Name: ________________________________ CompID: ________________

CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Administered in class friday november 15, 2019

Quiz 10

problem 1 *Arithmetic*

1. The prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$.
2. Re-write $\log_3(x) = y$ without a log function: $3^y = x$.
3. Re-write $\log_a(b)$ with a base-c log: $\dfrac{\log_c a}{\log_c b}$.
4. Re-write $\log_2(a^2 \cdot b)$ with no constants or operators in a log's argument: $2\log_2(a) + \log_2(b)$.
5. Suppose $\log_a(b) = \frac{2}{3}$. What is $\log_b(a)$? 1.5.

problem 2 *Proof by Contradiction*

Complete the following proof that $\log_2(3)$ is irrational.

Proof. Because $3 > 2$, $\log_2(3) > 1$. Assume that $\log_2(3)$ is rational. Then $\log_2(3) = \frac{a}{b}$, where a and b are positive integers. Re-writing this equation we get

$$\log_2(3) = \frac{a}{b}$$

$$\begin{aligned} \log_2(3) &= \frac{a}{b} \\ b\log_2(3) &= a \\ \log_2(3^b) &= a \\ 3^b &= 2^a \end{aligned}$$

Since a and b are positive integers, both sides of the last equation above are integers. But they clearly share no prime factors, which contradicts the fundamental theorem of arithmetic.

Because the assumption led to a contradiction, it must be false and $\log_2(3)$ must be irrational. □