



Set  $X$  closed under operation  $\odot$

$\equiv$

$$\forall x, y \in X, \quad x \odot y \in X$$

a

ab

aaa

ababa

aaaaa

aaaaab

reduced

'hello world

no

print

print("hello")

HW  $\leq$  python print

$$6 \sum_{i=0}^n i^3 - i = \binom{n+2}{4}$$

Proof: assume  $\exists n$ .  $6 \sum_{i=0}^n i^3 - i \neq \binom{n+2}{4}$ . Consider the smallest such  $n$ ; call it  $n_0$ . Either  $n_0$  is  $0$  or  $> 0$ .

Case  $n_0 = 0$ : we have  $6 \sum_{i=0}^0 i^3 - i \neq \binom{0+2}{4}$ . The LHS is  $0$  because it is the sum of  $0$  numbers.  $\binom{2}{4}$  has no term  $= 0$  but that means  $0 \neq 0$ , which is a contradiction.   
 $\binom{2}{4}$  has no term  $= 0$  is nonsense

Case  $n_0 > 0$ : consider  $x = n_0 - 1$ . because  $n_0 \in \mathbb{N}$  and  $n_0 > 0$ ,  $x \in \mathbb{N}$ . then  $6 \sum_{i=0}^x i^3 - i = \binom{x+2}{4}$  because  $n_0$  was the smallest where that was not true. add  $6(n_0^3 - n_0)$  to both sides to get

$$6 \left( n_0^3 - n_0 + \sum_{i=0}^{n_0-1} i^3 - i \right) = \binom{n_0-1+2}{4} + 6(n_0^3 - n_0)$$

$$\underbrace{6 \sum_{i=0}^{n_0} i^3 - i}_{= 6 \sum_{i=0}^{n_0} i^3 - i} = \frac{(n_0+1)(n_0+0)(n_0+1)(n_0-2)}{24} + 6(n_0^3 - n_0)$$

$$\frac{(n_0+2)(n_0+1)(n_0)(n_0-1)}{24}$$

$$\binom{n_0+2}{4}$$