Changelog

27 October 2020: correct quiz answer review slide to mark sets correctly

27 October 2020: counting misses: version 1: correct N^2 to $N^2 \div$ block size

29 October 2020: simple blocking — counting misses: correct off-by-factor-of-two error in misses for C

last time

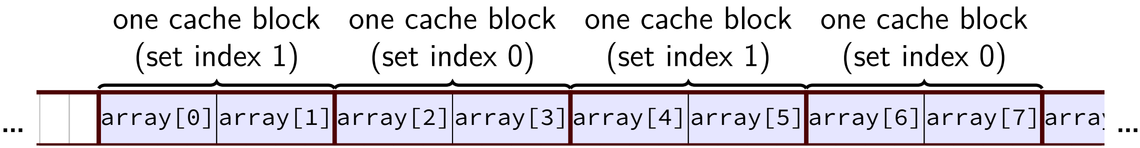
cache tradeoffs in terms of
- hit rate/miss rate
- types of misses mitigated/helped
- hit time
- miss penalty

what accesses use the cache?

alignment — avoid crossing cache lines

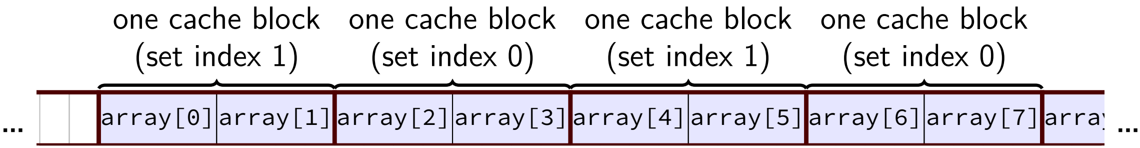
counting cache misses from C code

quiz exercise solution



memory access	set 0 afterwards	set 1 afterwards
—	(empty)	(empty)
read array[0] (miss)	{array[0], array[1]}	(empty)
read array[3] (miss)	{array[0], array[1]}	{array[2], array[3]}
read array[6] (miss)	{array[0], array[1]}	{array[6], array[7]}
read array[1] (hit)	{array[0], array[1]}	{array[6], array[7]}
read array[4] (miss)	{array[4], array[5]}	{array[6], array[7]}
read array[7] (hit)	{array[4], array[5]}	{array[6], array[7]}
read array[2] (miss)	{array[4], array[5]}	{array[2], array[3]}
read array[5] (hit)	{array[4], array[5]}	{array[6], array[7]}
read array[8] (miss)	{array[8], array[9]}	{array[6], array[7]}

quiz exercise solution



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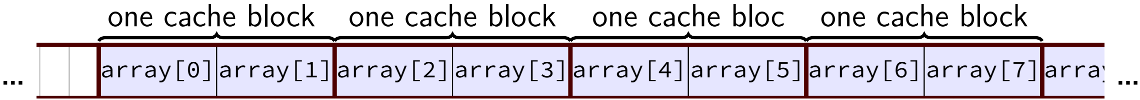
read array[1] (hit)	{array[0], array[1]}	{array[6], array[7]}
read array[4] (miss)	{array[4], array[5]}	{array[6], array[7]}
read array[7] (hit)	{array[4], array[5]}	{array[6], array[7]}
read array[2] (miss)	{array[4], array[5]}	{array[2], array[3]}
read array[5] (hit)	{array[4], array[5]}	{array[6], array[7]}
read array[8] (miss)	{array[8], array[9]}	{array[6], array[7]}

quiz exercise solution

	one cache block (set index 1)		one cache block (set index 0)		one cache block (set index 1)		one cache block (set index 0)		
...	array[0]	array[1]	array[2]	array[3]	array[4]	array[5]	array[6]	array[7]	arra ...

memory access	set 0 afterwards	**set 1 afterwards**
—	(empty)	(empty)
read array[0] (miss)	{array[0], array[1]}	(empty)
read array[3] (miss)	{array[0], array[1]}	{array[2], array[3]}
read array[6] (miss)	{array[0], array[1]}	{array[6], array[7]}
read array[1] (hit)	{array[0], array[1]}	{array[6], array[7]}
read array[4] (miss)	{array[4], array[5]}	{array[6], array[7]}
read array[7] (hit)	{array[4], array[5]}	{array[6], array[7]}
read array[2] (miss)	{array[4], array[5]}	{array[2], array[3]}
read array[5] (hit)	{array[4], array[5]}	{array[6], array[7]}
read array[8] (miss)	{array[8], array[9]}	{array[6], array[7]}

not the quiz problem



if 1-set 2-way cache instead of 2-set 1-way cache:

memory access	**single set with 2-ways, LRU first**
—	---, ---
read `array[0]` (miss)	`---, {array[0], array[1]}`
read `array[3]` (miss)	`{array[0], array[1]}, {array[2], array[3]}`
read `array[6]` (miss)	`{array[2], array[3]}, {array[6], array[7]}`
read `array[1]` (miss)	`{array[6], array[7]}, {array[0], array[1]}`
read `array[4]` (miss)	`{array[0], array[1]}, {array[3], array[4]}`
read `array[7]` (miss)	`{array[3], array[4]}, {array[6], array[7]}`
read `array[2]` (miss)	`{array[6], array[7]}, {array[2], array[3]}`
read `array[5]` (miss)	`{array[2], array[3]}, {array[5], array[6]}`
read `array[8]` (miss)	`{array[5], array[6]}, {array[8], array[9]}`

approximate miss analysis

very tedious to precisely count cache misses

- even more tedious when we take advanced cache optimizations into account

instead, approximations:

good or bad temporal/spatial locality

- good temporal locality: value stays in cache
- good spatial locality: use all parts of cache block

with nested loops: what does inner loop use?

- intuition: values used in inner loop loaded into cache once
- (that is, once each time the inner loop is run)
- …if they can all fit in the cache

approximate miss analysis

very tedious to precisely count cache misses

- even more tedious when we take advanced cache optimizations into account

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- good spatial locality: use all parts of cache block

with nested loops: what does inner loop use?

- intuition: values used in inner loop loaded into cache once
- (that is, once each time the inner loop is run)
- …if they can all fit in the cache

locality exercise (1)

```
/* version 1 */
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]

/* version 2 */
for (int j = 0; j < N; ++j)
    for (int i = 0; i < N; ++i)
        A[i] += B[j] * C[i * N + j];
```

exercise: which has better temporal locality in A? in B? in C?

how about spatial locality?

exercise: miss estimating (1)

```
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]
```

Assume: 4 array elements per block, N very large, nothing in cache at beginning.

Example: $N/4$ estimated misses for A accesses:

A[i] should always be hit on all but first iteration of inner-most loop.
first iter: A[i] should be hit about 3/4s of the time (same block as A[i-1] that often)

Exericse: estimate $\#$ of misses for B, C

a note on matrix storage

A — $N \times N$ matrix

represent as array

makes dynamic sizes easier:

```
float A_2d_array[N][N];
float *A_flat = malloc(N * N);
```

```
A_flat[i * N + j] === A_2d_array[i][j]
```

convertion re: rows/columns

going to call the first index rows

$A_{i,j}$ is A row i, column j

rows are stored together

this is an arbitrary choice

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

if array starts on cache block
first cache block = first elements
all together in one row!

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

second cache block:
1 from row 0
3 from row 1

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

5x5 array and 4-element cache blocks

array[0*5 + 0]	array[0*5 + 1]	array[0*5 + 2]	array[0*5 + 3]	array[0*5 + 4]
array[1*5 + 0]	array[1*5 + 1]	array[1*5 + 2]	array[1*5 + 3]	array[1*5 + 4]
array[2*5 + 0]	array[2*5 + 1]	array[2*5 + 2]	array[2*5 + 3]	array[2*5 + 4]
array[3*5 + 0]	array[3*5 + 1]	array[3*5 + 2]	array[3*5 + 3]	array[3*5 + 4]
array[4*5 + 0]	array[4*5 + 1]	array[4*5 + 2]	array[4*5 + 3]	array[4*5 + 4]

generally: cache blocks contain data from 1 or 2 rows
$\rightarrow$ better performance from reusing rows

matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j */
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i * N + j] += A[i * N + k] * B[k * N + j];
```

matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

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for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

loop orders and locality

loop body: $C_{ij}+ = A_{ik}B_{kj}$

kij order: C_{ij}, B_{kj} have spatial locality

kij order: A_{ik} has temporal locality

… better than …

ijk order: A_{ik} has spatial locality

ijk order: C_{ij} has temporal locality

loop orders and locality

loop body: $C_{ij} += A_{ik} B_{kj}$

$ki\color{red}{j}$ order: $C_{i\color{red}{j}}$, $B_{k\color{red}{j}}$ have spatial locality

kij order: A_{ik} has temporal locality

... better than ...

$ij\color{red}{k}$ order: $A_{i\color{red}{k}}$ has spatial locality

ijk order: C_{ij} has temporal locality

matrix multiply

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for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

which is better?

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
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/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```
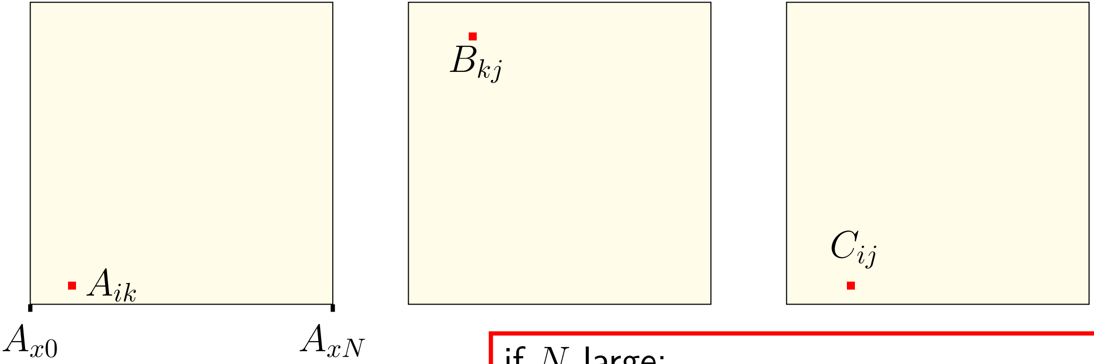

exercise: Which version has better spatial/temporal locality for...

...accesses to C? ...accesses to A? ...accesses to B?

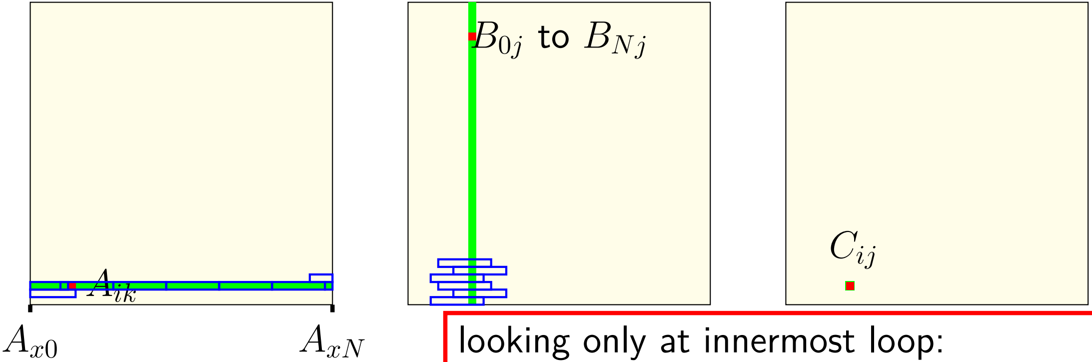
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

if N large:
using C_{ij} many times per load into cache
using A_{ik} once per load-into-cache
(but using $A_{i,k+1}$ right after)
using B_{kj} once per load into cache

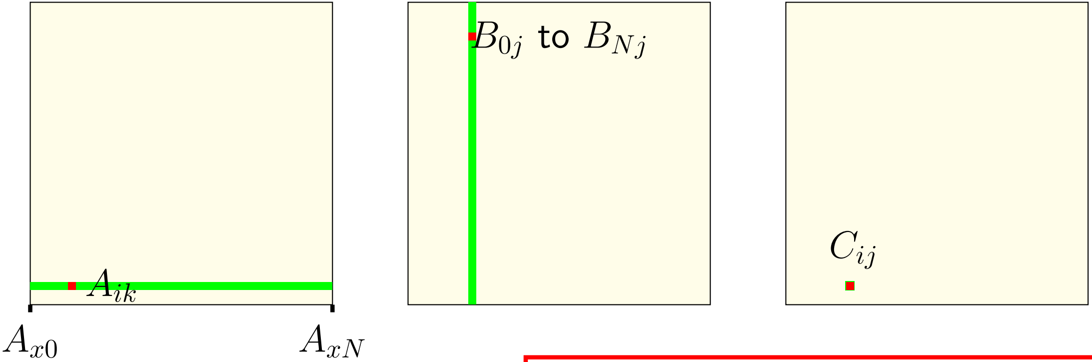
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
good spatial locality in A
(rows stored together = reuse cache blocks)
bad spatial locality in B
(use each cache block once)
no useful spatial locality in C

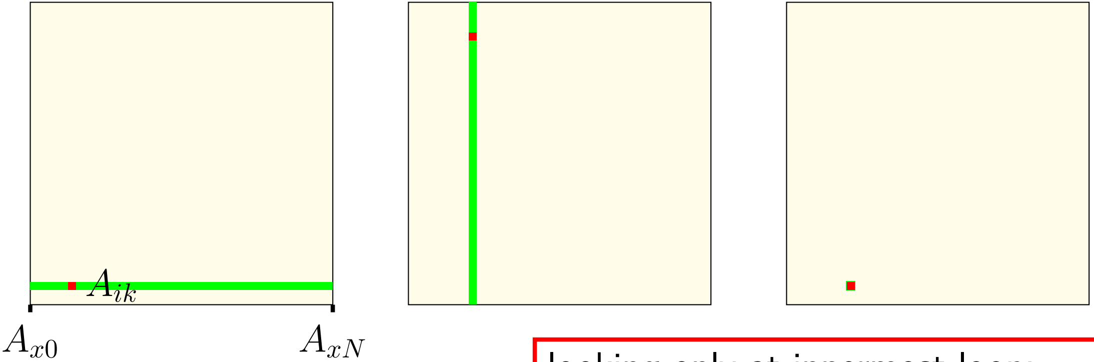
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
temporal locality in C
bad temporal locality in everything else
(everything accessed exactly once)

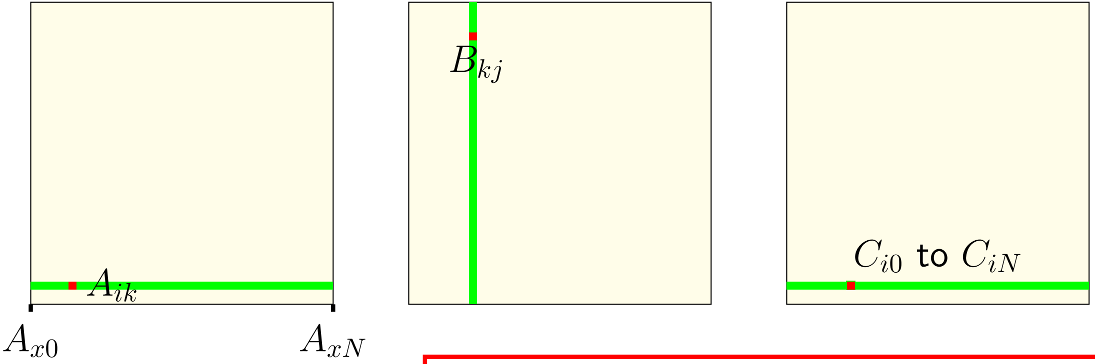
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
row of A (elements used once)
column of B (elements used once)
single element of C (used many times)

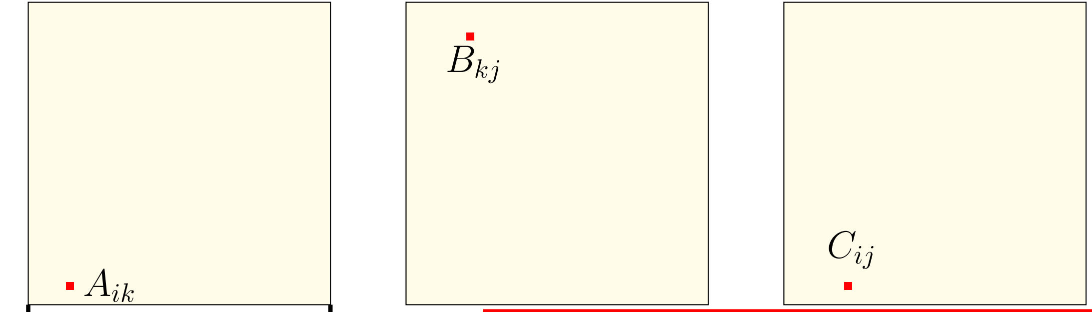
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at two innermost loops together:
some temporal locality in A (column reused)
some temporal locality in B (row reused)
some temporal locality in C (row reused)

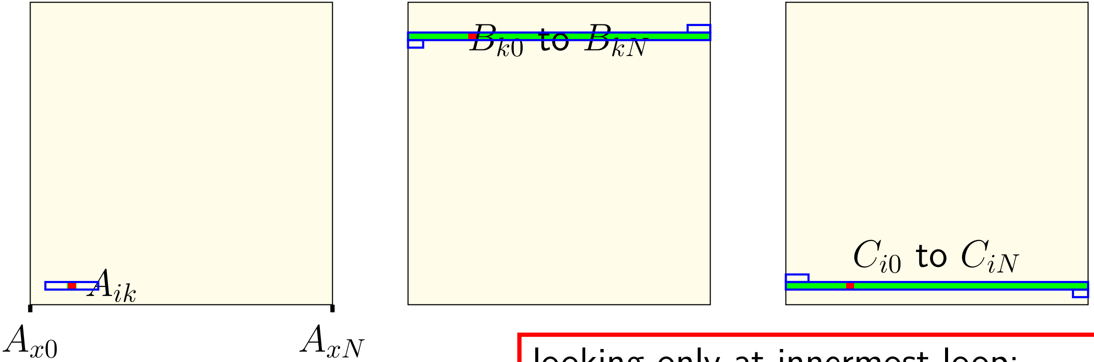
array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

if N large:
using C_{ij} once per load into cache
(but using $C_{i,j+1}$ right after)
using A_{ik} many times per load-into-cache
using B_{kj} once per load into cache
(but using $B_{k,j+1}$ right after)

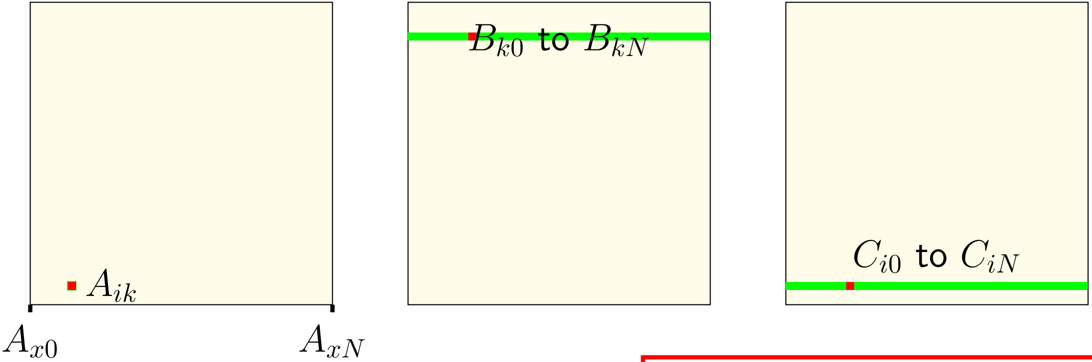
array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
spatial locality in B, C
(use most of loaded B, C cache blocks)
no useful spatial locality in A
(rest of A's cache block wasted)

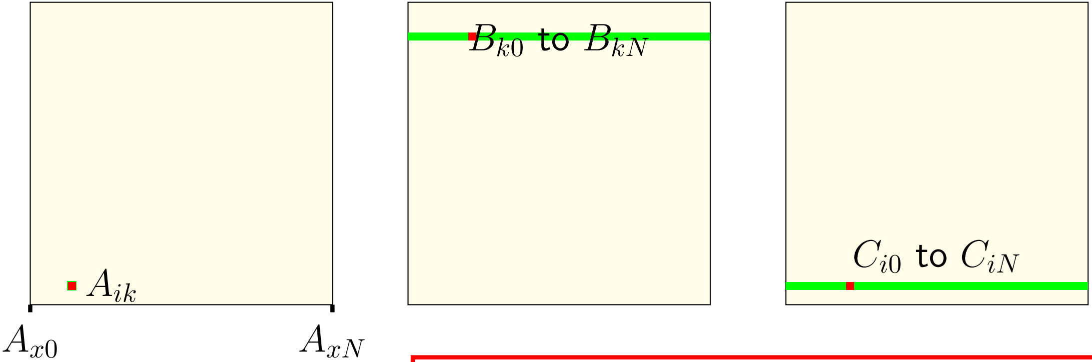
array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
temporal locality in A
no temporal locality in B, C
(B, C values used exactly once)

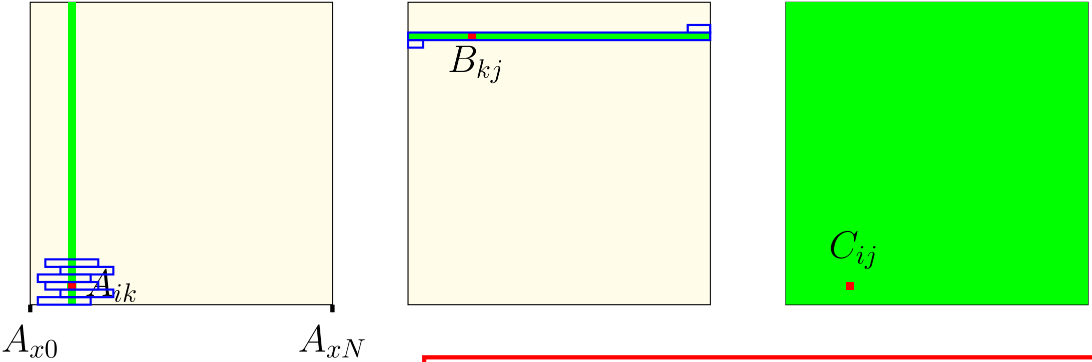
array usage: kij order



for all k:
    for all i:
        for all j:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at innermost loop:
processing one element of A (use many times)
row of B (each element used once)
column of C (each element used once)

array usage: kij order



for all k:
  for all i:
    for all j:
      $C_{ij} += A_{ik} \times B_{kj}$

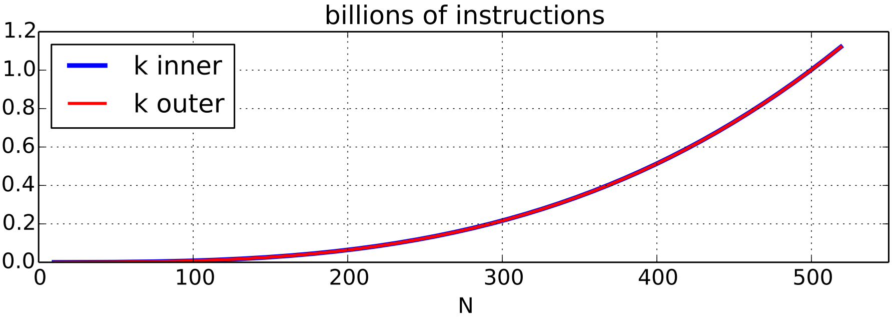
looking only at two innermost loops together:
good temporal locality in A (column reused)
good temporal locality in B (row reused)
bad temporal locality in C (nothing reused)

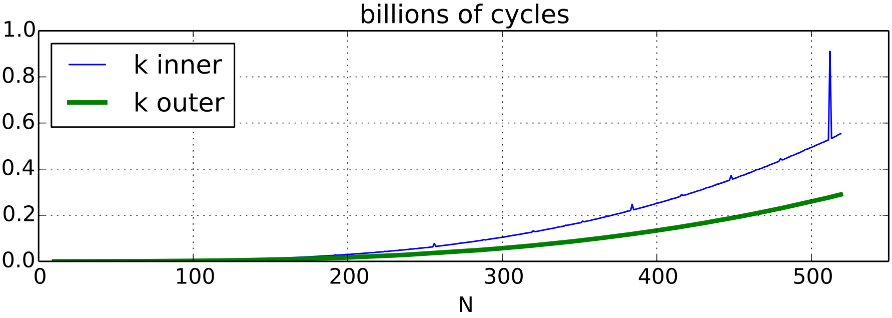
matrix multiply

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

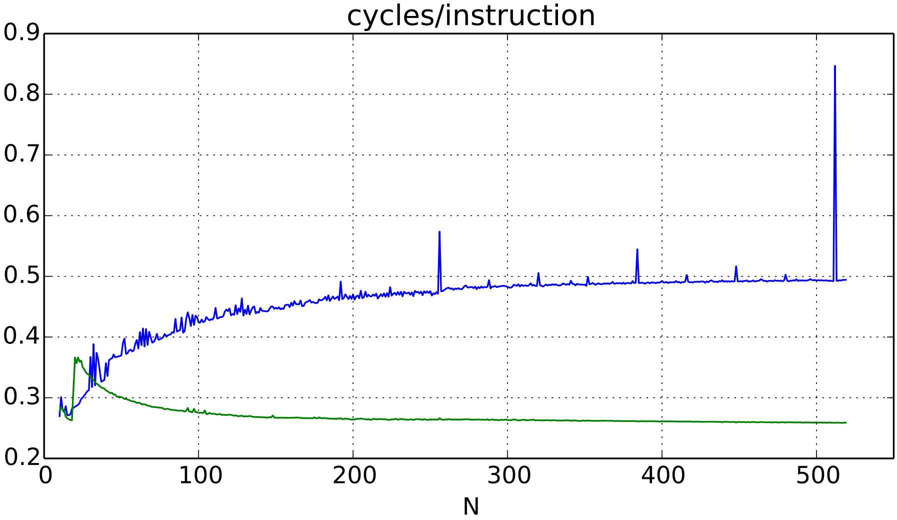
```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i*N+j] += A[i * N + k] * B[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i*N+j] += A[i * N + k] * B[k * N + j];
```

performance (with A=B)

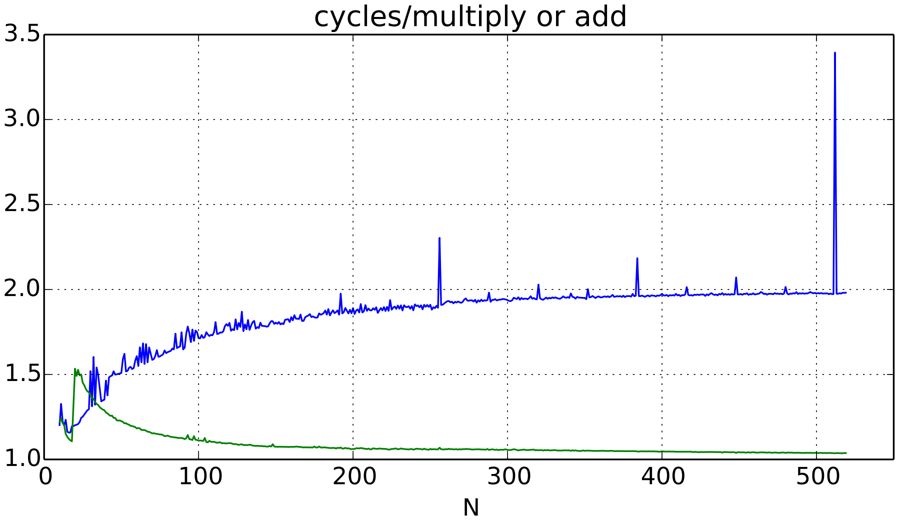




alternate view 1: cycles/instruction



alternate view 2: cycles/operation



counting misses: version 1

```
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      C[i * N + j] += A[i * N + k] * B[k * N + j];
```

if N really large

- assumption: can't get close to storing N values in cache at once

for A: about $N \div$ block size misses per k-loop

- total misses: $N^3 \div$ block size

for B: about N misses per k-loop

- total misses: N^3

for C: about $1 \div$ block size miss per k-loop

- total misses: $N^2 \div$ block size

counting misses: version 2

```
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      C[i * N + j] += A[i * N + k] * B[k * N + j];
```

for A: about 1 misses per j-loop

total misses: N^2

for B: about N $\div$ block size miss per j-loop

total misses: $N^3 \div$ block size

for C: about N $\div$ block size miss per j-loop

total misses: $N^3 \div$ block size

exercise: miss estimating (2)

```
for (int k = 0; k < 1000; k += 1)
    for (int i = 0; i < 1000; i += 1)
        for (int j = 0; j < 1000; j += 1)
            A[k*N+j] += B[i*N+j];
```

assuming: 4 elements per block

assuming: cache not close to big enough to hold 1K elements

estimate: *approximately* how many misses for A, B?

locality exercise (2)

```
/* version 2 */
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        A[i] += B[j] * C[i * N + j]

/* version 3 */
for (int ii = 0; ii < N; ii += 32)
    for (int jj = 0; jj < N; jj += 32)
        for (int i = ii; i < ii + 32; ++i)
            for (int j = jj; j < jj + 32; ++j)
                A[i] += B[j] * C[i * N + j];
```

exercise: which has better temporal locality in A? in B? in C?

how about spatial locality?

a transformation

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
          C[i*N+j] += A[i*N+k] * B[k*N+j];
```

split the loop over k — should be exactly the same

(assuming even N)

a transformation

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
          C[i*N+j] += A[i*N+k] * B[k*N+j];
```

split the loop over k — should be exactly the same
(assuming even N)

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; ++i)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            C[i*N+j] += A[i*N+k] * B[k*N+j];
```

now reorder split loop — same calculations

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; ++i)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            C[i*N+j] += A[i*N+k] * B[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; ++i)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            C[i*N+j] += A[i*N+k] * B[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

Temporal locality in C_{ij}s

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

More spatial locality in A_{ik}

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
  }
}
```

Still have good spatial locality in B_{kj}, C_{ij}

counting misses for A (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for A:

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)

A[1*N+0], A[0*N+1], A[0*N+0], A[1*N+1] …(repeats N times)

…

…

counting misses for A (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for A:

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)

A[1*N+0], A[0*N+1], A[0*N+0], A[1*N+1] …(repeats N times)

…

A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] …

A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] …

…

counting misses for A (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for A:

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)

A[1*N+0], A[0*N+1], A[0*N+0], A[1*N+1] …(repeats N times)

…

A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] …

A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] …

…

counting misses for A (2)

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] ...(repeats N times)
A[1*N+0], A[0*N+1], A[0*N+0], A[1*N+1] ...(repeats N times)

...

...

counting misses for A (2)

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] …(repeats N times)
A[1*N+0], A[0*N+1], A[0*N+0], A[1*N+1] …(repeats N times)
…
A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] …
A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] …
…

likely cache misses: only first iterations of j loop

how many cache misses per iteration? usually one

A[0*N+0] and A[0*N+1] usually in same cache block

counting misses for A (2)

A[0*N+0], A[0*N+1], A[0*N+0], A[0*N+1] ...(repeats N times)
A[1*N+0], A[0*N+1], A[0*N+0], A[1*N+1] ...(repeats N times)
...
A[(N-1)*N+0], A[(N-1)*N+1], A[(N-1)*N+0], A[(N-1)*N+1] ...
A[0*N+2], A[0*N+3], A[0*N+2], A[0*N+3] ...
...

likely cache misses: only first iterations of j loop

how many cache misses per iteration? usually one

- A[0*N+0] and A[0*N+1] usually in same cache block

about $\frac{N}{2} \cdot N$ misses total

counting misses for B (1)

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

access pattern for B:

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]

B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]

B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]

...

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]

...

counting misses for B (2)

access pattern for B:

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]

...

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]

...

counting misses for B (2)

access pattern for B:

B[0*N+0], B[1*N+0], …B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], …B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], …B[4*N+(N-1)], B[5*N+(N-1)]

…

B[0*N+0], B[1*N+0], …B[0*N+(N-1)], B[1*N+(N-1)]

…

likely cache misses: any access, each time

counting misses for B (2)

access pattern for B:

B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]
...
B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
...

likely cache misses: any access, each time

how many cache misses per iteration? equal to # cache blocks in 2 rows

counting misses for B (2)

access pattern for B:
B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
B[2*N+0], B[3*N+0], ...B[2*N+(N-1)], B[3*N+(N-1)]
B[4*N+0], B[5*N+0], ...B[4*N+(N-1)], B[5*N+(N-1)]
...
B[0*N+0], B[1*N+0], ...B[0*N+(N-1)], B[1*N+(N-1)]
...

likely cache misses: any access, each time

how many cache misses per iteration? equal to # cache blocks in 2 rows

about $\frac{N}{2} \cdot N \cdot \frac{2N}{\text{block size}} = N^3 \div$ block size misses

simple blocking – counting misses

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

$\frac{N}{2} \cdot N$ j-loop iterations, and (assuming N large):

about 1 misses from A per j-loop iteration

$N^2/2$ total misses (before blocking: N^2)

about $2N \div$ block size misses from B per j-loop iteration

$N^3 \div$ block size total misses (same as before blocking)

about $N \div$ block size misses from C per j-loop iteration

$N^3 \div (2 \cdot$ block size$)$ total misses (before: $N^3 \div$ block size)

simple blocking – counting misses

```
for (int kk = 0; kk < N; kk += 2)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
    }
```

$\frac{N}{2} \cdot N$ j-loop iterations, and (assuming N large):

about 1 misses from A per j-loop iteration

   $N^2/2$ total misses (before blocking: N^2)

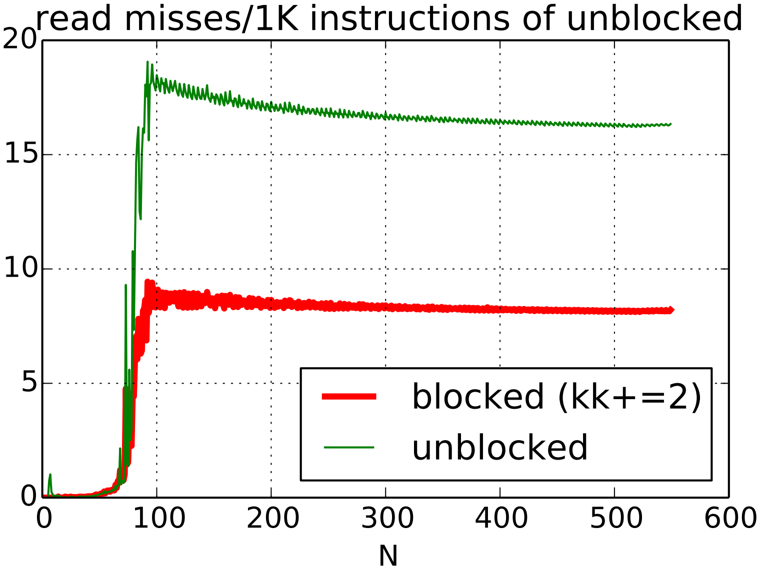
about $2N \div$ block size misses from B per j-loop iteration

   $N^3 \div$ block size total misses (same as before blocking)

about $N \div$ block size misses from C per j-loop iteration

   $N^3 \div (2 \cdot$ block size$)$ total misses (before: $N^3 \div$ block size)

improvement in read misses



simple blocking – with 3?

```
for (int kk = 0; kk < N; kk += 3)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
      C[i*N+j] += A[i*N+kk+2] * B[(kk+2)*N+j];
    }
```

$\frac{N}{3} \cdot N$ j-loop iterations, and (assuming N large):

about 1 misses from A per j-loop iteration

   $N^2/3$ total misses (before blocking: N^2)

about $3N \div$ block size misses from B per j-loop iteration

   $N^3 \div$ block size total misses (same as before)

about $3N \div$ block size misses from C per j-loop iteration

   $N^3 \div$ block size total misses (same as before)

simple blocking – with 3?

```
for (int kk = 0; kk < N; kk += 3)
  for (int i = 0; i < N; i += 1)
    for (int j = 0; j < N; ++j) {
      C[i*N+j] += A[i*N+kk+0] * B[(kk+0)*N+j];
      C[i*N+j] += A[i*N+kk+1] * B[(kk+1)*N+j];
      C[i*N+j] += A[i*N+kk+2] * B[(kk+2)*N+j];
    }
```

$\frac{N}{3} \cdot N$ j-loop iterations, and (assuming N large):

about 1 misses from A per j-loop iteration

$N^2/3$ total misses (before blocking: N^2)

about $3N \div$ block size misses from B per j-loop iteration

$N^3 \div$ block size total misses (same as before)

about $3N \div$ block size misses from C per j-loop iteration

$N^3 \div$ block size total misses (same as before)

more than 3?

can we just keep doing this increase from 3 to some large X? …

assumption: X values from A would stay in cache

- X too large — cache not big enough

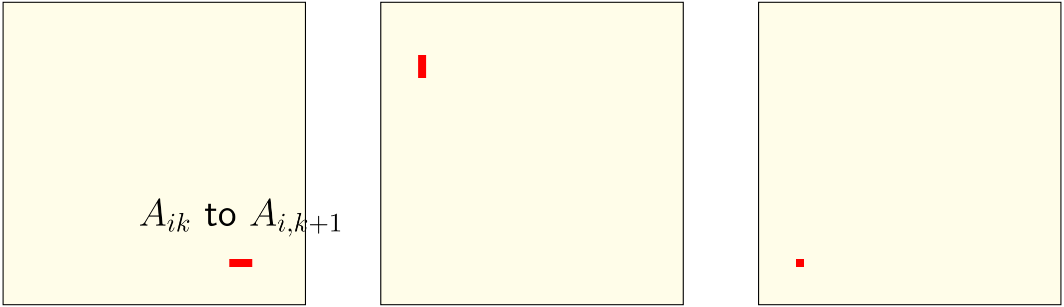
assumption: X blocks from B would help with spatial locality

- X too large — evicted from cache before next iteration

array usage (2 k at a time)

for each kk:
    for each i:
        for each j:
            for k=kk,kk+1:
                $C_{ij} += A_{ik} \cdot B_{kj}$

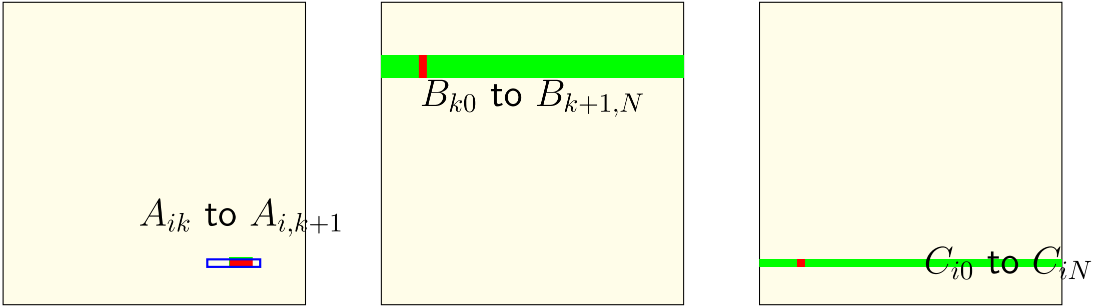
array usage (2 k at a time)



for each kk:
    for each i:
        for each j:
            for k=kk,kk+1:
                $C_{ij} += A_{ik} \cdot B_{kj}$

within innermost loop
good spatial locality in A
bad locality in B
good temporal locality in C

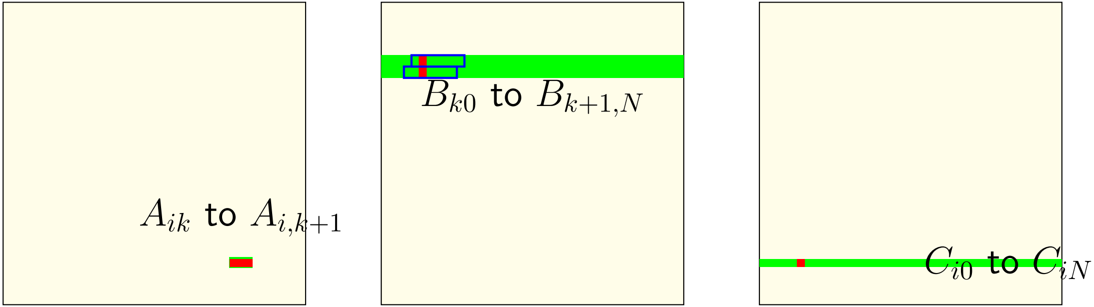
array usage (2 k at a time)



for each kk:
  for each i:
    for each j:
      for k=kk,kk+1:
        $C_{ij} += A_{ik} \cdot B_{kj}$

loop over j: better spatial locality over A than before; still good temporal locality for A

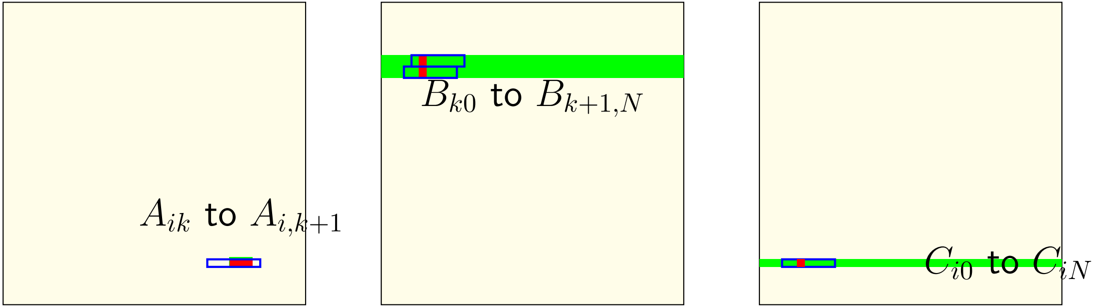
array usage (2 k at a time)



for each kk:
    for each i:
        for each j:
            for k=kk,kk+1:
                $C_{ij} += A_{ik} \cdot B_{kj}$

loop over j: spatial locality over B is worse
but probably not more misses
cache needs to keep two cache blocks
for next iter instead of one
(probably has the space left over!)

array usage (2 k at a time)



for each kk:
    for each i:
        for each j:
            for k=kk,kk+1:
                $C_{ij}+=A_{ik}\cdot$

right now: only really care about
keeping 4 cache blocks in j loop

have more than 4 cache blocks?
increasing kk increment would use more of them

simple blocking (2)

same thing for i in addition to k?

```
for (int kk = 0; kk < N; kk += 2) {
  for (int ii = 0; ii < N; ii += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      for (int k = kk; k < kk + 2; ++k)
        for (int i = 0; i < ii + 2; ++i)
            C[i*N+j] += A[i*N+k] * B[k*N+j];
    }
  }
}
```

simple blocking — locality

```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    /* load a block around Aik */
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
```

$$C_{i+0,j} \mathrel{+}= A_{i+0,k+0} \star B_{k+0,j}$$
$$C_{i+0,j} \mathrel{+}= A_{i+0,k+1} \star B_{k+1,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+0} \star B_{k+0,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+1} \star B_{k+1,j}$$

```
    }
  }
}
```

simple blocking — locality

```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    /* load a block around Aik */
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      C_{i+0,j} += A_{i+0,k+0} * B_{k+0,j}
      C_{i+0,j} += A_{i+0,k+1} * B_{k+1,j}
      C_{i+1,j} += A_{i+1,k+0} * B_{k+0,j}
      C_{i+1,j} += A_{i+1,k+1} * B_{k+1,j}
    }
  }
}
```

now: more temporal locality in B

previously: access B_{kj}, then don't use it again for a long time

simple blocking — counting misses for A

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
      C[i+0,j] += A[i+0,k+0] * B[k+0,j]
      C[i+0,j] += A[i+0,k+1] * B[k+1,j]
      C[i+1,j] += A[i+1,k+0] * B[k+0,j]
      C[i+1,j] += A[i+1,k+1] * B[k+1,j]
    }
```

$\frac{N}{2} \cdot \frac{N}{2}$ iterations of j loop

likely 2 misses per loop with A (2 cache blocks)

total misses: $\frac{N^2}{2}$ (same as only blocking in K)

simple blocking — counting misses for B

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
      C[i+0][j] += A[i+0][k+0] * B[k+0][j]
      C[i+0][j] += A[i+0][k+1] * B[k+1][j]
      C[i+1][j] += A[i+1][k+0] * B[k+0][j]
      C[i+1][j] += A[i+1][k+1] * B[k+1][j]
    }
```

$\frac{N}{2} \cdot \frac{N}{2}$ iterations of j loop

likely $2 \div$ block size misses per iteration with B

total misses: $\frac{N^3}{2 \cdot \text{block size}}$ (before: $\frac{N^3}{\text{block size}}$)

simple blocking — counting misses for C

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
```
$$
\begin{aligned}
C_{i+0,j} &\mathrel{+}= A_{i+0,k+0} * B_{k+0,j} \\
C_{i+0,j} &\mathrel{+}= A_{i+0,k+1} * B_{k+1,j} \\
C_{i+1,j} &\mathrel{+}= A_{i+1,k+0} * B_{k+0,j} \\
C_{i+1,j} &\mathrel{+}= A_{i+1,k+1} * B_{k+1,j}
\end{aligned}
$$
```
    }
```

$\frac{N}{2} \cdot \frac{N}{2}$ iterations of j loop

likely $\frac{2}{\text{block size}}$ misses per iteration with C

total misses: $\frac{N^3}{2 \cdot \text{block size}}$ (same as blocking only in K)

simple blocking — counting misses (total)

```
for (int k = 0; k < N; k += 2)
  for (int i = 0; i < N; i += 2)
    for (int j = 0; j < N; ++j) {
      C[i+0][j] += A[i+0][k+0] * B[k+0][j]
      C[i+0][j] += A[i+0][k+1] * B[k+1][j]
      C[i+1][j] += A[i+1][k+0] * B[k+0][j]
      C[i+1][j] += A[i+1][k+1] * B[k+1][j]
    }
```

$$C_{i+0,j} \mathrel{+}= A_{i+0,k+0} * B_{k+0,j}$$
$$C_{i+0,j} \mathrel{+}= A_{i+0,k+1} * B_{k+1,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+0} * B_{k+0,j}$$
$$C_{i+1,j} \mathrel{+}= A_{i+1,k+1} * B_{k+1,j}$$

before:

A: $\frac{N^2}{2}$; B: $\frac{N^3}{1 \cdot \text{block size}}$; C $\frac{N^3}{1 \cdot \text{block size}}$

after:

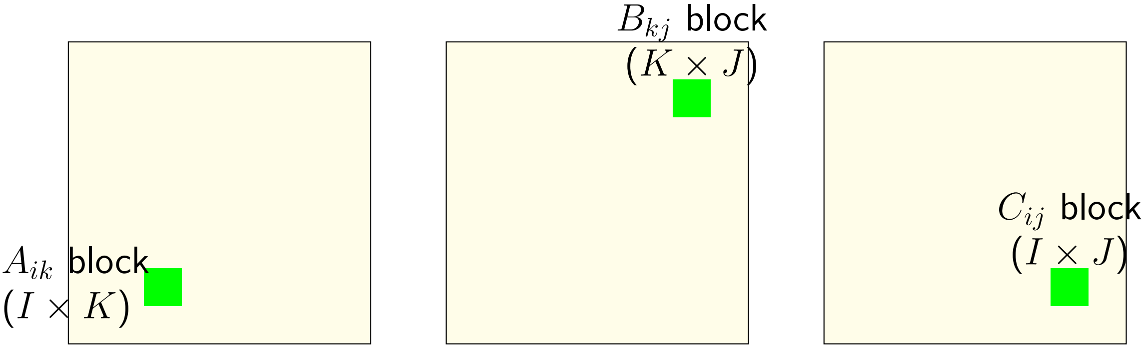
A: $\frac{N^2}{2}$; B: $\frac{N^3}{2 \cdot \text{block size}}$; C $\frac{N^3}{2 \cdot \text{block size}}$

generalizing: divide and conquer

```
partial_matrixmultiply(float *A, float *B, float *C
                int startI, int endI, ...) {
  for (int i = startI; i < endI; ++i) {
    for (int j = startJ; j < endJ; ++j) {
      for (int k = startK; k < endK; ++k) {
        ...
}
matrix_multiply(float *A, float *B, float *C, int N) {
  for (int ii = 0; ii < N; ii += BLOCK_I)
    for (int jj = 0; jj < N; jj += BLOCK_J)
      for (int kk = 0; kk < N; kk += BLOCK_K)
         ...
         /* do everything for segment of A, B, C
            that fits in cache! */
         partial_matmul(A, B, C,
                ii, ii + BLOCK_I, jj, jj + BLOCK_J,
                kk, kk + BLOCK_K)
}
```

array usage: matrix block

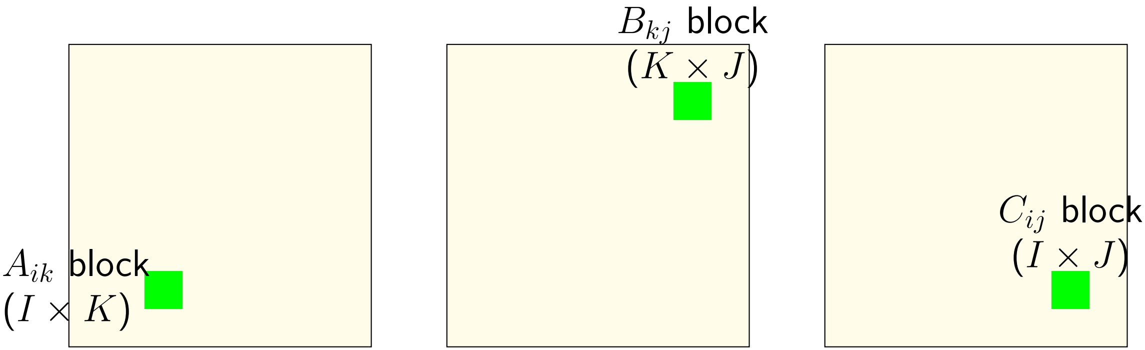
$$\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$$



inner loops work on "matrix block" of A, B, C
rather than rows of some, little blocks of others
blocks fit into cache (b/c we choose I, K, J)
where previous rows might not

array usage: matrix block

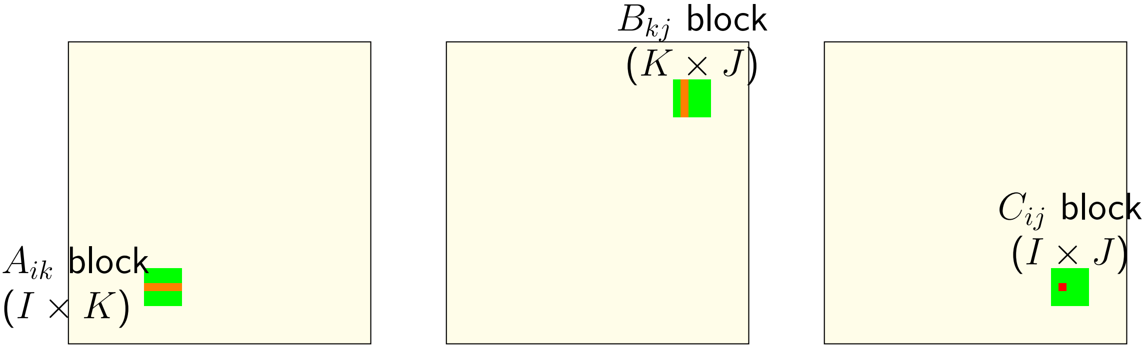
$$\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$$



now (versus loop ordering example)
some spatial locality in A, B, and C
some temporal locality in A, B, and C

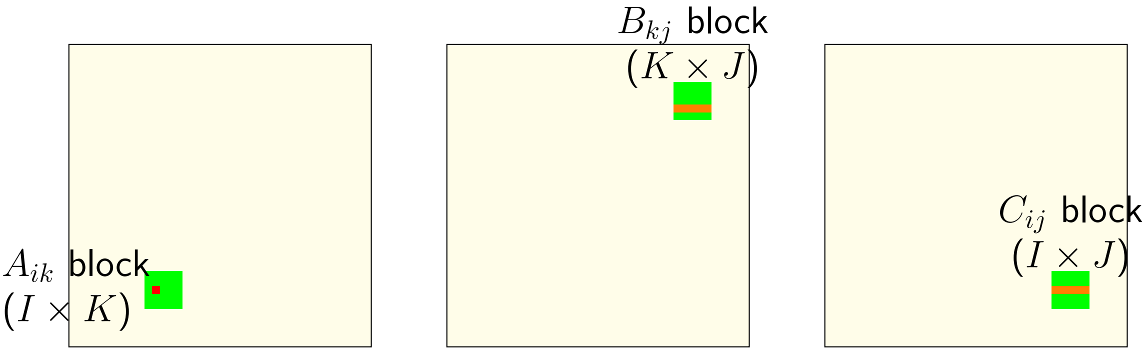
array usage: matrix block

$$\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$$



C_{ij} calculation uses strips from A, B
K calculations for one cache miss
good temporal locality!

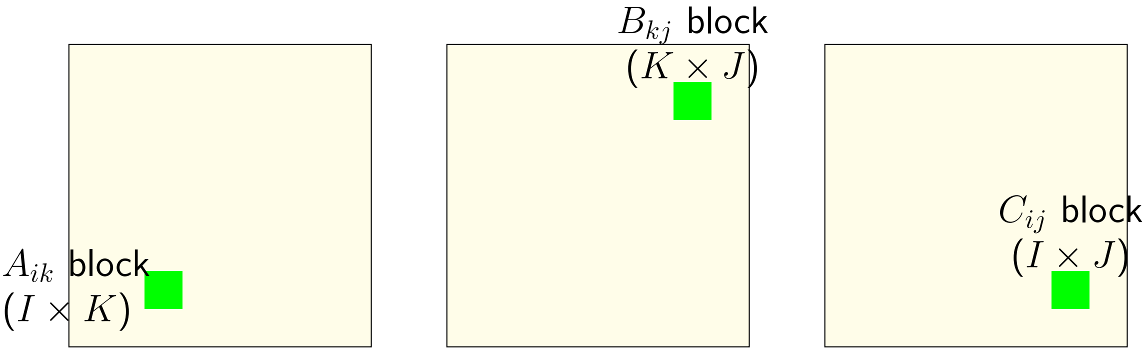
array usage: matrix block $\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$



A_{ik} used with entire strip of B J calculations for one cache miss

good temporal locality!

array usage: matrix block $\mathbf{C_{ij} \mathrel{+}= A_{ik} \cdot B_{kj}}$



(approx.) KIJ fully cached calculations
for $KI + IJ + KJ$ loads
(assuming everything stays in cache)

cache blocking efficiency

for each of N^3/IJK matrix blocks:

load $I \times K$ elements of A_{ik}:

$\approx IK \div$ block size misses per matrix block

$\approx N^3/(J \cdot \text{blocksize})$ misses total

load $K \times J$ elements of A_{kj}:

$\approx N^3/(I \cdot \text{blocksize})$ misses total

load $I \times J$ elements of B_{ij}:

$\approx N^3/(K \cdot \text{blocksize})$ misses total

bigger blocks — more work per load!

catch: $IK + KJ + IJ$ elements must fit in cache

otherwise estimates above don't work

cache blocking rule of thumb

fill the most of the cache with useful data

and do as much work as possible from that

example: my desktop 32KB L1 cache

$I = J = K = 48$ uses $48^2 \times 3$ elements, or 27KB.

assumption: conflict misses aren't important

systematic approach

```
for (int k = 0; k < N; ++k) {
  for (int i = 0; i < N; ++i) {
    A_ik loaded once in this loop:
    for (int j = 0; j < N; ++j)
      C_ij, B_kj loaded each iteration (if N big):
      B[i*N+j] += A[i*N+k] * A[k*N+j];
```

values from A_{ik} used N times per load

values from B_{kj} used 1 times per load

but good spatial locality, so cache block of B_{kj} together

values from C_{ij} used 1 times per load

but good spatial locality, so cache block of C_{ij} together

exercise: miss estimating (3)

```
for (int kk = 0; kk < 1000; kk += 10)
    for (int jj = 0; jj < 1000; jj += 10)
        for (int i = 0; i < 1000; i += 1)
            for (int j = jj; j < jj+10; j += 1)
                for (int k = kk; k < kk + 10; k += 1)
                    A[k*N+j] += B[i*N+j];
```

assuming: 4 elements per block

assuming: cache not close to big enough to hold 1K elements, but big enough to hold 500 or so

estimate: *approximately* how many misses for A, B?

hint 1: part of A, B loaded in two inner-most loops only needs to be loaded once

loop ordering compromises

loop ordering forces compromises:

for k: for i: for j: c[i,j] += a[i,k] * b[j,k]

perfect temporal locality in a[i,k]

bad temporal locality for c[i,j], b[j,k]

perfect spatial locality in c[i,j]

bad spatial locality in b[j,k], a[i,k]

loop ordering compromises

loop ordering forces compromises:

for k: for i: for j: c[i,j] += a[i,k] * b[j,k]

perfect temporal locality in a[i,k]

bad temporal locality for c[i,j], b[j,k]

perfect spatial locality in c[i,j]

bad spatial locality in b[j,k], a[i,k]

cache blocking: work on blocks rather than rows/columns

    have some temporal, spatial locality in everything

cache blocking pattern

no perfect loop order? work on rectangular matrix blocks

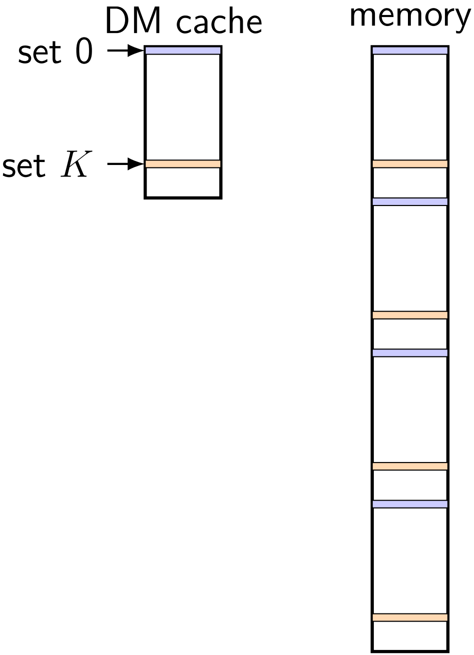
size amount used in inner loops based on cache size

in practice:

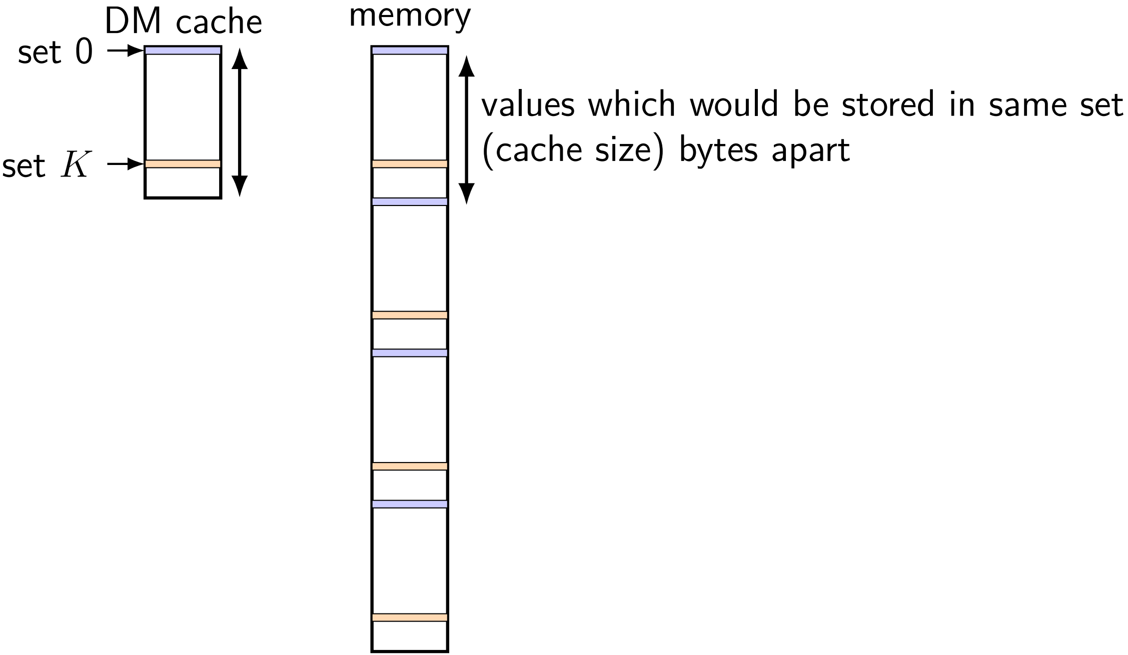
- test performance to determine ‘size’ of blocks

backup slides

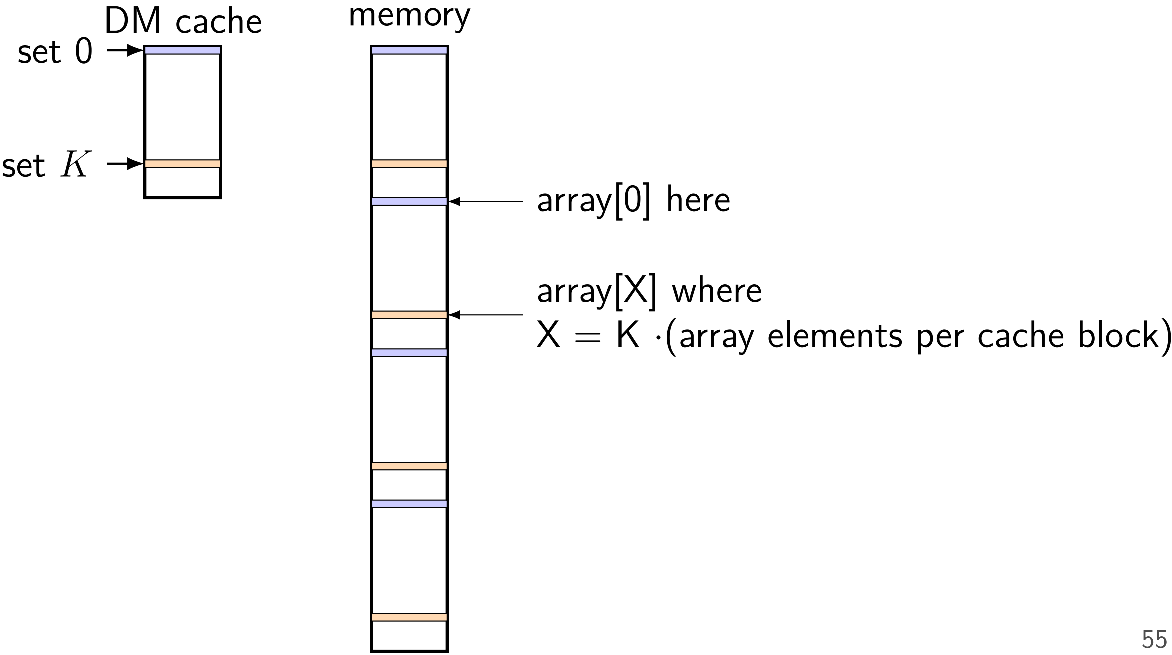
mapping of sets to memory (direct-mapped)



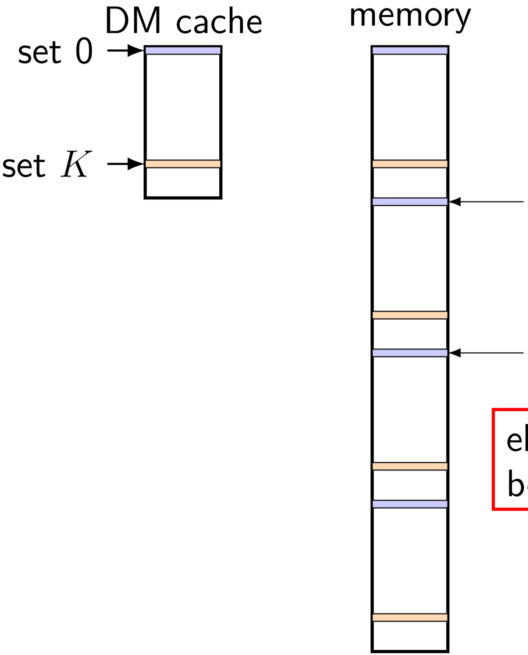
mapping of sets to memory (direct-mapped)



mapping of sets to memory (direct-mapped)



mapping of sets to memory (direct-mapped)

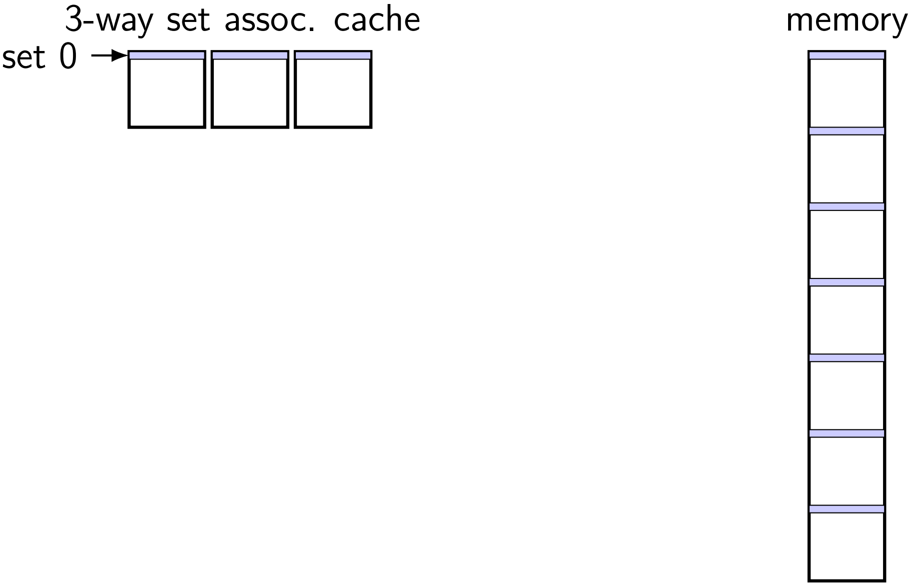


array[0] here

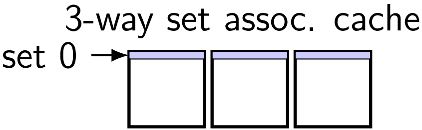
array[X]
X = (cache size / array element size)

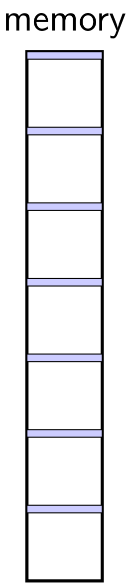
elements (cache size) bytes apart in array
beware conflict misses!

mapping of sets to memory (3-way)

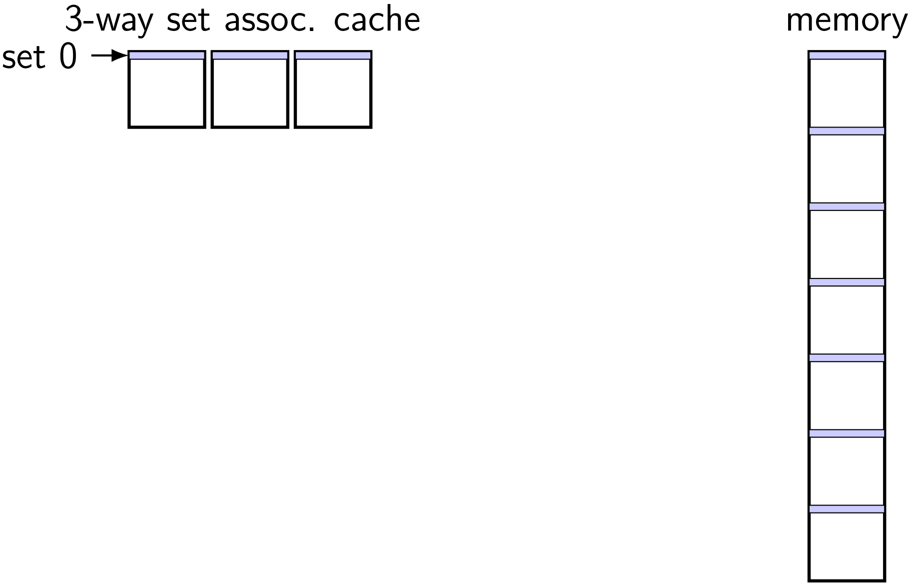


mapping of sets to memory (3-way)

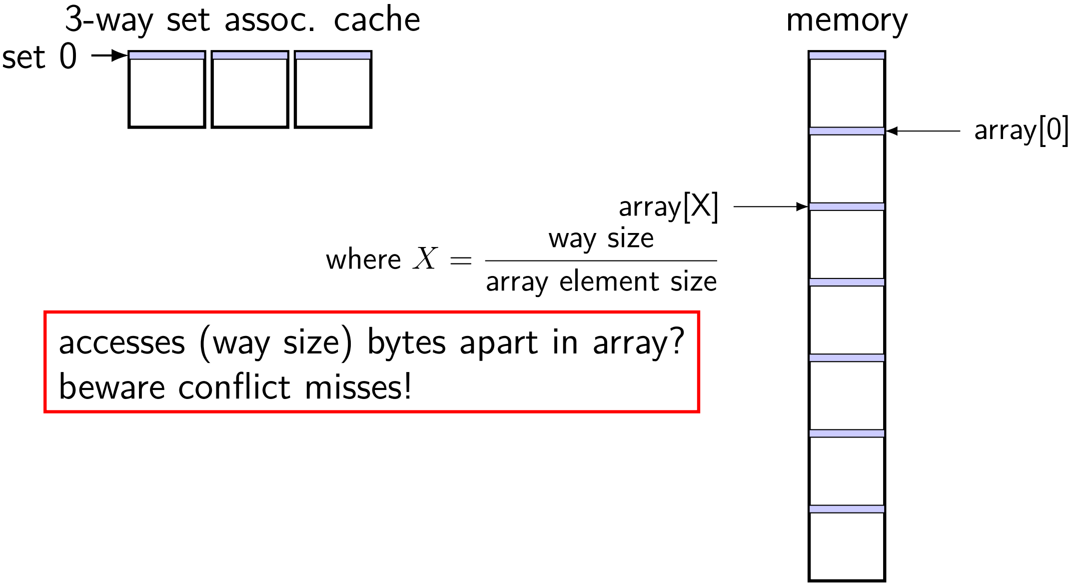




mapping of sets to memory (3-way)



mapping of sets to memory (3-way)



where $X = \dfrac{\text{way size}}{\text{array element size}}$

accesses (way size) bytes apart in array?
beware conflict misses!

C and cache misses (4)

```
typedef struct {
    int a_value, b_value;
    int other_values[6];
} item;
item items[5];
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 5; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 5; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

C and cache misses (4, rewrite)

```
int array[40]
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 40; i += 8)
    a_sum += array[i];
for (int i = 1; i < 40; i += 8)
    b_sum += array[i];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny) and array starts at beginning of cache block.

How many *data cache misses* on a 2-way set associative 128B cache with 16B cache blocks and LRU replacement?

C and cache misses (4, solution pt 1)

ints 4 byte $\rightarrow$ array[0 to 3] and array[16 to 19] in same cache set

- 64B $=$ 16 ints stored per way
- 4 sets total

accessing 0, 8, 16, 24, 32, 1, 9, 17, 25, 33

C and cache misses (4, solution pt 1)

ints 4 byte $\rightarrow$ array[0 to 3] and array[16 to 19] in same cache set

- 64B $=$ 16 ints stored per way
- 4 sets total

accessing 0, 8, 16, 24, 32, 1, 9, 17, 25, 33

0 (set 0), 8 (set 2), 16 (set 0), 24 (set 2), 32 (set 0)

1 (set 0), 9 (set 2), 17 (set 0), 25 (set 2), 33 (set 0)

C and cache misses (4, solution pt 2)

access	set 0 after (LRU first)	result	
—	—, —		
array[0]	—, array[0 to 3]	miss	
array[16]	array[0 to 3], array[16 to 19]	miss	6 misses for set 0
array[32]	array[16 to 19], array[32 to 35]	miss	
array[1]	array[32 to 35], array[0 to 3]	miss	
array[17]	array[0 to 3], array[16 to 19]	miss	
array[32]	array[16 to 19], array[32 to 35]	miss	

C and cache misses (4, solution pt 3)

access	set 2 after (LRU first)	result	
—	—, —		
array[8]	—, array[8 to 11]	miss	2 misses for set 1
array[24]	array[8 to 11], array[24 to 27]	miss	
array[9]	array[8 to 11], array[24 to 27]	hit	
array[25]	array[16 to 19], array[32 to 35]	hit	

arrays and cache misses (1)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2) {
    even_sum += array[i + 0];
    odd_sum +=  array[i + 1];
}
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

arrays and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 0; i < 1024; i += 2)
    odd_sum +=  array[i + 1];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks? Would a set-associtiave cache be better?

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int other_values[10];
} item;
item items[5];
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 5; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 5; ++i)
    b_sum += items[i].b_value;
```

observation: 12 ints in struct: only first two used

equivalent to accessing array[0], array[12], array[24], etc.

...then accessing array[1], array[13], array[25], etc.

C and cache misses (3, rewritten?)

```
int array[60];
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 60; i += 12)
    a_sum += array[i];
for (int i = 1; i < 60; i += 12)
    b_sum += array[i];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny) and array at beginning of cache block.

How many *data cache misses* on a 128B two-way set associative cache with 16B cache blocks and LRU replacement?

observation 1: first loop has 5 misses — first accesses to blocks

observation 2: array[0] and array[1], array[12] and array[13], etc. in same cache block

C and cache misses (3, solution)

ints 4 byte → array[0 to 3] and array[16 to 19] in same cache set

- 64B = 16 ints stored per way
- 4 sets total

accessing array indices 0, 12, 24, 36, 48, 1, 13, 25, 37, 49

so access to 1, 21, 41, 61, 81 all hits:

- set 0 contains block with array[0 to 3]
- set 5 contains block with array[20 to 23]
- etc.

C and cache misses (3, solution)

ints 4 byte → array[0 to 3] and array[16 to 19] in same cache set

- 64B = 16 ints stored per way
- 4 sets total

accessing array indices 0, 12, 24, 36, 48, 1, 13, 25, 37, 49

so access to 1, 21, 41, 61, 81 all hits:

- set 0 contains block with array[0 to 3]
- set 5 contains block with array[20 to 23]
- etc.

C and cache misses (3, solution)

ints 4 byte → array[0 to 3] and array[16 to 19] in same cache set

- 64B = 16 ints stored per way
- 4 sets total

accessing array indices 0, 12, 24, 36, 48, 1, 13, 25, 37, 49

0 (set 0, array[0 to 3]), 12 (set 3), 24 (set 2), 36 (set 1), 48 (set 0)

- each set used at most twice
- no replacement needed

so access to 1, 21, 41, 61, 81 all hits:

- set 0 contains block with array[0 to 3]
- set 5 contains block with array[20 to 23]
- etc.

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (3, rewritten?)

```
item array[1024]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 1024; i += 128)
    a_sum += array[i];
for (int i = 1; i < 1024; i += 128)
    b_sum += array[i];
```

C and cache misses (4)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 4-way set associative 2KB direct-mapped cache with 16B cache blocks?

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

block at 0: array[0] through array[3]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

block at 16: array[4] through array[7]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

block at 2032: array[508] through array[511]

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

block at 0: array[0] through array[3]

block at 0+2KB: array[512] through array[515]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

block at 16: array[4] through array[7]

block at 16+2KB: array[516] through array[519]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

block at 2032: array[508] through array[511]

block at 2032+2KB: array[1020] through array[1023]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, $0 + 2$KB, $0 + 4$KB, ...

set 1: address 16, $16 + 2$KB, $16 + 4$KB, ...

...

set 63: address 1008, $2032 + 2$KB, $2032 + 4$KB ...

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 2KB, 0 + 4KB, ...

    block at 0: array[0] through array[3]

set 1: address 16, 16 + 2KB, 16 + 4KB, ...

    address 16: array[4] through array[7]

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...

    address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, $0 + 2$KB, $0 + 4$KB, …

- block at 0: array[0] through array[3]
- block at 0+1KB: array[256] through array[259]
- block at 0+2KB: array[512] through array[515]
- …

set 1: address 16, $16 + 2$KB, $16 + 4$KB, …

- address 16: array[4] through array[7]

…

set 63: address 1008, $2032 + 2$KB, $2032 + 4$KB …

- address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, $0 + 2\text{KB}$, $0 + 4\text{KB}$, ...

- block at 0: array[0] through array[3]
- block at 0+1KB: array[256] through array[259]
- block at 0+2KB: array[512] through array[515]
- ...

set 1: address 16, $16 + 2\text{KB}$, $16 + 4\text{KB}$, ...

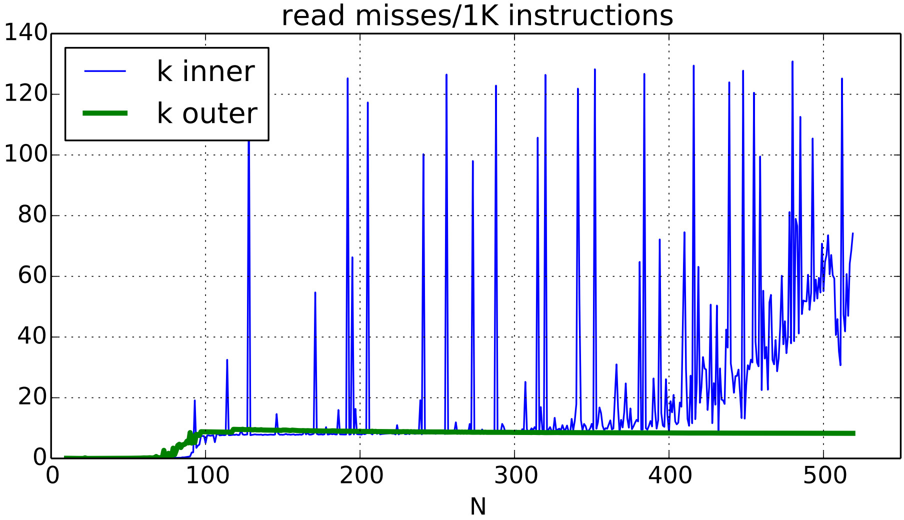
- address 16: array[4] through array[7]

...

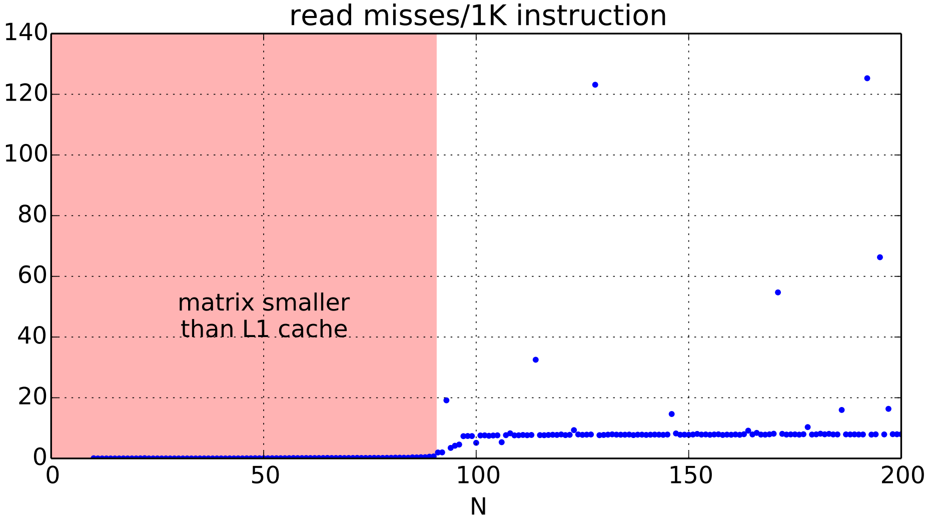
set 63: address 1008, $2032 + 2\text{KB}$, $2032 + 4\text{KB}$ ...

- address 1008: array[252] through array[255]

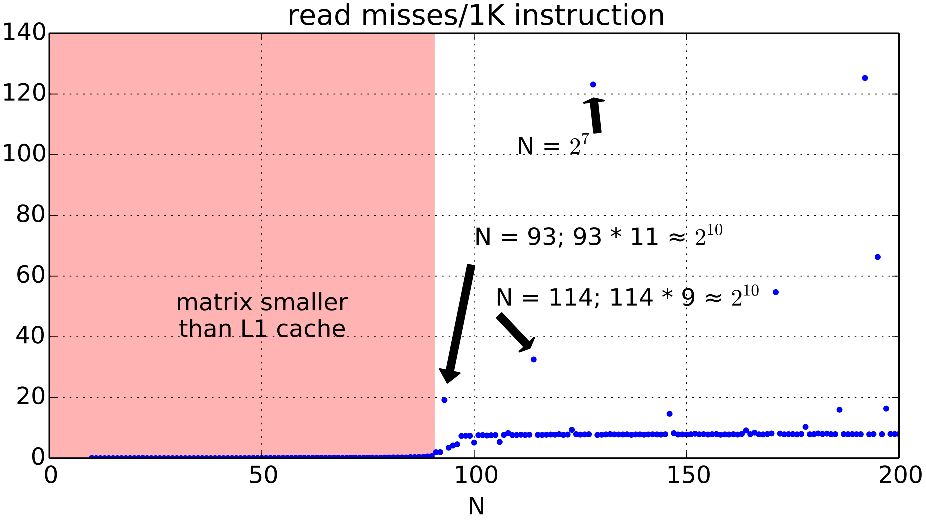
L1 misses (with A=B)



L1 miss detail (1)



L1 miss detail (2)



addresses

`B[k*114+j]`	is at	`10 0000 0000 0100`
`B[k*114+j+1]`	is at	`10 0000 0000 1000`
`B[(k+1)*114+j]`	is at	`10 0011 1001 0100`
`B[(k+2)*114+j]`	is at	`10 0101 0101 1100`
…		
`B[(k+9)*114+j]`	is at	`11 0000 0000 1100`

addresses

```
B[k*114+j]        is at  10 0000 0000 0100
B[k*114+j+1]      is at  10 0000 0000 1000
B[(k+1)*114+j]    is at  10 0011 1001 0100
B[(k+2)*114+j]    is at  10 0101 0101 1100
…
B[(k+9)*114+j]    is at  11 0000 0000 1100
```

test system L1 cache: 6 index bits, 6 block offset bits

conflict misses

powers of two — lower order bits unchanged

B[k*93+j] and B[(k+11)*93+j]:

  1023 elements apart (4092 bytes; 63.9 cache blocks)

64 sets in L1 cache: usually maps to same set

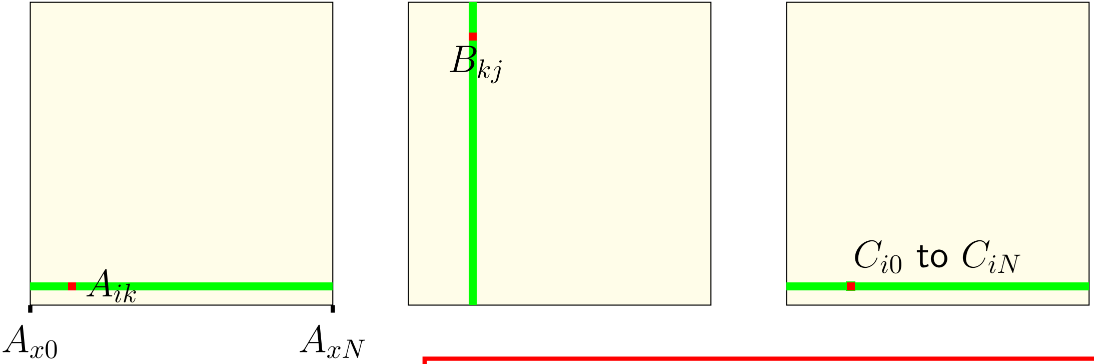
B[k*93+(j+1)] will not be cached (next i loop)

even if in same block as B[k*93+j]

how to fix? improve spatial locality

  (maybe even if it requires copying)

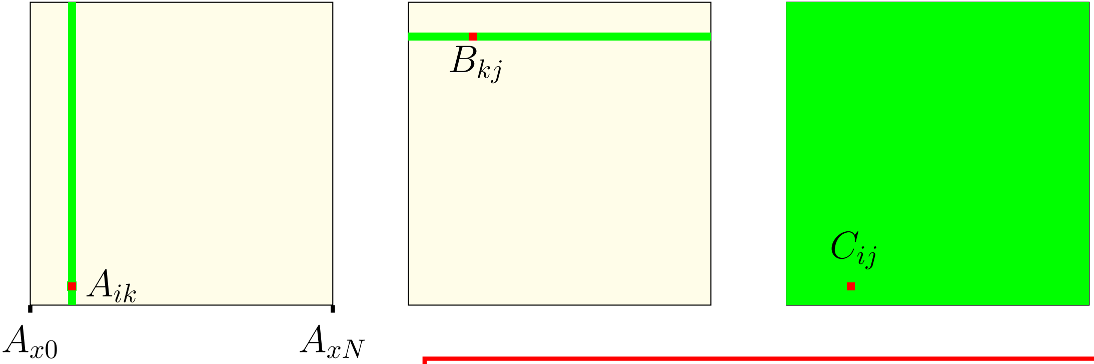
array usage: ijk order



for all i:
    for all j:
        for all k:
            $C_{ij} += A_{ik} \times B_{kj}$

looking only at two innermost loops together:
good spatial locality in A
poor spatial locality in B
good spatial locality in C

array usage: kij order



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looking only at two innermost loops together:
poor spatial locality in A
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keeping values in cache

can't *explicitly* ensure values are kept in cache

...but reusing values *effectively* does this

- cache will try to keep recently used values

cache optimization ideas: choose what's in the cache

- for thinking about it: load values explicitly
- for implementing it: access only values we want loaded