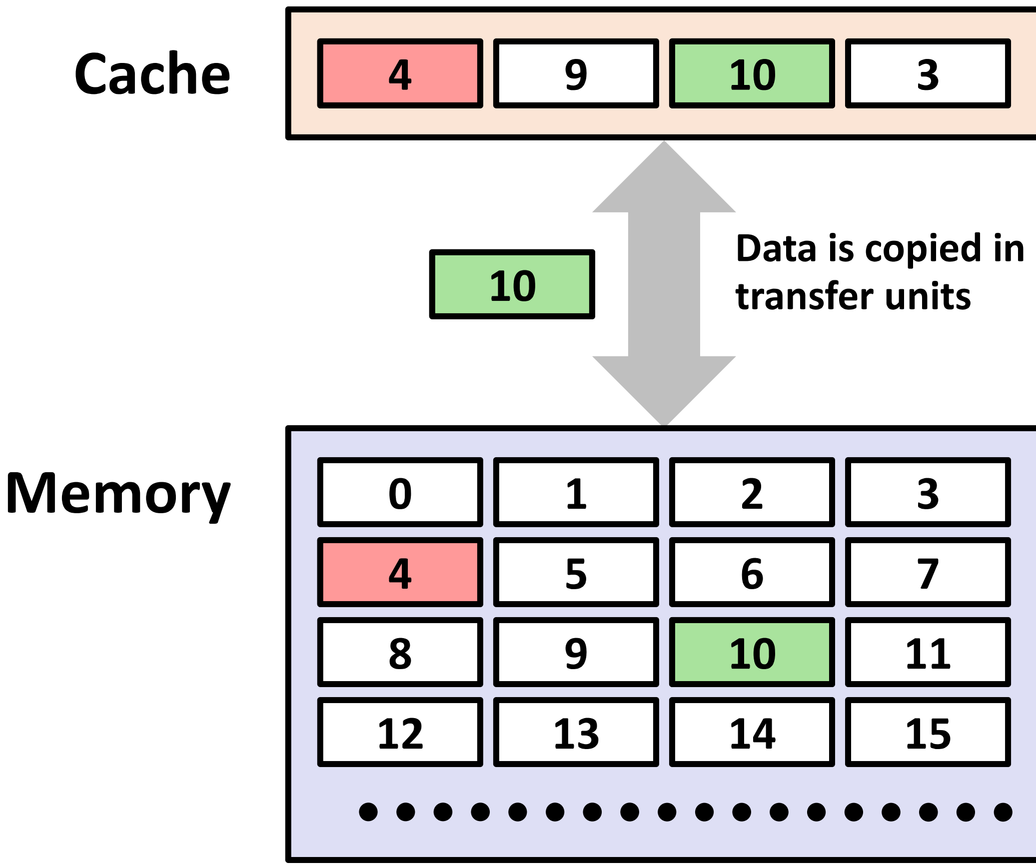
Cache Memories

Lecture, Oct. 30, 2018

General Cache Concept



Smaller, faster, more expensive memory caches a subset of the blocks

Data is copied in block-sized transfer units

Larger, slower, cheaper memory viewed as partitioned into "blocks"

C and cache misses (1)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2) {
    even_sum += array[i + 0];
    odd_sum +=  array[i + 1];
}
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 0; i < 1024; i += 2)
    odd_sum +=  array[i + 1];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks? Would a set-associtiave cache be better?

3

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

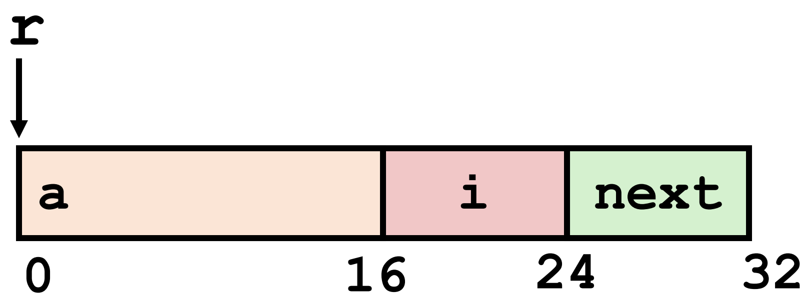
Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

6

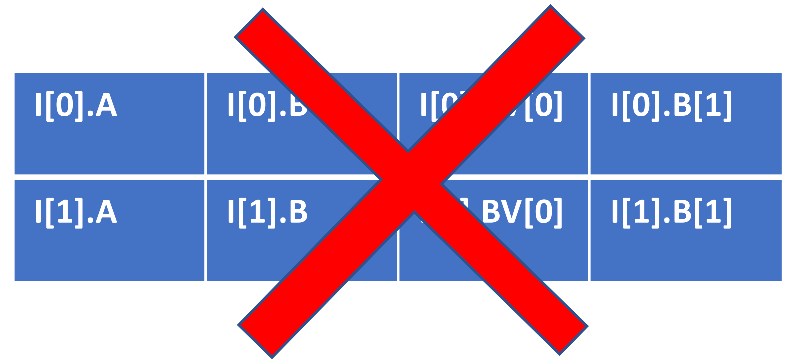
Structure Representation

```
struct rec {
    int a[4];
    size_t i;
    struct rec *next;
};
```



C and cache misses (3)

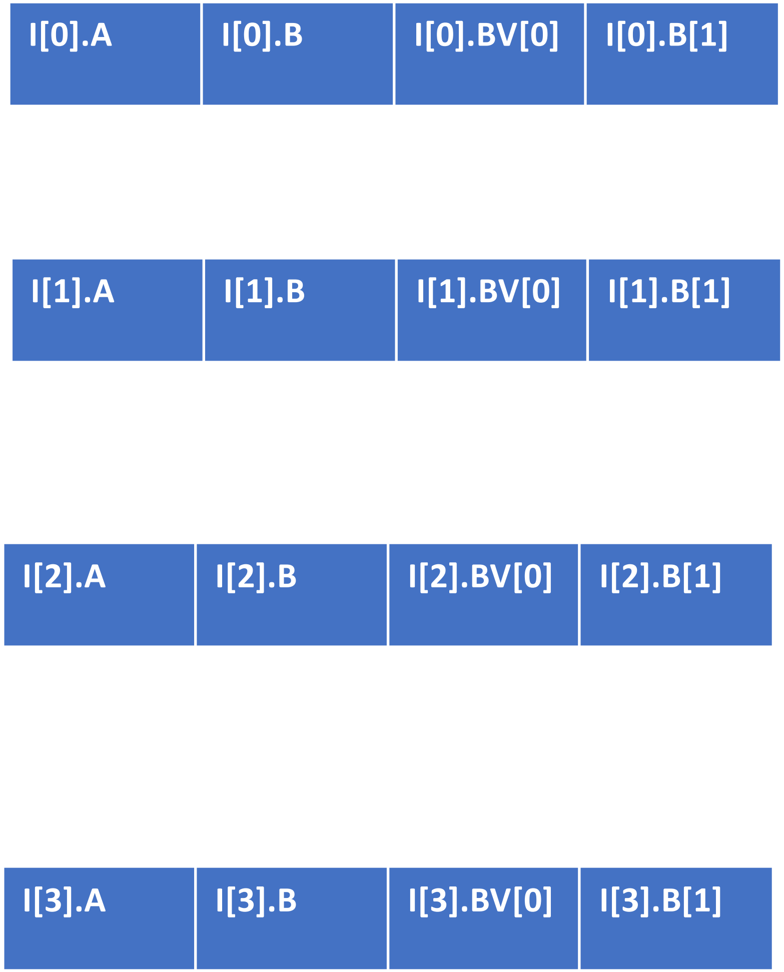
```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```



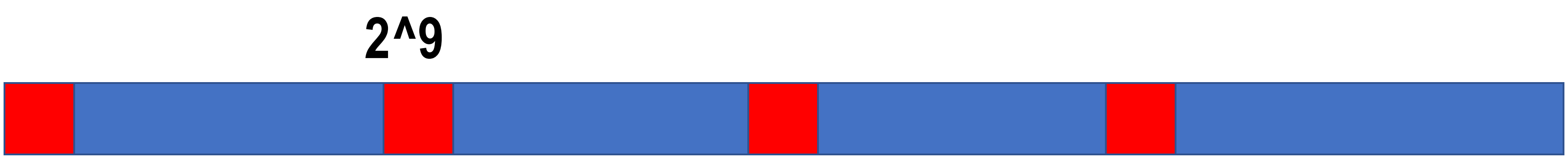
Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

6



Each block associated the first half of the array has a unique spot in memory



Cache Optimization Techniques

```
for (j = 0; j < 3: j = j+1){
          for( i = 0; i < 3; i = i + 1){
                    x[i][j] = 2*x[i][j];
          }
}
```

```
for (i = 0; i < 3: i = i+1){
          for( j = 0; j < 3; j = j + 1){
                    x[i][j] = 2*x[i][j];
          }
}
```

These two loops compute the same result

Inner loop analysis

Array in row major order

X[0][0]	X[0][1]	X[0][2]
X[1][0]	X[1][1]	X[1][2]
X[2][0]	X[2][1]	X[2][2]

0x0 – 0x3	0x4 - 0x7	0x8-0x11	0x12–0x15	0x16 - 0x19	0x20-0x23

X[0][0]	X[0][1]	X[0][2]	X[1][0]	X[1][1]	X[1][2]	X[2][0]	X[2][1]	X[2][2]

Cache Optimization Techniques

```
for (j = 0; j < 3: j = j+1){
          for( i = 0; i < 3; i = i + 1){
                    x[i][j] = 2*x[i][j];
          }
}
```

```
for (i = 0; i < 3: i = i+1){
          for( j = 0; j < 3; j = j + 1){
                    x[i][j] = 2*x[i][j];
          }
}
```

These two loops compute the same result

Array in row major order

X[0][0]	X[0][1]	X[0][2]
X[1][0]	X[1][1]	X[1][2]
X[2][0]	X[2][1]	X[2][2]

```
int *x = malloc(N*N);
for (i = 0; i < 3: i = i+1){
          for( j = 0; j < 3; j = j + 1){
                    x[i*N +j] = 2*x[i*N + j];
          }
}
```

0x0 – 0x3	0x4 - 0x7	0x8-0x11	0x12–0x15	0x16 - 0x19	0x20-0x23

X[0][0]	X[0][1]	X[0][2]	X[1][0]	X[1][1]	X[1][2]	X[2][0]	X[2][1]	X[2][2]

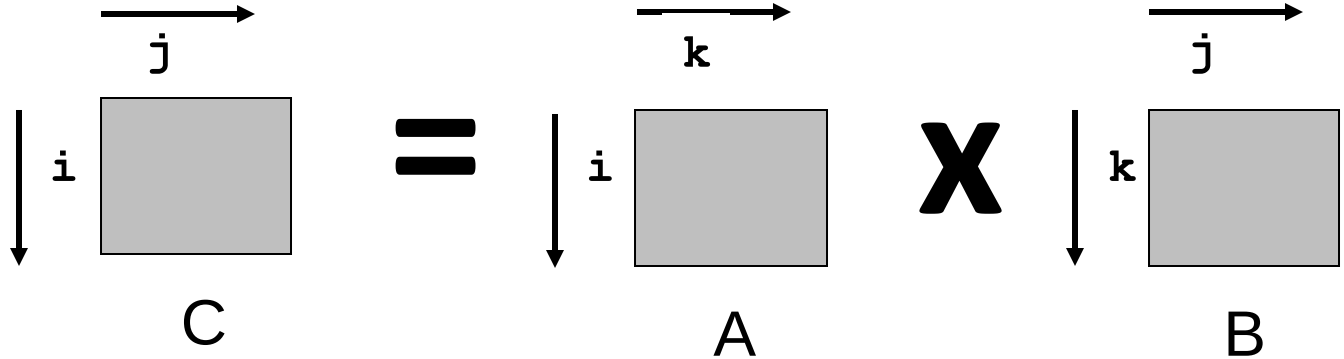
Matrix Multiplication Refresher

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \\ & \end{bmatrix}$$

$$1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 = 58$$

Miss Rate Analysis for Matrix Multiply

- Assume:
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
  - Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



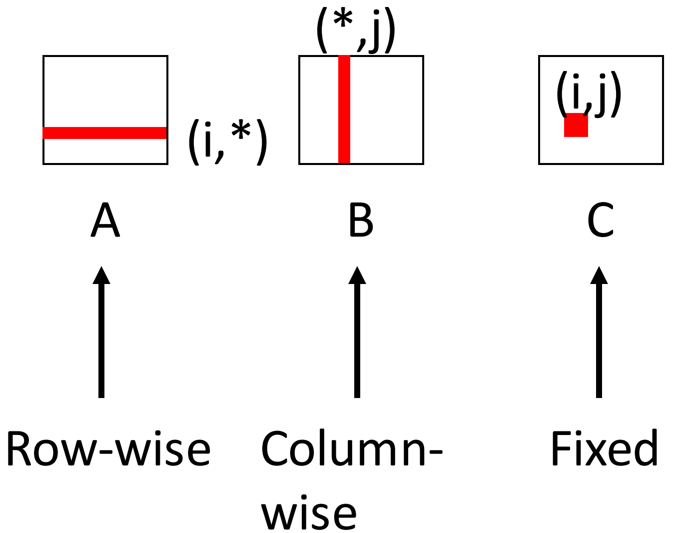
Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through columns in one row:
  - `for (i = 0; i < N; i++)`
    `sum += a[0][i];`
  - accesses successive elements
  - if block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B
- Stepping through rows in one column:
  - `for (i = 0; i < n; i++)`
    `sum += a[i][0];`
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                       matmult/mm.c
```

Inner loop:



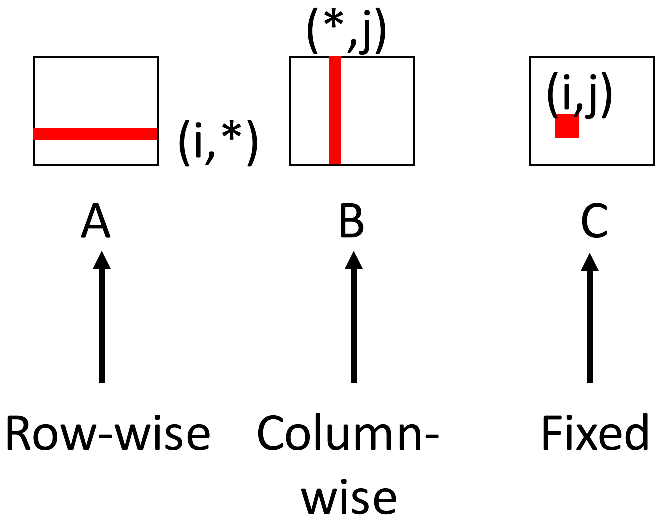
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}                              matmult/mm.c
```

Inner loop:

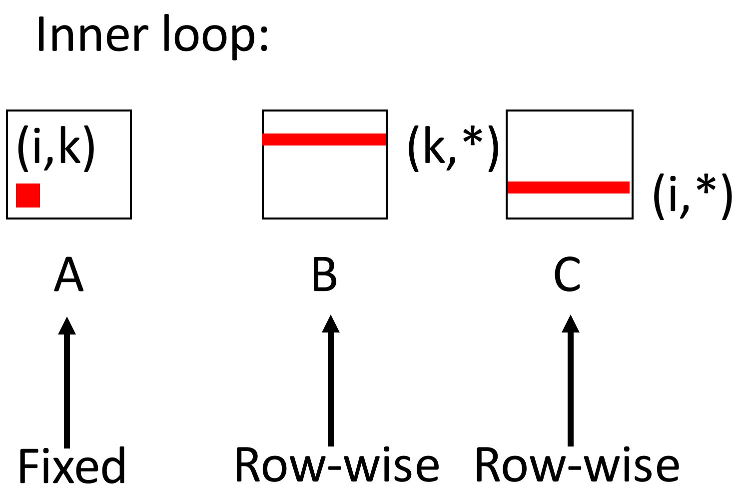


Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}                          matmult/mm.c
```

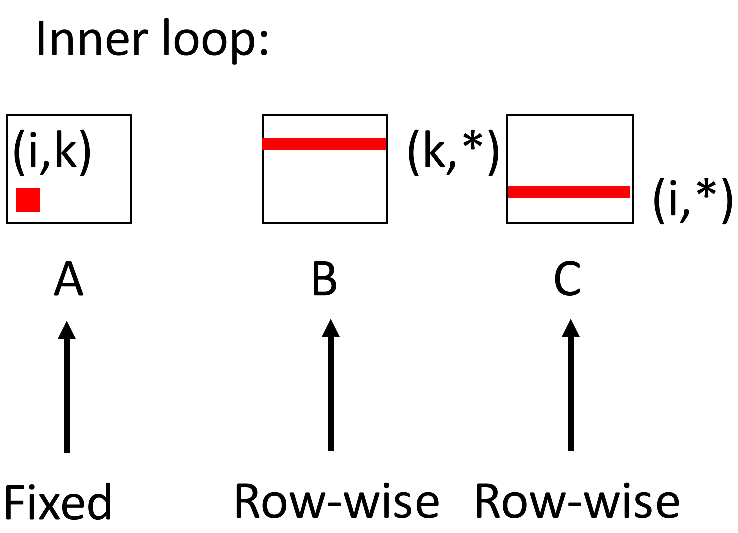


<u>Misses per inner loop iteration:</u>

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
                          matmult/mm.c
```



Misses per inner loop iteration:

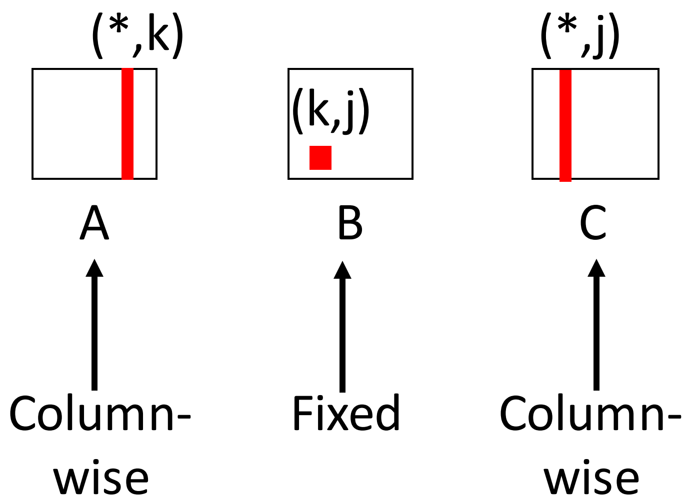
A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

matmult/mm.c

Inner loop:



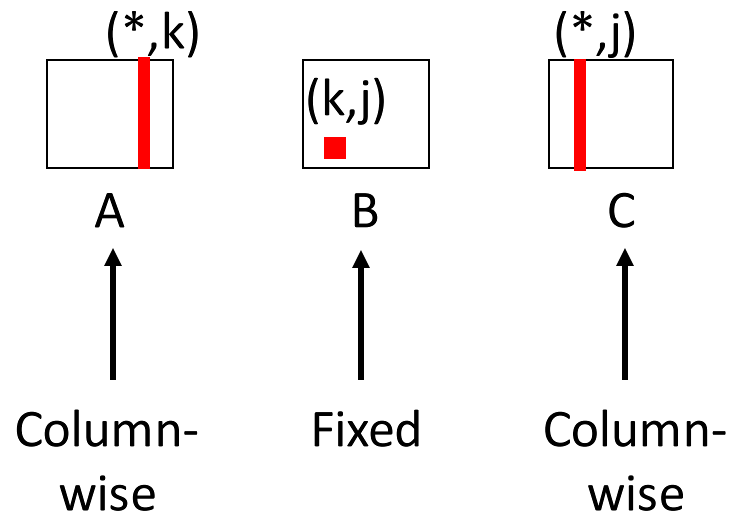
<u>Misses per inner loop iteration:</u>

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}                                matmult/mm.c
```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++) {
     sum += a[i][k] * b[k][j];}
   c[i][j] = sum;
 }
}
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

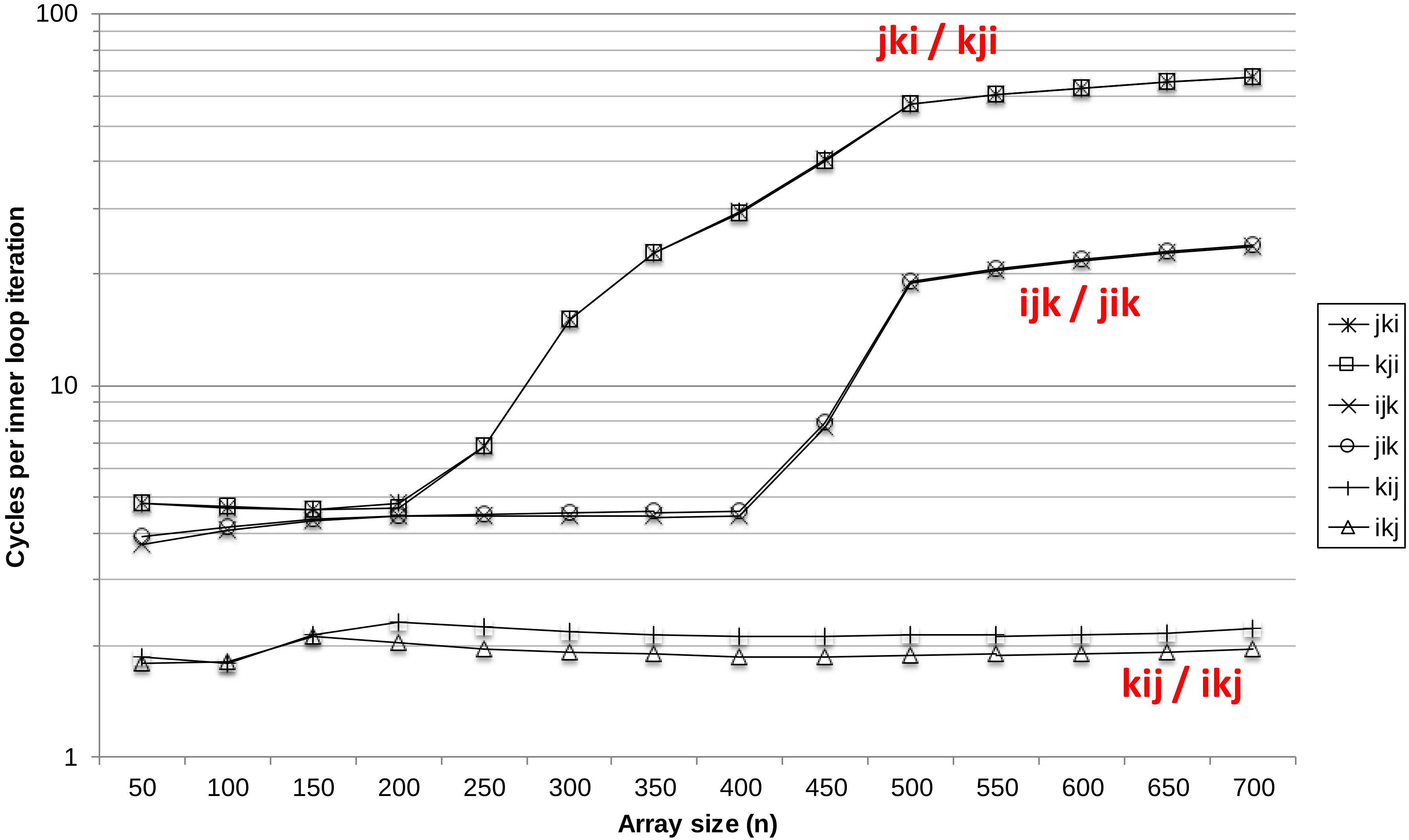
```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++){
   c[i][j] += r * b[k][j];}
 }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++){
    c[i][j] += a[i][k] * r;}
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

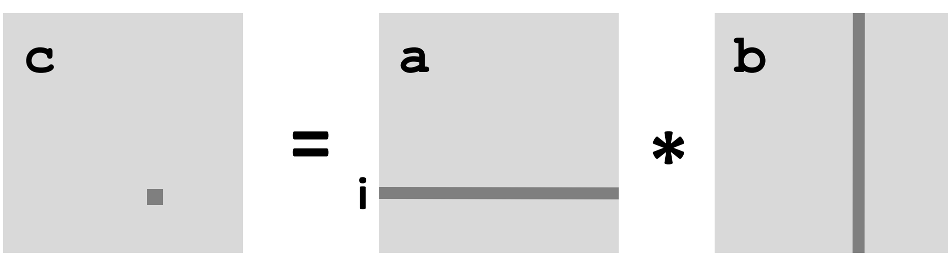
Core i7 Matrix Multiply Performance



Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```

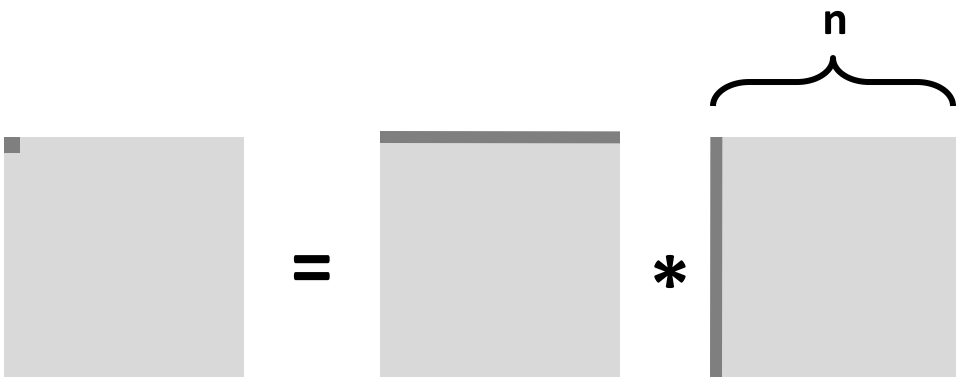


Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Assume the matrix is square
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)

- First iteration:
  - n/8 + n = 9n/8 misses

  - Afterwards in cache:
    (schematic)

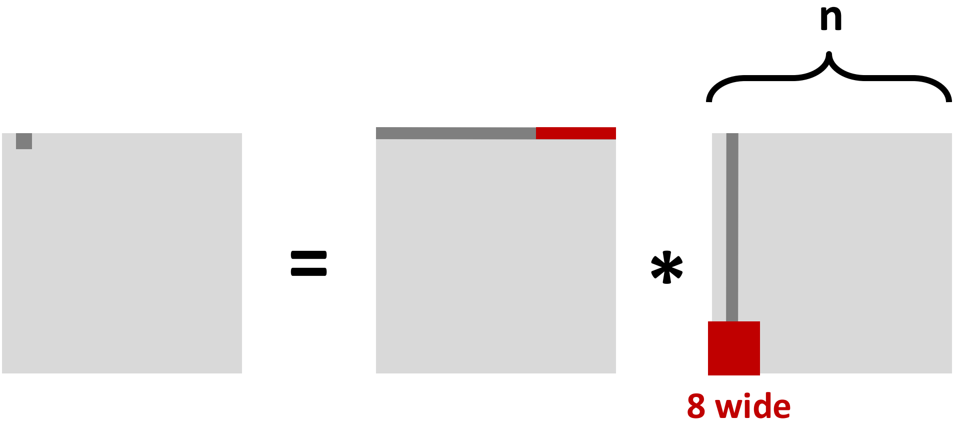


Cache Miss Analysis

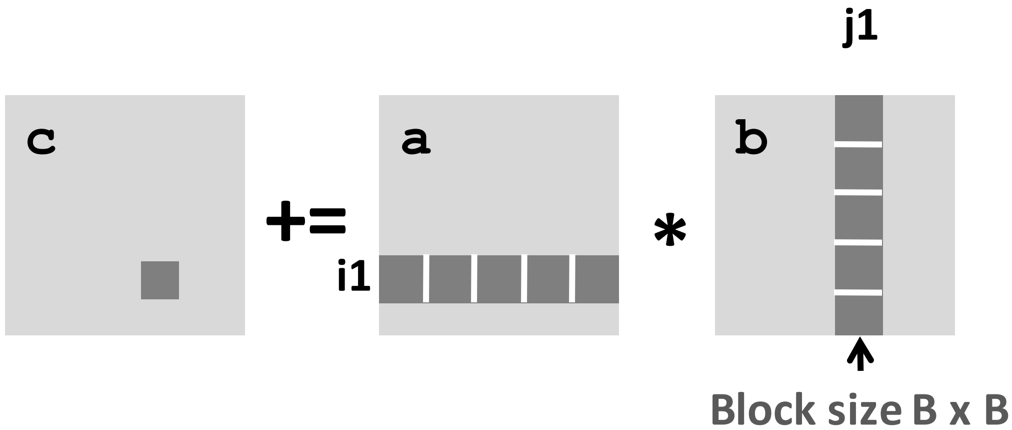
- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)

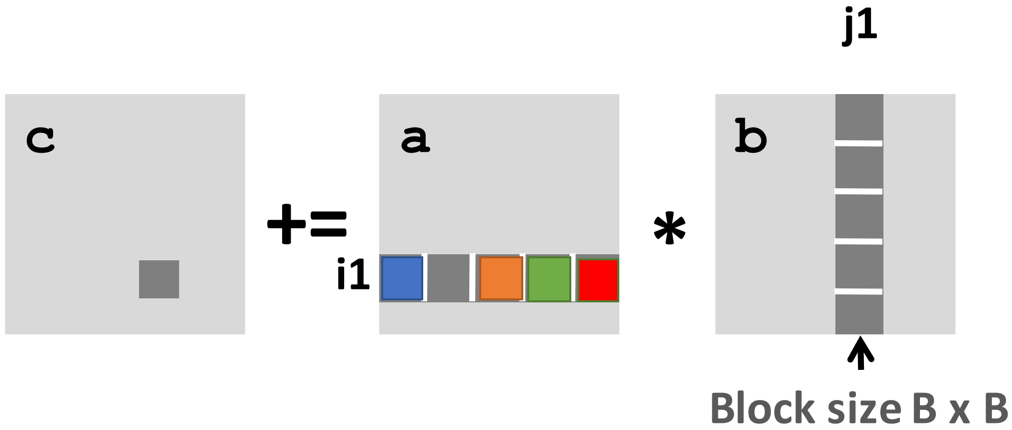
- Second iteration:
  - Again:
    n/8 + n = 9n/8 misses

- Total misses:
  - $9n/8 * n^2 = (9/8) * n^3$

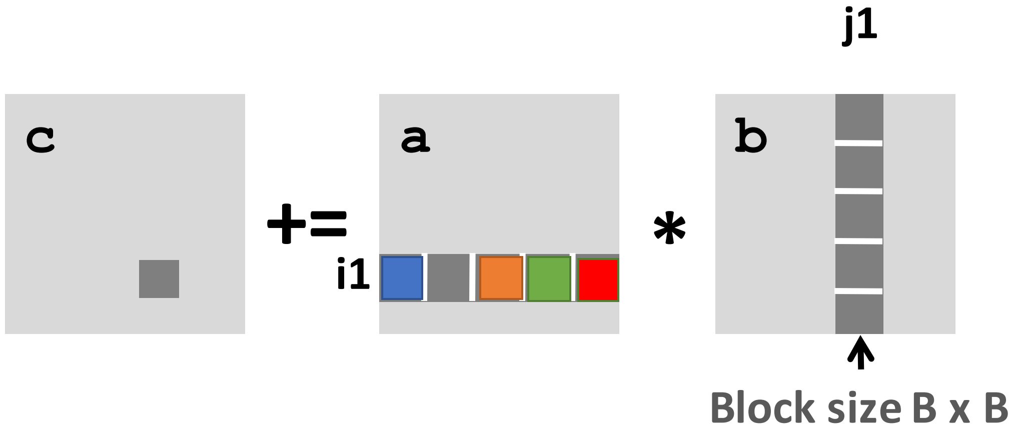


Blocked Matrix Multiplication



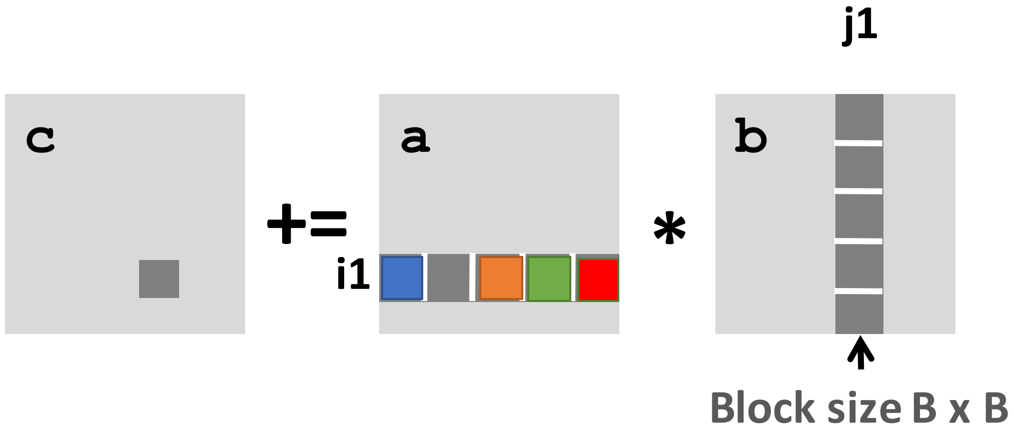


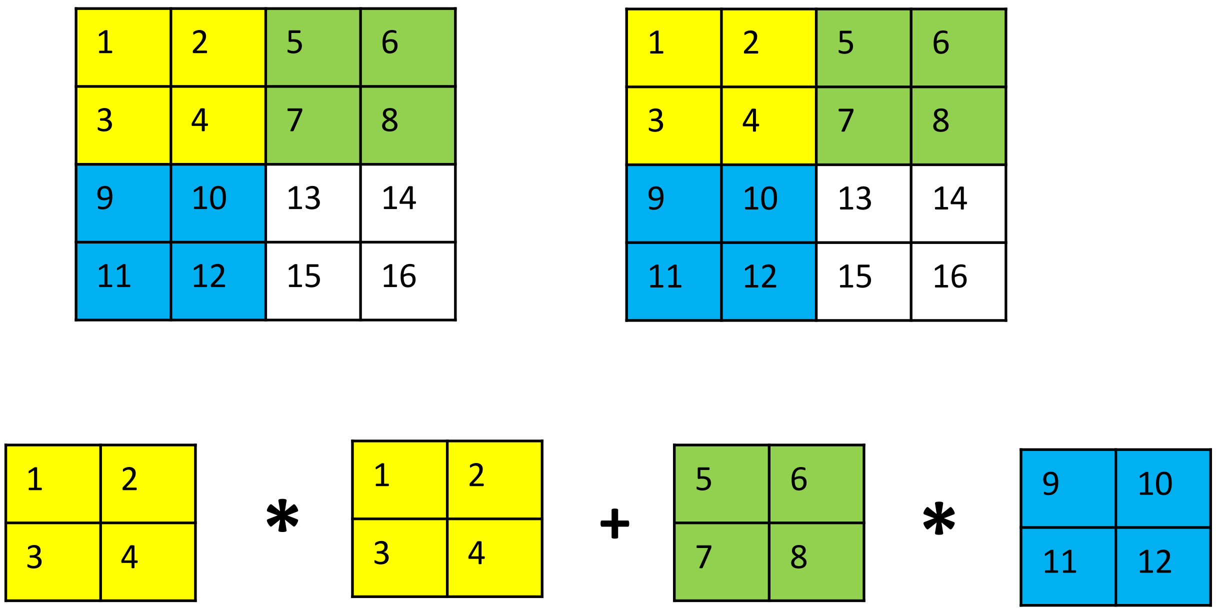
j1
c
+=
a
i1
*
b
Block size B x B

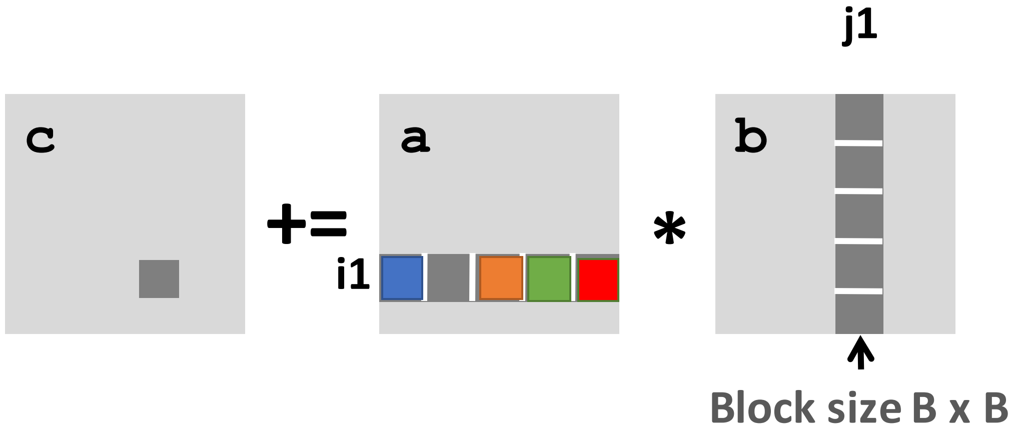


1	2	5	6
3	4	7	8
9	10	13	14
11	12	15	16

1	2	5	6
3	4	7	8
9	10	13	14
11	12	15	16


j1
c
+=
a
i1
*
b
Block size B x B


1 2 5 6
3 4 7 8
9 10 13 14
11 12 15 16
1 2 5 6
3 4 7 8
9 10 13 14
11 12 15 16
1 2
3 4
*
1 2
3 4
+
5 6
7 8
*
9 10
11 12



1	2	5	6
3	4	7	8
9	10	13	14
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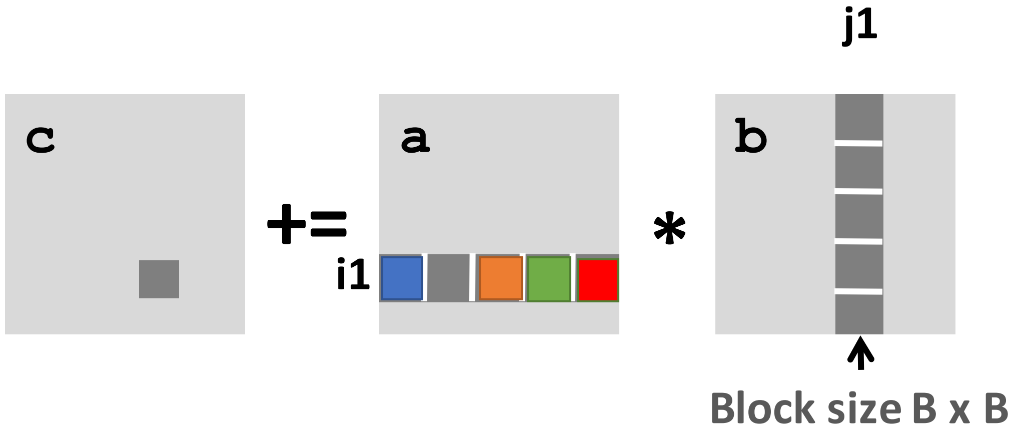
1	2	5	6
3	4	7	8
9	10	13	14
11	12	15	16

$$\begin{bmatrix} 118 & 132 \\ 166 & 188 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} * \begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix}$$



118	132		
166	188		

1	2	5	6
3	4	7	8
9	10	13	14
11	12	15	16

1	2	5	6
3	4	7	8
9	10	13	14
11	12	15	16

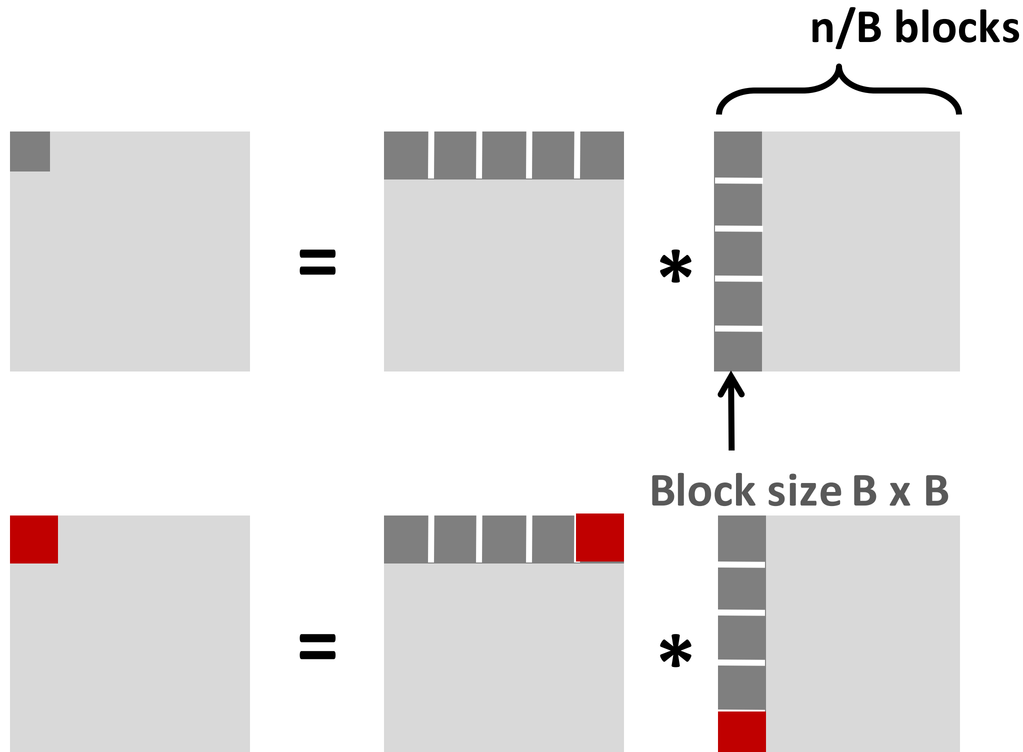
$$\begin{bmatrix} 118 & 132 \\ 166 & 188 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} * \begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix}$$

Cache Miss Analysis

- Assume:
  - Square Matrix
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$ (Where B^2 is the size of B x B block)

- First (block) iteration:
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$
    (omitting matrix c)

  - Afterwards in cache
    (schematic)

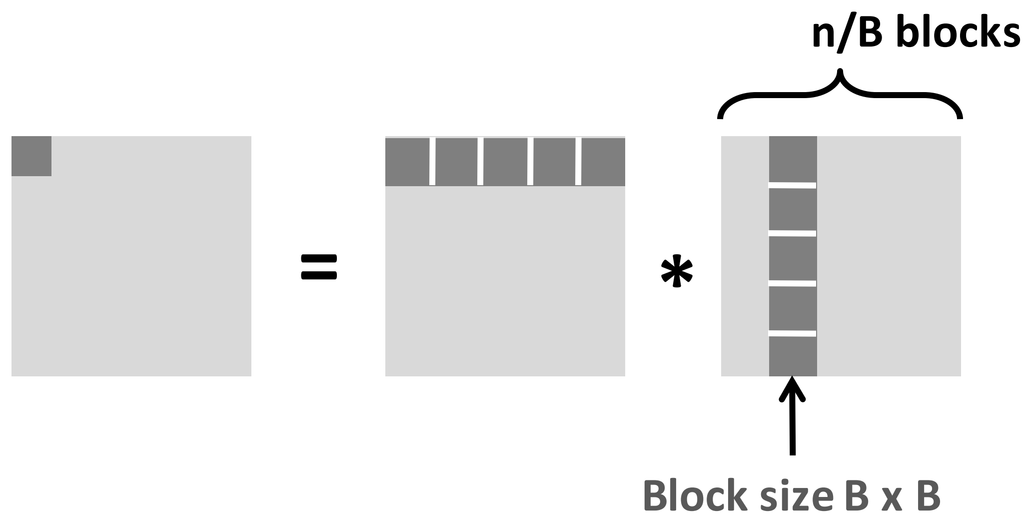


Cache Miss Analysis

- Assume:
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$

- Second (block) iteration:
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$

- Total misses:
  - $nB/4 * (n/B)^2 = n^3/(4B)$



Blocking Summary

- No blocking: $(9/8) * n^3$
- Blocking: $1/(4B) * n^3$

- Suggest largest possible block size B, but limit $3B^2 < C$!

- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

Cache Summary

- Cache memories can have significant performance impact

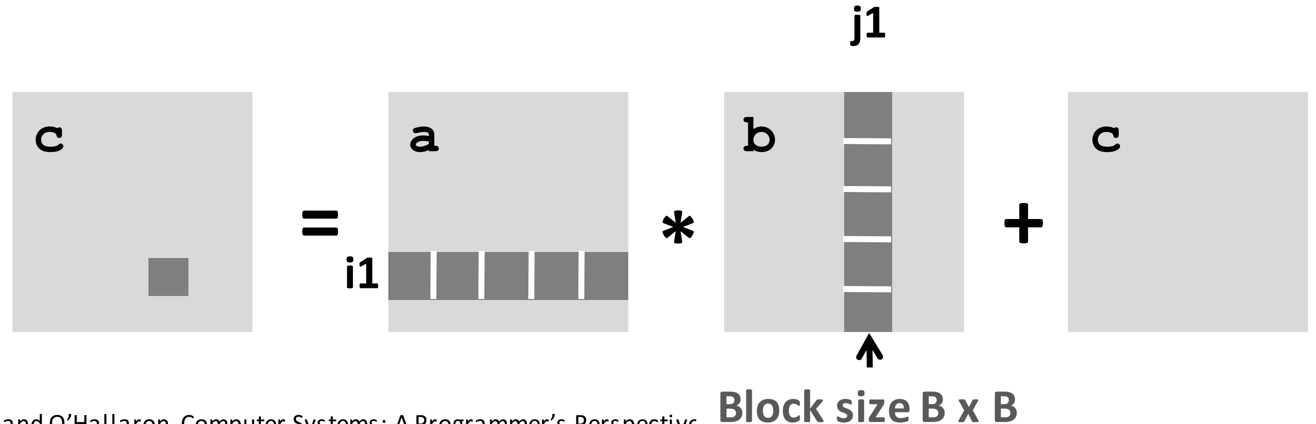
- You can write your programs to exploit this!
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

matmult/bmm.c



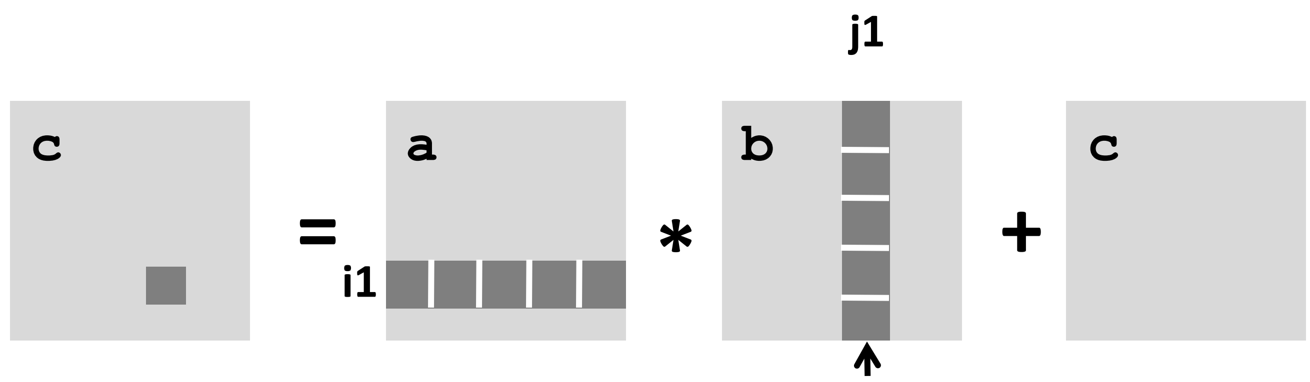
Program Optimization

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
             for (k = 0; k < n; k+=B)
                 /* B x B mini matrix multiplications */
                   for (i1 = i; i1 < i+B; i++)
                       for (j1 = j; j1 < j+B; j++)
                           for (k1 = k; k1 < k+B; k++)
                               c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

matmult/bmm.c



Block size B x B

Compiler Optimizations

Optimizing Compilers

- Provide efficient mapping of program to machine
  - register allocation
  - code selection and ordering (scheduling)
  - dead code elimination
  - eliminating minor inefficiencies
- Don't (usually) improve asymptotic efficiency
  - up to programmer to select best overall algorithm
  - big-O savings are (often) more important than constant factors
    - but constant factors also matter
- Have difficulty overcoming "optimization blockers"
  - potential memory aliasing
  - potential procedure side-effects

Limitations of Optimizing Compilers

- Operate under fundamental constraint
  - Must not cause any change in program behavior
    - Except, possibly when program making use of nonstandard language features
  - Often prevents it from making optimizations that would only affect behavior under edge conditions.

- Most analysis is performed only within procedures
  - Whole-program analysis is too expensive in most cases
  - Newer versions of GCC do interprocedural analysis within individual files
    - But, not between code in different files

- Most analysis is based only on *static* information
  - Compiler has difficulty anticipating run-time inputs

- When in doubt, the compiler must be conservative

example assembly (unoptimized)

```
long sum(long *A, int N) {
    long result = 0;
    for (int i = 0; i < N; ++i)
        result += A[i];
    return result;
}
```

```
sum:    ...
the_loop:
        ...
        leaq    0(,%rax,8), %rdx// offset ← i * 8
        movq    −24(%rbp), %rax // get A from stack
        addq    %rdx, %rax      // add offset
        movq    (%rax), %rax    // get *(A+offset)
        addq    %rax, −8(%rbp)  // add to sum, on stack
        addl    $1, −12(%rbp)   // increment i
condition:
        movl    −12(%rbp), %eax
        cmpl    −28(%rbp), %eax
        jl      the_loop
```

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example assembly (gcc 5.4 -Os)

```
long sum(long *A, int N) {
    long result = 0;
    for (int i = 0; i < N; ++i)
        result += A[i];
    return result;
}
```

```
sum:
        xorl    %edx, %edx
        xorl    %eax, %eax
the_loop:
        cmpl    %edx, %esi
        jle     done
        addq    (%rdi,%rdx,8), %rax
        incq    %rdx
        jmp     the_loop
done:
        ret
```

%edx holds i

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example assembly (gcc 5.4 -O2)

```
long sum(long *A, int N) {
    long result = 0;
    for (int i = 0; i < N; ++i)
        result += A[i];
    return result;
}
sum:
        testl   %esi, %esi
        jle     return_zero
        leal    −1(%rsi), %eax
        leaq    8(%rdi,%rax,8), %rdx // rdx=end of A
        xorl    %eax, %eax
the_loop:
        addq    (%rdi), %rax // add to sum
        addq    $8, %rdi     // advance pointer
        cmpq    %rdx, %rdi
        jne     the_loop
        rep ret
return_zero:    ...
```

```
long *p = A;
long * end = A + N-1;
while( p!= end){
       result+ = p;
       p++;
}
```

Optimization removes i
Makes a more efficient compare
Because were are now testing for equivalence so we can use test.

Also makes the address calculation simpler

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Some categories of optimizations compilers are good at

Generally Useful Optimizations

- Optimizations that you or the compiler should do regardless of processor / compiler

- Code Motion
  - Reduce frequency with which computation performed
    - If it will always produce same result
    - Especially moving code out of loop

```
void set_row(double *a, double *b,
   long i, long n)
{
    long j;
    for (j = 0; j < n; j++)
         a[n*i+j] = b[j];
}
```

→

```
    long j;
    int ni = n*i;
    for (j = 0; j < n; j++)
         a[ni+j] = b[j];
```

Reduction in Strength

- Replace costly operation with simpler one
- Shift, add instead of multiply or divide

```
16*x   -->    x << 4
```

  - Depends on cost of multiply or divide instruction
    - On Intel Nehalem, integer multiply requires 3 CPU cycles
    - https://www.agner.org/optimize/instruction_tables.pdf
- Recognize sequence of products

```
for (i = 0; i < n; i++) {
  int ni = n*i;
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
}
```

→

```
int ni = 0;
for (i = 0; i < n; i++) {
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
  ni += n;
}
```

We can replace multiple operation with and add

Share Common Subexpressions

- Reuse portions of expressions
- GCC will do this with –O1

```
/* Sum neighbors of i,j */
up =    val[(i-1)*n + j  ];
down =  val[(i+1)*n + j  ];
left =  val[i*n     + j-1];
right = val[i*n     + j+1];
sum = up + down + left + right;
```

```
long inj = i*n + j;
up =    val[inj - n];
down =  val[inj + n];
left =  val[inj - 1];
right = val[inj + 1];
sum = up + down + left + right;
```

3 multiplications: i*n, (i–1)*n, (i+1)*n

1 multiplication: i*n

```
leaq   1(%rsi), %rax   # i+1
leaq   -1(%rsi), %r8   # i-1
imulq  %rcx, %rsi      # i*n
imulq  %rcx, %rax      # (i+1)*n
imulq  %rcx, %r8       # (i-1)*n
addq   %rdx, %rsi      # i*n+j
addq   %rdx, %rax      # (i+1)*n+j
addq   %rdx, %r8       # (i-1)*n+j
```

```
imulq    %rcx, %rsi   # i*n
addq     %rdx, %rsi   # i*n+j
movq     %rsi, %rax   # i*n+j
subq     %rcx, %rax   # i*n+j-n
leaq     (%rsi,%rcx), %rcx # i*n+j+n
```

Share Common Subexpressions

- Reuse portions of expressions
- GCC will do this with –O1

Distribute the N

```
/* Sum neighbors of i,j */
up =    val[(i-1)*n + j  ];
down =  val[(i+1)*n + j  ];
left =  val[i*n     + j-1];
right = val[i*n     + j+1];
sum = up + down + left + right;
```

3 multiplications: i*n, (i–1)*n, (i+1)*n

```
long inj = i*n + j;
up =    val[inj - n];
down =  val[inj + n];
left =  val[inj - 1];
right = val[inj + 1];
sum = up + down + left + right;
```

1 multiplication: i*n

Write Compiler Friendly code: Times when the compilers need help

Optimization Blocker #1: Procedure Calls

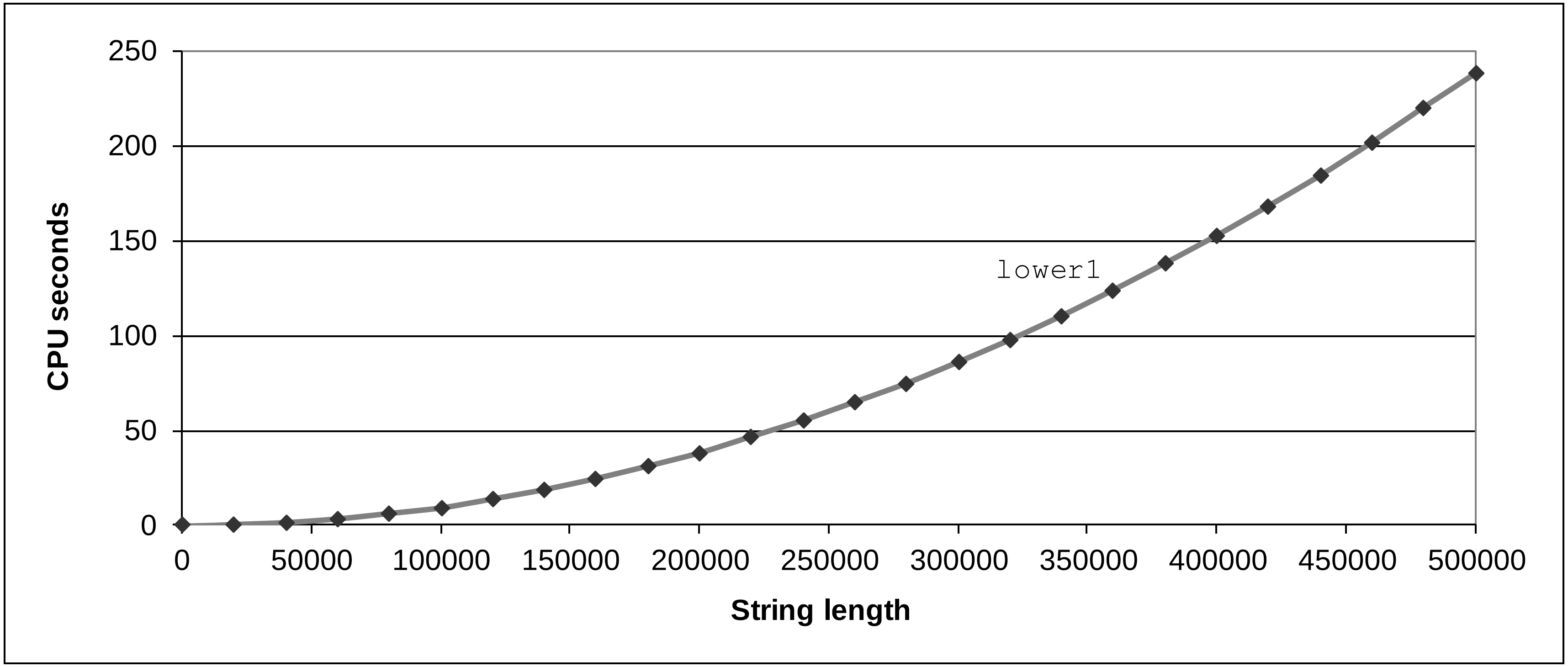
- Procedure to Convert String to Lower Case

```
void lower(char *s)
{
  size_t i;
  for (i = 0; i < strlen(s); i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

A = 65
Z = 90
a = 97
z = 122

Lower Case Conversion Performance

- Time quadruples when double string length
- Quadratic performance



Convert Loop To Goto Form

```
void lower(char *s)
{
   size_t i = 0;
   if (i >= strlen(s))
     goto done;
 loop:
   if (s[i] >= 'A' && s[i] <= 'Z')
       s[i] -= ('A' - 'a');
   i++;
   if (i < strlen(s))
     goto loop;
 done:
}
```

- `strlen` executed every iteration

Calling Strlen

```
/* My version of strlen */
size_t strlen(const char *s)
{
    size_t length = 0;
    while (*s != '\0') {
       s++;
       length++;
    }
    return length;
}
```

- Strlen performance
  - Only way to determine length of string is to scan its entire length, looking for null character.
- Overall performance, string of length N
  - N calls to strlen
  - Require times N, N-1, N-2, …, 1
  - Overall $O(N^2)$ performance

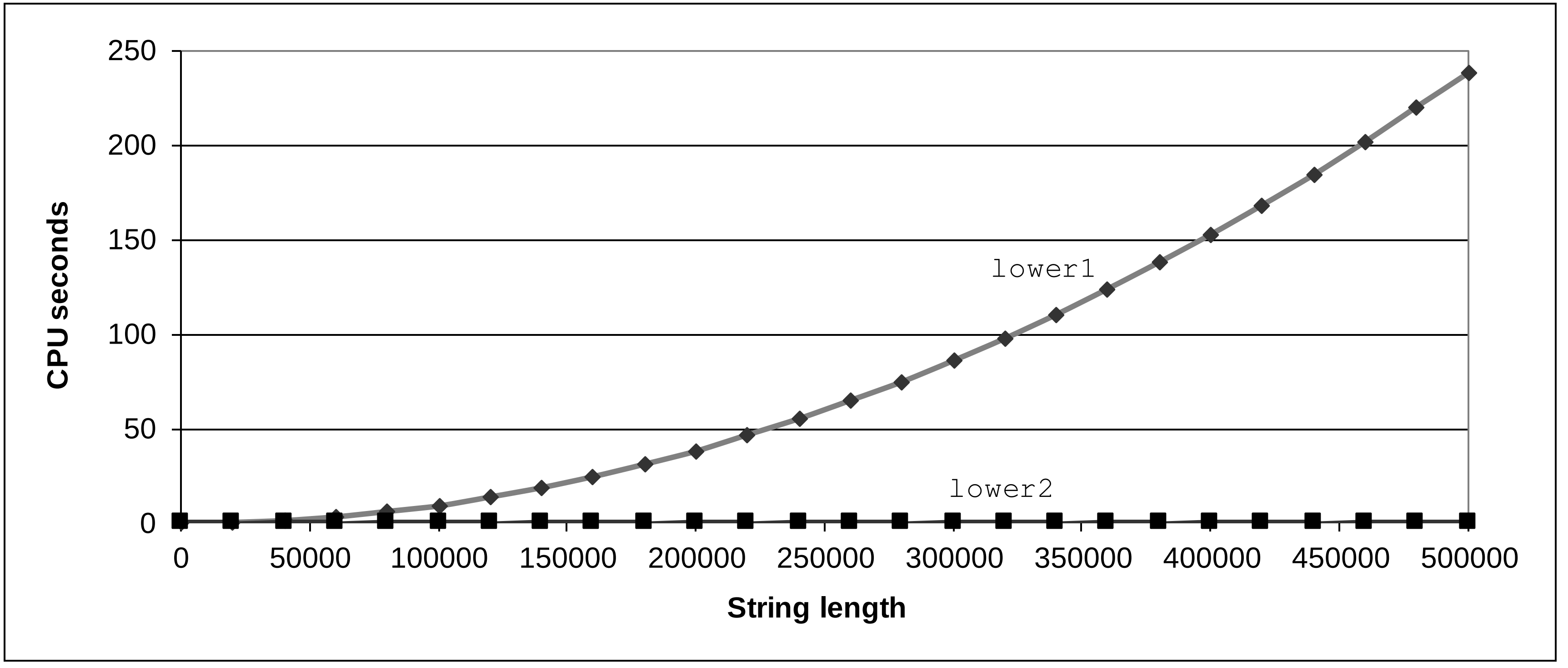
Improving Performance

```
void lower(char *s)
{
  size_t i;
  size_t len = strlen(s);
  for (i = 0; i < len; i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

- Move call to `strlen` outside of loop
- Since result does not change from one iteration to another
- Form of code motion

Lower Case Conversion Performance

- Time doubles when double string length
- Linear performance of lower2



Optimization Blocker: Procedure Calls

- *Why couldn't compiler move* `strlen` *out of inner loop?*
  - Procedure may have side effects
    - Alters global state each time called
  - Function may not return same value for given arguments
    - Depends on other parts of global state
    - Procedure `lower` could interact with `strlen`
- Warning:
  - Compiler treats procedure call as a black box
- Remedies:
  - Do your own code motion

```
size_t lencnt = 0;
size_t strlen(const char *s)
{
    size_t length = 0;
    while (*s != '\0') {
       s++; length++;
    }
    lencnt += length;
    return length;
}
```

loop with a function call

```
int addWithLimit(int x, int y) {
    int total = x + y;
    if (total > 10000)
        return 10000;
    else
        return total;
}
...
int sum(int *array, int n) {
    int sum = 0;
    for (int i = 0; i < n; i++)
        sum = addWithLimit(sum, array[i]);
    return sum;
}
```

3

function call assembly

```
movl (%rbx), %esi // mov array[i]
movl %eax, %edi   // mov sum
call addWithLimit
```

extra instructions executed: two moves, a call, and a ret

manual inlining

```
int sum(int *array, int n) {
    int sum = 0;
    for (int i = 0; i < n; i++) {
        sum = sum + array[i];
        if (sum > 10000)
            sum = 10000;
    }
    return sum;
}
```

compiler inlining

compilers will inline, but...

will usually avoid making code much bigger

- heuristic: inline if function is small enough
- heuristic: inline if called exactly once

will usually not inline across .o files

some compilers allow hints to say "please inline/do not inline this function"

Memory Aliasing

aliasing

```
void twiddle(long *px, long *py) {
    *px += *py;
    *px += *py;
}
```

the compiler **cannot** generate this:

```
twiddle: // BROKEN // %rsi = px, %rdi = py
        movq    (%rdi), %rax // rax ← *py
        addq    %rax, %rax   // rax ← 2 * *py
        addq    %rax, (%rsi) // *px ← 2 * *py
        ret
```

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aliasing problem

```
void twiddle(long *px, long *py) {
    *px += *py;
    *px += *py;
    // NOT the same as *px += 2 * *py;
}
...
    long x = 1;
    twiddle(&x, &x);
    // result should be 4, not 3
```

```
twiddle: // BROKEN // %rsi = px, %rdi = py
        movq    (%rdi), %rax // rax ← *py
        addq    %rax, %rax   // rax ← 2 * *py
        addq    %rax, (%rsi) // *px ← 2 * *py
        ret
```

Another example of Aliasing

Memory Aliasing

```
/* Sum rows is of n X n matrix a
   and store in vector b  */
void sum_rows1(double *a, double *b, long n) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```

- Code updates `b[i]` on every iteration
- Why couldn't compiler optimize this away?

Memory Aliasing

```
/* Sum rows is of n X n matrix a
   and store in vector b  */
void sum_rows1(double *a, double *b, long n) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```

```
double A[9] =
  { 0,   1,   2,
    4,   8,  16},
   32,  64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

Value of B:

```
init:   [4, 8, 16]
```

```
i = 0: [3, 8, 16]
```

```
i = 1: [3, 22, 16]
```

```
i = 2: [3, 22, 224]
```

- Code updates `b[i]` on every iteration
- Must consider possibility that these updates will affect program behavior

Memory Aliasing

```
double A[9] =
  { 0,   1,   2,
    4,   8,  16},
   32,  64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

```
double A[9] =
  { 0,   1,   2,
    3,   8,  16},
   32,  64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

```
double A[9] =
  { 0,   1,   2,
    3,   3,  16},
   32,  64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

```
double A[9] =
  { 0,   1,   2,
    3,   6,  16},
   32,  64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

```
double A[9] =
  { 0,   1,   2,
    3,   6,  22},
   32,  64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

Value of B:

```
init:   [4, 8, 16]
```

```
i = 0: [3, 8, 16]
```

```
i = 1: [3, 22, 16]
```

```
i = 2: [3, 22, 224]
```

- Code updates `b[i]` on every iteration
- Must consider possibility that these updates will affect program behavior

Memory Matters

```
/* Sum rows is of n X n matrix a
   and store in vector b  */
void sum_rows1(double *a, double *b, long n) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```

- Code updates `b[i]` on every iteration
- Why couldn't compiler optimize this away?

```
/* Sum rows is of n X n matrix a
   and store in vector b  */
void sum_rows1(double *a, double *b, long n) {
    long i, j;
    for (i = 0; i < n; i++) {
        sum = 0;
        for (j = 0; j < n; j++)
            sum += a[i*n + j];
        b[i] = sum
    }
}
```

Optimization Blocker: Memory Aliasing

- Aliasing
  - Two different memory references specify single location
  - Easy to have happen in C
    - Since allowed to do address arithmetic
    - Direct access to storage structures
  - Get in habit of introducing local variables
    - Accumulating within loops
    - Your way of telling compiler not to check for aliasing

Loop unrolling

loop unrolling (C)

```c
for (int i = 0; i < N; ++i)
    sum += A[i];
```

```c
int i;
for (i = 0; i + 1 < N; i += 2) {
    sum += A[i];
    sum += A[i+1];
}
// handle leftover, if needed
if (i < N)
    sum += A[i];
```

loop unrolling (ASM)

```
loop:
        cmpl    %edx, %esi
        jle     endOfLoop
        addq    (%rdi,%rdx,8), %rax
        incq    %rdx
        jmp
endOfLoop:
```

```
loop:
        cmpl    %edx, %esi
        jle     endOfLoop
        addq    (%rdi,%rdx,8), %rax
        addq    8(%rdi,%rdx,8), %rax
        addq    $2, %rdx
        jmp     loop
        // plus handle leftover?
endOfLoop:
```

more loop unrolling (C)

```
int i;
for (i = 0; i + 4 <= N; i += 4) {
    sum += A[i];
    sum += A[i+1];
    sum += A[i+2];
    sum += A[i+3];
}
// handle leftover, if needed
for (; i < N; i += 1)
    sum += A[i];
```

automatic loop unrolling

loop unrolling is easy for compilers

…but often not done or done very much

why not?

slower if small number of iterations

larger code — could exceed instruction cache space

loop unrolling performance

on my laptop with 992 elements (fits in L1 cache)

times unrolled	cycles/element	instructions/element
1	1.33	4.02
2	1.03	2.52
4	1.02	1.77
8	1.01	1.39
16	1.01	1.21
32	1.01	1.15

1.01 cycles/element — latency bound