Problem 1 Equivalence rules

Consider the following equivalence rules:

- the associativity and commutativity of \( \land, \lor, \) and \( \oplus \)
- double negation: \( \neg
eg \neg P \equiv P \)
- simplification: \( P \land \bot \equiv \bot \), \( P \land \top \equiv P \), \( P \lor \bot \equiv P \), and \( P \lor \top \equiv \top \)
- distribution: \( A \land (B \lor C) \equiv (A \land B) \lor (A \land C) \) and \( A \lor (B \land C) \equiv (A \lor B) \land (A \lor C) \)
- De Morgan: \( \neg(A \land B) \equiv (\neg A) \lor (\neg B) \) and \( \neg(A \lor B) \equiv (\neg A) \land (\neg B) \)
- definitions: \( A \rightarrow B \equiv (\neg A) \lor B \), \( (A \leftrightarrow B) \equiv (A \rightarrow B) \land (B \rightarrow A) \) and \( (A \oplus B) \equiv (A \lor B) \land \neg(A \land B) \)

Prove that \( (P \land \neg Q) \equiv \neg(P \rightarrow Q) \) by writing out a series of steps, one per line, where the first line is \( (P \land \neg Q) \), the last line is \( \neg(P \rightarrow Q) \), and each line other than the first is an application of one of those equivalences to the line above it.

You do not need to name your steps, though doing so might help us grade more easily

\begin{align*}
(P \land \neg Q) \\
(\neg \neg P \land \neg Q) \\
\neg(\neg P \lor Q) \\
\neg(P \rightarrow Q)
\end{align*}
PROBLEM 2  Prose proof by case analysis

Write a prose proof of \((P \land Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)\) by completing the provided template.

Proof. Either \(P\) is true or it is false.

Case 1: \(P\) is true  The expression \((P \land Q) \rightarrow M\) in this case

\[\text{can be simplified to } Q \rightarrow M \text{ by the equivalence of } \top \land Q \text{ and } Q.\]

The expression \(P \rightarrow (Q \rightarrow M)\) in this case

\[\text{can be simplified to } Q \rightarrow M \text{ by the equivalence of } \top \rightarrow Q \text{ and } Q.\]

Because the two are equivalent to the same thing, they are equivalent to each other.

Case 2: \(P\) is false  The expression \((P \land Q) \rightarrow M\) in this case

\[\text{can be simplified to } \bot \rightarrow M \text{ by the equivalence of } \bot \land Q \text{ and } \bot, \text{ which in turn is just } \top \text{ regardless of the value of } M.\]

The expression \(P \rightarrow (Q \rightarrow M)\) in this case

\[\text{is } \bot \rightarrow (Q \rightarrow M), \text{ which is } \top \text{ regardless of the values of } Q \text{ and } M.\]

Because the two are equivalent to the same thing, they are equivalent to each other.

Since \((P \land Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)\) is true in both cases, it is true in general. \(\square\)