1, 2, 3, 4, 5\}$  are there?
3. \_\_\_\_\_ My passphrase is a six-word extract taken randomly from the 5-billion-word string created by concatenating all Wikipedia articles. If no six-word string is repeated twice in that corpus, how many passwords can be created in this method?
4. \_\_\_\_\_ My passphrase is an eight-character string made up of a random collection of lower-case letters (from the 26 letters a through z), without repeating any letter. How many passwords can be created in this method?
5. \_\_\_\_\_ My passphrase is an eight-character string made up of a random collection of lower-case letters (from the 26 letters a through z), allowing letter repetitions. How many passwords can be created in this method?
6. \_\_\_\_\_ I roll four fair six-sided dice and total the result. How many possible numbers could I roll?
7. \_\_\_\_\_ I roll four fair six-sided dice and total the result. What is the chance the total will be 4?
8. \_\_\_\_\_ I roll four fair six-sided dice and total the result. What is the chance the total will be 14?

PROBLEM 2 *Problems about Bogosort*

Bogosort sorts a list by shuffling it, checking to see if it is in order, and then shuffling again if not. We have two versions: version **R** shuffles randomly each time; version **U** shuffles in a way that guarantees each shuffling will be unique (i.e., it never checks the same permutation twice).

9. \_\_\_\_\_ If given a list of 20 distinct numbers, what is the chance **R** will get the sorted list after just one shuffle?
10. \_\_\_\_\_ If given a list of 20 numbers consisting of ten 1s and ten 2s, what is the chance **R** will get the sorted list after just one shuffle?
11. \_\_\_\_\_ If given a list of 20 distinct numbers, 0 through 18 with 0 repeated in the list twice; what is the chance **U** will get the sorted list after just one shuffle?
12. \_\_\_\_\_ How likely **R** to get the right answer after no more than three tries given a list of 20 distinct numbers?
13. \_\_\_\_\_ How likely is **U** to get the right answer after no more than three tries given a list of 20 distinct numbers?
14. \_\_\_\_\_ If I know nothing about the contents of the list, but know it contains  $n$  values, how many times could **U** shuffle the list in the worst case before it gets the list sorted?
15. \_\_\_\_\_  
(continuing from the previous problem) Describe that worst-case list.