CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Practice exercise in class friday november 15, 2019

Practice 10

problem 1 *Arithmetic*

1. The prime factorization of 18^2 is $2^2 \cdot 3^4$.
2. Re-write $p^r = w$ without a exponent function: $\log_p(w) = r$.
3. Simplify $\dfrac{\log_3(7)}{\log_3(5)}$: $\log_5(7)$.
4. Re-write $\log_2(16x^3)$ with no constants or operators in a log's argument: $4 + 3\log_2(x)$.
5. What is $\log_3(5)\log_5(3)$? 1.

problem 2 *Proof by Contradiction*

Complete the following proof that $\forall x \in \mathbb{Z}^+ \,.\, \big(\log_3(x) \in \mathbb{Q}^+\big) \rightarrow \big(\exists n \in \mathbb{N} \,.\, x = 3^n\big)$.

Proof. Assume that the implication does not hold; that is, that $\big(\log_3(x) \in \mathbb{Q}^+\big) \wedge \big(\nexists n \in \mathbb{N} \,.\, x = 3^n\big)$. Since $\log_3(x) \in \mathbb{Q}^+$, there are positive integers a and b such that $\log_3(x) = \frac{a}{b}$. Re-writing that equation,

$$\begin{aligned} \log_3(x) &= \frac{a}{b} \\ b\log_3(x) &= a \\ \log_3(x^b) &= a \\ x^b &= 3^a \end{aligned}$$

Since a and b are positive integers, both sides of the last equation above are integers. By the fundamental theorem of arithmetic, both sides must have the same prime factors, meaning that all of x's factors must be 3. But that contradicts our assumption that $\nexists n \in \mathbb{N} \,.\, x = 3^n$.

Because the assumption led to a contradiction, it must be false; thus,

$$\big(\log_3(x) \in \mathbb{Q}^+\big) \rightarrow \big(\exists n \in \mathbb{N} \,.\, x = 3^n\big)$$

□

Want additional practice? Try the following:
Simplify (show your work)

- $\log_3(5) + \log_3(2)$
- $\log_3(5) + \log_9(0.2)$
- $\log_3\left(\frac{5}{27}\right)$
- $7^{\frac{\log_3(5)}{\log_3(7)}}$

Complete

- $\log_{\sqrt{3}}(5) = \log_3\left(\qquad\qquad \right)$
- $\log_a(b)\log_a\left(\qquad\qquad \right) = 1$
- $\log_a(b)\log_b\left(\qquad\qquad \right) = 1$
- $\log_3(13) = \log_3(5) + \log_3\left(\qquad\qquad \right)$
- $3^{\log_5(7)} = 7^{\log_\square(\square)}$

Prove that

- there is no largest prime number. Use contradiction, with 1 + the product of all primes as part of how you get the contradiction.
- $\big(\log_a(b) = \log_b(a)\big) \rightarrow \big(a = b\big)$. Both direct proof and contradiction should be able to work here.
- "$\forall n \in \left\{i \mid i \in \mathbb{Z} \wedge 1 < i < x\right\} .\ \log_i(x) \notin \mathbb{Q}$" is true for all prime numbers x. Use contradiction.
- $3\log_2(10) < 10$. Direct proof should be enough.
- $\log_3(10) > 2$. Direct proof should be enough.