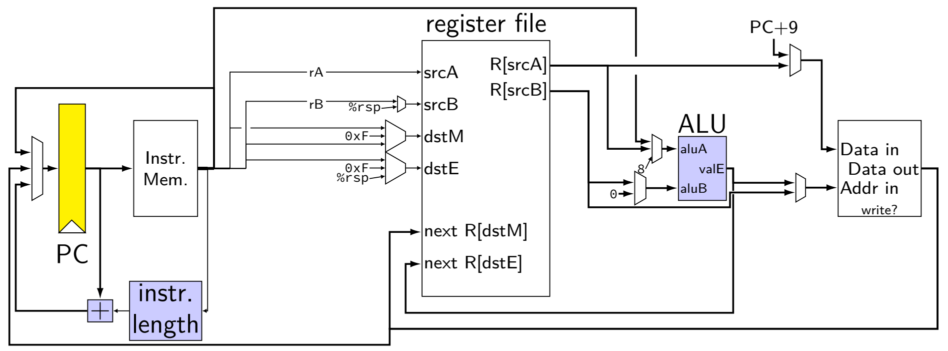
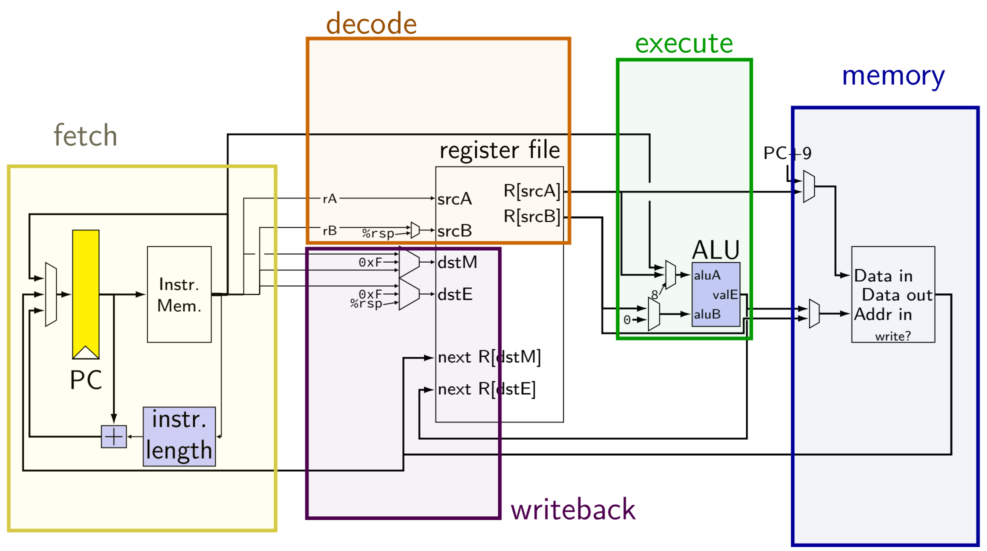
Exam Review

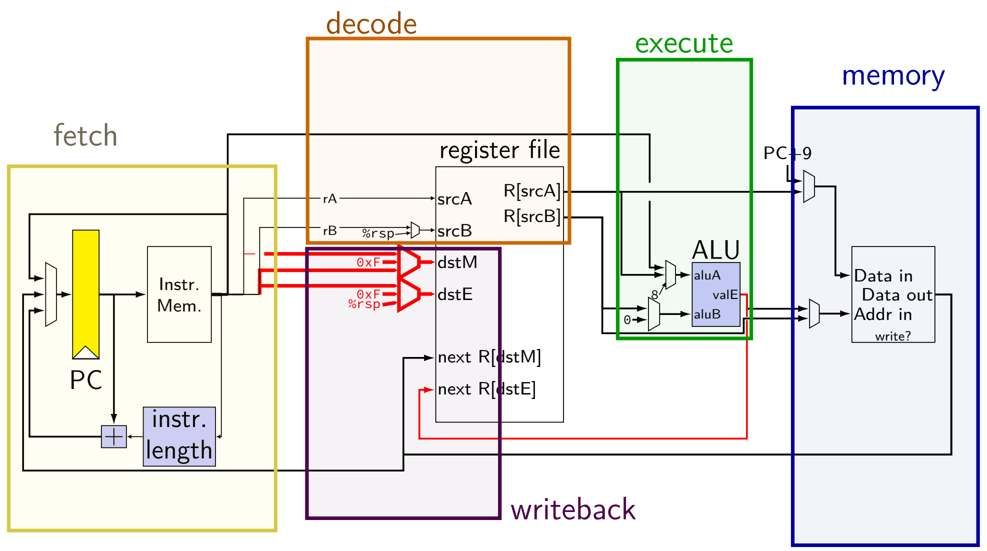
SEQ without stages



SEQ with stages

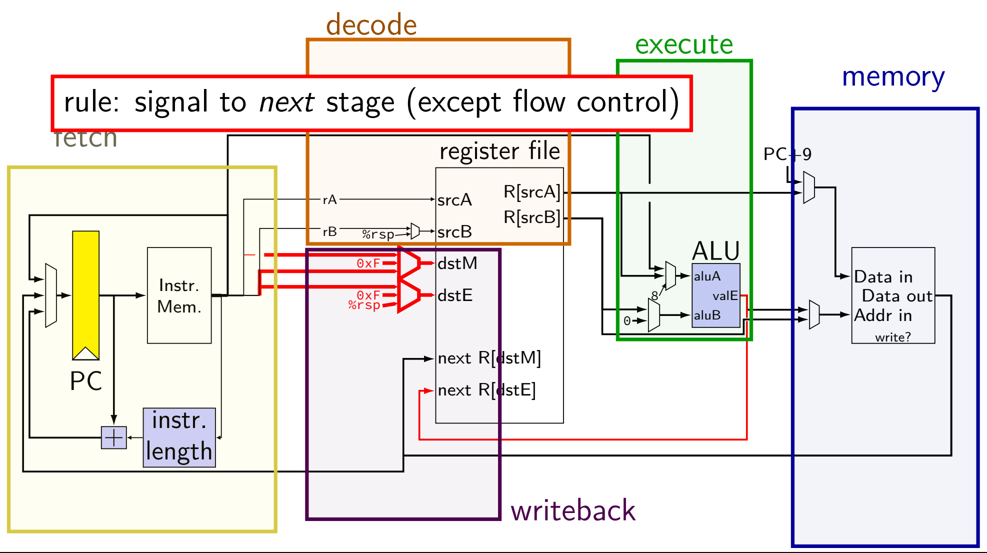


SEQ with stages

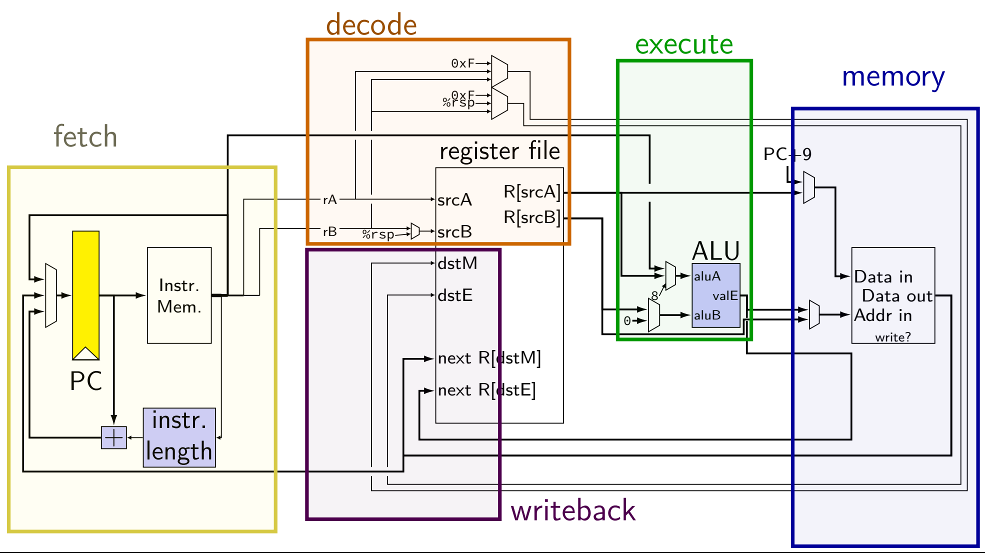


SEQ with stages

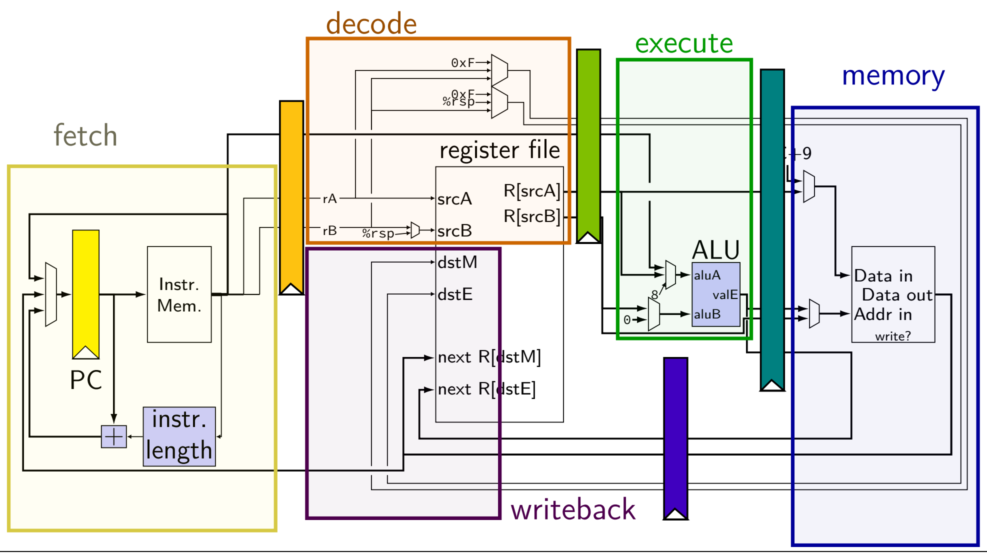
rule: signal to *next* stage (except flow control)



SEQ with stages (actually sequential)

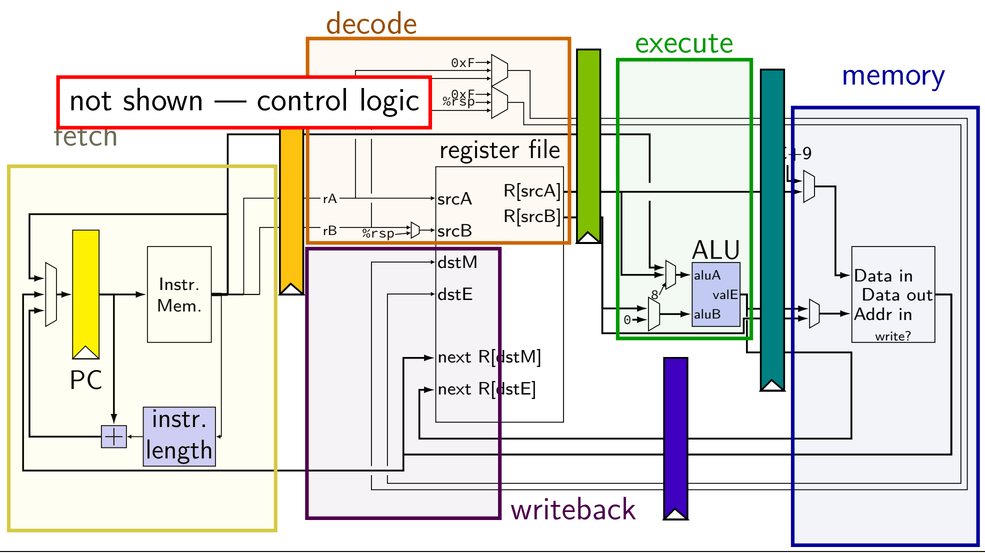


adding pipeline registers



adding pipeline registers

not shown — control logic



pipeline with different hazards

example: 4-stage pipeline:
fetch/decode/execute+memory/writeback

```
                 // 4 stage   // 5 stage
addq %rax, %r8   //           // W
subq %rax, %r9   // W         // M
xorq %rax, %r10  // EM        // E
andq %r8,  %r11  // D         // D
```

pipeline with different hazards

example: 4-stage pipeline:
fetch/decode/execute+memory/writeback

```
                 // 4 stage   // 5 stage
addq %rax, %r8   //           // W
subq %rax, %r9   // W         // M
xorq %rax, %r10  // EM        // E
andq %r8,  %r11  // D         // D
```

addq/andq is hazard with 5-stage pipeline

addq/andq is **not** a hazard with 4-stage pipeline

exercise: different pipeline

split execute into two stages: F/D/E1/E2/M/W

result only available after second execute stage

where does forwarding, stalls occur?

cycle #	0	1	2	3	4	5	6	7	8
`addq %rcx, %r9`	F	D	E1	E2	M	W			
`addq %r9, %rbx`									
`addq %rax, %r9`									
`rmmovq %r9, (%rbx)`									

exercise: different pipeline

split execute into two stages: F/D/E1/E2/M/W

cycle #	0	1	2	3	4	5	6	7	8
`addq %rcx, %r9`	F	D	E1	E2	M	W			
`addq %r9, %rbx`									
`addq %rax, %r9`									
`rmmovq %r9, (%rbx)`									

exercise: different pipeline

split execute into two stages: F/D/E1/E2/M/W

cycle #	0	1	2	3	4	5	6	7	8
`addq %rcx, %r9`	F	D	E1	E2	M	W			
`addq %r9, %rbx`		F	D	E1	E2	M	W		
`addq %rax, %r9`			F	D	E1	E2	M	W	
`rmmovq %r9, (%rbx)`				F	D	E1	E2	M	W

exercise: different pipeline

split execute into two stages: F/D/E1/E2/M/W

cycle #	0	1	2	3	4	5	6	7	8	
`addq %rcx, %r9`	F	D	E1	E2	M	W				
`addq %r9, %rbx`		F	D	E1	E2	M	W			
`addq %r9, %rbx`		F	D	D	E1	E2	M	W		
`addq %rax, %r9`			F	D	E1	E2	M	W		
`addq %rax, %r9`			F	F	D	E1	E2	M	W	
`rmmovq %r9, (%rbx)`				F	D	E1	E2	M	W	
`rmmovq %r9, (%rbx)`					F	D	E1	E2	M	W

exercise: different pipeline

split execute into two stages: F/D/E1/E2/M/W

cycle #	0	1	2	3	4	5	6	7	8	
`addq %rcx, %r9`	F	D	E1	E2	M	W				
`addq %r9, %rbx`		F	D	E1	E2	M	W			
`addq %r9, %rbx`		F	D	D	E1	E2	M	W		
`addq %rax, %r9`			F	D	E1	E2	M	W		
`addq %rax, %r9`			F	F	D	E1	E2	M	W	
`rmmovq %r9, (%rbx)`				F	D	E1	E2	M	W	
`rmmovq %r9, (%rbx)`					F	D	E1	E2	M	W

exercise: different pipeline

split execute into two stages: F/D/E1/E2/M/W

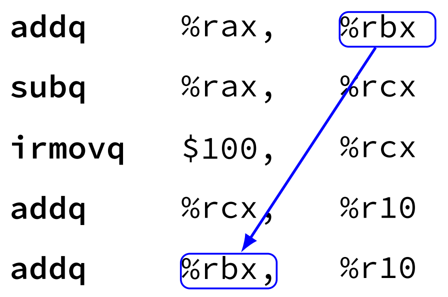
cycle #	0	1	2	3	4	5	6	7	8	
`addq %rcx, %r9`	F	D	E1	E2	M	W				
`addq %r9, %rbx`		F	D	E1	E2	M	W			
`addq %r9, %rbx`		F	D	D	E1	E2	M	W		
`addq %rax, %r9`			F	D	E1	E2	M	W		
`addq %rax, %r9`			F	F	D	E1	E2	M	W	
`rmmovq %r9, (%rbx)`				F	D	E1	E2	M	W	
`rmmovq %r9, (%rbx)`					F	D	E1	E2	M	W

ex.: dependencies and hazards (1)

```
addq     %rax,   %rbx
subq     %rax,   %rcx
irmovq   $100,   %rcx
addq     %rcx,   %r10
addq     %rbx,   %r10
```

where are dependencies?
which are hazards in our pipeline?
which are resolved with forwarding?

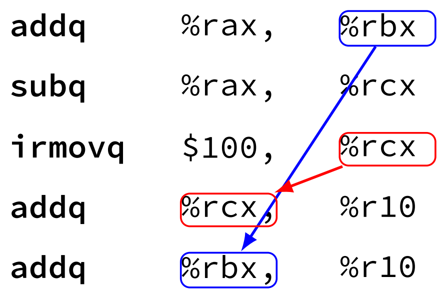
ex.: dependencies and hazards (1)



```
addq     %rax,   %rbx
subq     %rax,   %rcx
irmovq   $100,   %rcx
addq     %rcx,   %r10
addq     %rbx,   %r10
```

where are dependencies?
which are hazards in our pipeline?
which are resolved with forwarding?

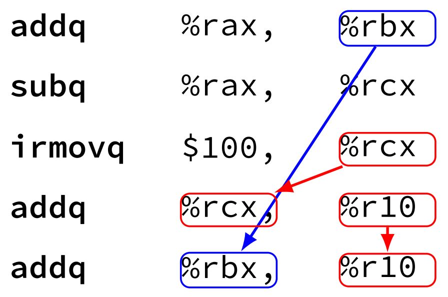
ex.: dependencies and hazards (1)



```
addq     %rax,   %rbx
subq     %rax,   %rcx
irmovq   $100,   %rcx
addq     %rcx,   %r10
addq     %rbx,   %r10
```

where are dependencies?
which are hazards in our pipeline?
which are resolved with forwarding?

ex.: dependencies and hazards (1)



```
addq     %rax,   %rbx
subq     %rax,   %rcx
irmovq   $100,   %rcx
addq     %rcx,   %r10
addq     %rbx,   %r10
```

where are dependencies?
which are hazards in our pipeline?
which are resolved with forwarding?

ex.: dependencies and hazards (2)

```
      mrmovq   0(%rax)  %rbx
      addq     %rbx     %rcx
      jne      foo
foo:  addq     %rcx     %rdx
      mrmovq   (%rdx)   %rcx
```

where are dependencies?
which are hazards in our pipeline?
which are resolved with forwarding?

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

multiple places to put values with same index
avoid conflict misses

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0		set 0	0		
1	0		set 1	0		

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

way 0
way 1

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

$m = 8$ bit addresses
$S = 2 = 2^s$ sets
$s = 1$ (set) index bits

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 6$ tag bits

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	`mem[0x00]` `mem[0x01]`	0		
1	0			0		

address (hex)	result
`00000000` (00)	miss
`00000001` (01)	
`01100011` (63)	
`01100001` (61)	
`01100010` (62)	
`00000000` (00)	
`01100100` (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	`mem[0x00]` `mem[0x01]`	0		
1	0			0		

address (hex)	result
`00000000` (00)	miss
`00000001` (01)	hit
`01100011` (63)	
`01100001` (61)	
`01100010` (62)	
`00000000` (00)	
`01100100` (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	`mem[0x00]` `mem[0x01]`	0		
1	1	011000	`mem[0x62]` `mem[0x63]`	0		

address (hex)	result
`00000000` (00)	miss
`00000001` (01)	hit
`01100011` (63)	miss
`01100001` (61)	
`01100010` (62)	
`00000000` (00)	
`01100100` (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

needs to replace block in set 0!

tag index offset

adding associativity

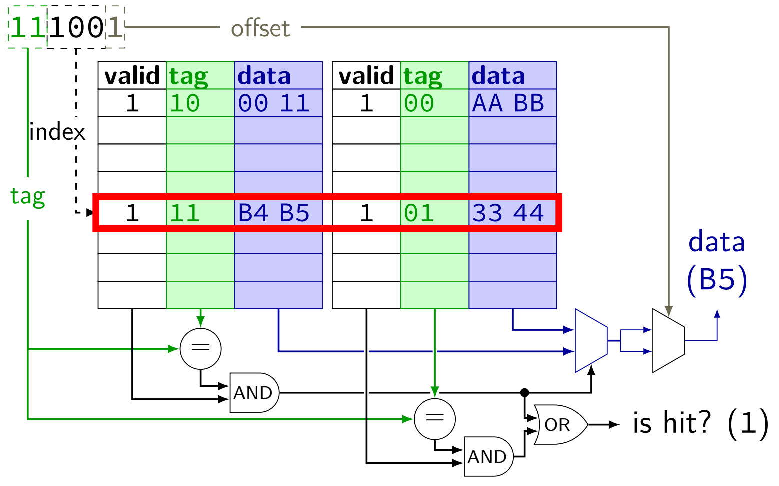
2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

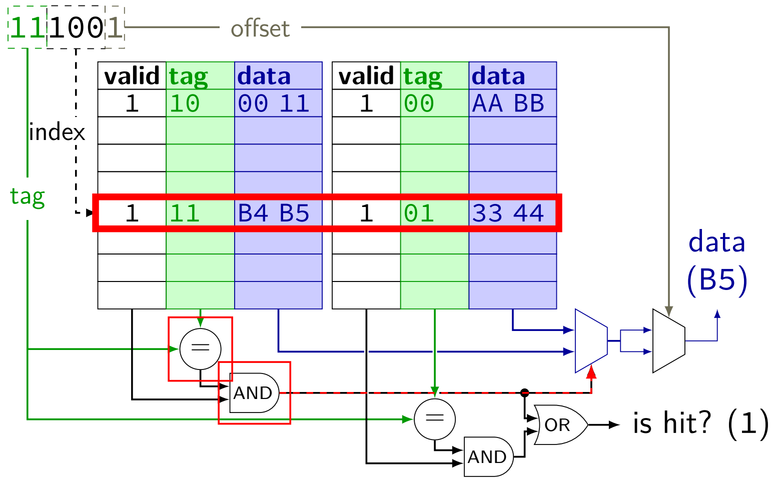
address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

tag index offset

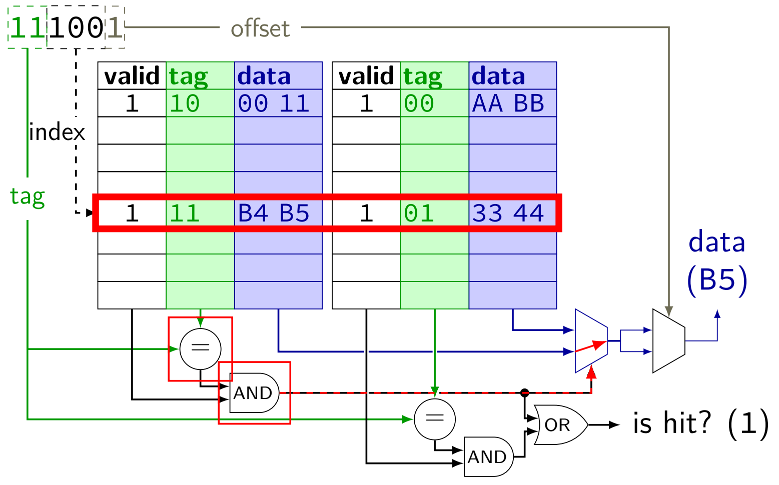
cache operation (associative)



cache operation (associative)



cache operation (associative)



associative lookup possibilities

none of the blocks for the index are valid

none of the valid blocks for the index match the tag
- something else is stored there

one of the blocks for the index is valid and matches the tag

Tag-Index-Offset formulas (complete)

m	memory addreses bits (Y86-64: 64)
E	number of blocks per set (“ways”)
$S = 2^s$	number of sets
s	(set) index bits
$B = 2^b$	block size
b	(block) offset bits
$t = m - (s + b)$	tag bits
$C = B \times S \times E$	cache size (excluding metadata)

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
000	
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

how to decide where to insert 0x64?

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value	LRU
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]	1
1	1	011000	mem[0x62] mem[0x63]	0			1

address (hex)	result
00000000 (00)	mi
00000001 (01)	hit
01100011 (63)	mi
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

track which block was read least recently
updated on every access

example replacement policies

least recently used and approximations

- take advantage of temporal locality
- exact: $\lceil \log_2(E!) \rceil$ bits per set for E-way cache
- good approximations: E to $2E$ bits

first-in, first-out

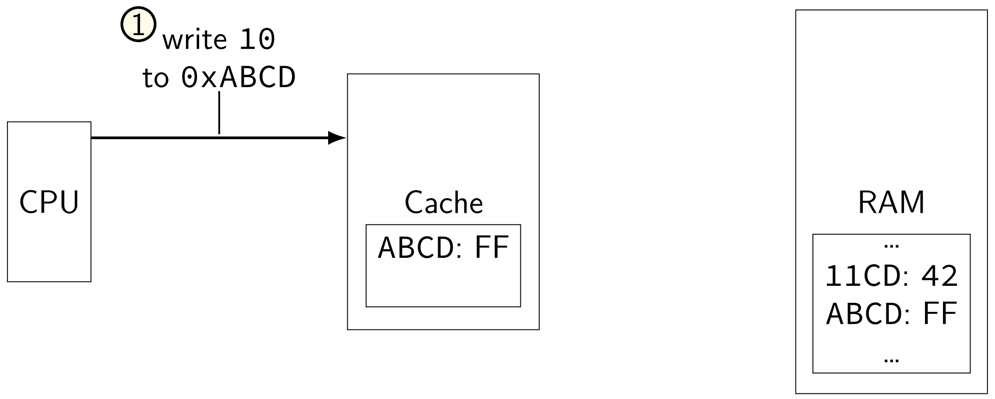
- counter per set — where to replace next

(pseudo-)random

- no extra information!

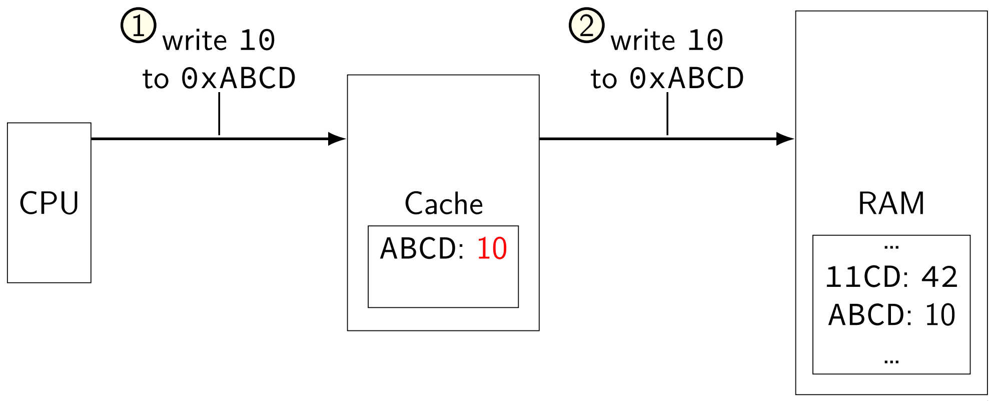
write-through v. write-back

option 1: write-through



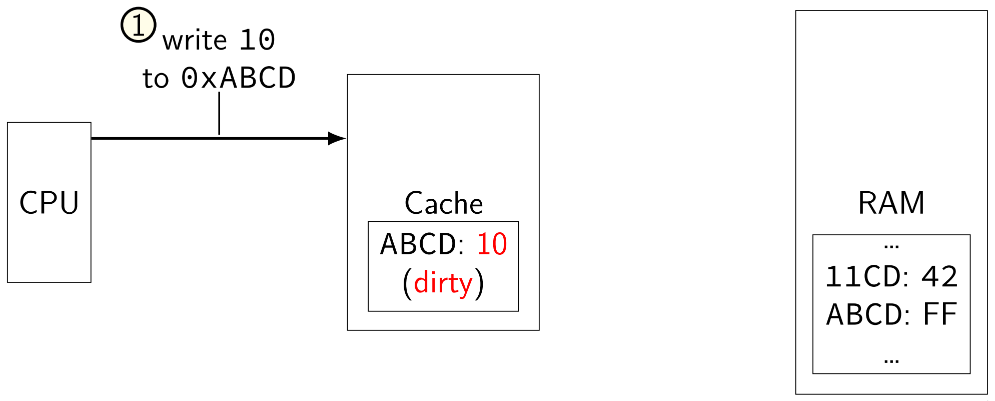
write-through v. write-back

option 1: write-through



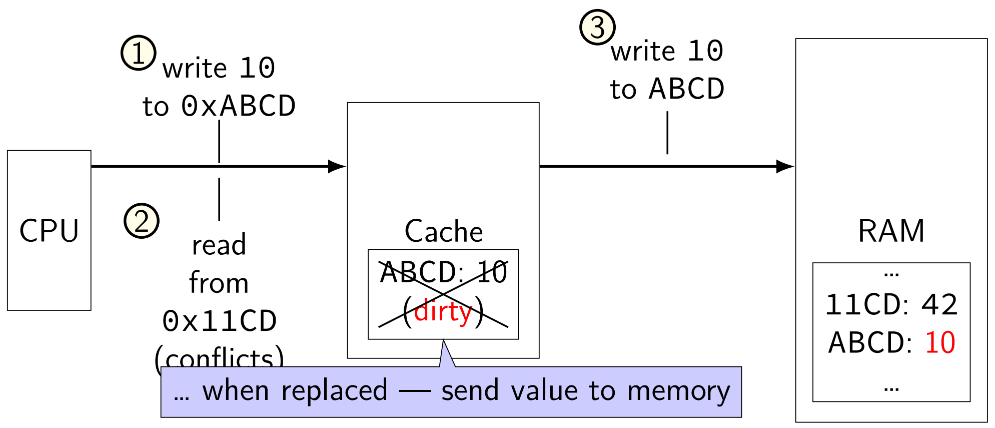
write-through v. write-back

option 2: write-back

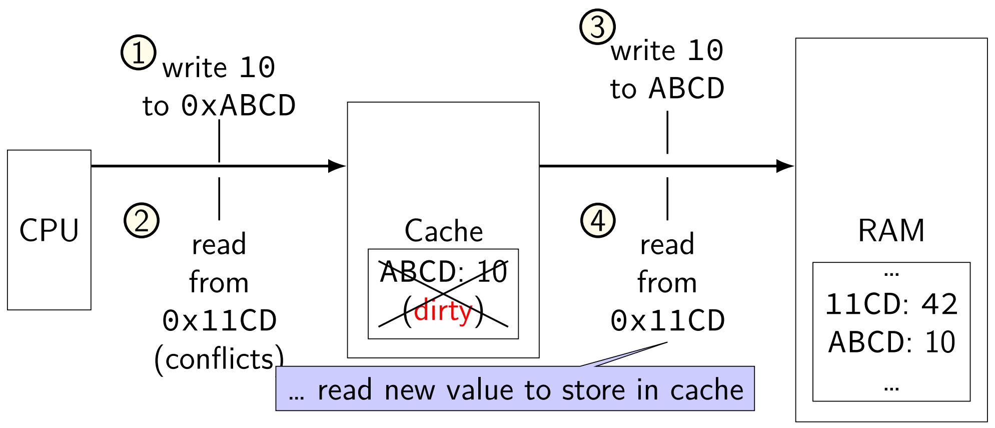


write-through v. write-back

option 2: write-back



write-through v. write-back



writeback policy

changed value!

2-way set associative, 4 byte blocks, 2 sets

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

1 = dirty (different than memory)
needs to be written if evicted

allocate on write?

processor writes less than whole cache block

block not yet in cache

two options:

write-allocate
- fetch rest of cache block, replace written part

write-no-allocate
- send write through to memory
- guess: not read soon?

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0}$xFF into address `0x04`?
index 0, tag 000001

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0}$xFF into address `0x04`?
index 0, tag 000001
step 1: find least recently used block

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	~~1~~	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0}$xFF into address `0x04`?
index 0, tag 000001
step 1: find least recently used block
step 2: possibly writeback old block

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	0xFF mem[0x05]	1	0
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing $\widehat{0}$xFF into address `0x04`?
index 0, tag 000001
step 1: find least recently used block
step 2: possibly writeback old block
step 3a: read in new block – to get mem[0x05]
step 3b: update LRU information

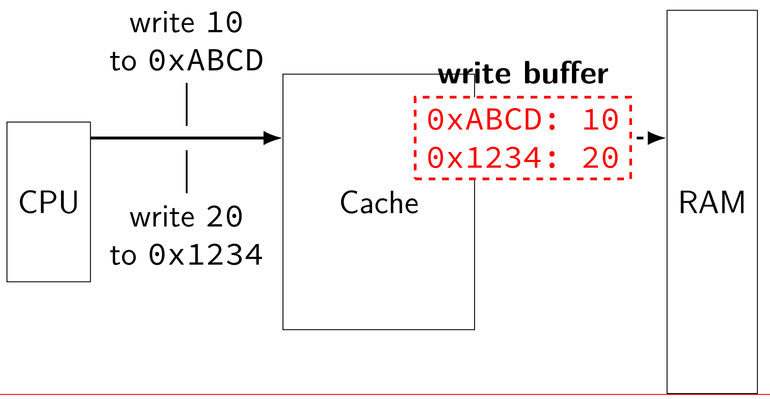
write-no-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address `0x04`?
step 1: is it in cache yet?
step 2: no, just send it to memory

fast writes



write appears to complete immediately when placed in buffer
memory can be much slower

exercise (1)

initial cache: 64-byte blocks, 64 sets, 8 ways/set

If we leave the other parameters listed above unchanged, which will probably reduce the number of capacity misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte blocks, 64 sets, 8 ways/set)
B. quadrupling the number of sets
C. quadrupling the number of ways/set

exercise (2)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of capacity misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cach
B. quadrupling the number of ways/set
C. quadrupling the cache size

exercise (3)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of conflict misses in a typical program? (Multiple may be correct.)

A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cach
B. quadrupling the number of ways/set
C. quadrupling the cache size

C and cache misses (1)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2) {
    even_sum += array[i + 0];
    odd_sum +=  array[i + 1];
}
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 1; i < 1024; i += 2)
    odd_sum +=  array[i + 1];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks? Would a set-associtiave cache be better?

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, ...

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, ...

...

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, ...

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …
  block at 0: array[0] through array[3]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …
  block at 16: array[4] through array[7]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …
  block at 2032: array[508] through array[511]

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …
  block at 0: array[0] through array[3]
  block at 0+2KB: array[512] through array[515]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …
  block at 16: array[4] through array[7]
  block at 16+2KB: array[516] through array[519]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …
  block at 2032: array[508] through array[511]
  block at 2032+2KB: array[1020] through array[1023]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses —

set 0: address 0, 0 + 2KB, 0 + 4KB, …

set 1: address 16, 16 + 2KB, 16 + 4KB, …

…

set 63: address 1008, 2032 + 2KB, 2032 + 4KB …

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses —

set 0: address 0, 0 + 2KB, 0 + 4KB, ...
    block at 0: array[0] through array[3]

set 1: address 16, 16 + 2KB, 16 + 4KB, ...
    address 16: array[4] through array[7]

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...
    address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses —

set 0: address 0, 0 + 2KB, 0 + 4KB, ...
    block at 0: array[0] through array[3]
    block at 0+1KB: array[256] through array[259]
    block at 0+2KB: array[512] through array[515]
    ...

set 1: address 16, 16 + 2KB, 16 + 4KB, ...
    address 16: array[4] through array[7]

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...
    address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses —

set 0: address 0, 0 + 2KB, 0 + 4KB, ...
    block at 0: array[0] through array[3]
    block at 0+1KB: array[256] through array[259]
    block at 0+2KB: array[512] through array[515]
    ...

set 1: address 16, 16 + 2KB, 16 + 4KB, ...
    address 16: array[4] through array[7]

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...
    address 1008: array[252] through array[255]

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (3, rewritten?)

```
item array[1024]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 1024; i += 128)
    a_sum += array[i];
for (int i = 1; i < 1024; i += 128)
    b_sum += array[i];
```

C and cache misses (4)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 4-way set associative 2KB direct-mapped cache with 16B cache blocks?

a note on matrix storage

A — $N \times N$ matrix

represent as array

makes dynamic sizes easier:

```
float A_2d_array[N][N];
float *A_flat = malloc(N * N);

A_flat[i * N + j] === A_2d_array[i][j]
```

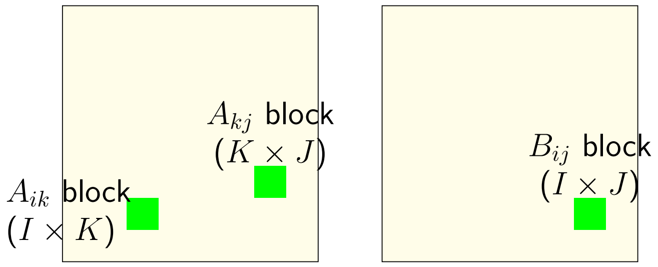
locality exercise (1)

```
/* version 1 */
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]

/* version 2 */
for (int j = 0; j < N; ++j)
    for (int i = 0; i < N; ++i)
        A[i] += B[j] * C[i * N + j];
```

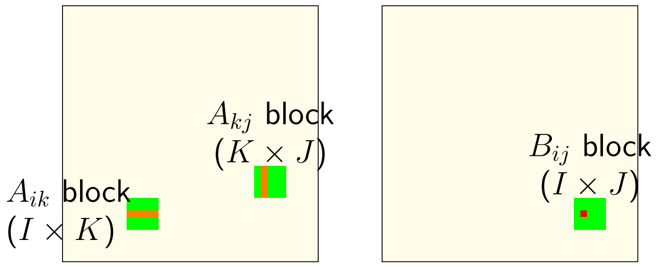
exercise: which has better temporal locality in A? in B? in C?
how about spatial locality?

array usage: block



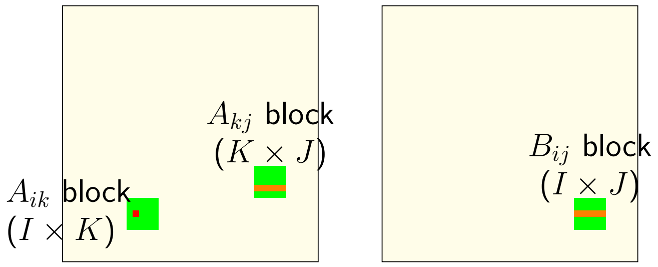
inner loop keeps "blocks" from A, B in cache

array usage: block



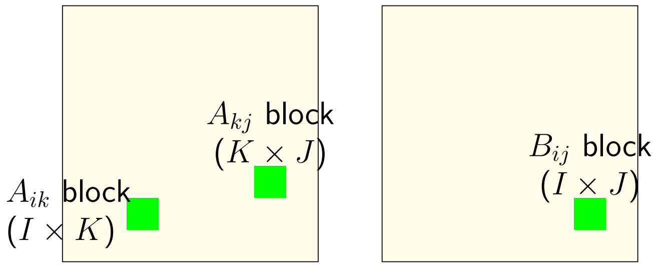
B_{ij} calculation uses strips from A
K calculations for one load (cache miss)

array usage: block



A_{ik} calculation uses strips from A, B
J calculations for one load (cache miss)

array usage: block



(approx.) KIJ fully cached calculations
for $KI + IJ + KJ$ loads
(assuming everything stays in cache)

cache blocking efficiency

load $I \times K$ elements of A_{ik}:
- do $> J$ multiplies with each

load $K \times J$ elements of A_{kj}:
- do I multiplies with each

load $I \times J$ elements of B_{ij}:
- do K adds with each

bigger blocks — more work per load!

catch: $IK + KJ + IJ$ elements must fit in cache

cache blocking rule of thumb

fill the most of the cache with useful data

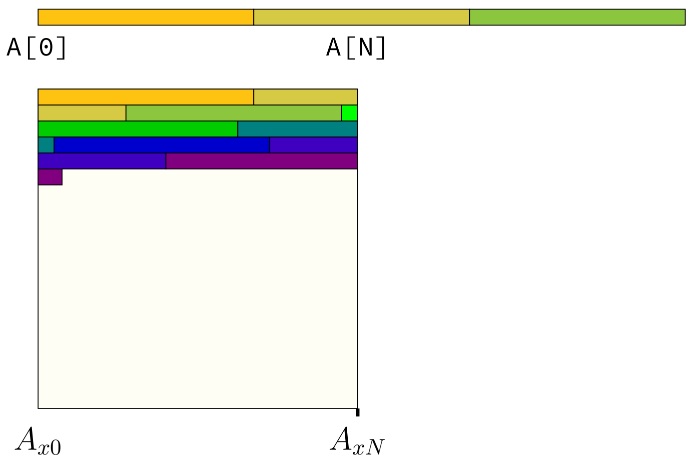
and do as much work as possible from that

example: my desktop 32KB L1 cache

$I = J = K = 48$ uses $48^2 \times 3$ elements, or 27KB.

assumption: conflict misses aren't important

'flat' 2D arrays and cache blocks


A[0]
A[N]
A_{x0}
A_{xN}