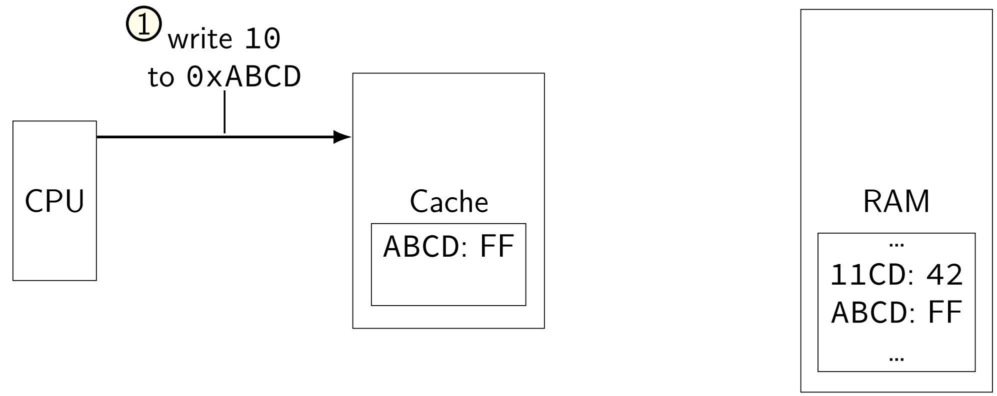
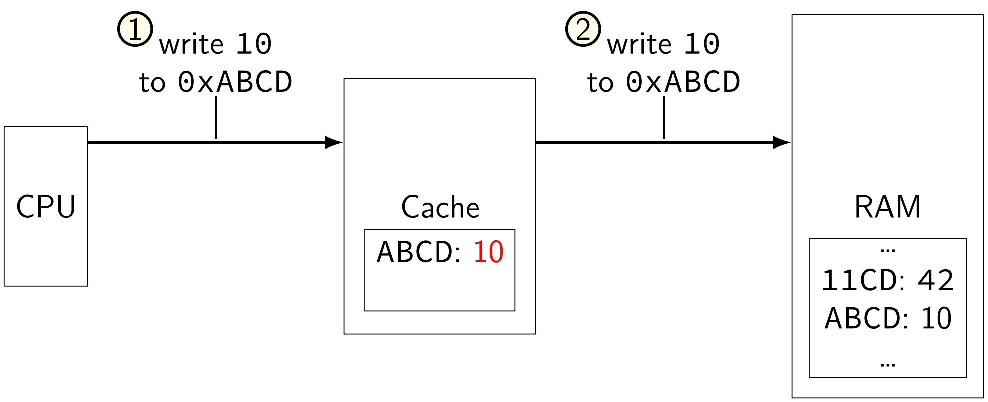
write-through v. write-back

option 1: write-through



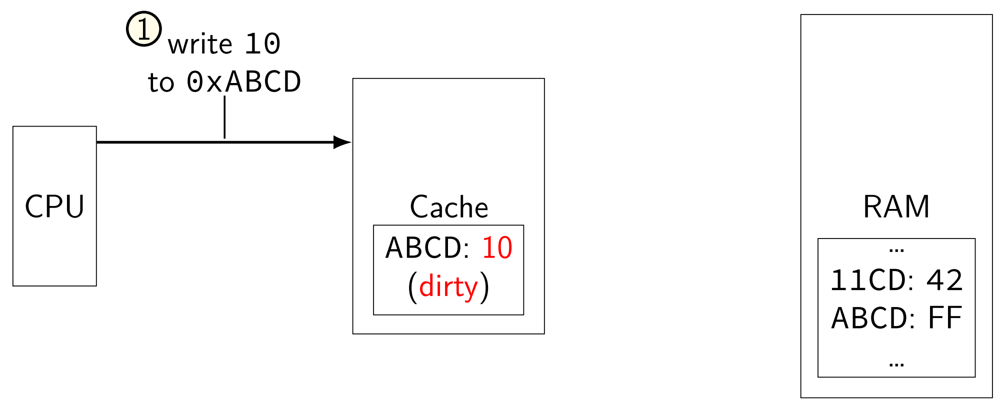
write-through v. write-back

option 1: write-through

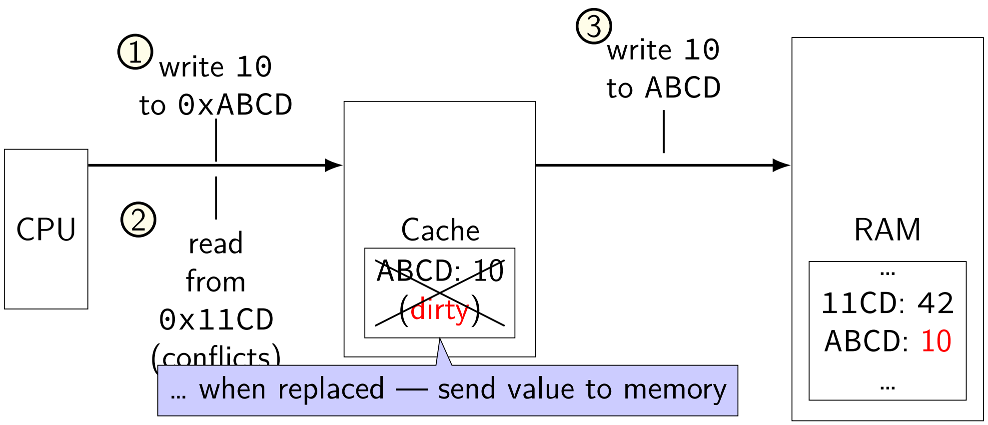


write-through v. write-back

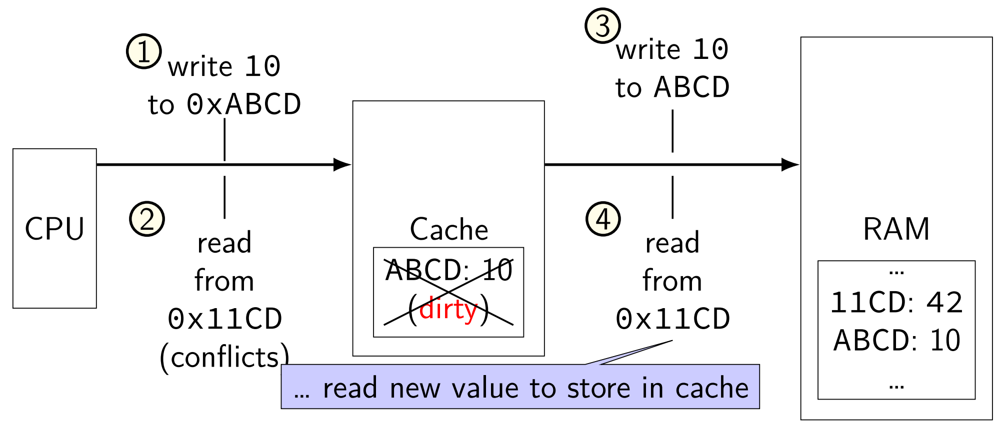
option 2: write-back



write-through v. write-back



write-through v. write-back



writeback policy

changed value!

2-way set associative, 4 byte blocks, 2 sets

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

1 = dirty (different than memory)
needs to be written if evicted

allocate on write?

processor writes less than whole cache block

block not yet in cache

two options:

write-allocate
- fetch rest of cache block, replace written part

write-no-allocate
- send write through to memory
- guess: not read soon?

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address `0x04`?
index 0, tag 000001

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address `0x04`?
index 0, tag 000001
step 1: find least recently used block

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	~~1~~	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address `0x04`?
index 0, tag 000001
step 1: find least recently used block
step 2: possibly writeback old block

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	0xFF mem[0x05]	1	0
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address `0x04`?
index 0, tag 000001
step 1: find least recently used block
step 2: possibly writeback old block
step 3a: read in new block – to get mem[0x05]
step 3b: update LRU information

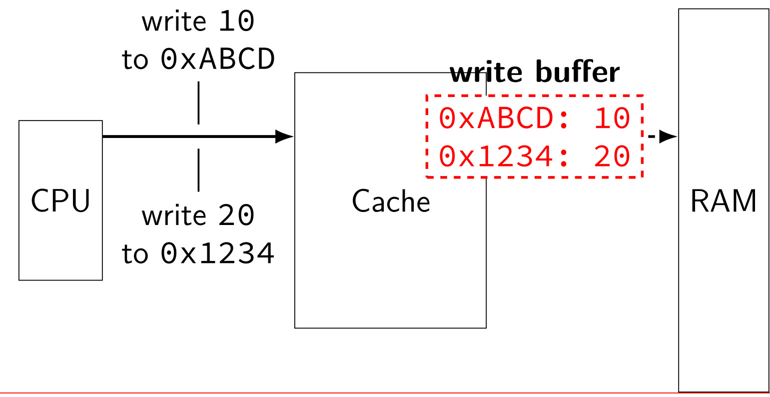
write-no-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?
step 1: is it in cache yet?
step 2: no, just send it to memory

fast writes



write appears to complete immediately when placed in buffer
memory can be much slower

cache organization and miss rate

depends on program; one example:

SPEC CPU2000 benchmarks, 64B block size

LRU replacement policies

data cache miss rates:

Cache size	direct-mapped	2-way	8-way	fully assoc.
1KB	8.63%	6.97%	5.63%	5.34%
2KB	5.71%	4.23%	3.30%	3.05%
4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

Data: Cantin and Hill, "Cache Performance for SPEC CPU2000 Benchmarks"
http://research.cs.wisc.edu/multifacet/misc/spec2000cache-data/

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64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

Data: Cantin and Hill, "Cache Performance for SPEC CPU2000 Benchmarks"
http://research.cs.wisc.edu/multifacet/misc/spec2000cache-data/

reasoning about cache performance

hit time: time to lookup and find value in cache

L1 cache — typically 1 cycle?

miss rate: portion of hits (value in cache)

miss penalty: extra time to get value if there's a miss

time to access next level cache or memory

miss time: hit time + miss penalty

average memory access time

AMAT = hit time + miss penalty × miss rate

effective speed of memory

what cache parameters are better?

can write a program to make a cache look bad:

1. access enough blocks, to fill the cache
2. access an additional block, replacing something
3. access last block replaced
4. access last block replaced
5. access last block replaced

...

but — typical real programs have locality

cache optimizations

	miss rate	hit time	miss penalty
increase cache size	better	worse	—
increase associativity	better	worse	worse?
increase block size	depends	worse	worse
add secondary cache	—	—	better
write-allocate	better	—	worse?
writeback	better	—	worse?
LRU replacement	better	?	worse?

average time = hit time + miss rate × miss penalty

cache optimizations by miss type

	capacity	conflict	compulsory
increase cache size	fewer misses	—	—
increase associativity	—	fewer misses	—
increase block size	—	more misses	fewer misses

exercise (1)

initial cache: 64-byte blocks, 64 sets, 8 ways/set

If we leave the other parameters listed above unchanged, which will probably reduce the number of capacity misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte blocks, 64 sets, 8 ways/set)
B. quadrupling the number of sets
C. quadrupling the number of ways/set

exercise (2)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of capacity misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache
B. quadrupling the number of ways/set
C. quadrupling the cache size

exercise (3)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of conflict misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache
B. quadrupling the number of ways/set
C. quadrupling the cache size

a note on matrix storage

A — $N \times N$ matrix

represent as array

makes dynamic sizes easier:

```
float A_2d_array[N][N];
float *A_flat = malloc(N * N);

A_flat[i * N + j] === A_2d_array[i][j]
```


matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

```
/* version 1: inner loop is k, middle is j */
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i * N + j] += A[i * N + k] * A[k * N + j];
```


matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i*N+j] += A[i * N + k] * A[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```


matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

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/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
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for (int k = 0; k < N; ++k)
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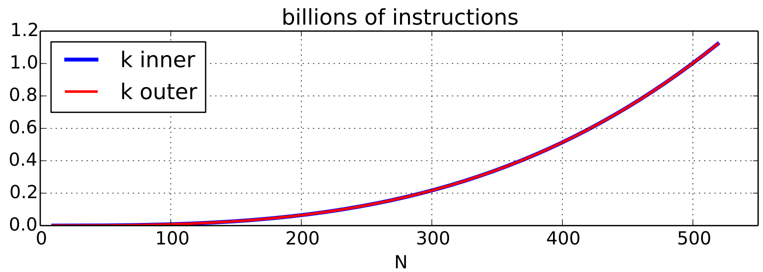
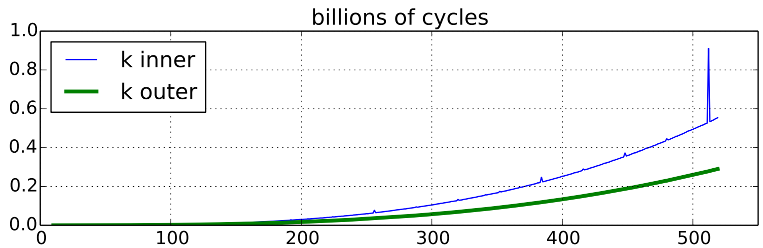
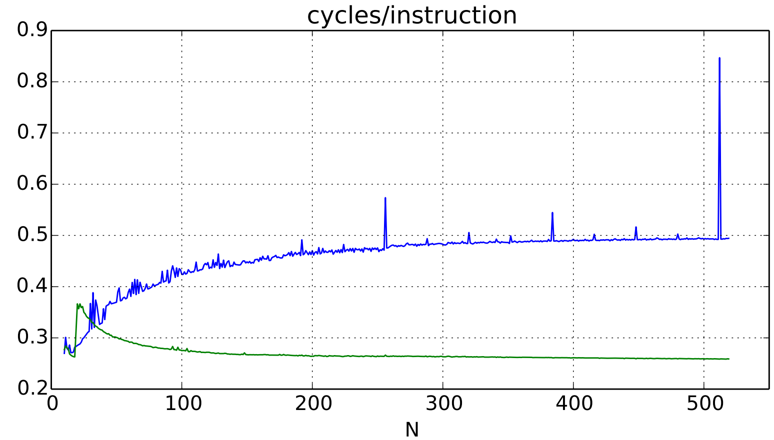
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

performance

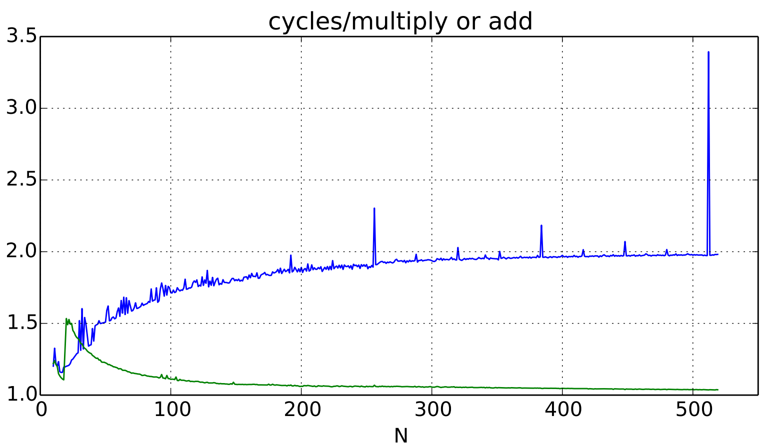




alternate view 1: cycles/instruction



alternate view 2: cycles/operation



loop orders and locality

loop body: $B_{ij} += A_{ik}A_{kj}$

kij order: B_{ij}, A_{kj} have spatial locality

kij order: A_{ik} has temporal locality

... better than ...

ijk order: A_{ik} has spatial locality

ijk order: B_{ij} has temporal locality

loop orders and locality

loop body: $B_{ij} += A_{ik}A_{kj}$

kij order: B_{ij}, A_{kj} have spatial locality

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... better than ...

ijk order: A_{ik} has spatial locality

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matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

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/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i*N+j] += A[i * N + k] * A[k * N + j];

/* version 2: outer loop is k, middle is i */
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      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

exercise: which should perform better? why?

matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

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/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
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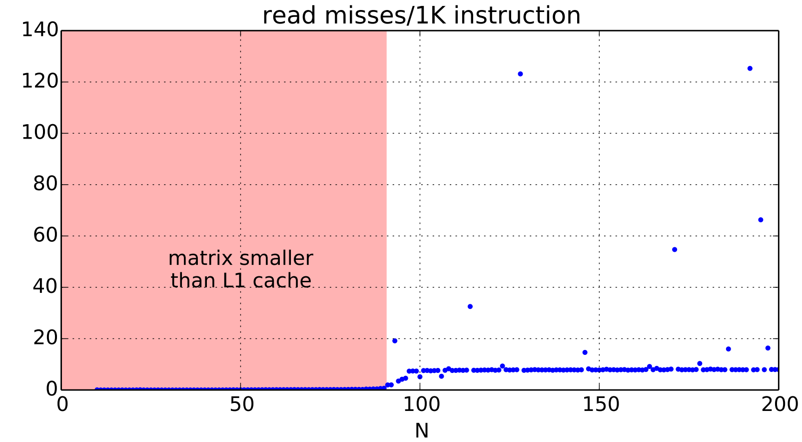
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

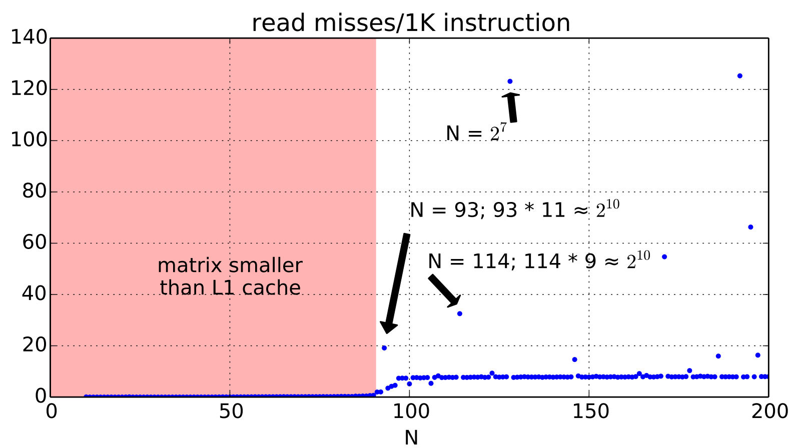
exercise: which should perform better? why?

L1 misses

L1 miss detail (1)



L1 miss detail (2)



addresses

`A[k*114+j]`	is at	`10 0000 0000 0100`
`A[k*114+j+1]`	is at	`10 0000 0000 1000`
`A[(k+1)*114+j]`	is at	`10 0011 1001 0100`
`A[(k+2)*114+j]`	is at	`10 0101 0101 1100`
…		
`A[(k+9)*114+j]`	is at	`11 0000 0000 1100`

addresses

```
A[k*114+j]       is at  10 0000 0000 0100
A[k*114+j+1]     is at  10 0000 0000 1000
A[(k+1)*114+j]   is at  10 0011 1001 0100
A[(k+2)*114+j]   is at  10 0101 0101 1100
...
A[(k+9)*114+j]   is at  11 0000 0000 1100
```

recall: 6 index bits, 6 block offset bits (L1)

conflict misses

powers of two — lower order bits unchanged

`A[k*93+j]` and `A[(k+11)*93+j]`:
- 1023 elements apart (4092 bytes; 63.9 cache blocks)

64 sets in L1 cache: usually maps to same set

`A[k*93+(j+1)]` will not be cached (next i loop)

even if in same block as `A[k*93+j]`

reasoning about loop orders

changing loop order changed locality

how do we tell which loop order will be best?
- besides running each one?

systematic approach (1)

```
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i*N+k] * A[k*N+j];
```

goal: get most out of each cache miss

if N is larger than the cache:

miss for B_{ij} — 1 comptuation

miss for A_{ik} — N computations

miss for A_{kj} — 1 computation

effectively caching just 1 element

locality exercise (1)

```
/* version 1 */
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]

/* version 2 */
for (int j = 0; j < N; ++j)
    for (int i = 0; i < N; ++i)
        A[i] += B[j] * C[i * N + j];
```

exercise: which has better temporal locality in A? in B? in C?
how about spatial locality?

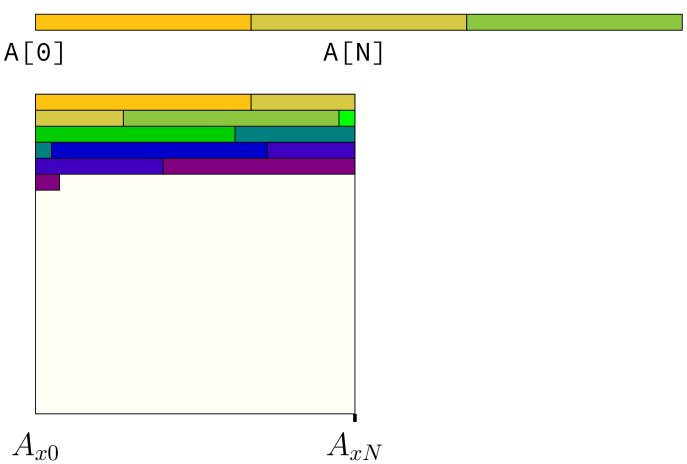
keeping values in cache

can't *explicitly* ensure values are kept in cache

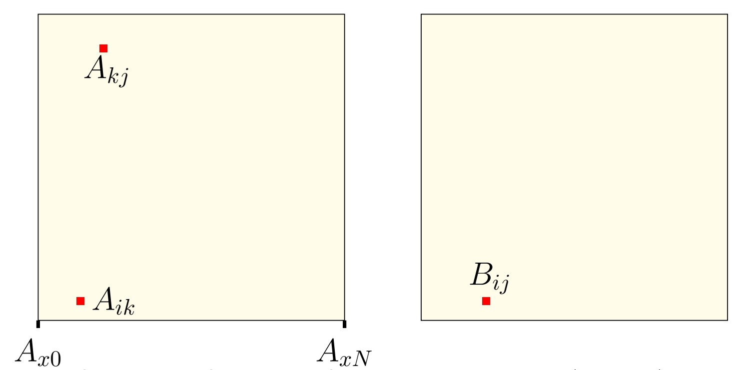
…but reusing values *effectively* does this
- cache will try to keep recently used values

cache optimization ideas: choose what's in the cache
- for thinking about it: load values explicitly
- for implementing it: access only values we want loaded

'flat' 2D arrays and cache blocks

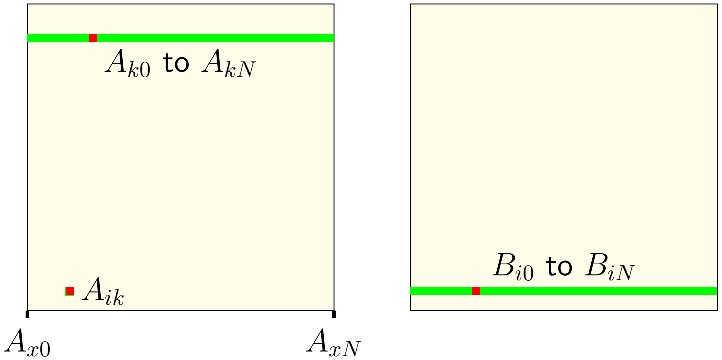


array usage: kij order



for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$
N calculations for A_{ik}
1 for A_{kj}, B_{ij}

array usage: kij order



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N calculations for A_{ik}

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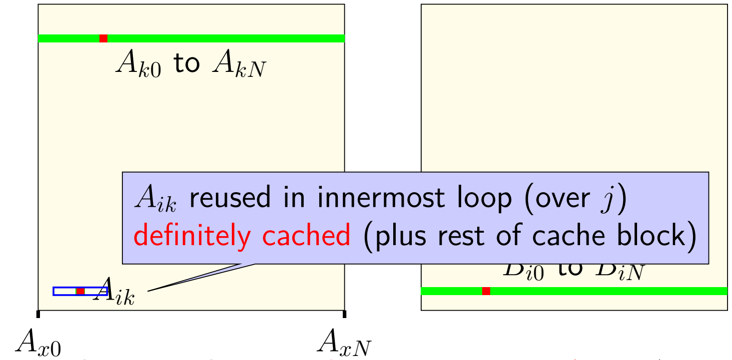
array usage: kij order



for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

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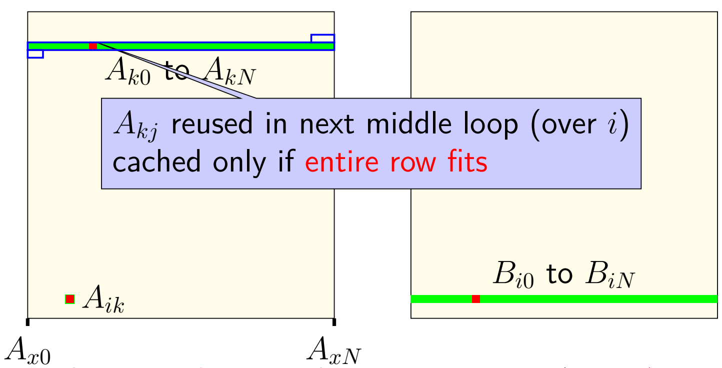
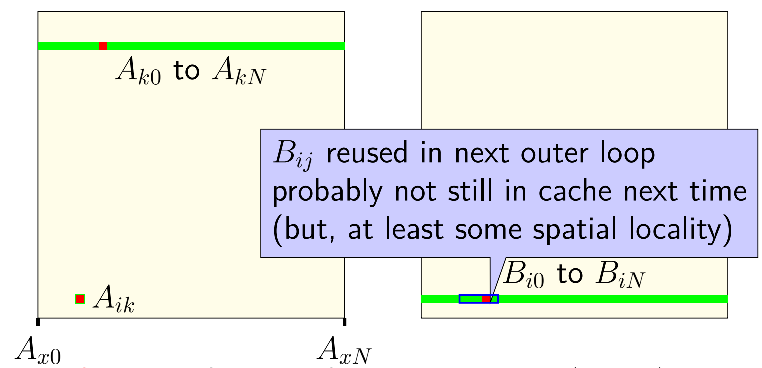


for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

N calculations for A_{ik}

1 for A_{kj}, B_{ij}

array usage: kij order



for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

N calculations for A_{ik}

1 for A_{kj}, B_{ij}

inefficiencies

if a row doesn't fit in cache —
cache effectively holds one element

- everything else — too much other stuff between accesses

if a row does fit in cache —
cache effectively holds one row + one element

- everything else — too much other stuff between accesses

systematic approach (2)

```
for (int k = 0; k < N; ++k) {
  for (int i = 0; i < N; ++i) {
    A_ik loaded once in this loop (N^2 times):
    for (int j = 0; j < N; ++j)
      B_ij, A_kj loaded each iteration (if N big):
      B[i*N+j] += A[i*N+k] * A[k*N+j];
```

N^3 multiplies, N^3 adds

about 1 load per operation

a transformation

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; i += 2)
        for (int j = 0; j < N; ++j)
          B[i*N+j] += A[i*N+k] * A[k*N+j];
```

split the loop over k — should be exactly the same

- (assuming even N)

a transformation

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; i += 2)
        for (int j = 0; j < N; ++j)
          B[i*N+j] += A[i*N+k] * A[k*N+j];
```

split the loop over k — should be exactly the same

- (assuming even N)

38

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; i += 2)
      for (int j = 0; j < N; ++j)
        for (int k = kk; k < kk + 2; ++k)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
```

now reorder split loop — same calculations

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; i += 2)
      for (int j = 0; j < N; ++j)
        for (int k = kk; k < kk + 2; ++k)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
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      for (int j = 0; j < N; ++j)
        for (int k = kk; k < kk + 2; ++k)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

Temporal locality in B_{ij}s

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

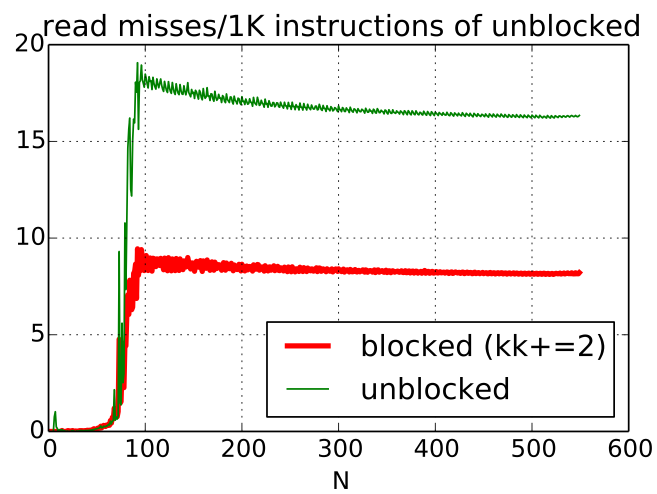
More spatial locality in A_{ik}

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

Still have good spatial locality in A_{kj}, B_{ij}

improvement in read misses



simple blocking (2)

same thing for i in addition to k?

```
for (int kk = 0; kk < N; kk += 2) {
  for (int ii = 0; ii < N; ii += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      for (int k = kk; k < kk + 2; ++k)
        for (int i = 0; i < ii + 2; ++i)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
    }
  }
}
```


simple blocking — expanded

```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
```

$$B_{i+0,j} \mathrel{+}= A_{i+0,k+0} * A_{k+0,j}$$
$$B_{i+0,j} \mathrel{+}= A_{i+0,k+1} * A_{k+1,j}$$
$$B_{i+1,j} \mathrel{+}= A_{i+1,k+0} * A_{k+0,j}$$
$$B_{i+1,j} \mathrel{+}= A_{i+1,k+1} * A_{k+1,j}$$

```
    }
  }
}
```


simple blocking — expanded

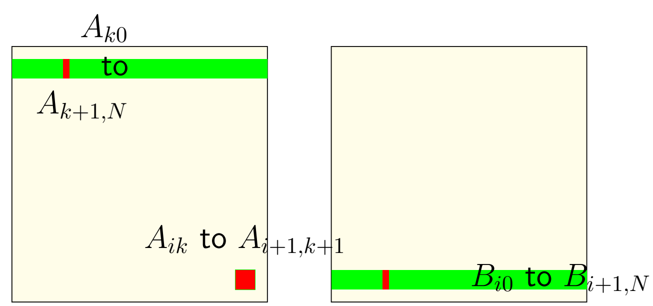
```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
```

$$B_{i+0,j} \mathrel{+}= A_{i+0,k+0} * A_{k+0,j}$$
$$B_{i+0,j} \mathrel{+}= A_{i+0,k+1} * A_{k+1,j}$$
$$B_{i+1,j} \mathrel{+}= A_{i+1,k+0} * A_{k+0,j}$$
$$B_{i+1,j} \mathrel{+}= A_{i+1,k+1} * A_{k+1,j}$$

```
    }
  }
}
```

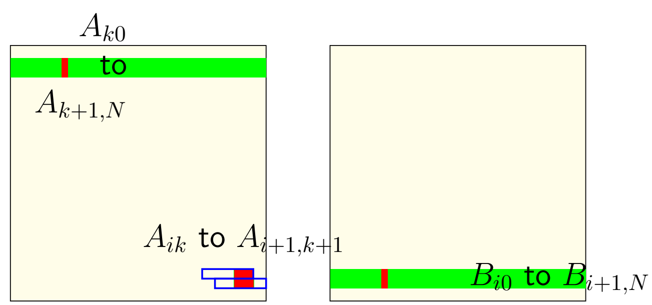

Now A_{kj} reused in inner loop — more calculations per load!

array usage (better)



more temporal locality:
N calculations for each A_{ik}
2 calculations for each B_{ij} (for k, $k+1$)
2 calculations for each A_{kj} (for k, $k+1$)

array usage (better)



more spatial locality:
calculate on each $A_{i,k}$ and $A_{i,k+1}$ together
both in same cache block — same amount of cache loads

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                          * A[k * N + j];
```

B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

A_{kj} used I times for one miss — N^2/I misses

catch: $IK + KJ + IJ$ elements must fit in cache

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                          * A[k * N + j];
```

B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

A_{kj} used I times for one miss — N^2/I misses

catch: $IK + KJ + IJ$ elements must fit in cache

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                          * A[k * N + j];
```

B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

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catch: $IK + KJ + IJ$ elements must fit in cache

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                          * A[k * N + j];
```

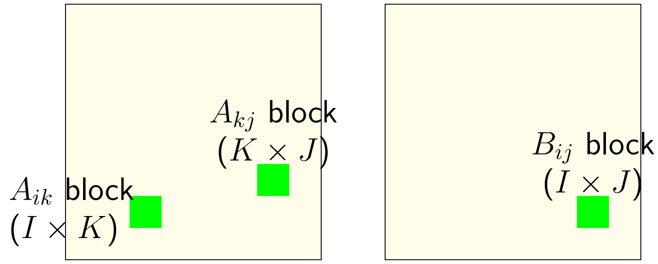
B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

A_{kj} used I times for one miss — N^2/I misses

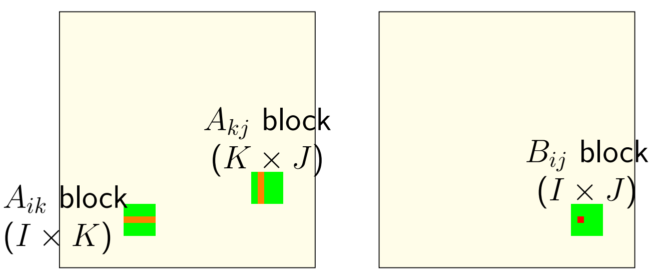
catch: $IK + KJ + IJ$ elements must fit in cache

array usage: block



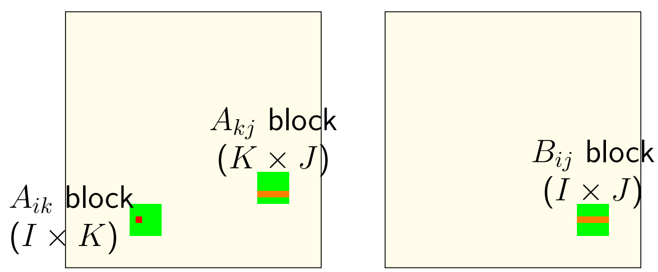
inner loop keeps "blocks" from A, B in cache

array usage: block



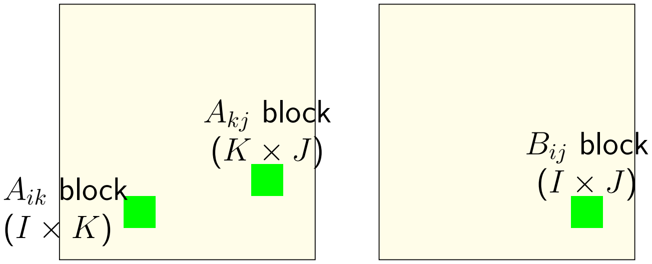
B_{ij} calculation uses strips from A
K calculations for one load (cache miss)

array usage: block



A_{ik} calculation uses strips from A, B
J calculations for one load (cache miss)

array usage: block



(approx.) KIJ fully cached calculations
for $KI + IJ + KJ$ loads
(assuming everything stays in cache)

cache blocking efficiency

load $I \times K$ elements of A_{ik}:
- do $> J$ multiplies with each

load $K \times J$ elements of A_{kj}:
- do I multiplies with each

load $I \times J$ elements of B_{ij}:
- do K adds with each

bigger blocks — more work per load!

catch: $IK + KJ + IJ$ elements must fit in cache

cache blocking rule of thumb

fill the most of the cache with useful data

and do as much work as possible from that

example: my desktop 32KB L1 cache

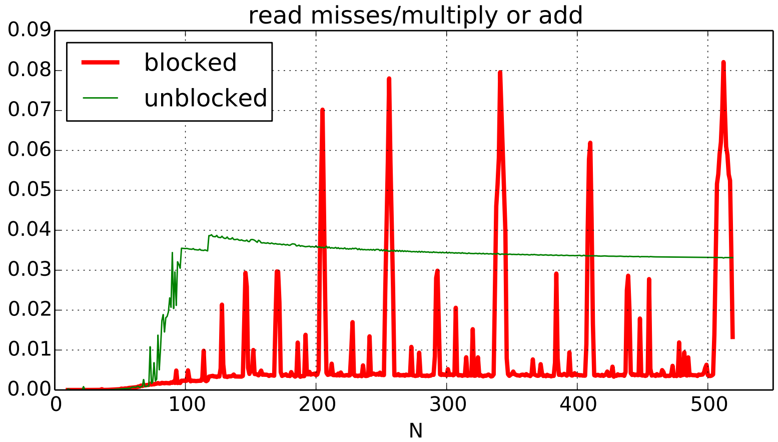
$I = J = K = 48$ uses $48^2 \times 3$ elements, or 27KB.

assumption: conflict misses aren't important

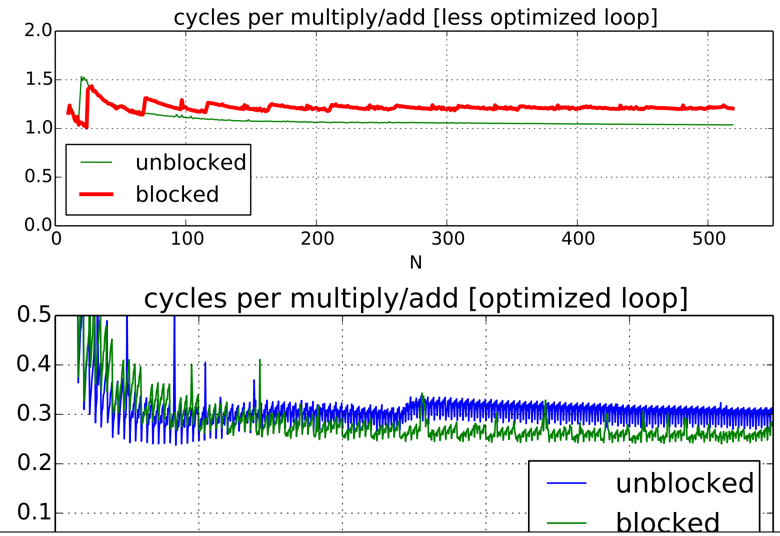
view 2: divide and conquer

```
partial_square(float *A, float *B,
               int startI, int endI, ...) {
  for (int i = startI; i < endI; ++i) {
    for (int j = startJ; j < endJ; ++j) {
      ...
}
square(float *A, float *B, int N) {
  for (int ii = 0; ii < N; ii += BLOCK)
    ...
      /* segment of A, B in use fits in cache! */
      partial_square(
            A, B,
            ii, ii + BLOCK,
            jj, jj + BLOCK, ...);
}
```

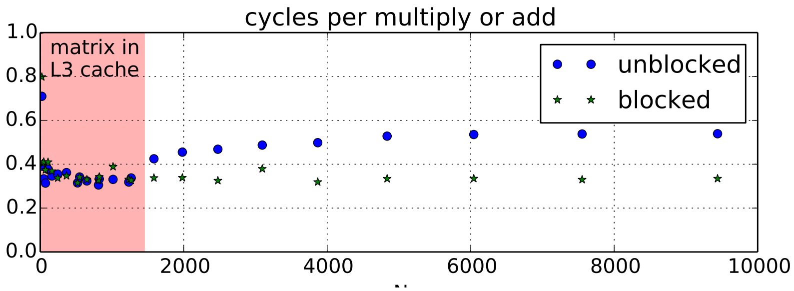
cache blocking and miss rate



what about performance?



performance for big sizes



optimized loop???

performance difference wasn't visible at small sizes

until I optimized arithmetic in the loop

(mostly by supplying better options to GCC)

1: reducing number of loads

2: doing adds/multiplies/etc. with less instructions

3: simplifying address computations

optimized loop???

performance difference wasn't visible at small sizes

until I optimized arithmetic in the loop

(mostly by supplying better options to GCC)

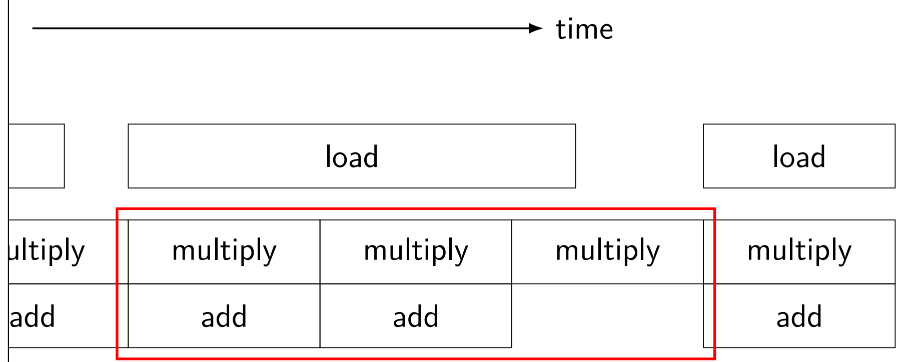
1: reducing number of loads

2: doing adds/multiplies/etc. with less instructions

3: simplifying address computations

but… how can that make cache blocking better???

overlapping loads and arithmetic



speed of load might not matter if these are slower

optimization and bottlenecks

arithmetic/loop efficiency was the bottleneck

after fixing this, cache performance was the bottleneck

common theme when optimizing:
  X may not matter until Y is optimized

cache blocking: summary

reorder calculation to reduce cache misses:

make explicit choice about what is in cache

perform calculations in cache-sized blocks

get more spatial and temporal locality

temporal locality — reuse values in many calculations
  before they are replaced in the cache

spatial locality — use adjacent values in calculations
  before cache block is replaced

cache blocking ugliness — fringe

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    for (int jj = 0; jj < N; jj += J) {
      for (int k = kk; k < min(kk + K, N) ; ++k) {
          // ...
      }
    }
  }
}
```


cache blocking ugliness — fringe

```
for (kk = 0; kk + K <= N; kk += K) {
  for (ii = 0; ii + I <= N; ii += I) {
    for (jj = 0; jj + J <= N; ii += J) {
      // ...
    }
    for (; jj < N; ++jj) {
      // handle remainder
    }
  }
  for (; ii < N; ++ii) {
    // handle remainder
  }
}
for (; kk < N; ++kk) {
  // handle remainder
}
```


avoiding conflict misses

problem — array is scattered throughout memory

observation: 32KB cache can store 32KB contiguous array

- contiguous array is split evenly among sets

solution: copy block into contiguous array

avoiding conflict misses (code)

```
process_block(ii, jj, kk) {
  float B_copy[I * J];
  /* pseudocode for loop to save space */
  for i = ii to ii + I, j = jj to jj + J:
    B_copy[i * J + j] = B[i * N + j];
  for i = ii to ii + I, j = jj to jj + J, k:
    B_copy[i * J + j] += A[k * N + j] * A[i * N + k];
  for all i, j:
    B[i * N + j] = B_copy[i * J + j];
}
```

register reuse

```
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i*N+k] * A[k*N+j];
// optimize into:
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i) {
    float Aik = A[i*N+k]; // hopefully keep in register!
                          // faster than even cache hit!
    for (int j = 0; j < N; ++j)
      B[i*N+j] += Aik * A[k*N+j];
  }
}
```

can compiler do this for us?

can compiler do register reuse?

Not easily — What if `A=B`? What if `A=&B[10]`

```
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i) {
    // want to preload A[i*N+k] here!
    for (int j = 0; j < N; ++j) {
      // but if A = B, modifying here!
      B[i*N+j] += A[i*N+k] * A[k*N+j];
    }
  }
}
```


automatic register reuse

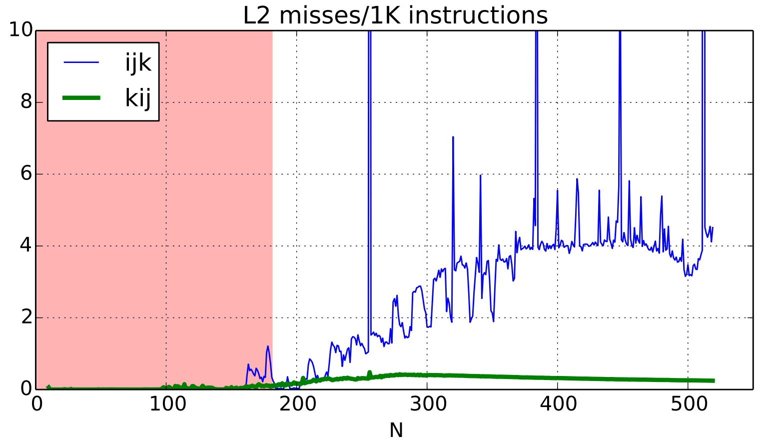
Compiler would need to generate overlap check:

```
if ((B > A + N * N || B < A) &&
    (B + N * N > A + N * N ||
     B + N * N < A)) {
  for (int k = 0; k < N; ++k) {
    for (int i = 0; i < N; ++i) {
      float Aik = A[i*N+k];
      for (int j = 0; j < N; ++j) {
        B[i*N+j] += Aik * A[k*N+j];
      }
    }
  }
} else { /* other version */ }
```


“register blocking”

```
  for (int k = 0; k < N; ++k) {
    for (int i = 0; i < N; i += 2) {
      float Ai0k = A[(i+0)*N + k];
      float Ai1k = A[(i+1)*N + k];
      for (int j = 0; j < N; j += 2) {
        float Akj0 = A[k*N + j+0];
        float Akj1 = A[k*N + j+1];
        B[(i+0)*N + j+0] += Ai0k * Akj0;
        B[(i+1)*N + j+0] += Ai1k * Akj0;
        B[(i+0)*N + j+1] += Ai0k * Akj1;
        B[(i+1)*N + j+1] += Ai1k * Akj1;
      }
    }
  }
```

L2 misses



L2 misses/1K instructions
10
8
6
4
2
0
0
100
200
300
400
500
N
ijk
kij