Virtual Memory (2)

Changelog

Changes made in this version not seen in first lecture:

- 21 November 2017: 1-level example: added final answer of memory value, not just location
- 21 November 2017: two-level example: answer was `0x0A` not `0xBA`
- 21 November 2017: do we really need a complete copy?: even size of memory regions between copies
- 21 November 2017: swapping components: add "(if modified)" as condition on when we swap out

exercise: 64-bit system

my desktop: 39-bit physical addresses; 48-bit virtual addresses

4096 byte pages

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4096 byte pages

top 16 bits of address not used for translation

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4096 byte pages

exercise: how many page table entries?

exercise: how large are physical page numbers?

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4096 byte pages

exercise: how many page table entries? $2^{48}/2^{12} = 2^{36}$ entries

exercise: how large are physical page numbers? $39 - 12 = 27$ bits

exercise: 64-bit system

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4096 byte pages

exercise: how many page table entries? $2^{48}/2^{12} = 2^{36}$ entries

exercise: how large are physical page numbers? $39 - 12 = 27$ bits

page table entries are 8 bytes (room for expansion, metadata)

would take up 2^{39} bytes?? (512GB??)

exam and conflicts

final exam 7 December 2017

Gilmer 130

fill out conflict form (today, please?) if you can't make it

- linked off schedule
- also Collab announcement

rotate HW and cache blocking

many of you seemed to think you were doing just loop unrolling...

but generally changed the order of memory accesses in the process

basically cache blocking

```
// unroll + cache blocking
for (int i = ...; i += 2)
    for (int j = ...) {
        f(i + 0, j);
        f(i + 1, j);
    }
```

```
// unroll loop in i only
// (probably not very helpful)
for (int i = ...; i += 2) {
    for (int j = ...)
        f(i, j);
    for (int j = ...)
        f(i + 1, j);
}
```

page table tricks

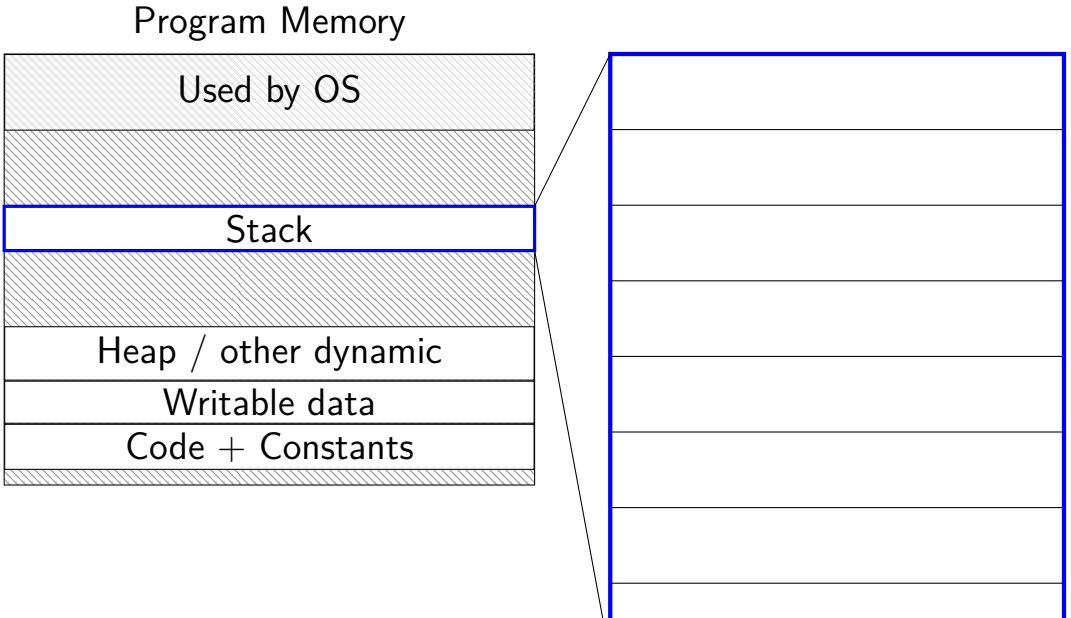
page tables let operating systems do ‘magic’ with memory

key idea: OS doesn’t need to crash on a fault!

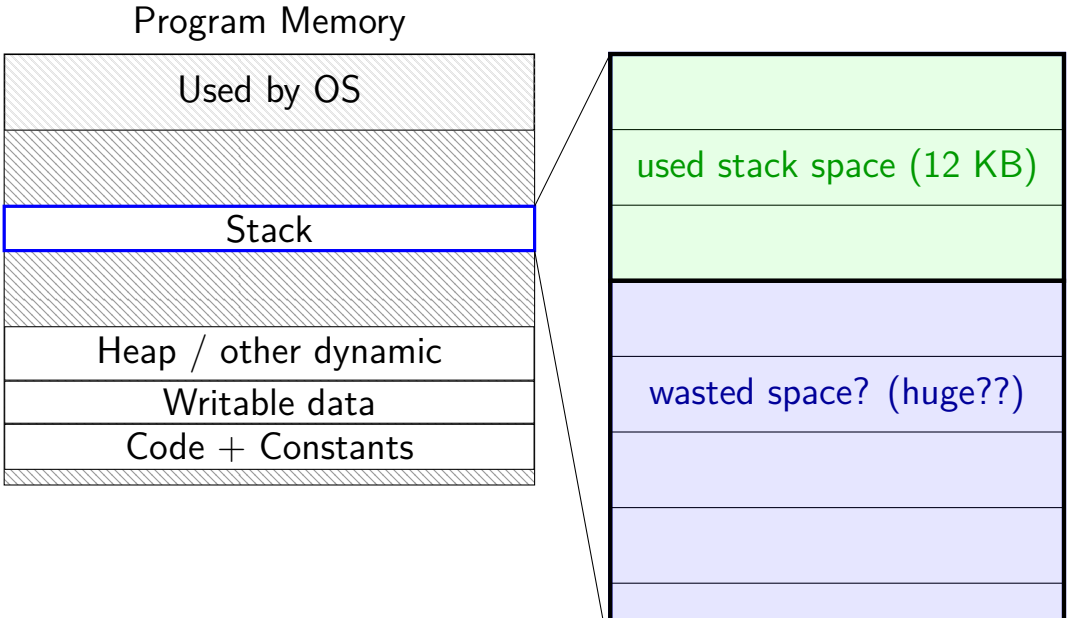
instead: change page tables and rerun memory access

- similar idea to trap-and-emulate

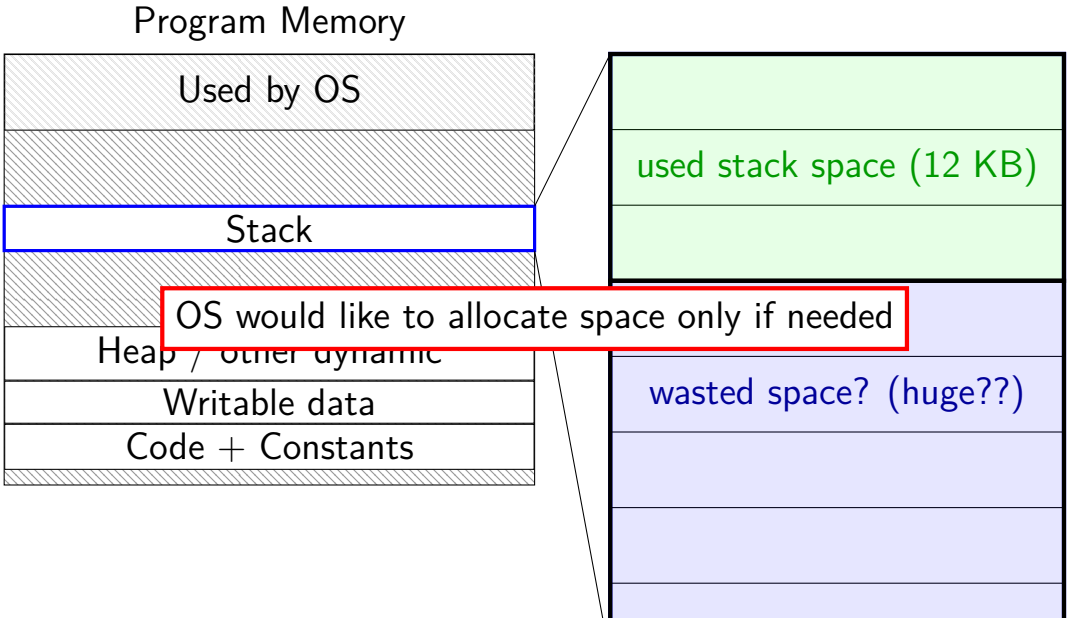
space on demand



space on demand



space on demand

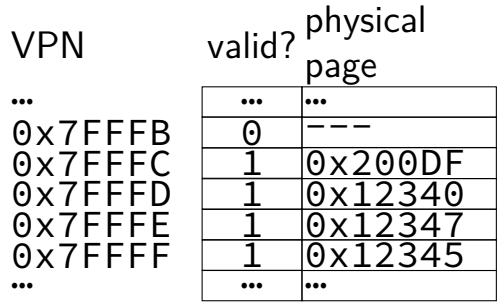


allocating space on demand

`%rsp = 0x7FFFC000`

```
...
// requires more stack space
A: pushq %rbx

B: movq 8(%rcx), %rbx
C: addq %rbx, %rax
...
```



VPN	valid?	physical page
…	…	…
0x7FFFB	0	---
0x7FFFC	1	0x200DF
0x7FFFD	1	0x12340
0x7FFFE	1	0x12347
0x7FFFF	1	0x12345
…	…	…

allocating space on demand

`%rsp = 0x7FFFC000`

```
...
// requires more stack space
A: pushq %rbx
B: movq 8(%rcx), %rbx
C: addq %rbx, %rax
...
```

page fault!

VPN	valid?	physical page
…	…	…
0x7FFFB	0	---
0x7FFFC	1	0x200DF
0x7FFFD	1	0x12340
0x7FFFE	1	0x12347
0x7FFFF	1	0x12345
…	…	…

pushq triggers exception
hardware says "accessing address `0x7FFFBFF8`"
OS looks up what's should be there — "stack"

allocating space on demand

%rsp = 0x7FFFC000

```
...
// requires more stack space
A: pushq %rbx          restarted

B: movq 8(%rcx), %rbx
C: addq %rbx, %rax
...
```

VPN	valid?	physical page
…	…	…
0x7FFFB	1	0x200D8
0x7FFFC	1	0x200DF
0x7FFFD	1	0x12340
0x7FFFE	1	0x12347
0x7FFFF	1	0x12345
…	…	…

in exception handler, OS allocates more stack space
OS updates the page table
then returns to retry the instruction

allocating space on demand

note: the space doesn't have to be initially empty

only change: load from file, etc. instead of allocating empty page

loading program can be merely creating empty page table

everything else can be handled in response to page faults

- no time/space spent loading/allocating unneeded space

page tricks generally

deliberately make program trigger page/protection fault

but don't assume page/protection fault is an error

have seperate data structures represent logically allocated memory

> e.g. "addresses `0x7FFF8000` to `0x7FFFFFFF` are the stack"
> might talk about Linux data structures later (book section 9.7)

page table is for the hardware and not the OS

hardware help for page table tricks

information about the address causing the fault

- e.g. special register with memory address accessed
- harder alternative: OS disassembles instruction, look at registers

(by default) rerun faulting instruction when returning from exception

precise exceptions: no side effects from faulting instruction or after

- e.g. `pushq` that caused did not change `%rsp` before fault
- e.g. instructions reordered after faulting instruction not visible

swapping

our textbook presents virtual memory to support swapping

using disk (or SSD, ...) as the next level of the memory hierarchy

OS allocates space on disk

DRAM is a cache for disk

swapping versus caching

"cache block" $\approx$ physical page

fully associative

- every virtual page can be stored in any physical page

replacement is managed by the OS

normal cache hits happen without OS

- common case that needs to be fast

swapping components

"swap in" a page — exactly like allocating on demand!

- OS gets page fault — invalid in page table
- check where page actually is (from virtual address)
- read from disk
- eventually restart process

"swap out" a page

- OS marks as invalid in the page table(s)
- copy to disk (if modified)

HDD/SDDs are slow

HDD reads and writes: milliseconds to tens of milliseconds

- minimum size: 512 bytes
- writing tens of kilobytes basically as fast as writing 512 bytes

SSD writes and writes: hundreds of microseconds

- designed for writes/reads of kilobytes (not much smaller)

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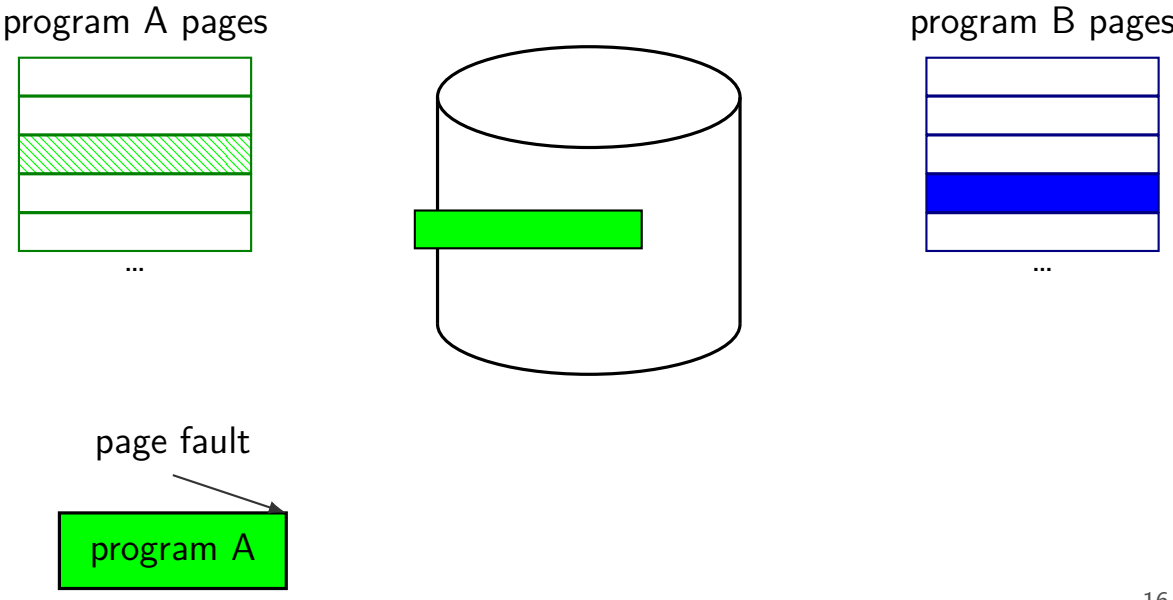
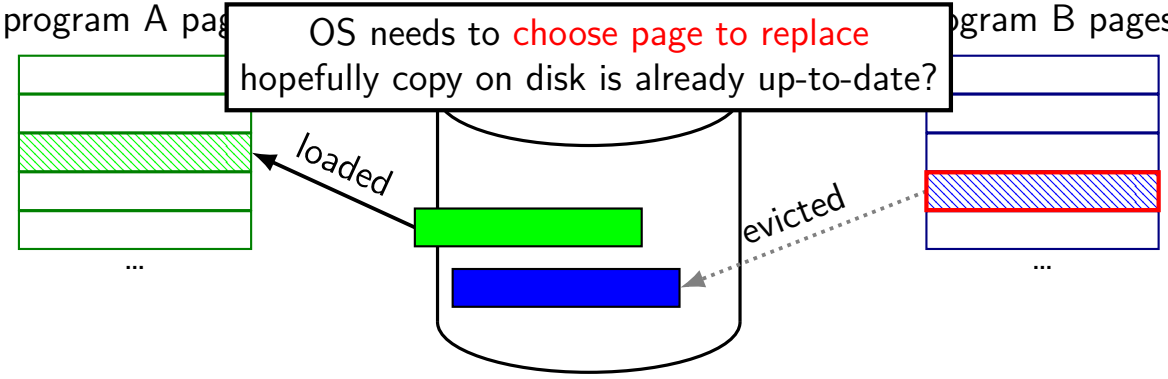
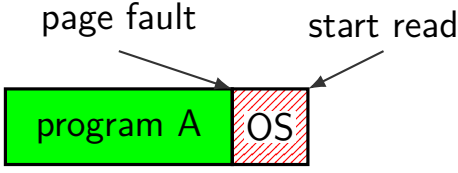
swapping timeline



swapping timeline





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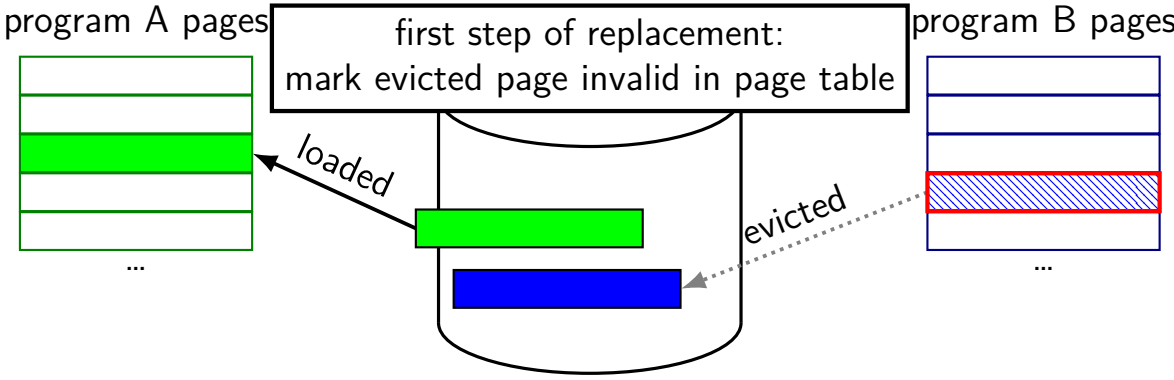
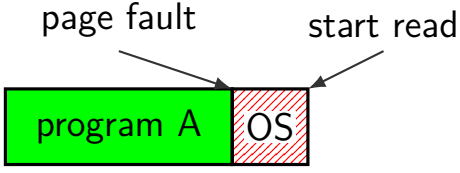
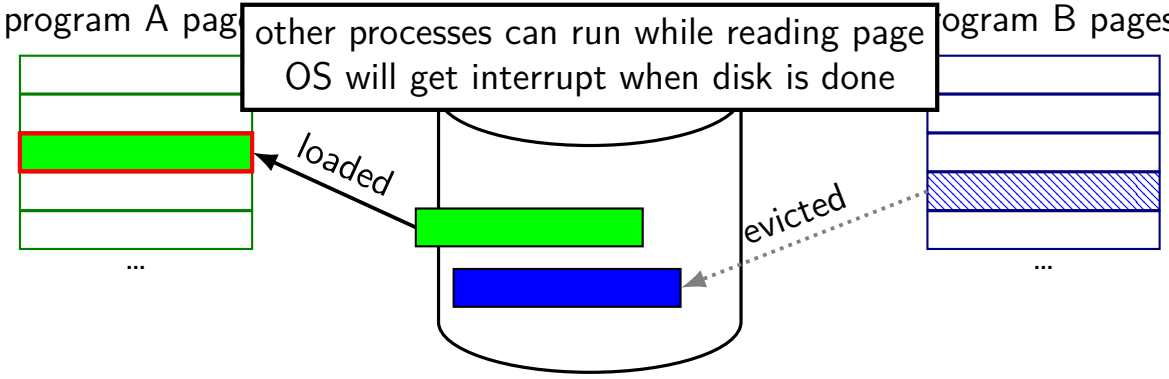
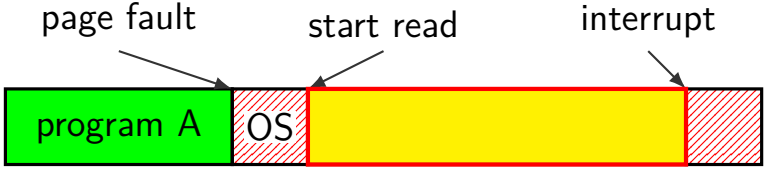


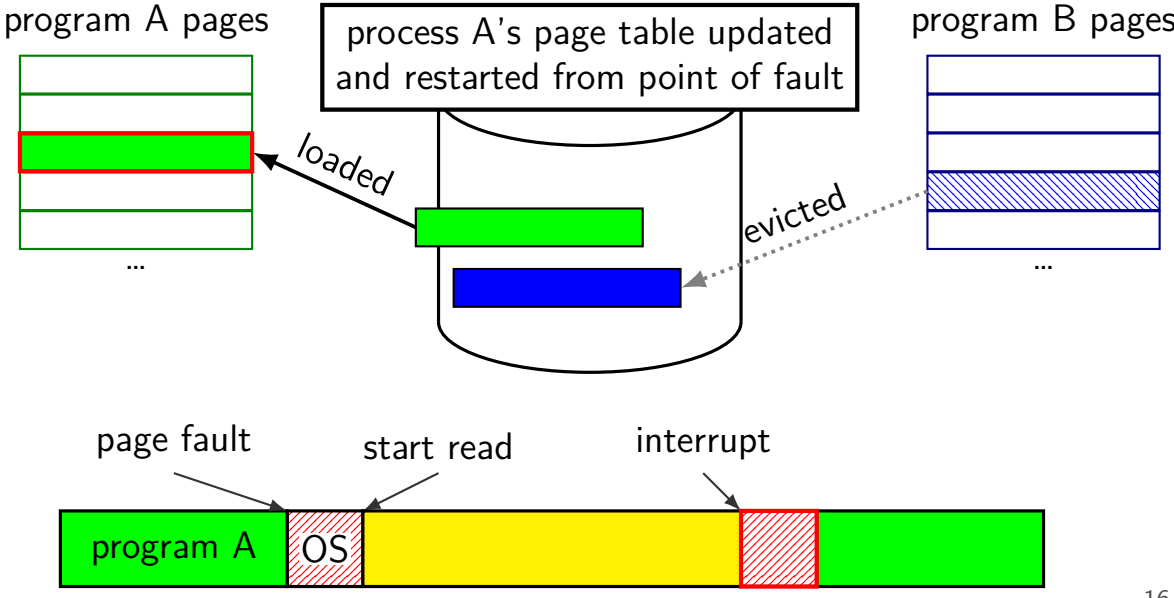


swapping timeline





swapping timeline



swapping decisions

write policy

replacement policy

swapping decisions

write policy

replacement policy

swapping is writeback

implementing write-through is hard

- when fault happens — physical page not written
- when OS resumes process — no chance to forward write
- HW itself doesn't know how to write to disk

write-through would also be really slow

- HDD/SSD perform best if one writes at least a whole page at a time

implementing writeback

need a *dirty bit* per page ("was page modified")

often kept in the page table!

option 1 (most common): hardware sets dirty bit in page table entry (on write)

bit means "physical page was modified using this PTE"

option 2: OS sets page read-only, flips read-only+dirty bit on fault

swapping decisions

write policy

replacement policy

replacement policies really matter

huge cost for "miss" on swapping (milliseconds!)

- many millions of computations on modern processor
- much much worse than even L3 caches

replacement policy implemented in software

- a lot more room for fancy policies

LRU replacement?

problem: need to identify when pages are used

- ideally every single time

not practical to do this exactly

- HW would need to keep a list of when each page was accessed, or
  SW would need to force every access to trigger a fault

approximating LRU

one policy: "not recently used"

OS periodically marks all pages as unreadable/writeable

when page fault happens:

- make page accessible again
- put on list of "used" pages

OS replaces pages not on "used" list

- tiebreaker: when was page last on used list?

hardware help for not-recently-used

hardware usually implements accessed bit in page table entry

whenever page is read/written — hardware sets this bit (if not set)

makes it easier (faster?) for OS to maintain "used " list

OS can periodically scan/clear "accessed" bit

instead of marking pages invalid temporarily

construct lists of "used" pages

accessed/dirty bits

information about how a page table entry was used

indirectly about underlying physical page

kept in page table entries themselves

accessed/dirty bits

information about how a page table entry was used

- *indirectly* about underlying physical page

kept in page table entries themselves

multiple page table entries refer to same page

- separate valid/accessed bits for each
- OS needs to consolidate them all

mmap

Linux/Unix has a function to “map” a file to memory

```
int file = open("somefile.dat", O_RDWR);

    // data is region of memory that represents file
char *data = mmap(..., file, 0);

    // read byte 6 from somefile.dat
char seventh_char = data[6];

   // modifies byte 100 of somefile.dat
data[100] = 'x';
    // can continue to use 'data' like an array
```

swapping almost mmap

access mapped file for first time, read from disk

(like swapping when memory was swapped out)

write "mapped" memory, write to disk eventually

(like writeback policy in swapping)

use "dirty" bit

extra detail: other processes should see changes

all accesses to file use same physical memory

fast copies

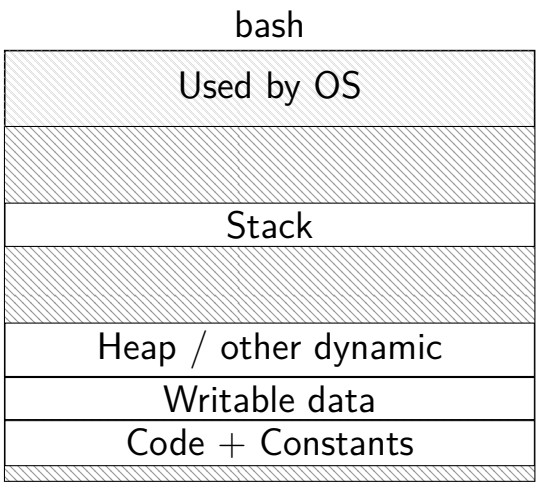
Unix mechanism for starting a new process: `fork()`

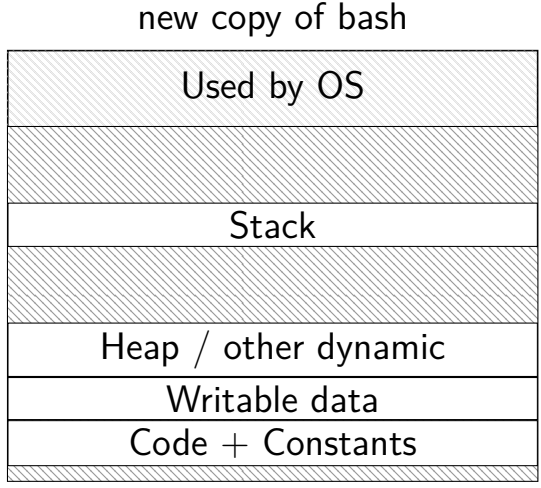
creates a copy of an entire program!

(usually, the copy then calls `execve` — replaces itself with another program)

how isn't this really slow?

do we really need a complete copy?





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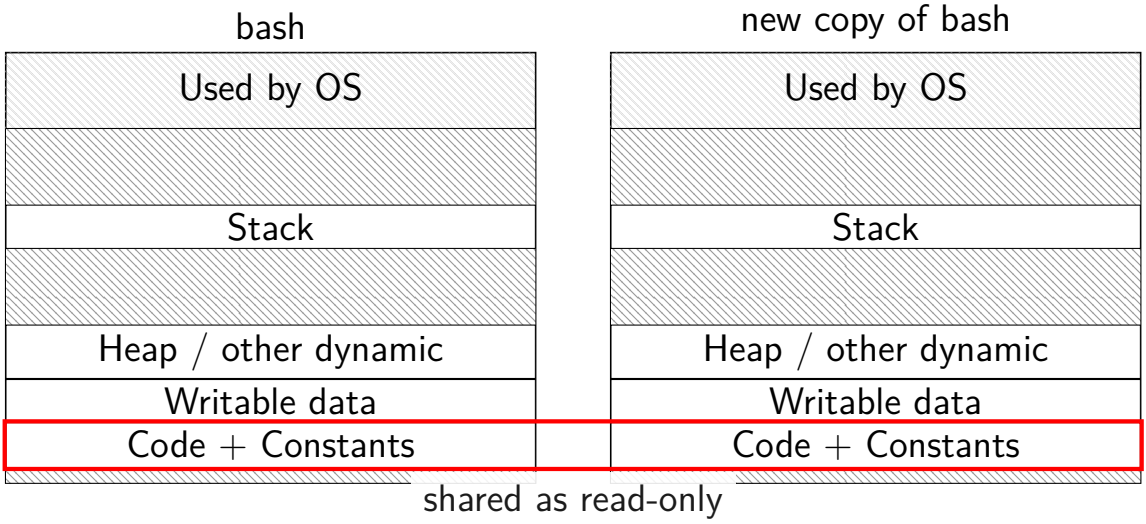

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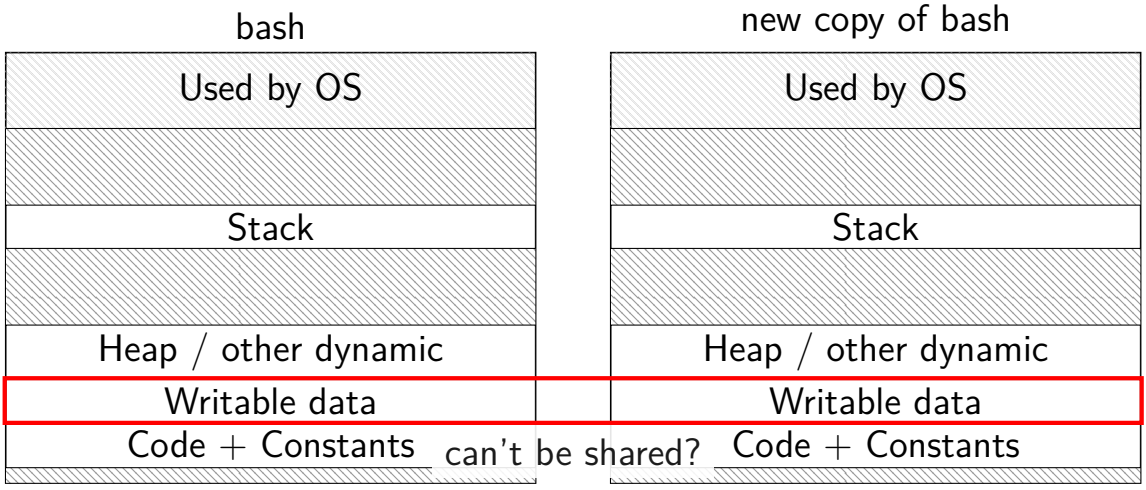

trick for extra sharing

sharing writeable data is fine — until either process modifies the copy

can we detect modifications?

trick: tell CPU (via page table) shared part is read-only

processor will trigger a fault when it's written

copy-on-write and page tables

VPN	valid?	write?	physical page
…	…	…	…
0x00601	1	1	0x12345
0x00602	1	1	0x12347
0x00603	1	1	0x12340
0x00604	1	1	0x200DF
0x00605	1	1	0x200AF
…	…	…	…

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0x00605	1	0	0x200AF
…	…	…	…

copy operation actually duplicates page table
both processes share all physical pages
but marks pages in both copies as read-only

copy-on-write and page tables

VPN	valid?	write?	physical page
…	…	…	…
0x00601	1	0	0x12345
0x00602	1	0	0x12347
0x00603	1	0	0x12340
0x00604	1	0	0x200DF
0x00605	1	0	0x200AF
…	…	…	…

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0x00601	1	0	0x12345
0x00602	1	0	0x12347
0x00603	1	0	0x12340
0x00604	1	0	0x200DF
0x00605	1	0	0x200AF
…	…	…	…

when either process tries to write read-only page triggers a fault — OS actually copies the page

copy-on-write and page tables

VPN	valid?	write?	physical page
…	…	…	…
0x00601	1	0	0x12345
0x00602	1	0	0x12347
0x00603	1	0	0x12340
0x00604	1	0	0x200DF
0x00605	1	0	0x200AF
…	…	…	…

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0x00602	1	0	0x12347
0x00603	1	0	0x12340
0x00604	1	0	0x200DF
0x00605	1	1	0x300FD
…	…	…	…

after allocating a copy, OS reruns the write instruction

exercise: 64-bit system

my desktop: 39-bit physical addresses; 48-bit virtual addresses

4096 byte pages

exercise: how many page table entries? $2^{48}/2^{12} = 2^{36}$ entries

exercise: how large are physical page numbers? $39 - 12 = 27$ bits

page table entries are 8 bytes (room for expansion, metadata)

would take up 2^{39} bytes?? (512GB??)

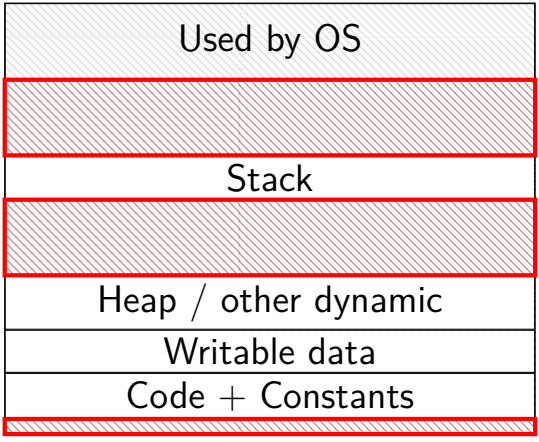
huge page tables

huge virtual address spaces!

impossible to store PTE for every page

how can we save space?

holes



most pages are invalid

saving space

basic idea: don't store (most) invalid page table entries

use a data structure other than a flat array

- want a map — lookup key (virtual page number), get value (PTE)

options?

saving space

basic idea: don't store (most) invalid page table entries

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options?

hashtable

- actually used by some historical processors
- but never common

saving space

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use a data structure other than a flat array

- want a map — lookup key (virtual page number), get value (PTE)

options?

hashtable

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- but never common

tree data structure

- but not quite a search tree

search tree tradeoffs

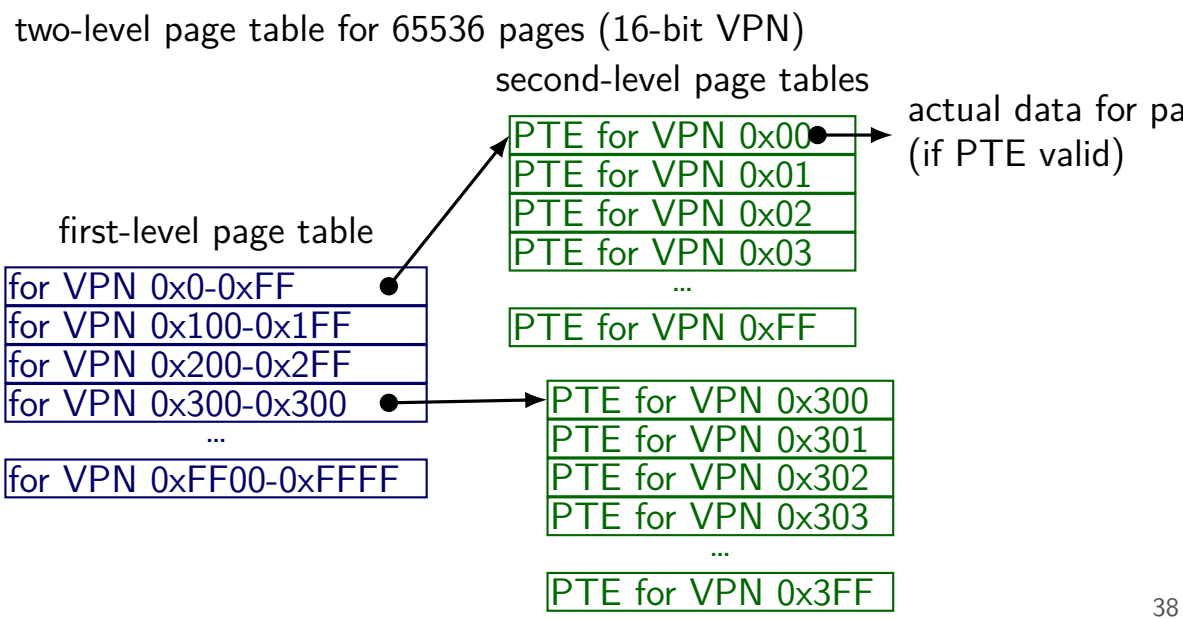
lookup usually implemented in hardware

- lookup should be simple

lookup should not involve many memory accesses

- doing two memory accesses is already very slow

two-level page tables



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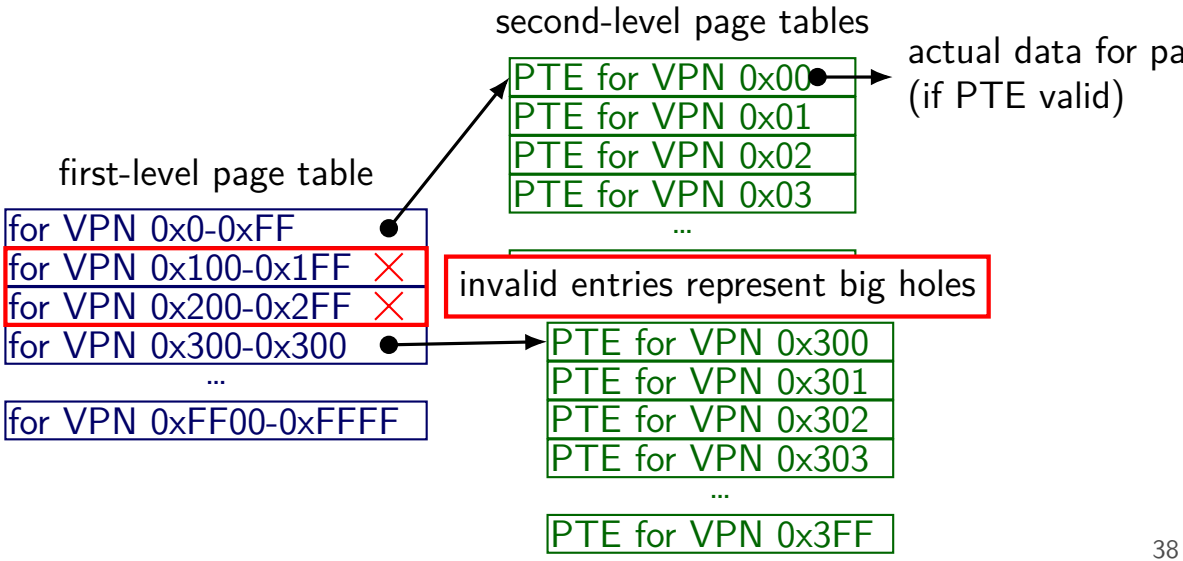
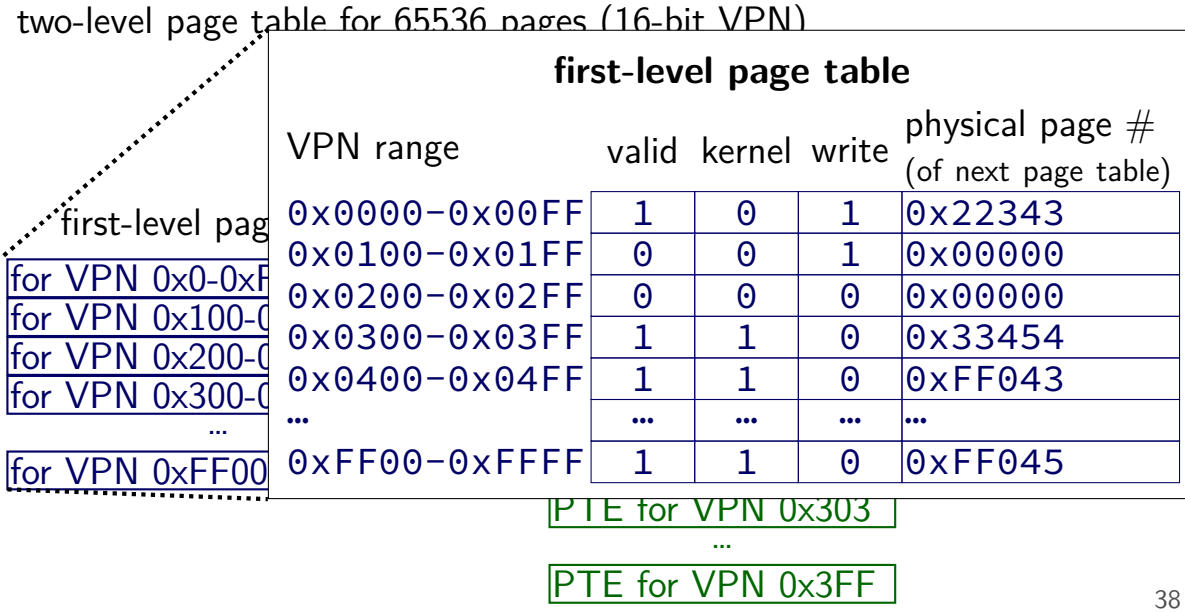


two-level page tables



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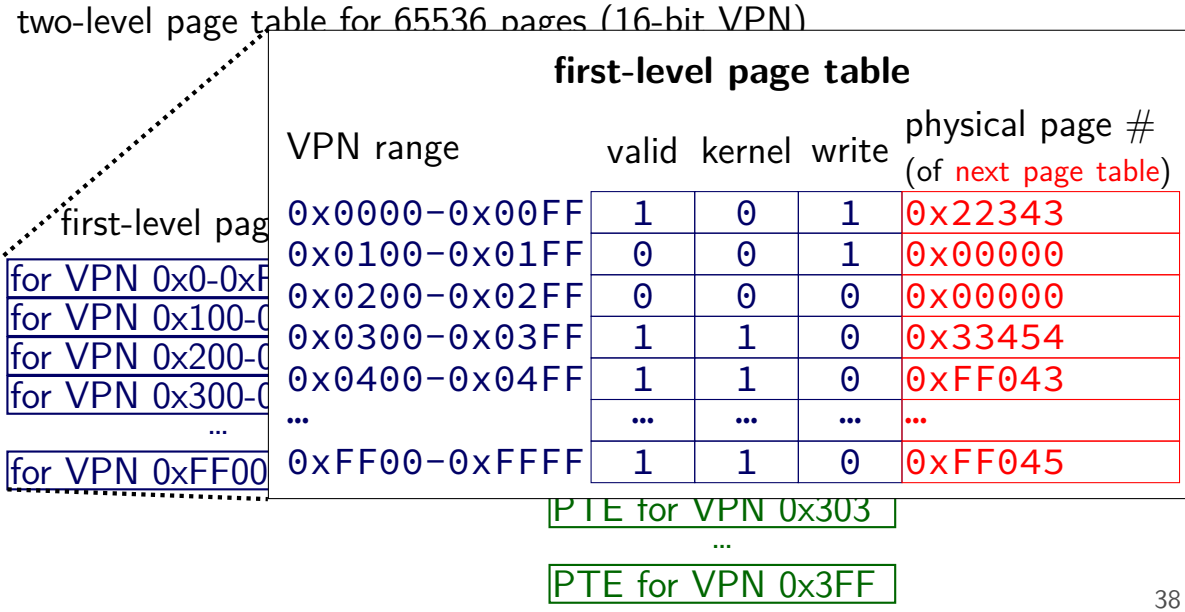
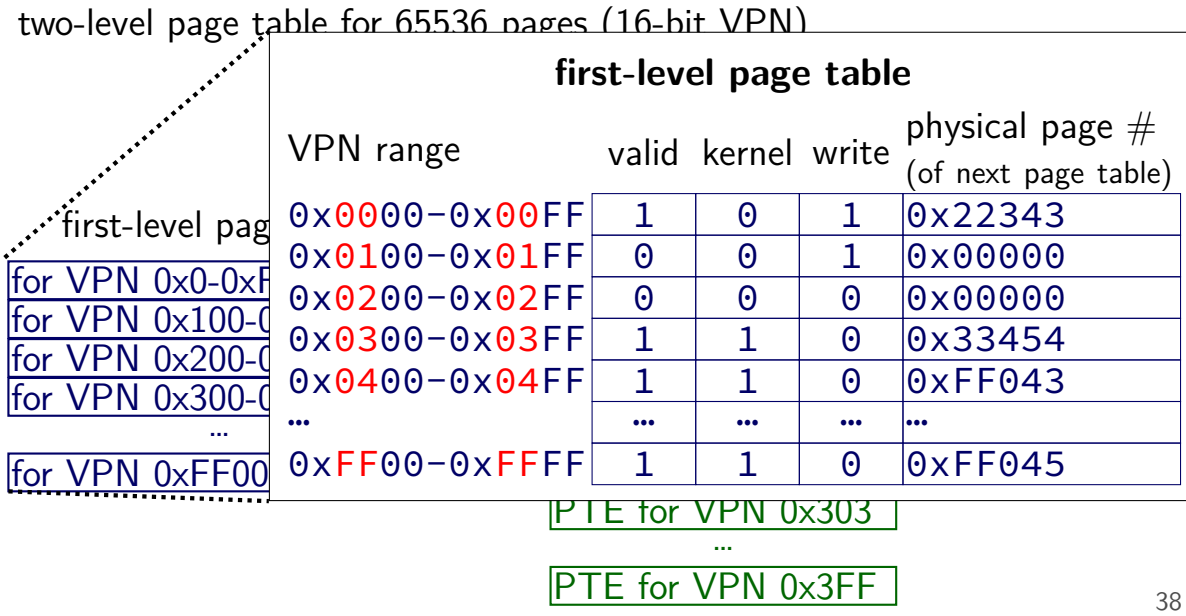
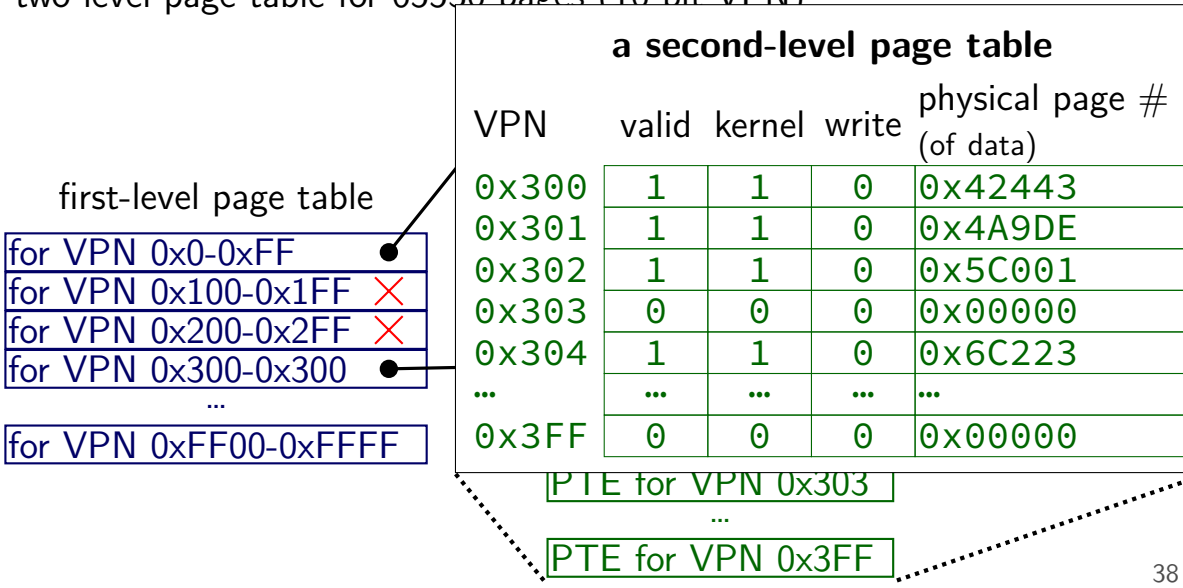


two-level page tables



two-level page tables

two-level page table for 65536 pages (16-bit VPN)



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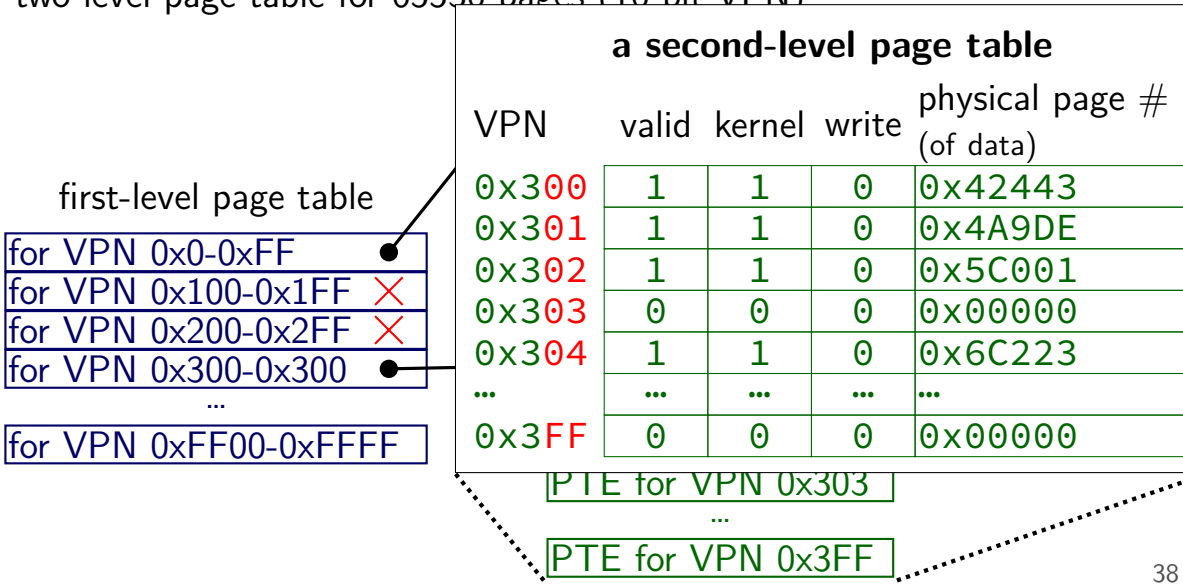
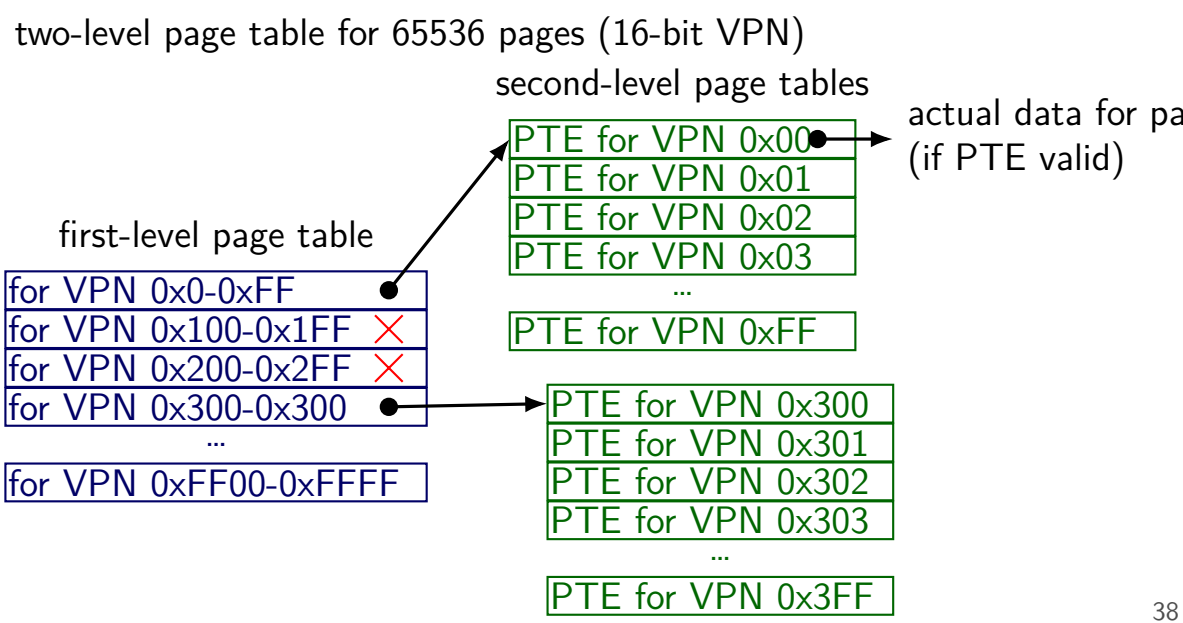
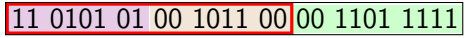


two-level page tables



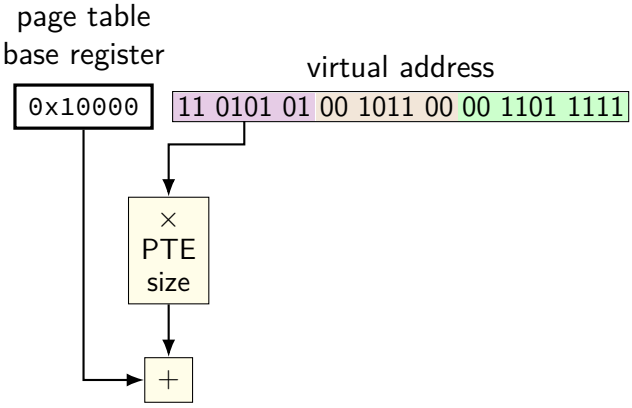
two-level page table lookup

virtual address



VPN — split into two parts (one per level)

two-level page table lookup



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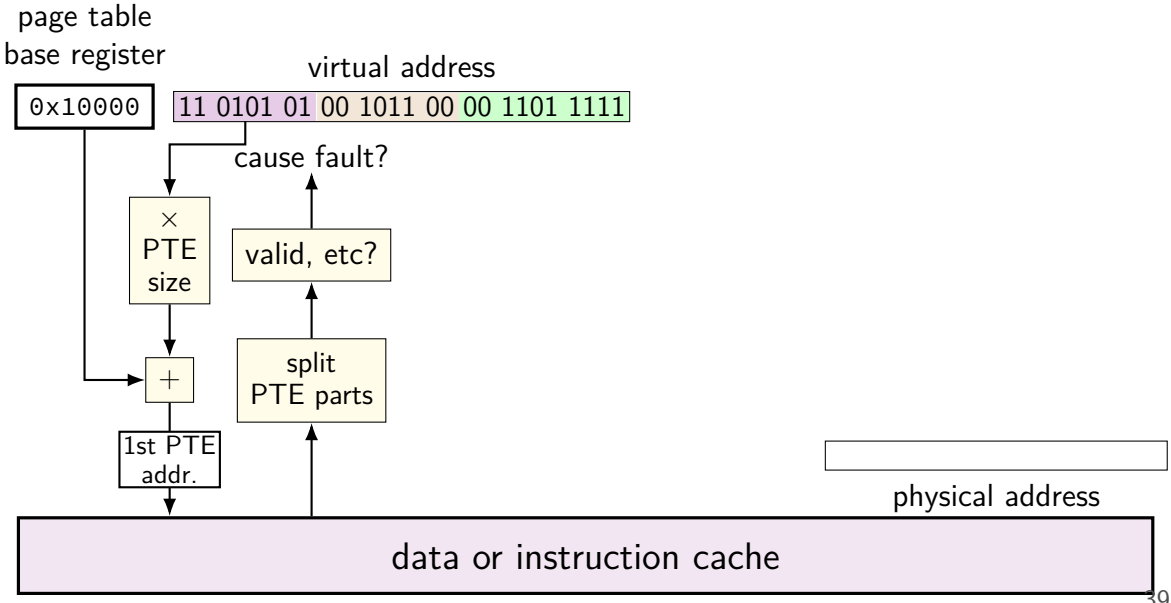

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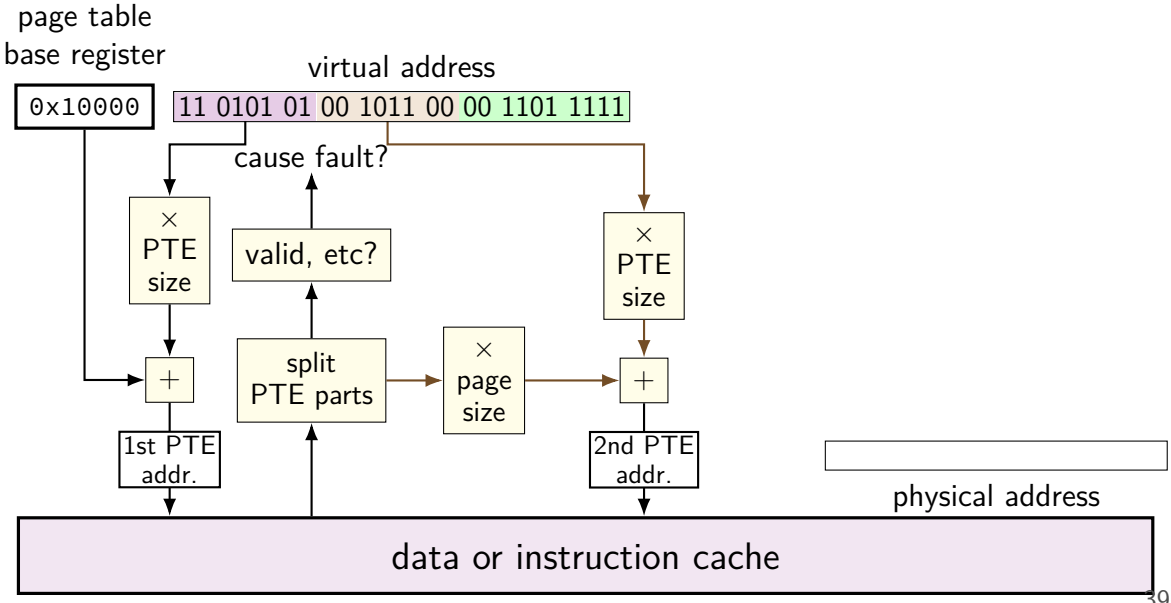
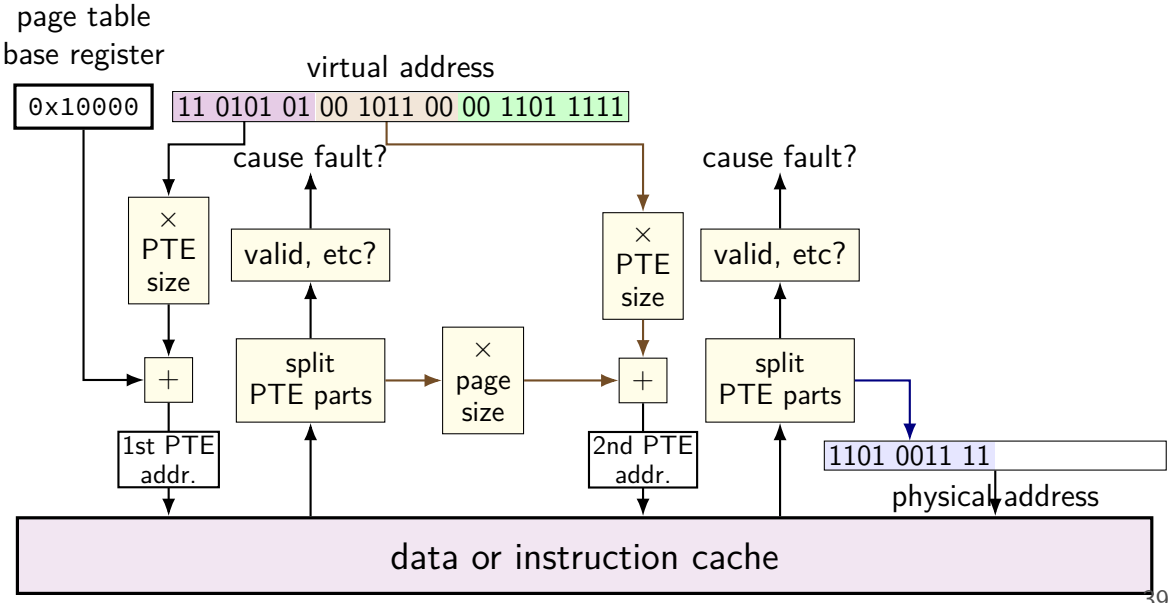
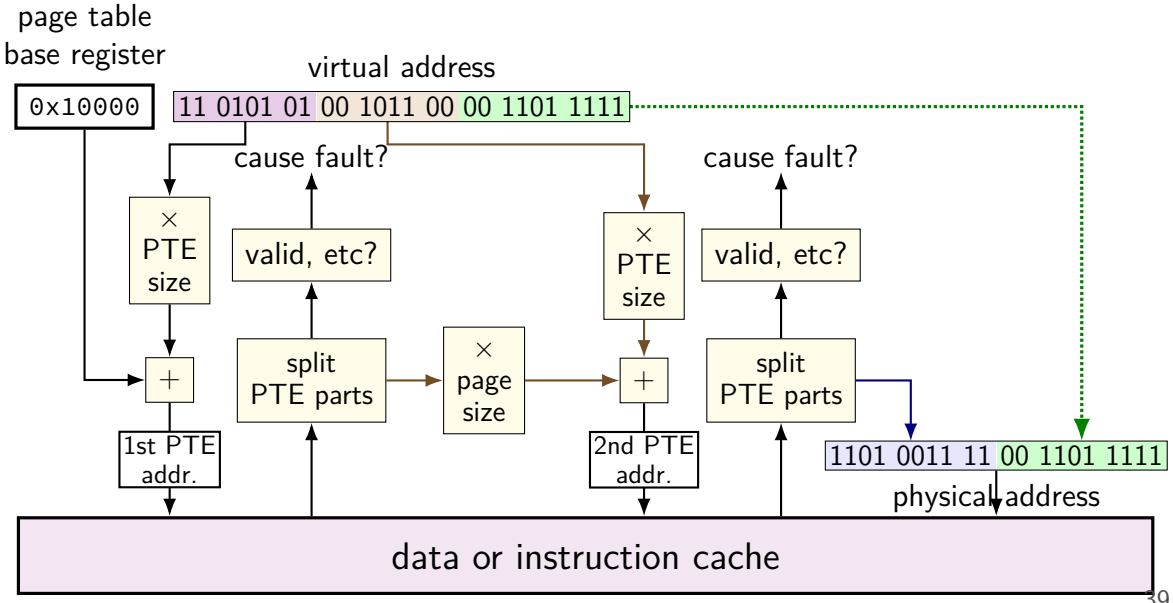


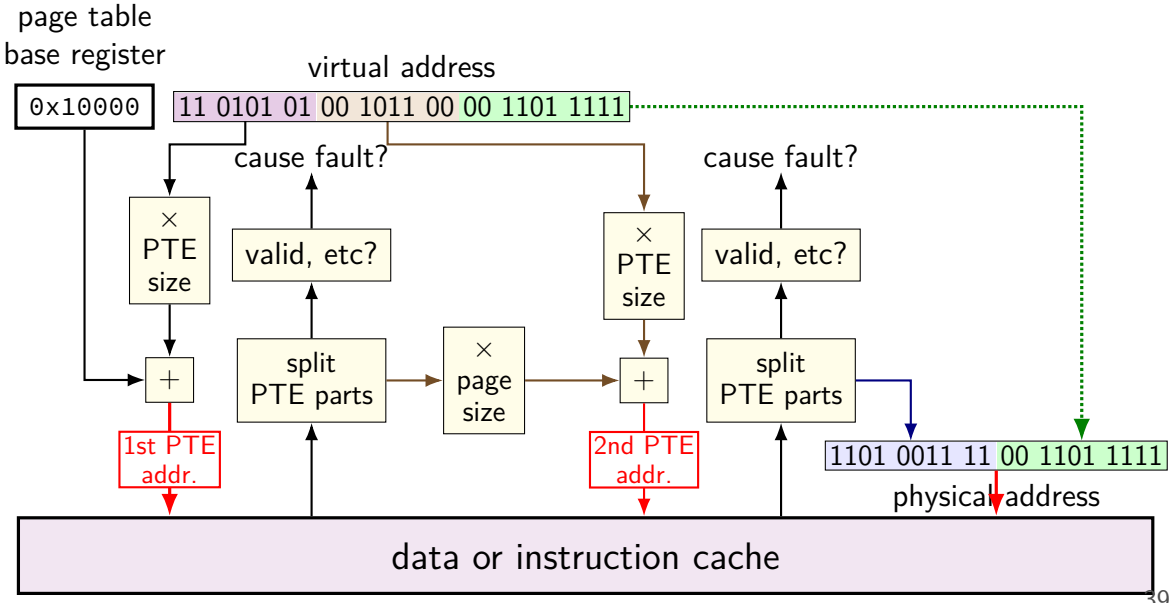
two-level page table lookup



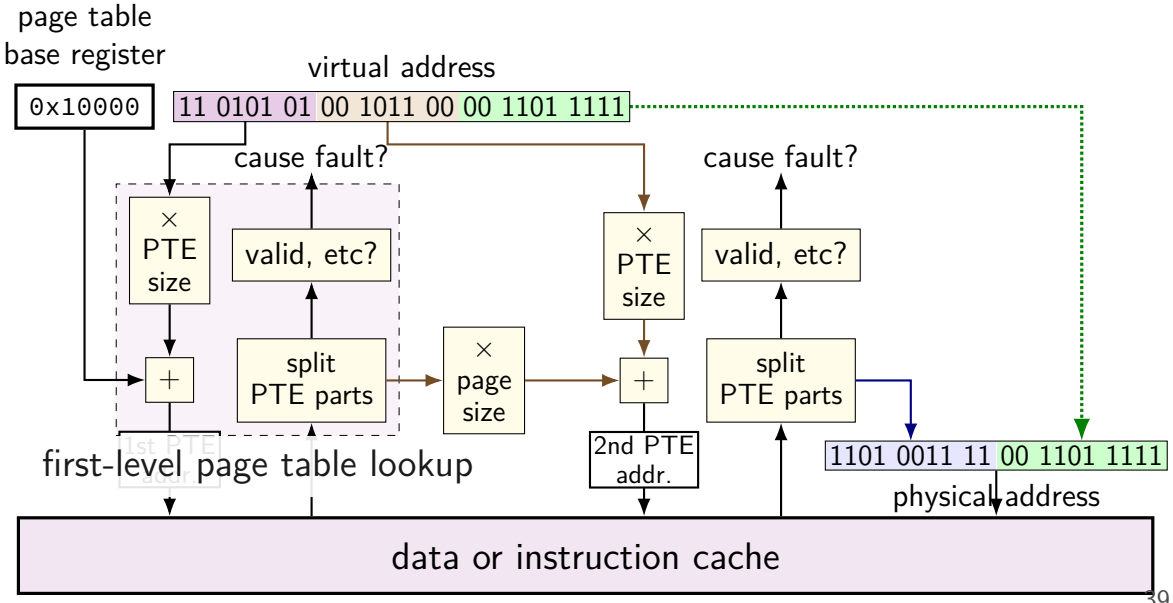
two-level page table lookup



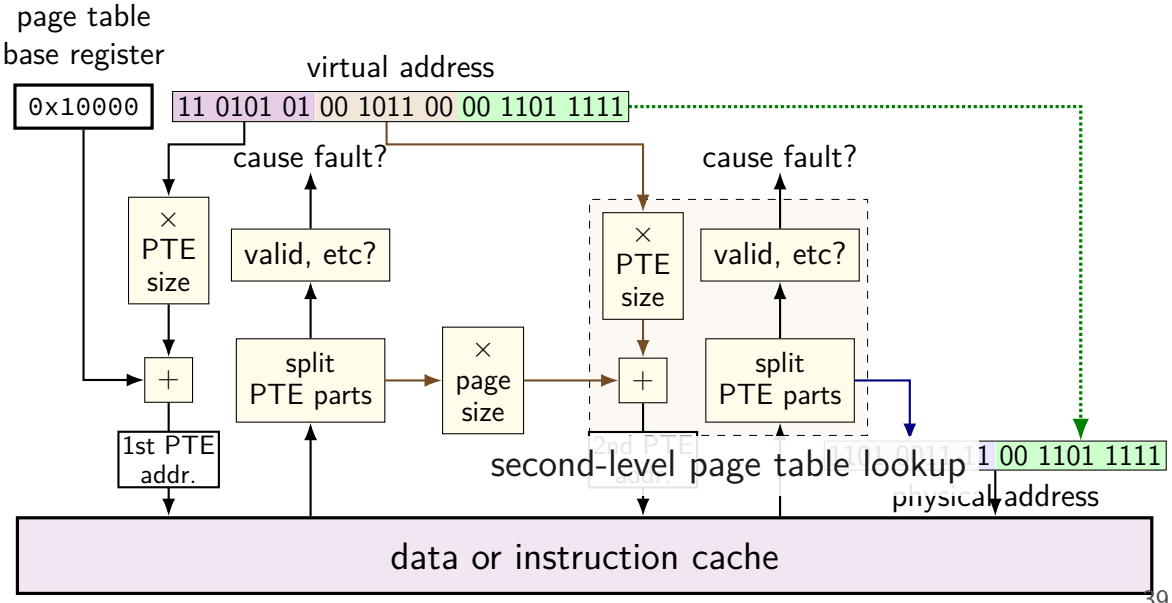
two-level page table lookup



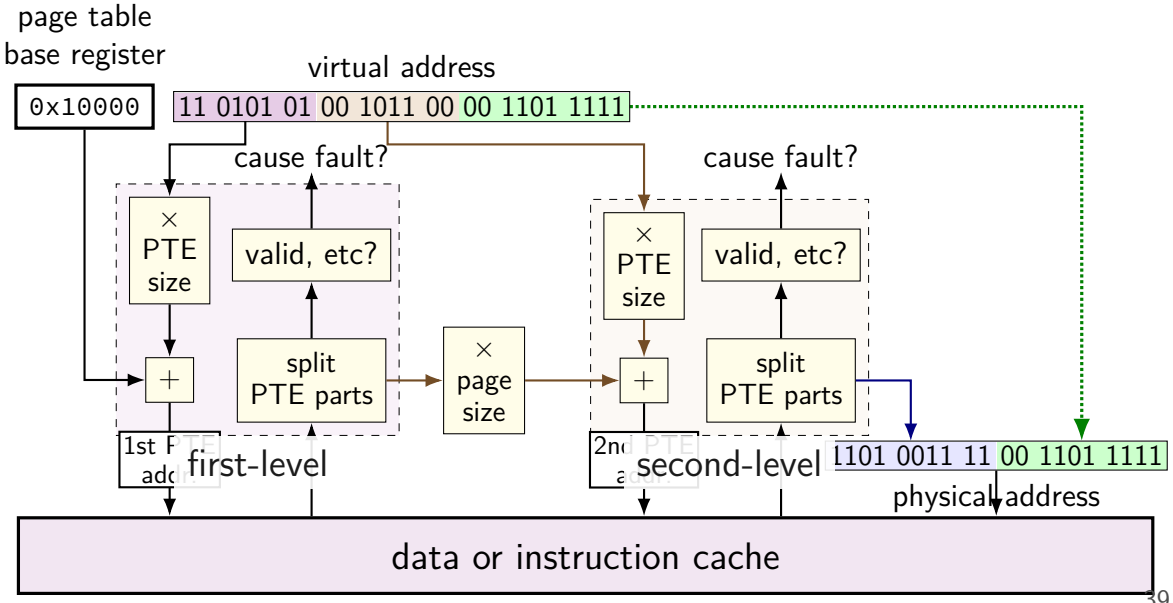
two-level page table lookup



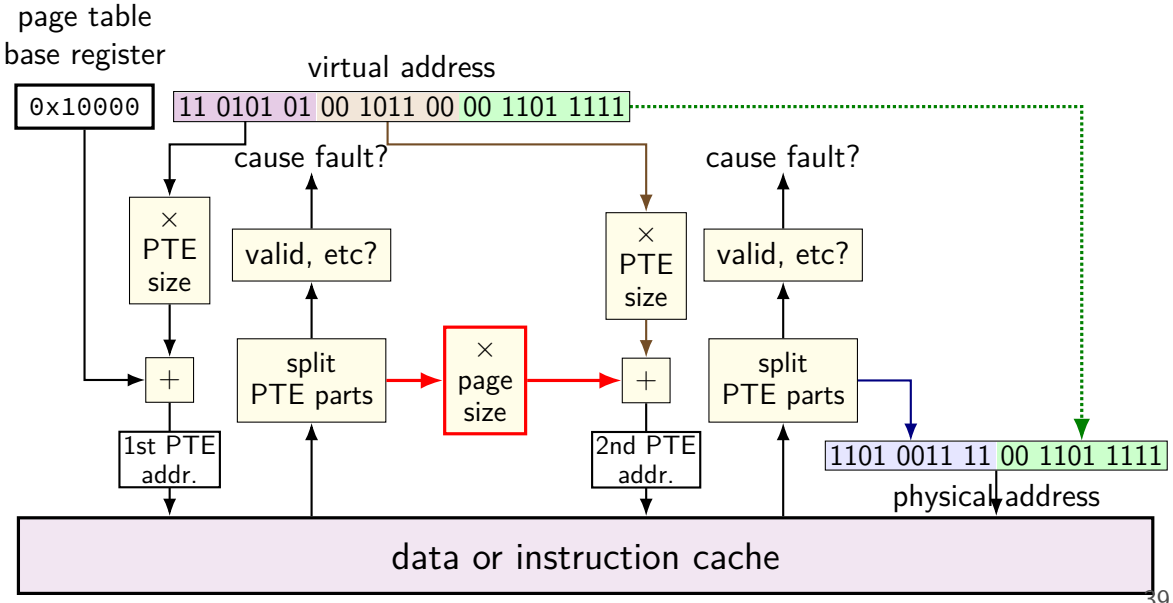
two-level page table lookup



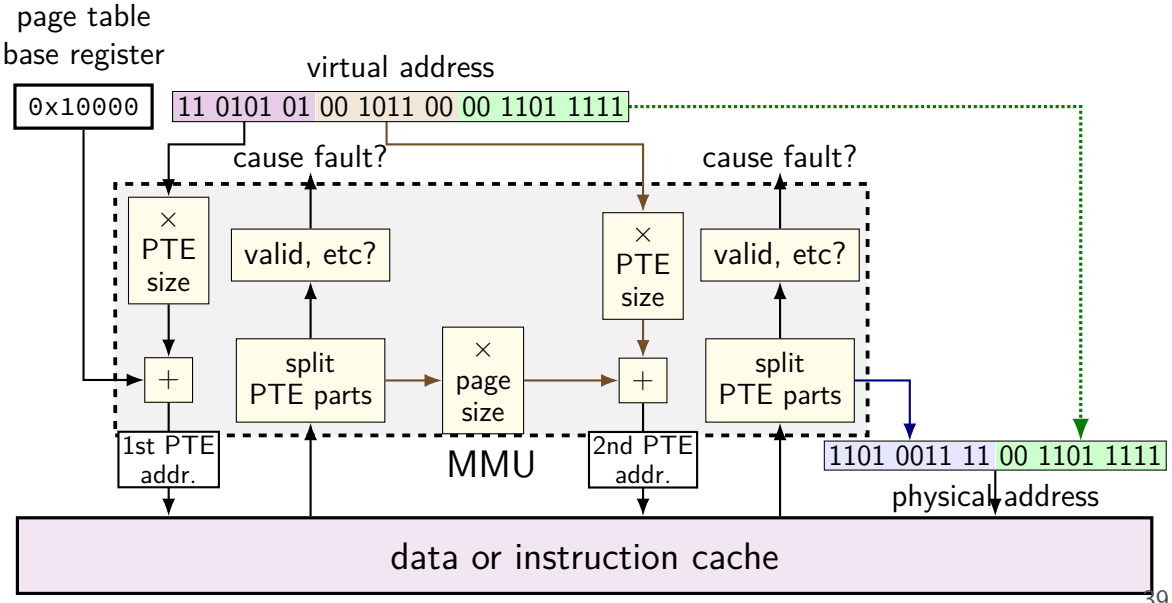
two-level page table lookup



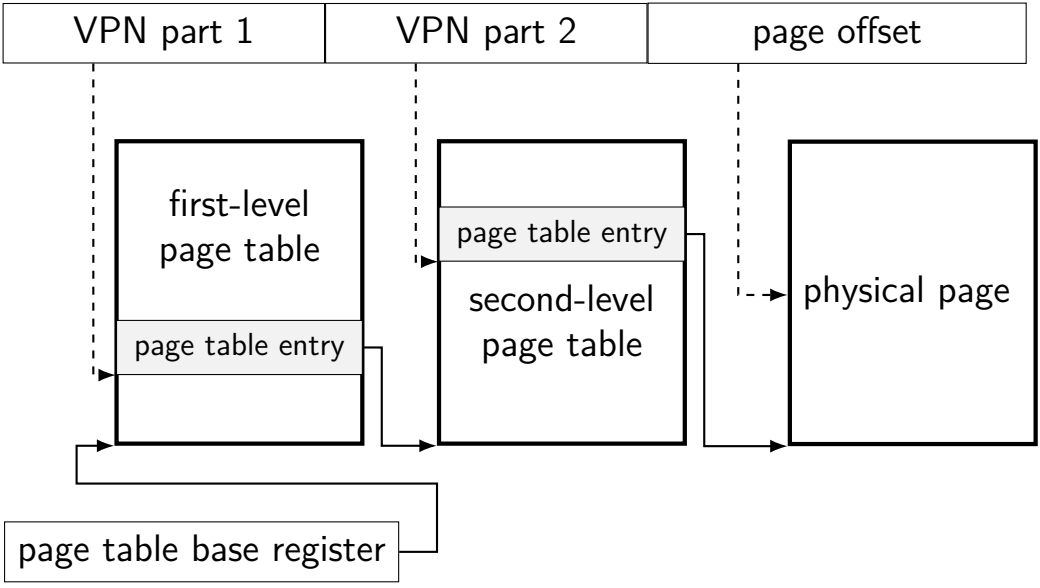
two-level page table lookup



two-level page table lookup



another view



multi-level page tables

VPN split into pieces for each level of page table

top levels: page table entries point to next page table

usually using physical page number of next page table

bottom level: page table entry points to destination page

validity and permission checks at each level

note on VPN splitting

textbook labels it ‘VPN 1’ and ‘VPN 2’ and so on

these are parts of the virtual page number

(there are not multiple VPNs)

splitting addresses for levels

x86-32

32-bit physical address; 32-bit virtual address

2^{12} byte page size

2-levels of page tables; each page table is one page

4 byte page table entries

how is address `0x12345678` split up?

splitting addresses for levels

x86-32

32-bit physical address; 32-bit virtual address

2^{12} byte page size

12-bit page offset

2-levels of page tables; each page table is one page

4 byte page table entries

how is address `0x12345678` split up?

splitting addresses for levels

x86-32

32-bit physical address; 32-bit virtual address

2^{12} byte page size

12-bit page offset

2-levels of page tables; each page table is one page

4 byte page table entries

$2^{12}/4 = 2^{10}$ *PTEs/page table; 10-bit VPN parts*

how is address `0x12345678` split up?

splitting addresses for levels

x86-32

32-bit physical address; 32-bit virtual address

2^{12} byte page size
12-bit page offset

2-levels of page tables; each page table is one page

4 byte page table entries
$2^{12}/4 = 2^{10}$ *PTEs/page table; 10-bit VPN parts*

how is address `0x12345678` split up?
10-bit VPN part 1: `0001 0010 00` (`0x48`);
10-bit VPN part 2: `11 0100 0101` (`0x345`);
12-bit page offset: `0x678`

1-level example

6-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 other bits;

page table base register `0x20`; translate virtual address `0x30`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	CB 0B CB 0B
0x38–B	DC 0C DC 0C
0x3C–F	EC 0C EC 0C

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0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	CB 0B CB 0B
0x38–B	DC 0C DC 0C
0x3C–F	EC 0C EC 0C

0x30 = 11 0000

PTE addr:

0x20 + 6 ×1 = 0x26

PTE value:

0xD6 = 1101 0110

PPN 110, valid 1

M[110 000] = **M[**0x30]

1-level example

6-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 other bits;

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0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	CB 0B CB 0B
0x38–B	DC 0C DC 0C
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physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	CB 0B CB 0B
0x38–B	DC 0C DC 0C
0x3C–F	EC 0C EC 0C

0x30 = 11 0000

PTE addr:

0x20 + 6 ×1 = 0x26

PTE value:

0xD6 = 1101 0110

PPN 110, valid 1

M[110 000] = **M[**0x30]

2-level example

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused

page table base register `0x20`; translate virtual address `0x131`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
0x3C–F	FC 0C FC 0C

2-level example

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused

page table base register `0x20`; translate virtual address `0x131`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
0x3C–F	FC 0C FC 0C

0x131 = 1 0011 0001

0x20 + 4 ×1 = 0x24

PTE 1 value:

0xD4 = 1101 0100

PPN 110, valid 1

2-level example

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused

page table base register `0x20`; translate virtual address `0x131`

physical addresses	bytes
0x00-3	00 11 22 33
0x04-7	44 55 66 77
0x08-B	88 99 AA BB
0x0C-F	CC DD EE FF
0x10-3	1A 2A 3A 4A
0x14-7	1B 2B 3B 4B
0x18-B	1C 2C 3C 4C
0x1C-F	1C 2C 3C 4C

physical addresses	bytes
0x20-3	D0 D1 D2 D3
0x24-7	D4 D5 D6 D7
0x28-B	89 9A AB BC
0x2C-F	CD DE EF F0
0x30-3	BA 0A BA 0A
0x34-7	DB 0B DB 0B
0x38-B	EC 0C EC 0C
0x3C-F	FC 0C FC 0C

`0x131` = `1 0011 0001`

`0x20` + 4 × 1 = `0x24`

PTE 1 value:

`0xD4` = `1101 0100`

PPN `110`, valid `1`

PTE 2 addr:

`110 000` + `110` = `0x36`

PTE 2 value: `0xDB`

2-level example

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused

page table base register `0x20`; translate virtual address `0x131`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
0x3C–F	FC 0C FC 0C

`0x131` = `1 0011 0001`

`0x20` + 4 × 1 = `0x24`

PTE 1 value:

`0xD4` = `1101 0100`

PPN `110`, valid 1

PTE 2 addr:

`110 000` + `110` = `0x36`

PTE 2 value: `0xDB`

PPN `110`; valid 1

M[`110 001` (`0x31`)] = `0x0A`

2-level example

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused

page table base register `0x20`; translate virtual address `0x131`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
0x3C–F	FC 0C FC 0C

`0x131` = `1 0011 0001`

`0x20` + 4 × `1` = `0x24`

PTE 1 value:

`0xD4` = `1101 0100`

PPN `110`, valid `1`

PTE 2 addr:

`110 000` + `110` = `0x36`

PTE 2 value: `0xDB`

PPN `110`; valid `1`

M[`110 001` (`0x31`)] = `0x0A`

2-level example

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused

page table base register `0x20`; translate virtual address `0x131`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
0x3C–F	FC 0C FC 0C

0x131 = 1 0011 0001

0x20 + 4 ×1 = 0x24

PTE 1 value:

0xD4 = 1101 0100

PPN 110, valid 1

PTE 2 addr:

110 000 + 110 = 0x36

PTE 2 value: 0xDB

PPN 110; valid 1

M[110 001 (0x31)] = 0x0A

2-level exercise (1)

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused;

page table base register `0x08`; translate virtual address `0x0FB`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
0x3C–F	FC 0C FC 0C

2-level exercise (1)

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused;

page table base register `0x08`; translate virtual address `0x0FB`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
0x3C–F	FC 0C FC 0C

`0x0F3` = `011 111 011`

PTE 1: `0xBB` at `0x0B`

PTE 1: PPN `101` (5) valid `1`

PTE 2: `0xF0` at `0x2F`

PTE 2: PPN `111` (7) valid `1`

`111 011` = `0x3B` → `0x0C`

2-level exercise (1)

9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused;

page table base register `0x08`; translate virtual address `0x0FB`

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0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
0x24–7	D4 D5 D6 D7
0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
0x30–3	BA 0A BA 0A
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0x00–3	00 11 22 33
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0x18–B	1C 2C 3C 4C
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9-bit virtual addresses, 6-bit physical; 8 byte pages, 1 byte PTE

page tables 1 page; PTE: 3 bit PPN (MSB), 1 valid bit, 4 unused;

page table base register `0x08`; translate virtual address `0x0FB`

physical addresses	bytes
0x00–3	00 11 22 33
0x04–7	44 55 66 77
0x08–B	88 99 AA BB
0x0C–F	CC DD EE FF
0x10–3	1A 2A 3A 4A
0x14–7	1B 2B 3B 4B
0x18–B	1C 2C 3C 4C
0x1C–F	1C 2C 3C 4C

physical addresses	bytes
0x20–3	D0 D1 D2 D3
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0x28–B	89 9A AB BC
0x2C–F	CD DE EF F0
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0x34–7	DB 0B DB 0B
0x38–B	EC 0C EC 0C
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`0x0F3` = `011 111 011`

PTE 1: `0xBB` at `0x0B`

PTE 1: PPN `101` (5) valid `1`

PTE 2: `0xF0` at `0x2F`

PTE 2: PPN `111` (7) valid `1`

`111 011` = `0x3B` → `0x0C`