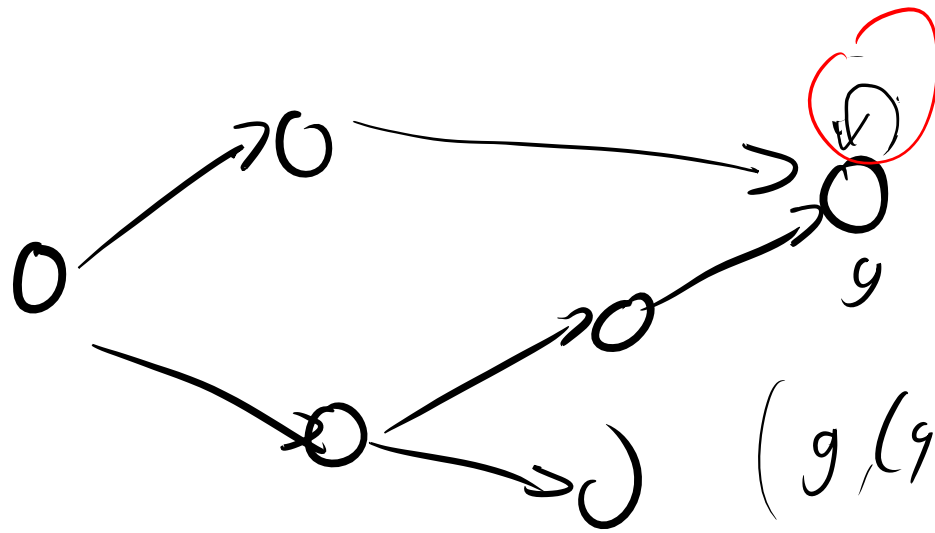


DAG



non-DAG

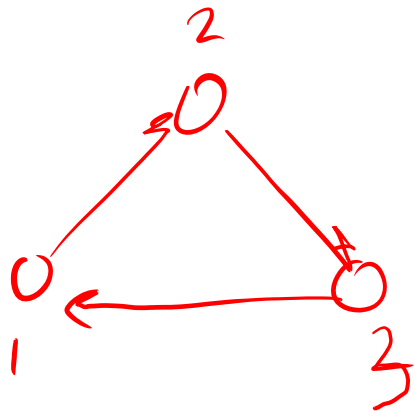
$(g, (g, g), g)$

(g)

walk
path
cycle
DAG

$v e v e \dots v$

$| \cdot |$ length of walk
degree # edges in
in out



$(1, (1, 2), 2, (2, 3), 3, (3, 1), 1)$

2	3	1	2
3	1	2	3

$\left. \begin{array}{l} \text{minimum} \\ \text{at} \end{array} \right\} \text{vertex in DAG}$
 $\left. \begin{array}{l} \text{Maximum} \\ \text{at} \end{array} \right\}$

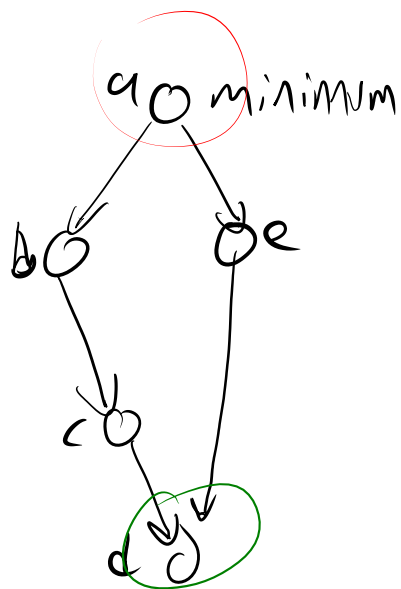
2 minimum m_0, m_1

\exists walk $w_0: m_0 \dots m_1$

\exists walk $w_1: m_1 \dots m_0$

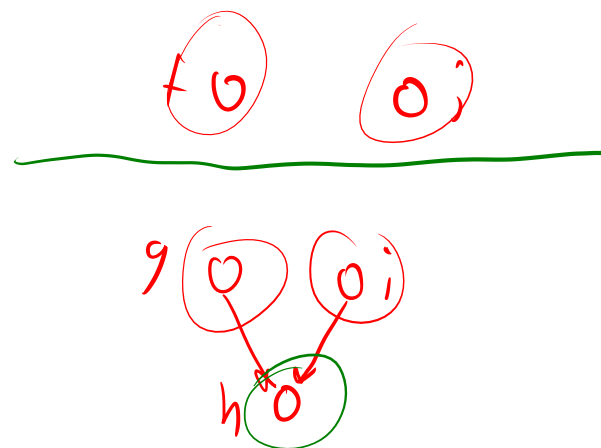
$w_0 \wedge w_1 \rightarrow \text{cycle}$

$m_0 \dots m_1 \dots m_0$



Maximum = reached by all
 Maximal = cannot reach any

minimal



relations, graph of

$$R: A \rightarrow B$$

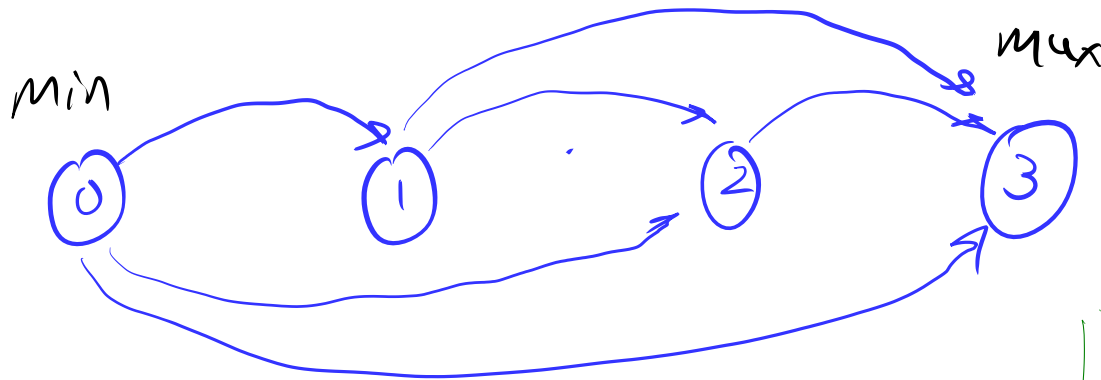
$$\forall a \in A, b \in B. R(a, b) \in \{T, F\}$$

$$\boxed{<} : \mathbb{R} \rightarrow \mathbb{R}$$

$$2.7 \boxed{<} \frac{\pi}{e^{13}} \equiv \perp$$

$$<(2.7, \frac{\pi}{e^{13}})$$

$<$ $\{0, 1, 2, 3\}$ (a, b) iff $R(a, b)$



minimum

maximum

$$R(a, b) \wedge R(b, c) \rightarrow R(a, c)$$

Transitive
 \exists walk from x to y , \rightarrow
 \exists walk of length 1 from x to y

Symmetric



$$R(a, b) \leftrightarrow R(b, a)$$

asymmetric

antisymmetric

$$a \neq b \rightarrow R(a, b) \rightarrow \neg R(b, a)$$

<

asymmetric

irreflexive

≤

antisymmetric

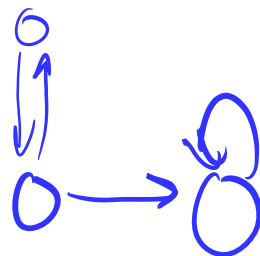
reflexive

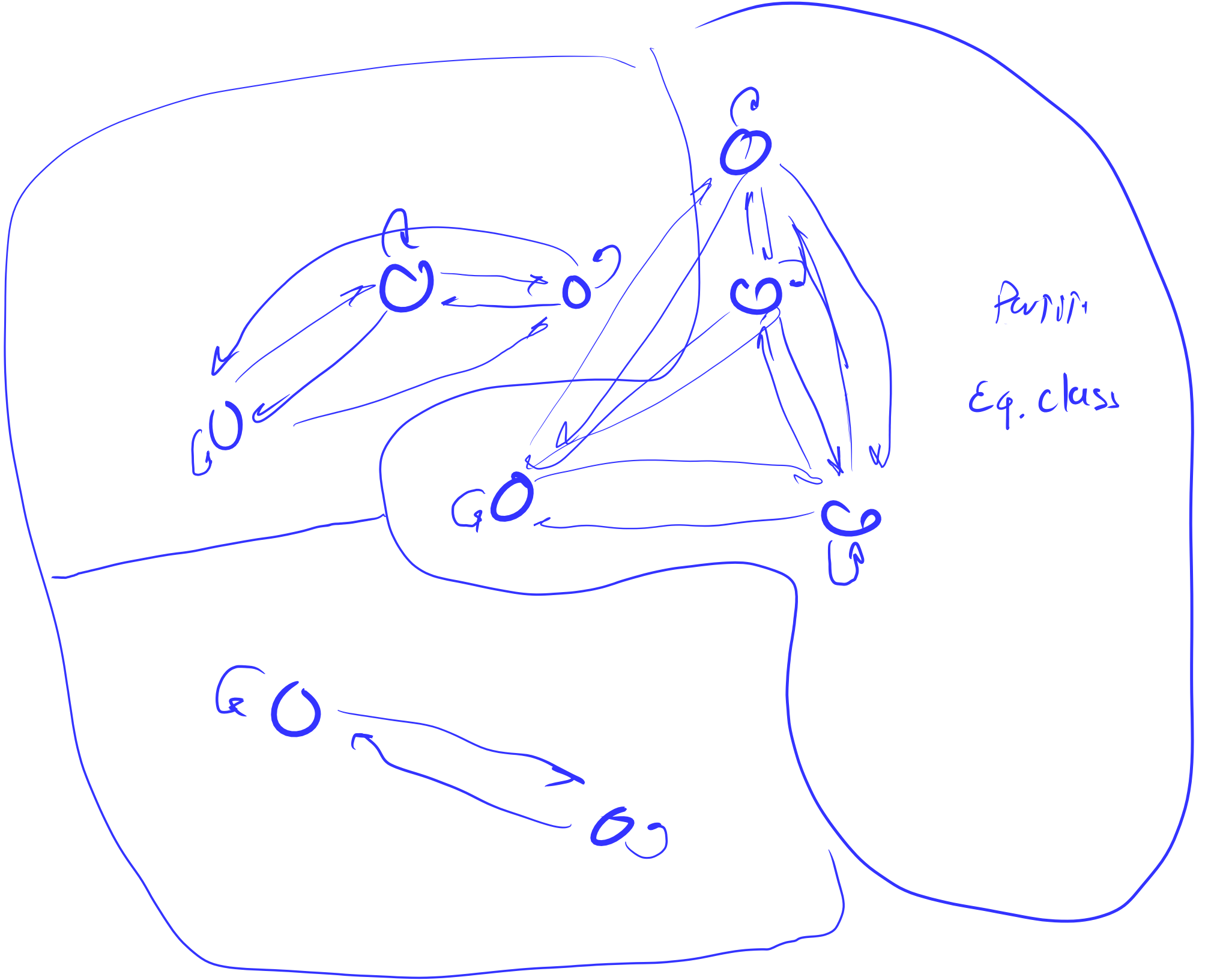
reflexive

irreflexive

$$\forall a. R(a, a)$$

$$\forall a. \neg R(a, a)$$





person attends school

