

last time (1)

out-of-order execution

- new hazards: computing old value of register while reading new value?

architectural versus physical registers

register renaming

- preprocessing step for instructions
- new physical register for each version of architectural register
- separate names for inputs and outputs
- might also do other instruction translations here

instruction dispatch

- queue of waiting renamed instructions
- each physical register: ready or not ready yet
- run instructions whose inputs are ready
- different types of 'slots' for instructions

last time (2)

data flow diagrams

reassociation

$(A \text{ op } B) \text{ op } (C \text{ op } D)$ often faster than $(A \text{ op } (B \text{ op } (C \text{ op } D)))$

more operations can be done in parallel

multiple accumulators

reassociation for loops

use two or more accumulators (sums, products)

combine at end

quiz Q1 (1)

I messed up

version A, B meant to be cache blocked versions

...

but I wrote $i = 0$ instead of $i = ii$

(also missed $A[i+1]$ in version D)

quiz Q1 version A

(with $i = ii$, $j = jj$ correction)

```
for (int ii = 0; ii < N; ii += 2) {
    for (int i = ii; i < ii + 2 && i < N; i += 1) {
        for (int jj = 0; jj < N; jj += 2) {
            for (int j = jj; j < jj + 2 && j < i; j += 1) {
                A[i] += B[i * N + j] * C[j * N + i];
            }
        }
    }
}
```

does $(i, j) = (0, 0), (0, 1), (0, 2), \dots$

then $(1, 0), (1, 1), (1, 2), \dots$

→ same order as original!

quiz Q1 version B

(with $i = ii$, $j = jj$ correction)

```
for (int ii = 0; ii < N; ii += 2) {
    for (int jj = j; jj < N; jj += 2) {
        for (int i = ii; i < ii + 2 && i < N; i += 1) {
            for (int j = jj; j < jj + 2 && j < i; j += 1) {
                A[i] += B[i * N + j] * C[j * N + i];
            }
        }
    }
}
```

$(0, 0), (0, 1), (1, 0), (1, 1), (0, 2), (0, 3), (1, 2), (1, 3), \dots$

→ more spatial locality in A!

quiz Q1 version C

```
for (int i = 0; i < N; i += 1) {  
    int j = 0;  
    for (; j + 1 < i; j += 2) {  
        A[i] += B[i * N + j] * C[j * N + i];  
        A[i] += B[i * N + j+1] * C[(j+1) * N + i];  
    }  
    if (j < i)  
        A[i] += B[i * N + j] * C[j * N + i];  
}
```

(0, 0), (0, 1), (0, 2), (0, 3), ...

(1, 0), (1, 1), (1, 2), (1, 3), ...

→ same order of i, j as original

so same data cache accesses (assuming i, j, etc. in regs)

quiz Q1 version D

```
for (int j = 0; j < N - 1; j += 1) {  
    int i;  
    for (i = 0; i + 1 <= j ; i += 2) {  
        A[i] += B[i * N + j] * C[j * N + i];  
        A[i+1] += B[(i+1) * N + j] * C[j * N + i+1];  
    }  
    if (i <= j)  
        A[i] += B[i * N + j] * C[j * N + i];  
}
```

iterating in i, not j

so worse temporal locality in A, spatial locality in B, C

→ worse performance

quiz Q2

init. rbx = x0, r8 = x1, r9 = x2

xorq %rbx, %r8		xorq %x0, %x1	->	%x3
andq %rbx, %r9		xorq %x0, %x2	->	%x4
subq %r8, %rbx		xorq %x3, %x4	->	%x5
addq %rbx, %r9		xorq %x5, %x4	->	%x6

quiz Q3, 5

```
movq    (%rdx,%rcx,8), %r9 /* E */  
movq    (%rsi,%rax,8), %r10 /* F */  
addq    %r10, %r9           /* G */
```

new 'version' of r9, r10 each time

so no dependencies (for the add) between loop iterations!

can be done in parallel

since already done in parallel, no multiple accumulators benefit

quiz Q4

```
// (setup code)
inner_loop:
    movq    (%rdx,%rcx,8), %r9 /* E */
    movq    (%rsi,%rax,8), %r10 /* F */
    addq    %r10, %r9           /* G */
    movq    %r9, (%r8,%rcx,8)  /* H */
    incq    %rcx                /* I */
    cmpq    $1024, %rcx         /* J */
    jne     inner_loop          /* K */
// (end of outer loop)
```

E through I duplicated

one comparision + cond jump per iteration

rotate/smooth-checkpoint due time

rotate + smooth-checkpoint originally were due next Wednesday

now due just before Thanksgiving break

strongly recommend doing early

...especially if you don't want to work on Thanksgiving week

performance HWs

assignment 1: rotate an image

assignment 2: smooth (blur) an image

two parts

part 1: due with rotate — optimizations we've mostly talked about

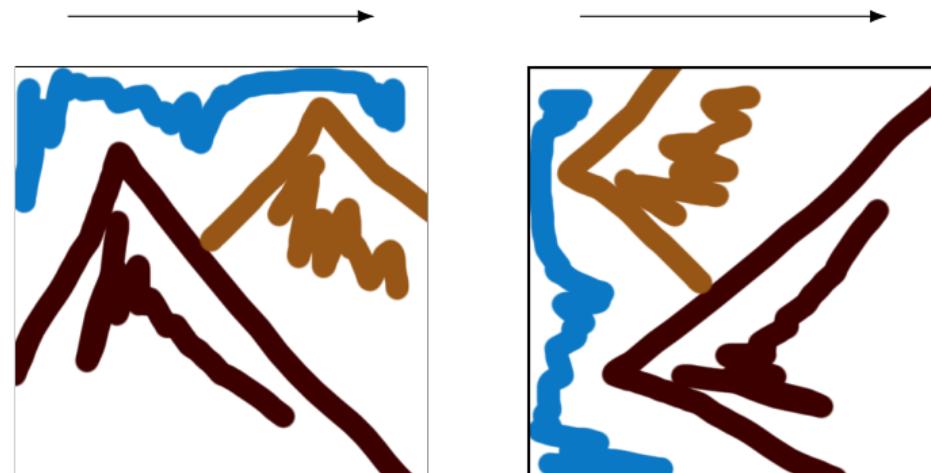
part 2: due later — part with vector instructions

image representation

```
typedef struct {  
    unsigned char red, green, blue, alpha;  
} pixel;  
pixel *image = malloc(dim * dim * sizeof(pixel));  
  
image[0]           // at (x=0, y=0)  
image[4 * dim + 5] // at (x=5, y=4)  
...
```

rotate assignment

```
void rotate(pixel *src, pixel *dst, int dim) {  
    int i, j;  
    for (i = 0; i < dim; i++)  
        for (j = 0; j < dim; j++)  
            dst[RIDX(dim - 1 - j, i, dim)] =  
                src[RIDX(i, j, dim)];  
}
```



preprocessor macros

```
#define DOUBLE(x) x*2
```

```
int y = DOUBLE(100);
```

// expands to:

```
int y = 100*2;
```

macros are text substitution (1)

```
#define BAD_DOUBLE(x) x*2  
  
int y = BAD_DOUBLE(3 + 3);  
// expands to:  
int y = 3+3*2;  
// y == 9, not 12
```

macros are text substitution (2)

```
#define FIXED_DOUBLE(x) (x)*2
```

```
int y = DOUBLE(3 + 3);
```

// expands to:

```
int y = (3+3)*2;  
// y == 9, not 12
```

RIDX?

```
#define RIDX(x, y, n) ((x) * (n) + (y))
```

```
dst[RIDX(dim - 1 - j, 1, dim)]
```

*// becomes *at compile-time*:*

```
dst[((dim - 1 - j) * (dim) + (1))]
```

performance grading

you can submit multiple variants in one file

grade: best performance

don't delete stuff that works!

we will measure speedup on **my machine**

web viewer for results (with some delay — has to run)

grade: achieving certain speedup on my machine

thresholds based on results with certain optimizations

general advice

(for when we don't give specific advice)

try techniques from book/lecture that seem applicable

vary numbers (e.g. cache block size)

often — too big/small is worse

some techniques combine well

loop unrolling and cache blocking

loop unrolling and reassociation/multiple accumulators

example assembly (unoptimized)

```
long sum(long *A, int N) {  
    long result = 0;  
    for (int i = 0; i < N; ++i)  
        result += A[i];  
    return result;  
}  
  
sum: ...  
the_loop:  
    ...  
    leaq    0(%rax,8), %rdx // offset <- i * 8  
    movq    -24(%rbp), %rax // get A from stack  
    addq    %rdx, %rax     // add offset  
    movq    (%rax), %rax   // get *(A+offset)  
    addq    %rax, -8(%rbp) // add to sum, on stack  
    addl    $1, -12(%rbp)  // increment i  
  
condition:  
    movl    -12(%rbp), %eax  
    cmpl    -28(%rbp), %eax  
    jl     the_loop  
    ...
```

example assembly (gcc 5.4 -Os)

```
long sum(long *A, int N) {
    long result = 0;
    for (int i = 0; i < N; ++i)
        result += A[i];
    return result;
}

sum:
    xorl    %edx, %edx
    xorl    %eax, %eax
the_loop:
    cmpl    %edx, %esi
    jle     done
    addq    (%rdi,%rdx,8), %rax
    incq    %rdx
    jmp     the_loop
done:
    ret
```

example assembly (gcc 5.4 -O2)

```
long sum(long *A, int N) {  
    long result = 0;  
    for (int i = 0; i < N; ++i)  
        result += A[i];  
    return result;  
}  
  
sum:  
    testl    %esi, %esi  
    jle     return_zero  
    leal    -1(%rsi), %eax  
    leaq    8(%rdi,%rax,8), %rdx // rdx=end of A  
    xorl    %eax, %eax  
  
the_loop:  
    addq    (%rdi), %rax // add to sum  
    addq    $8, %rdi      // advance pointer  
    cmpq    %rdx, %rdi  
    jne     the_loop  
    rep ret  
  
return_zero: ...
```

example assembly (gcc 9.2 -O3)

sum:

```
    testl  %esi, %esi
    ... /* approx 10 lines omitted */
```

the_loop:

```
    movdqu (%rax), %xmm2 /* <-- load 16 bytes from array */
    addq   $16, %rax
    paddq  %xmm2, %xmm0 /* <-- add 2 pairs of longs */
    cmpq   %rdx, %rax
    jne    the_loop
    ... /* approx 20 lines omitted */
    ret
```

example assembly (gcc 9.2 -O3 -march=skylake)

sum:

```
    testl  %esi, %esi
    ... /* approx 10 lines omitted */
```

the_loop:

```
    vpaddq (%rax), %ymm0, %ymm0 /* <- add 4 pairs of longs */
    addq   $32, %rax
    cmpq   %rdx, %rax
    jne    the_loop
    ... /* approx 20 lines omitted */
    ret
```

gcc 9.2 -O3 -funroll-loops -march=skylake

sum:

```
testl    %esi, %esi
... /* approx 60 lines omitted */
the_loop: /* loop unrolled 8 times + instrs that add 4 pairs at a
           vpaddq (%r8), %ymm0, %ymm1 /* <-- add 4 pairs of longs */
           addq   $256, %r8
           vpaddq -224(%r8), %ymm1, %ymm2
           vpaddq -192(%r8), %ymm2, %ymm3
           vpaddq -160(%r8), %ymm3, %ymm4
           vpaddq -128(%r8), %ymm4, %ymm5
           vpaddq -96(%r8), %ymm5, %ymm6
           vpaddq -64(%r8), %ymm6, %ymm7
           vpaddq -32(%r8), %ymm7, %ymm0
           cmpq   %rcx, %r8
           jne    .L4
... /* approx 20 lines omitted */
ret
```

example assembly (clang 9.0 -O -march=skylake)

sum:

```
    testl  %esi, %esi
    ... /* approx 35 lines omitted */
the_loop: /* loop unrolled + multiple accumulators + instrs that 4 pairs at a time */
    vpaddq (%rdi,%rsi,8), %ymm0, %ymm0
    vpaddq 32(%rdi,%rsi,8), %ymm1, %ymm1
    vpaddq 64(%rdi,%rsi,8), %ymm2, %ymm2
    vpaddq 96(%rdi,%rsi,8), %ymm3, %ymm3
    vpaddq 128(%rdi,%rsi,8), %ymm0, %ymm0
    vpaddq 160(%rdi,%rsi,8), %ymm1, %ymm1
    vpaddq 192(%rdi,%rsi,8), %ymm2, %ymm2
    vpaddq 224(%rdi,%rsi,8), %ymm3, %ymm3
    vpaddq 256(%rdi,%rsi,8), %ymm0, %ymm0
    vpaddq 288(%rdi,%rsi,8), %ymm1, %ymm1
    vpaddq 320(%rdi,%rsi,8), %ymm2, %ymm2
    vpaddq 352(%rdi,%rsi,8), %ymm3, %ymm3
    vpaddq 384(%rdi,%rsi,8), %ymm0, %ymm0
    vpaddq 416(%rdi,%rsi,8), %ymm1, %ymm1
    vpaddq 448(%rdi,%rsi,8), %ymm2, %ymm2
    vpaddq 480(%rdi,%rsi,8), %ymm3, %ymm3
    addq   $64, %rsi
    addq   $4, %rax
    jne   the_loop
```

optimizing compilers

these usually make your code fast

often not done by default

compilers and humans are good at **different kinds** of optimizations

compiler limitations

needs to generate code that does the same thing...

...even in corner cases that “obviously don’t matter”

often doesn't ‘look into’ a method

needs to assume it might do anything

can't predict what inputs/values will be

e.g. lots of loop iterations or few?

can't understand code size versus speed tradeoffs

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can't understand code size versus speed tradeoffs

aliasing

```
void twiddle(long *px, long *py) {  
    *px += *py;  
    *px += *py;  
}
```

the compiler **cannot** generate this:

```
twiddle: // BROKEN // %rsi = px, %rdi = py  
        movq    (%rdi), %rax // rax <- *py  
        addq    %rax, %rax   // rax <- 2 * *py  
        addq    %rax, (%rsi) // *px <- 2 * *py  
        ret
```

aliasing problem

```
void twiddle(long *px, long *py) {  
    *px += *py;  
    *px += *py;  
    // NOT the same as *px += 2 * *py;  
}  
...  
long x = 1;  
twiddle(&x, &x);  
// result should be 4, not 3
```

```
twiddle: // BROKEN // %rsi = px, %rdi = py  
        movq    (%rdi), %rax // rax <- *py  
        addq    %rax, %rax   // rax <- 2 * *py  
        addq    %rax, (%rsi) // *px <- 2 * *py  
        ret
```

non-contrived aliasing

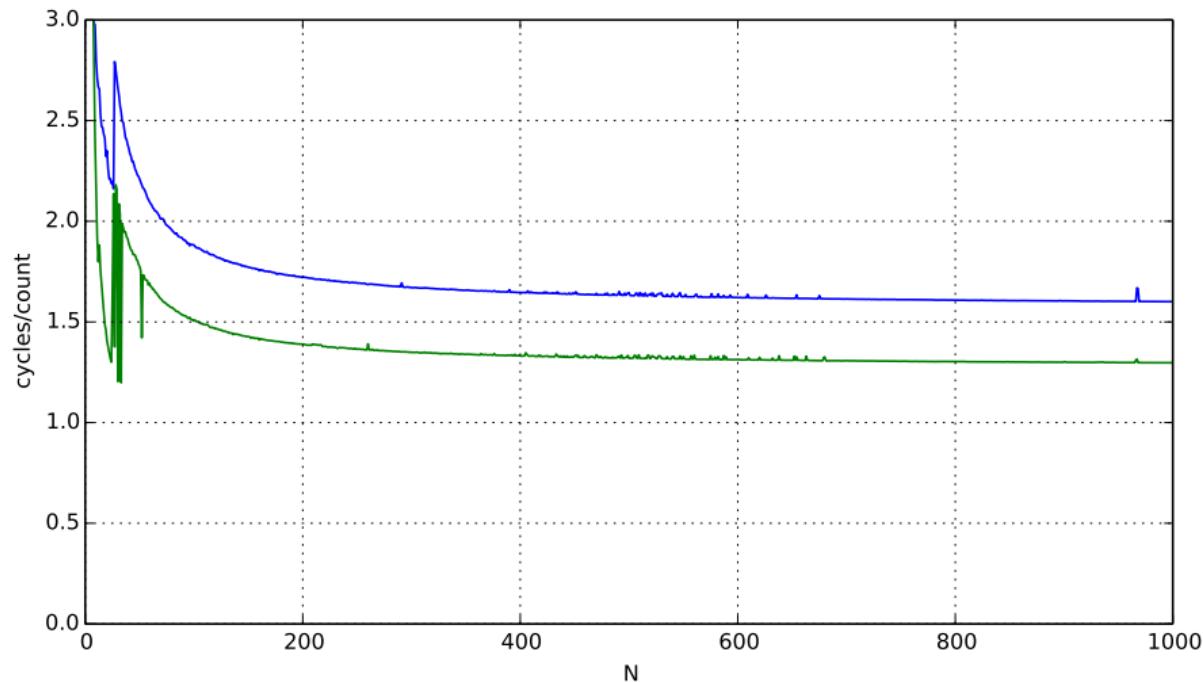
```
void sumRows1(int *result, int *matrix, int N) {  
    for (int row = 0; row < N; ++row) {  
        result[row] = 0;  
        for (int col = 0; col < N; ++col)  
            result[row] += matrix[row * N + col];  
    }  
}
```

non-contrived aliasing

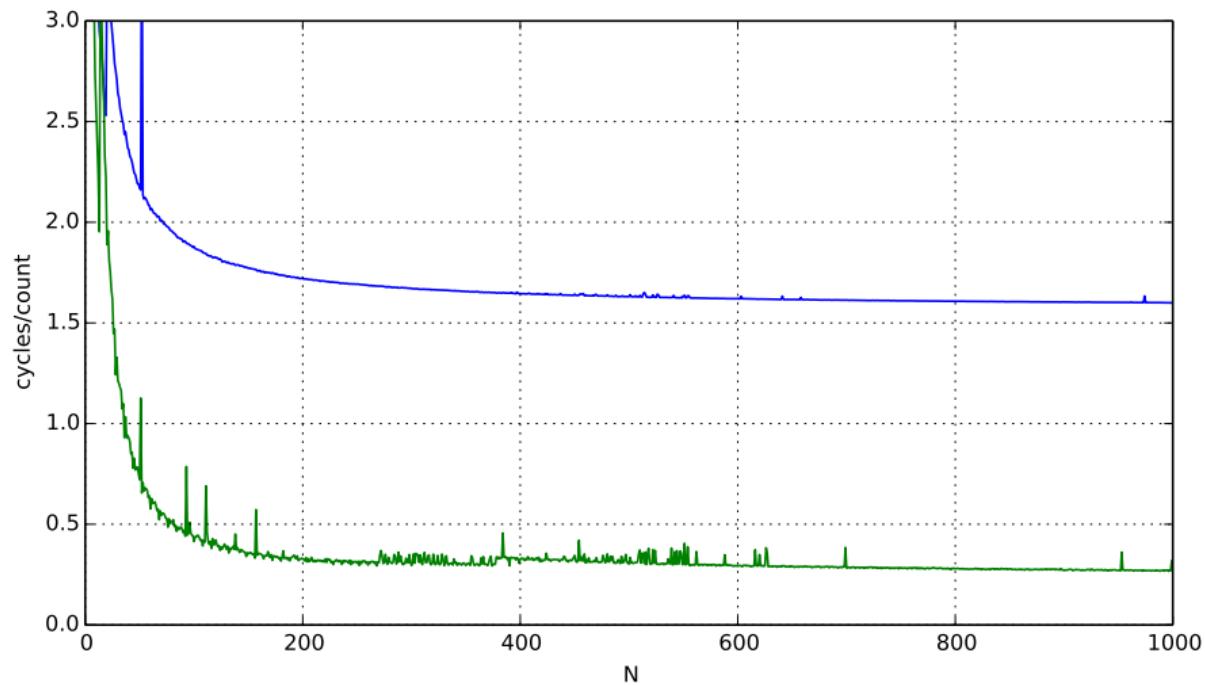
```
void sumRows1(int *result, int *matrix, int N) {  
    for (int row = 0; row < N; ++row) {  
        result[row] = 0;  
        for (int col = 0; col < N; ++col)  
            result[row] += matrix[row * N + col];  
    }  
}
```

```
void sumRows2(int *result, int *matrix, int N) {  
    for (int row = 0; row < N; ++row) {  
        int sum = 0;  
        for (int col = 0; col < N; ++col)  
            sum += matrix[row * N + col];  
        result[row] = sum;  
    }  
}
```

aliasing and performance (1) / GCC 5.4 -O2



aliasing and performance (2) / GCC 5.4 -O3



automatic register reuse

Compiler would need to generate overlap check:

```
if (result > matrix + N * N || result < matrix) {  
    for (int row = 0; row < N; ++row) {  
        int sum = 0; /* kept in register */  
        for (int col = 0; col < N; ++col)  
            sum += matrix[row * N + col];  
        result[row] = sum;  
    }  
} else {  
    for (int row = 0; row < N; ++row) {  
        result[row] = 0;  
        for (int col = 0; col < N; ++col)  
            result[row] += matrix[row * N + col];  
    }  
}
```

aliasing problems with cache blocking

```
for (int k = 0; k < N; k++) {  
    for (int i = 0; i < N; i += 2) {  
        for (int j = 0; j < N; j += 2) {  
            C[(i+0)*N + j+0] += A[i*N+k] * B[k*N+j];  
            C[(i+1)*N + j+0] += A[(i+1)*N+k] * B[k*N+j];  
            C[(i+0)*N + j+1] += A[i*N+k] * B[k*N+j+1];  
            C[(i+1)*N + j+1] += A[(i+1)*N+k] * B[k*N+j+1];  
        }  
    }  
}
```

can compiler keep $A[i*N+k]$ in a register?

“register blocking”

```
for (int k = 0; k < N; ++k) {  
    for (int i = 0; i < N; i += 2) {  
        float Ai0k = A[(i+0)*N + k];  
        float Ai1k = A[(i+1)*N + k];  
        for (int j = 0; j < N; j += 2) {  
            float Bkj0 = B[k*N + j+0];  
            float Bkj1 = B[k*N + j+1];  
            C[(i+0)*N + j+0] += Ai0k * Bkj0;  
            C[(i+1)*N + j+0] += Ai1k * Bkj0;  
            C[(i+0)*N + j+1] += Ai0k * Bkj1;  
            C[(i+1)*N + j+1] += Ai1k * Bkj1;  
        }  
    }  
}
```

aliasing exercise

```
void add(int *s1, int *s2, int *d) {  
    for (int i = 0; i < 1000; ++i)  
        d[i] = s1[i] + s2[i];  
}
```

The compiler **cannot** generate code equivalent to this:

```
void add(int *s1, int *s2, int *d) {  
    for (int i = 0; i < 1000; i += 2) {  
        int temp1 = s1[i] + s2[i];  
        int temp2 = s1[i+1] + s2[i+1];  
        d[i] = temp1; d[i+1] = temp2;  
    }  
}
```

Which is an example of a call where the results could disagree:

- A. add(&A[0], &A[1], &B[0]) B. add(&A[0], &A[0], &A[1])
 - C. add(&B[0], &A[10], &A[0]) D. add(&A[1000], &A[1001], &A[0])
- (assume A, B are distinct, large arrays)

aliasing exercise

recall: $s1=s2=A+0$; $d=A+1$

```
for (int i = 0; i < 1000; ++i)
    d[i] = s1[i] + s2[i];
```

```
/* i = 0: */ A[1] = A[0] + A[0];
/* i = 1: */ A[2] = A[1] + A[1];
```

```
for (int i = 0; i < 1000; i += 2) {
    temp1 = s1[i] + s2[i];
    temp2 = s1[i] + s2[i];
    d[i] = temp1;
    d[i+1] = temp2;
```

```
/* i = 0: */ temp1 = A[0] + A[0];
               temp2 = A[1] + A[1];
               A[1] = temp1;
               A[2] = temp2;
```

aliasing and cache optimizations

```
for (int k = 0; k < N; ++k)
    for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
            C[i*N+j] += A[i * N + k] * B[k * N + j];
```

```
for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
        for (int k = 0; k < N; ++k)
            C[i*N+j] += A[i * N + k] * B[k * N + j];
```

C = A? C = &A[10]?

compiler can't generate same code for both

loop unrolling downsides

bigger executables → instruction cache misses

slower if small number of loop iterations
extra code to handle leftovers, etc.

want to unroll loops that are run a lot and quick to execute

problem: compiler probably can't tell if this meets those criteria

```
for (int i = 0; i < some_variable; ++i) {  
    sum += some_function();  
}
```

figuring out how to unroll?

exercise: why can the compiler probably not do this transformation?

```
void foo() { int sum = 0;
    for (int i = 0; i < some_global_variable; ++i) {
        sum += some_function();
    }
}
```

```
void foo_transformed() { int sum = 0;
    int i = 0;
    if (some_global_variable % 2 == 1) {
        i += 1;
        sum += some_function();
    }
    for (; i < some_global_variable; i += 2) {
        sum += some_function();
        sum += some_function();
    }
}
```

multiple accumulators downsides

downsides of loop unrolling

- bigger executables, slower if small number of iterations

- + uses extra registers (can't use those regs for something else)

want to use multiple accumulators if latency likely bottleneck

problem: compiler probably can't tell if this meets those criteria

```
for (int i = 0; i < some_variable; ++i) {  
    sum += some_function();  
}
```

loop with a function call

```
int addWithLimit(int x, int y) {  
    int total = x + y;  
    if (total > 10000)  
        return 10000;  
    else  
        return total;  
}  
...  
int sum(int *array, int n) {  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        sum = addWithLimit(sum, array[i]);  
    return sum;  
}
```

loop with a function call

```
int addWithLimit(int x, int y) {  
    int total = x + y;  
    if (total > 10000)  
        return 10000;  
    else  
        return total;  
}  
...  
int sum(int *array, int n) {  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        sum = addWithLimit(sum, array[i]);  
    return sum;  
}
```

function call assembly

```
... loop stuff ...
movl (%rbx), %esi // mov array[i]
movl %eax, %edi // mov sum
call addWithLimit
... more loop stuff ...
...
addWithLimit:
... /* code here */
ret
```

extra instructions executed: two moves, a call, and a ret

function call assembly

```
... loop stuff ...
movl (%rbx), %esi // mov array[i]
movl %eax, %edi // mov sum
call addWithLimit
... more loop stuff ...
...
addWithLimit:
... /* code here */
ret
```

extra instructions executed: two moves, a call, and a ret

alternative: *inline* the call

```
... loop stuff ...
... /* code here (+ small changes for arguments
           being in different places) */
... more loop stuff ...
```

manual inlining

```
int sum(int *array, int n) {  
    int sum = 0;  
    for (int i = 0; i < n; i++) {  
        sum = sum + array[i];  
        if (sum > 10000)  
            sum = 10000;  
    }  
    return sum;  
}
```

inlining pro/con

avoids call, ret, extra move instructions

allows compiler to **use more registers**

no caller-saved register problems

but not always faster:

worse for instruction cache

(more copies of function body code)

compiler inlining

compilers will inline, but...

will usually **avoid making code much bigger**

heuristic: inline if function is small enough

heuristic: inline if called exactly once

will usually **not inline across .o files**

some compilers allow hints to say “please inline/do not inline this function”

compiler limitations

needs to generate code that does the same thing...
...even in corner cases that “obviously don’t matter”

often doesn't ‘look into’ a method

needs to assume it might do anything

can't predict what inputs/values will be
e.g. lots of loop iterations or few?

can't understand code size versus speed tradeoffs

remove redundant operations (1)

```
int number_of_As(const char *str) {
    int count = 0;
    for (int i = 0; i < strlen(str); ++i) {
        if (str[i] == 'a')
            count++;
    }
    return count;
}
```

remove redundant operations (1, fix)

```
int number_of_As(const char *str) {  
    int count = 0;  
    int length = strlen(str);  
    for (int i = 0; i < length; ++i) {  
        if (str[i] == 'a')  
            count++;  
    }  
    return count;  
}
```

call strlen once, not once per character!

Big-Oh improvement!

remove redundant operations (1, fix)

```
int number_of_As(const char *str) {  
    int count = 0;  
    int length = strlen(str);  
    for (int i = 0; i < length; ++i) {  
        if (str[i] == 'a')  
            count++;  
    }  
    return count;  
}
```

call strlen once, not once per character!

Big-Oh improvement!

remove redundant operations (2)

```
int shiftArray(int *source, int *dest, int N, int amount) {  
    for (int i = 0; i < N; ++i) {  
        if (i + amount < N)  
            dest[i] = source[i + amount];  
        else  
            dest[i] = source[N - 1];  
    }  
}
```

compare $i + amount$ to N many times

remove redundant operations (2, fix)

```
int shiftArray(int *source, int *dest, int N, int amount) {  
    int i;  
    for (i = 0; i + amount < N; ++i) {  
        dest[i] = source[i + amount];  
    }  
    for (; i < N; ++i) {  
        dest[i] = source[N - 1];  
    }  
}
```

eliminate comparisons

compiler limitations

needs to generate code that does the same thing...
...even in corner cases that “obviously don’t matter”

often doesn't ‘look into’ a method

needs to assume it might do anything

can't predict what inputs/values will be

e.g. lots of loop iterations or few?

can't understand code size versus speed tradeoffs

exercise: when optimizations backfire...

Which of these optimizations are likely to **increase** machine code size? (**Select all that apply.**)

Which of these optimizations are likely to **increase** number of instructions executed? (**Select all that apply.**)

- A. cache blocking
- B. function inlining
- C. loop unrolling
- D. moving a calculation outside a loop
- E. multiple accumulators (after loop unrolling)

backup slides

loop unrolling performance

on my laptop with 992 elements (fits in L1 cache)

work/loop iteration	cycles/element	instructions/element
1	1.33	4.02
2	1.03	2.52
4	1.02	1.77
8	1.01	1.39
16	1.01	1.21
32	1.01	1.15

1.01 cycles/element — latency bound

multiple accumulators

```
int i;
long sum1 = 0, sum2 = 0;
for (i = 0; i + 1 < N; i += 2) {
    sum1 += A[i];
    sum2 += A[i+1];
}
// handle leftover, if needed
if (i < N)
    sum1 += A[i];
sum = sum1 + sum2;
```

multiple accumulators performance

on my laptop with 992 elements (fits in L1 cache)

16x unrolling, variable number of accumulators

accumulators	cycles/element	instructions/element
1	1.01	1.21
2	0.57	1.21
4	0.57	1.23
8	0.59	1.24
16	0.76	1.57

starts hurting after too many accumulators

why?

multiple accumulators performance

on my laptop with 992 elements (fits in L1 cache)

16x unrolling, variable number of accumulators

accumulators	cycles/element	instructions/element
1	1.01	1.21
2	0.57	1.21
4	0.57	1.23
8	0.59	1.24
16	0.76	1.57

starts hurting after too many accumulators

why?

8 accumulator assembly

```
sum1 += A[i + 0];  
sum2 += A[i + 1];  
...  
...
```

```
addq    (%rdx), %rax          // sum1 +=  
addq    8(%rdx), %rcx         // sum2 +=  
subq    $-128, %rdx           // i +=  
addq    -112(%rdx), %rbx      // sum3 +=  
addq    -104(%rdx), %r11      // sum4 +=  
...  
....  
cmpq    %r14, %rdx
```

register for each of the sum1, sum2, ...variables:

16 accumulator assembly

compiler runs out of registers

starts to use the stack instead:

```
movq    32(%rdx), %rax // get A[i+13]
addq    %rax, -48(%rsp) // add to sum13 on stack
```

code does **extra cache accesses**

also — already using all the adders available all the time

so performance increase not possible

multiple accumulators performance

on my laptop with 992 elements (fits in L1 cache)

16x unrolling, variable number of accumulators

accumulators	cycles/element	instructions/element
1	1.01	1.21
2	0.57	1.21
4	0.57	1.23
8	0.59	1.24
16	0.76	1.57

starts hurting after too many accumulators

why?

maximum performance

2 additions per element:

- one to add to sum

- one to compute address (part of mov)

3/16 add/sub/cmp + 1/16 branch per element:

- over 16 because loop unrolled 16 times

loop overhead

- compiler not as efficient as it could have been

$$2 + \frac{3}{16} + \frac{1}{16} = 2 + \frac{1}{4} \text{ instructions per element}$$

hardware limits on my machine

4(?) register renamings per cycle

(Intel doesn't really publish exact numbers here...)

4-6 instructions decoded/cycle

(depending on instructions)

4(?) microinstructions committed/cycle

4 (add or cmp+branch executed)/cycle

hardware limits on my machine

4(?) register renamings per cycle

(Intel doesn't really publish exact numbers here...)

4-6 instructions decoded/cycle

(depending on instructions)

4(?) microinstructions committed/cycle

4 (add or cmp+branch executed)/cycle

$(2 + 1/4) \div 4 \approx 0.57$ cycles/element

getting over this limit

the $+1/4$ was from loop overhead

solution: more loop unrolling!

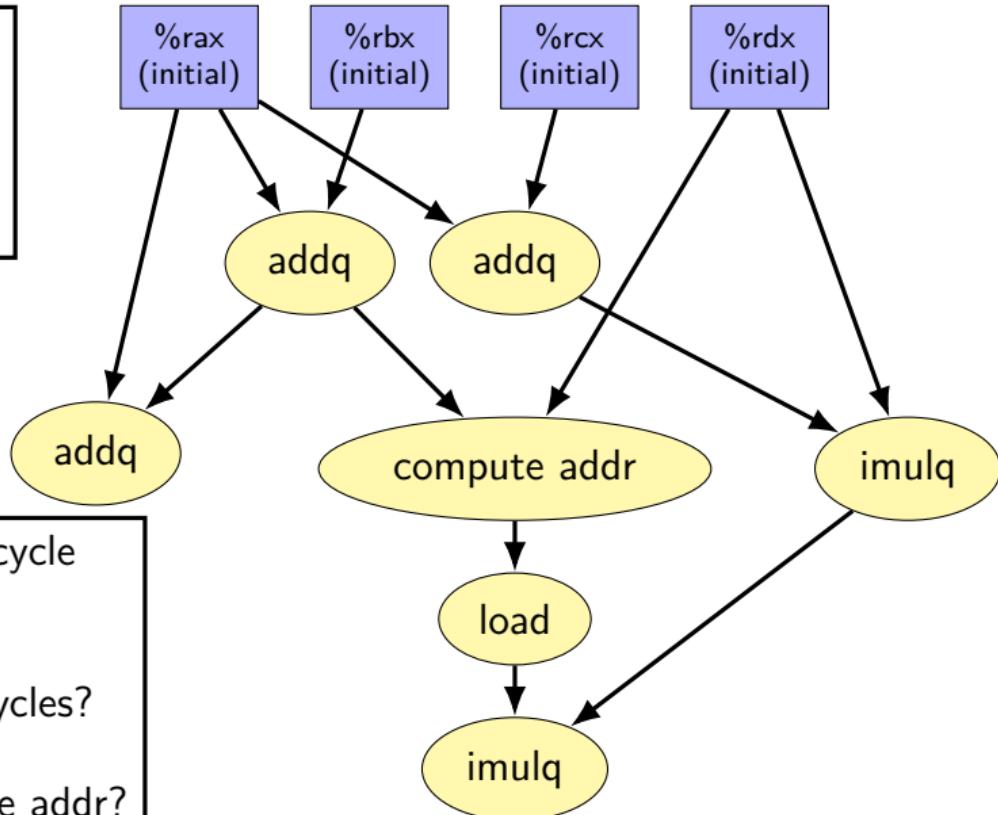
common theme with optimization:

fix one bottleneck (need to do adds one after the other)

find another bottleneck

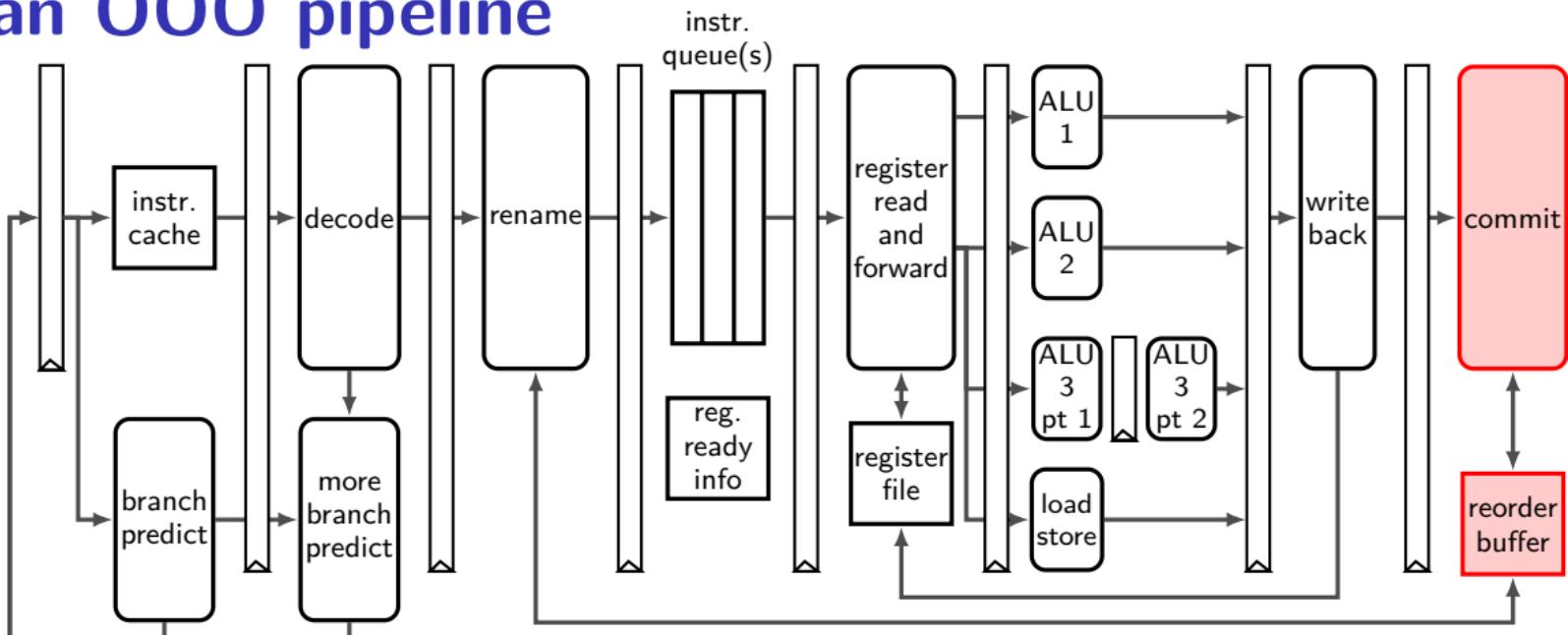
a data flow example

```
addq %rax, %rbx  
addq %rax, %rcx  
imulq %rdx, %rcx  
movq (%rbx, %rdx), %r8  
imulq %r8, %rcx  
addq %rax, %rbx
```



addq, compute addr: 1 cycle
imulq: 3 cycle latency
load: 3 cycle latency
Q1: latency bound on cycles?
Q2: what can be done
at same time as compute addr?

an OOO pipeline



reorder buffer: on rename

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07
...	...

free list

%x19
%x23
...
...

reorder buffer: on rename

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07
...	...

free list

%x19
%x23
...
...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x23		
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31		
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34		
19	0x1249	%rax / %x38		
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		

reorder buffer contains instructions started,
but not fully finished new entries created on rename
(not enough space? stall rename stage)

reorder buffer: on rename

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07
...	...

free list

%x19
%x23
...
...

remove here
when committed



reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x23		
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31		
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34		
19	0x1249	%rax / %x38		
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		

add here
on rename



place newly started instruction at end of buffer
remember at least its destination register
(both architectural and physical versions)

reorder buffer: on rename

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07 %x19
...	...

free list

%x19
%x23
...
...

remove here
when committed



reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x23		
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31		
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34		
19	0x1249	%rax / %x38		
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		
32	0x1230	%rdx / %x19		

add here
on rename



next renamed instruction goes in next slot, etc.

reorder buffer: on rename

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07 %x19
...	...

free list

%x19
%x23
...
...

remove here
when committed

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x23		
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31		
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34		
19	0x1249	%rax / %x38		
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		
32	0x1230	%rdx / %x19		

add here
on rename

reorder buffer: on commit

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07 %x19
...	...

free list

%x19
%x13
...
...

remove here
when committed

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24		
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31		
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34		
19	0x1249	%rax / %x38		
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		

reorder buffer: on commit

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07 %x19
...	...

remove here
when committed

free list

%x19
%x13
...
...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24		
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		✓

instructions marked done in reorder buffer
when result is computed
but not removed from reorder buffer ('committed') yet

reorder buffer: on commit

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07 %x19
...	...

free list

%x19
%x13
...
...

remove here
phys → arch. when committed
for committed

arch.	phys.
reg	reg
%rax	%x30
%rcx	%x28
%rbx	%x23
%rdx	%x21
...	...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24		
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		✓

commit stage tracks architectural to physical register map
for committed instructions

reorder buffer: on commit

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07 %x19
...	...

free list

%x19
%x13
...
%x23

remove here
phys → arch. when committed
for committed

arch.	phys.
reg	reg
%rax	%x30
%rcx	%x28
%rbx	%x23 %x24
%rdx	%x21
...	...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24	✓	
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		✓
32	0x1230	%rdx / %x19		

when next-to-commit instruction is done
update this register map and free register list
and remove instr. from reorder buffer

reorder buffer: on commit

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x07 %x19
...	...

free list

%x19
%x13
...
%x23

phys → arch. reg remove here
when committed

arch.	phys.
reg	reg
%rax	%x30
%rcx	%x28
%rbx	%x23 %x24
%rdx	%x21
...	...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24	✓	
15	0x1239	%rax / %x30		
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32		
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC		
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12		✓
32	0x1230	%rdx / %x19		

when next-to-commit instruction is done
update this register map and free register list
and remove instr. from reorder buffer

reorder buffer: commit mispredict (one way)

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x19
...	...

free list

%x19
%x13
...
...

phys → arch. reg
for committed

arch.	phys.
reg	reg
%rax	%x30 %x38
%rcx	%x31 %x32
%rbx	%x23 %x24
%rdx	%x21 %x34
...	...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24	✓	
15	0x1239	%rax / %x30	✓	
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32	✓	
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC	✓	✓
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12	✓	
32	0x1230	%rdx / %x19		



reorder buffer: commit mispredict (one way)

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x12
%rcx	%x17
%rbx	%x13
%rdx	%x19
...	...

phys → arch. reg
for committed

arch.	phys.
reg	reg
%rax	%x30 %x38
%rcx	%x31 %x32
%rbx	%x23 %x24
%rdx	%x21 %x34
...	...

free list

%x19
%x13
...
...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24	✓	
15	0x1239	%rax / %x30	✓	
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32	✓	
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC	✓	✓
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12	✓	
32	0x1230	%rdx / %x19		



when committing a mispredicted instruction...
this is where we undo mispredicted instructions

reorder buffer: commit mispredict (one way)

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x38
%rcx	%x32
%rbx	%x24
%rdx	%x34
...	...

phys → arch. reg
for committed

arch.	phys.
reg	reg
%rax	%x30 %x38
%rcx	%x31 %x32
%rbx	%x23 %x24
%rdx	%x21 %x34
...	...

free list

%x19
%x13
...
...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24	✓	
15	0x1239	%rax / %x30	✓	
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32	✓	
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC	✓	✓
21	0x1260	%rcx / %x17		
...
31	0x129f	%rax / %x12	✓	
32	0x1230	%rdx / %x19		

copy commit register map into rename register map
so we can start fetching from the correct PC

reorder buffer: commit mispredict (one way)

phys → arch. reg
for new instrs

arch.	phys.
reg	reg
%rax	%x38
%rcx	%x32
%rbx	%x24
%rdx	%x34
...	...

phys → arch. reg
for committed

arch.	phys.
reg	reg
%rax	%x30 %x38
%rcx	%x31 %x32
%rbx	%x23 %x24
%rdx	%x21 %x34
...	...

free list

%x19
%x13
...
...

reorder buffer (ROB)

instr num.	PC	dest. reg	done?	mispred? / except?
14	0x1233	%rbx / %x24	✓	
15	0x1239	%rax / %x30	✓	
16	0x1242	%rcx / %x31	✓	
17	0x1244	%rcx / %x32	✓	
18	0x1248	%rdx / %x34	✓	
19	0x1249	%rax / %x38	✓	
20	0x1254	PC	✓	✓
21	0x1260	%rcx / %x17		
...		
31	0x120f	%rax / %x12	✓	
32	0x1230	%rdx / %x19		

...and discard all the mispredicted instructions
(without committing them)

better? alternatives

can take snapshots of register map on each branch

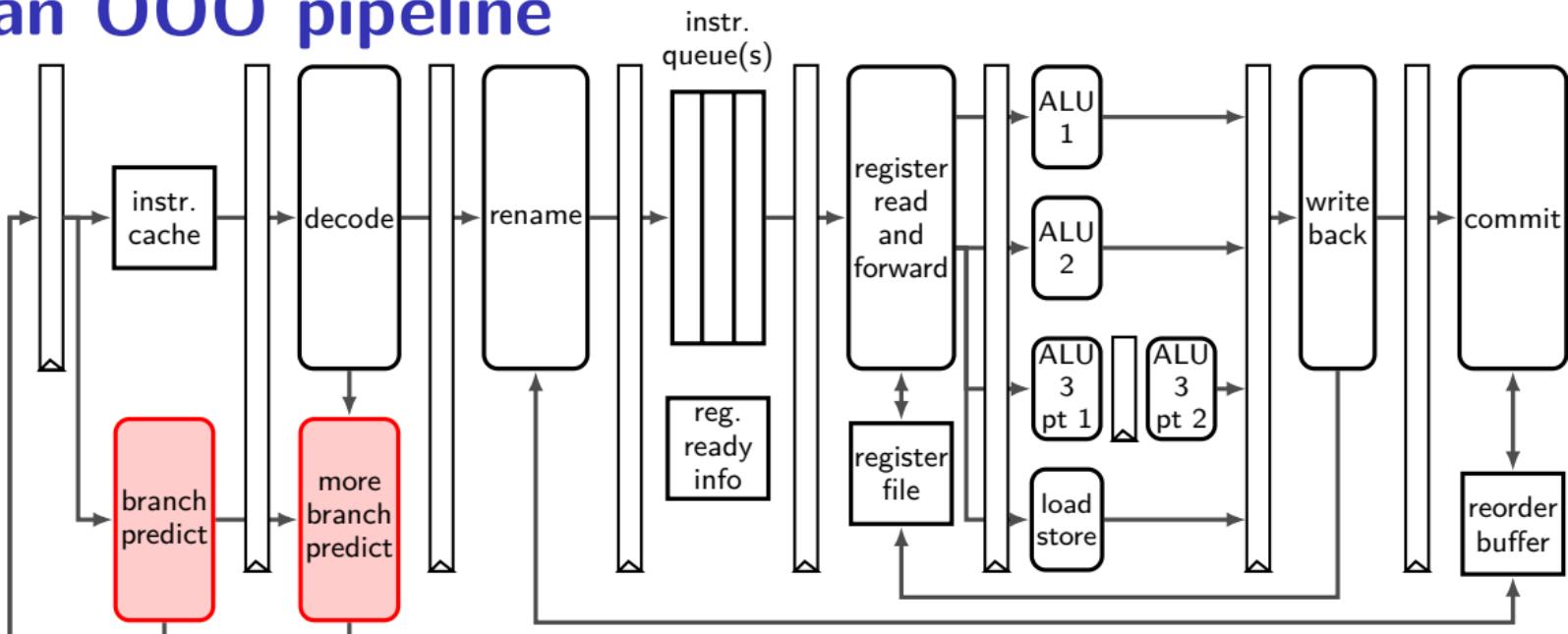
don't need to reconstruct the table
(but how to efficiently store them)

can reconstruct register map before we commit the branch instruction

need to let reorder buffer be accessed even more?

can track more/different information in reorder buffer

an OOO pipeline



branch target buffer

can take several cycles to fetch+decode jumps, calls, returns

still want 1-cycle prediction of next thing to fetch

BTB: cache for branches

idx	valid	tag	ofst	type	target	(more info?)
0x00	1	0x400	5	Jxx	0x3FFF3	...
0x01	1	0x401	C	JMP	0x401035	----
0x02	0	---	---	---	---	---
0x03	1	0x400	9	RET	---	...
...
0xFF	1	0x3FF	8	CALL	0x404033	...

valid	...
1	...
0	...
0	...
0	...
...	...
0	...

```
0x3FFF3:  movq %rax, %rsi
0x3FFF7:  pushq %rbx
0x3FFF8:  call 0x404033
0x400001:  popq %rbx
0x400003:  cmpq %rbx, %rax
0x400005:  jle 0x3FFF3
...
0x400031:  ret
...
```

BTB: cache for branches

idx	valid	tag	ofst	type	target	(more info?)	valid	...
0x00	1	0x400	5	Jxx	0x3FFF3	...	1	...
0x01	1	0x401	C	JMP	0x401035	----	0	...
0x02	0	---	---	----	---	----	0	...
0x03	1	0x400	9	RET	---	...	0	...
...
0xFF	1	0x3FF	8	CALL	0x404033	...	0	...

```
0x3FFF3:  movq %rax, %rsi
0x3FFF7:  pushq %rbx
0x3FFF8:  call 0x404033
0x400001:  popq %rbx
0x400003:  cmpq %rbx, %rax
0x400005:  jle 0x3FFF3
...
0x400031:  ret
...
```

BTB: cache for branches

idx	valid	tag	ofst	type	target	(more info?)	valid	...
0x00	1	0x400	5	Jxx	0x3FFF3	...	1	...
0x01	1	0x401	C	JMP	0x401035	----	0	...
0x02	0	---	---	----	---	----	0	...
0x03	1	0x400	9	RET	---	...	0	...
...
0xFF	1	0x3FF	8	CALL	0x404033	...	0	...

```
0x3FFF3:  movq %rax, %rsi
0x3FFF7:  pushq %rbx
0x3FFF8:  call 0x404033
0x400001:  popq %rbx
0x400003:  cmpq %rbx, %rax
0x400005:  jle 0x3FFF3
...
0x400031:  ret
...
```

aside on branch pred. and performance

modern branch predictors are very good

we might explore how later in semester (if time)

...usually can assume most branches will be predicted

but could be a problem if really no pattern

e.g. branch based on random number?

generally: measure and see

if branch prediction is bad...

avoiding branches — conditional move, etc.

replace multiple branches with single lookup?
one misprediction better than K ?

recall: shifts

we mentioned that compilers compile $x/4$ into a shift instruction
they are really good at these types of transformation...

“strength reduction”: replacing complicated op with simpler one

but can't do without seeing special case (e.g. divide by constant)

Intel Skylake OOO design

2015 Intel design — codename ‘Skylake’

94-entry instruction queue-equivalent

168 physical integer registers

168 physical floating point registers

4 ALU functional units

but some can handle more/different types of operations than others

2 load functional units

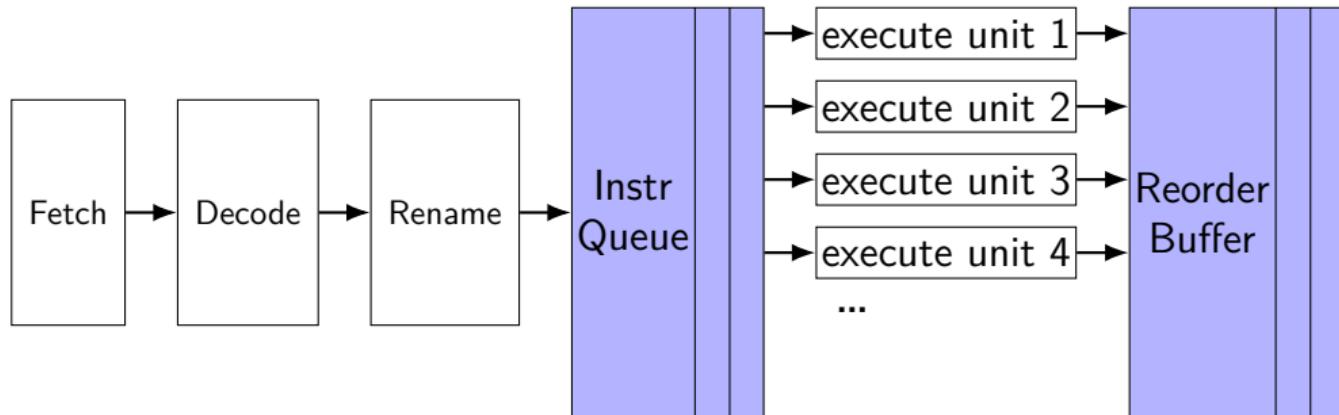
but pipelined: supports multiple pending cache misses in parallel

1 store functional unit

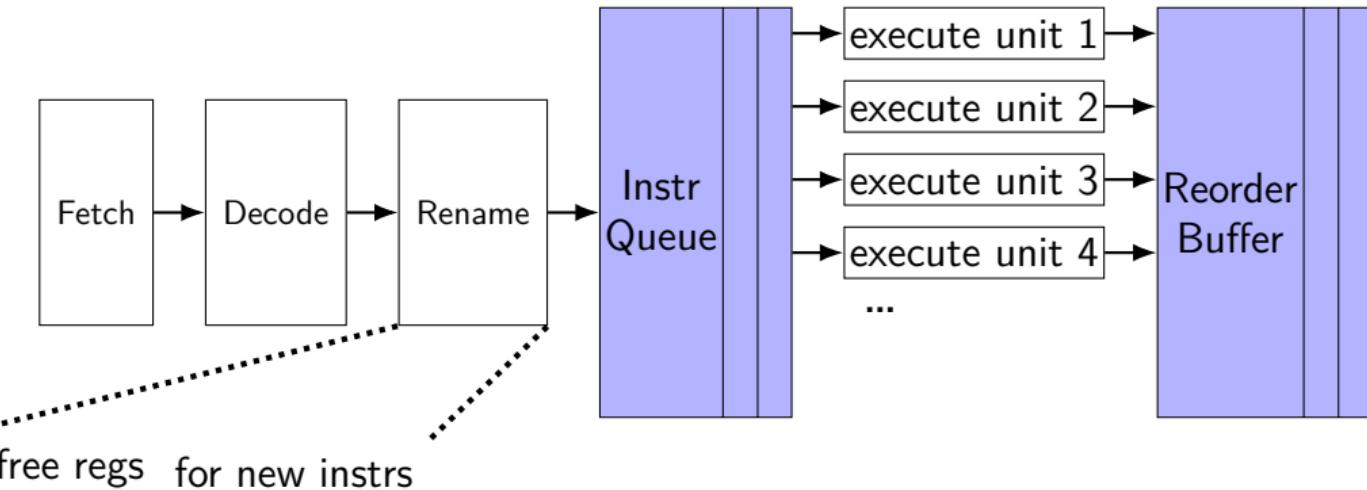
224-entry reorder buffer

determines how far ahead branch mispredictions, etc. can happen

exceptions and OOO (one strategy)



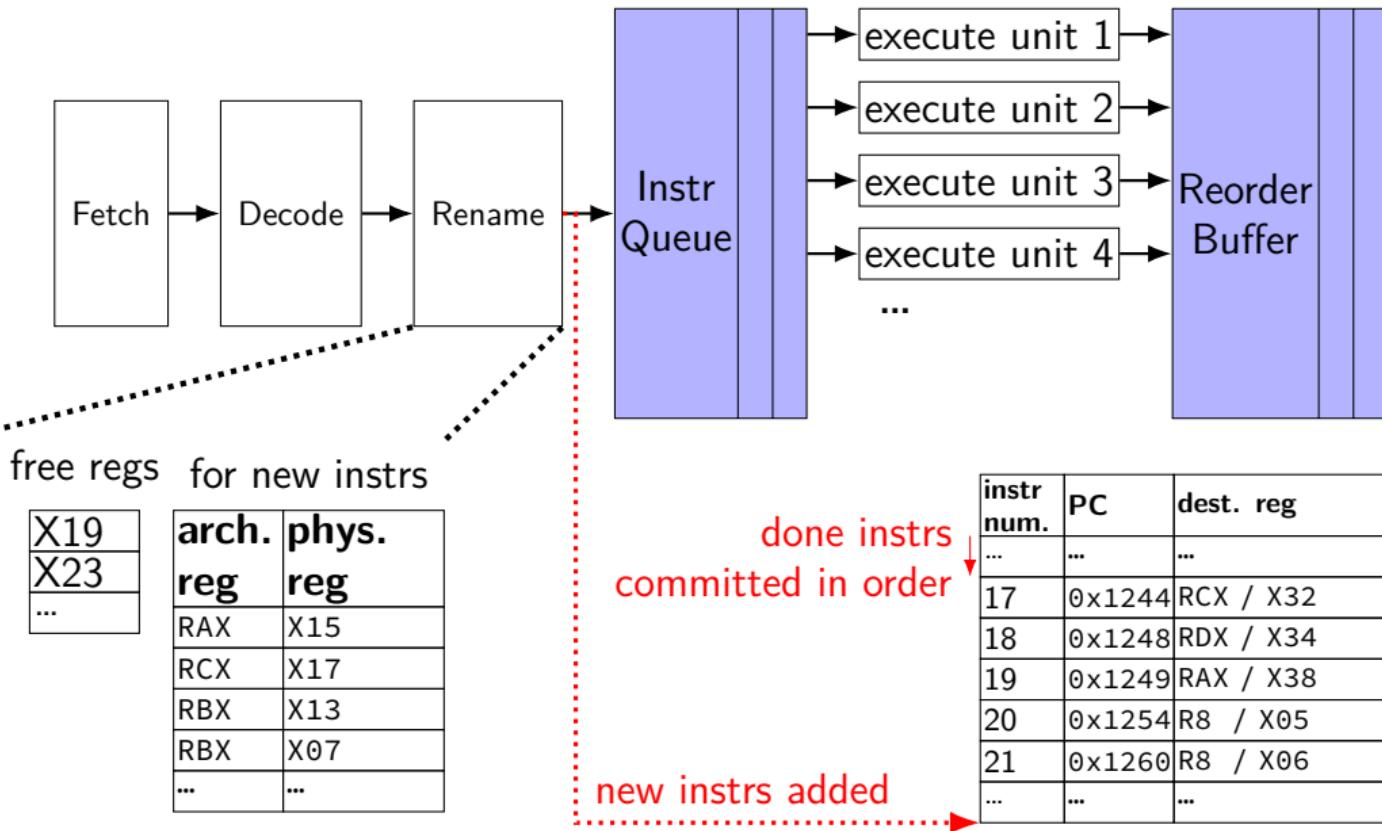
exceptions and OOO (one strategy)



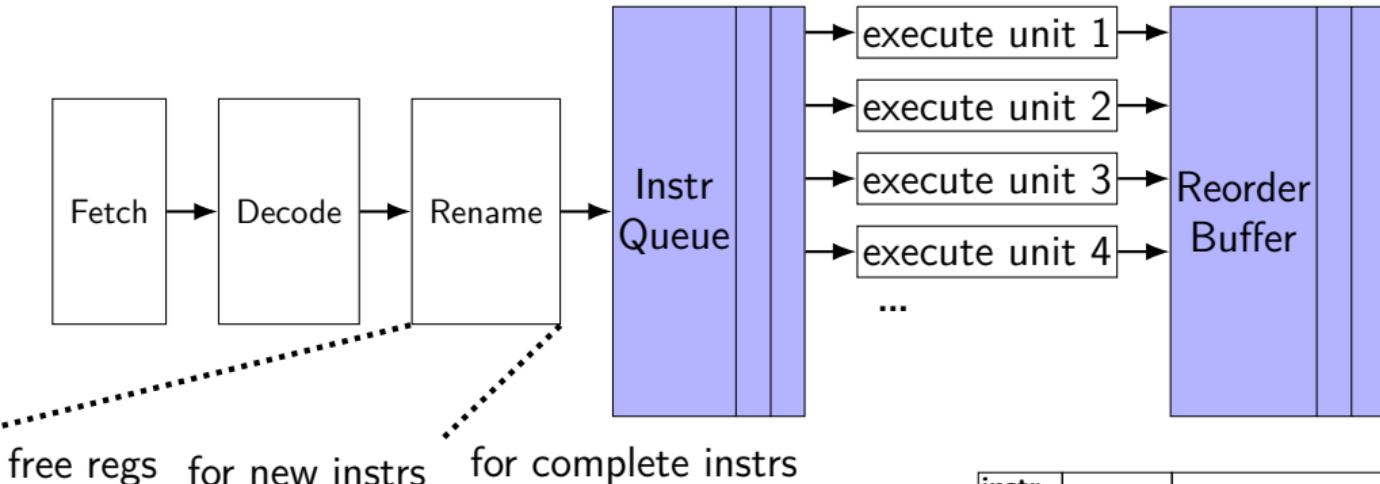
X19
X23
...

arch. reg	phys. reg
RAX	X15
RCX	X17
RBX	X13
RBX	X07
...	...

exceptions and OOO (one strategy)



exceptions and OOO (one strategy)



free regs for new instrs for complete instrs

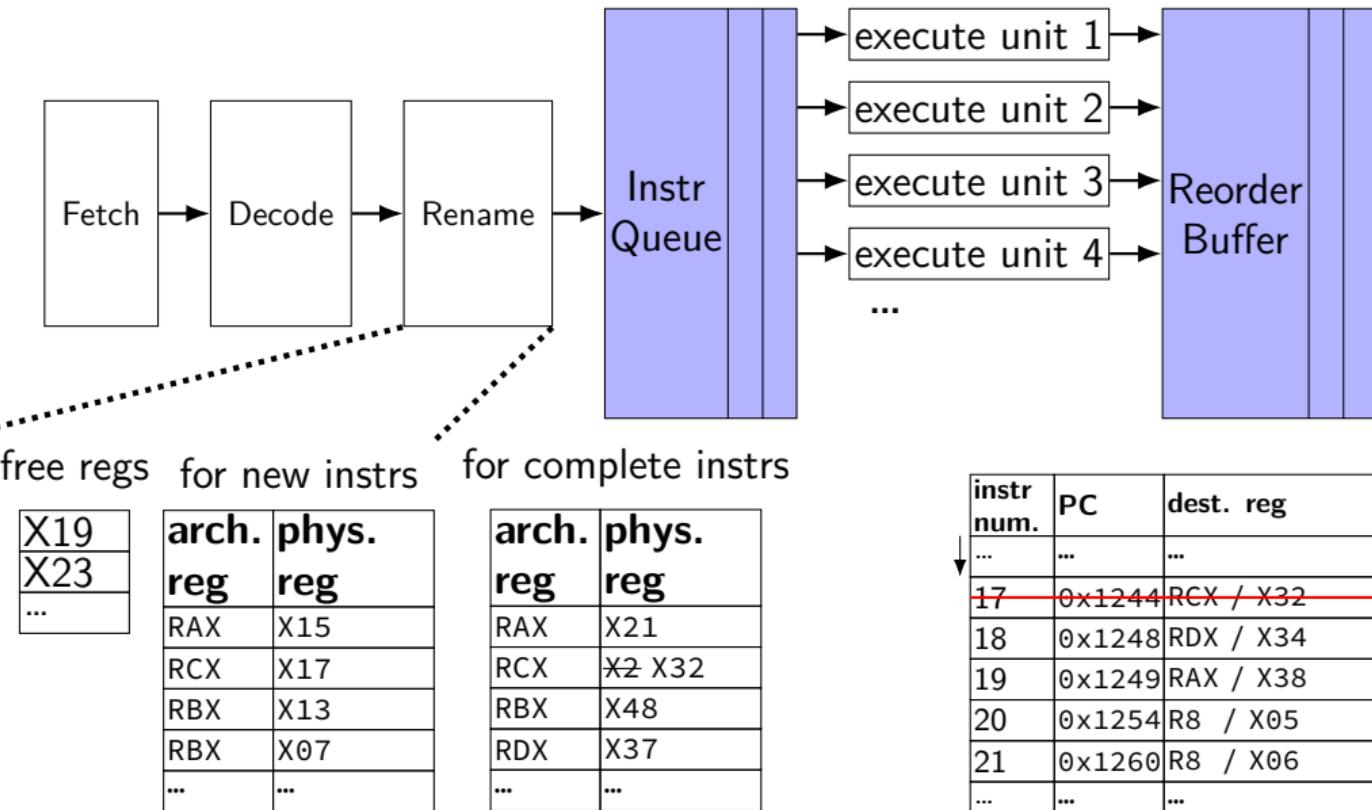
X19
X23
...

arch. reg	phys. reg
RAX	X15
RCX	X17
RBX	X13
RBX	X07
...	...

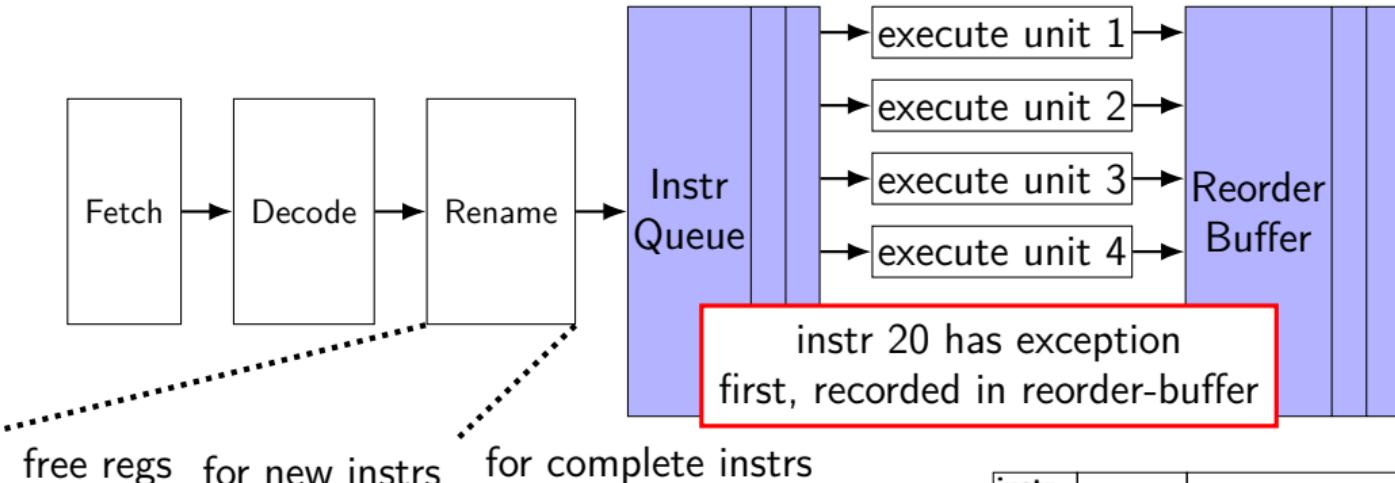
arch. reg	phys. reg
RAX	X21
RCX	X2 X32
RBX	X48
RDX	X37
...	...

instr num.	PC	dest. reg	done?	except?
...
17	0x1244	RCX / X32	✓	
18	0x1248	RDX / X34		
19	0x1249	RAX / X38	✓	
20	0x1254	R8 / X05		
21	0x1260	R8 / X06		
...

exceptions and OOO (one strategy)



exceptions and OOO (one strategy)



free regs for new instrs for complete instrs

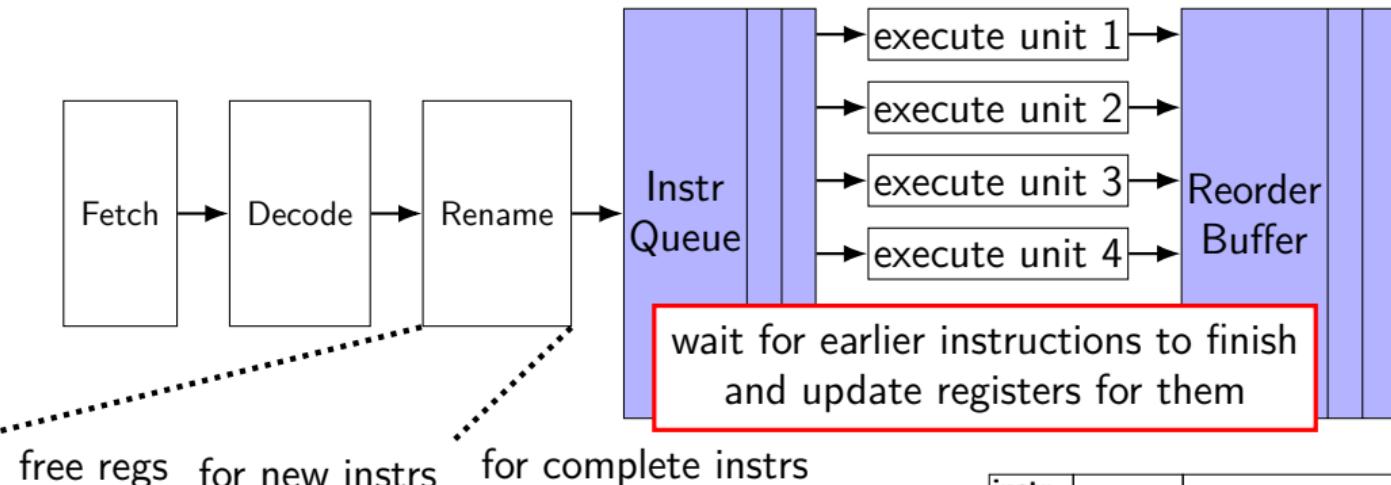
X19
X23
...

arch. reg	phys. reg
RAX	X15
RCX	X17
RBX	X13
RBX	X07
...	...

arch. reg	phys. reg
RAX	X21
RCX	X2 X32
RBX	X48
RDX	X37
...	...

instr num.	PC	dest. reg	done?	except?
...
17	0x1244	RCX / X32	✓	
18	0x1248	RDX / X34		
19	0x1249	RAX / X38	✓	
20	0x1254	R8 / X05	✓	✓
21	0x1260	R8 / X06		
...

exceptions and OOO (one strategy)



free regs for new instrs for complete instrs

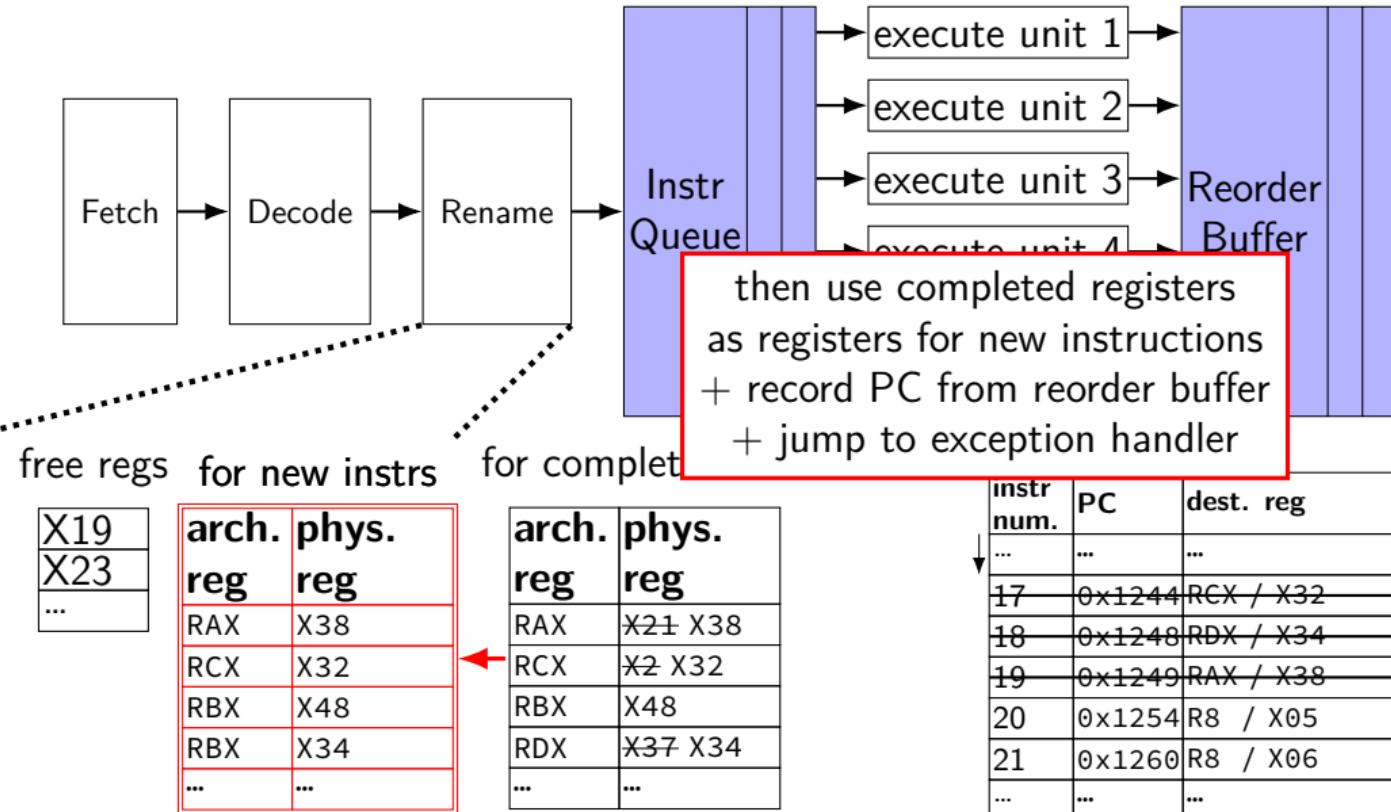
X19
X23
...

arch. reg	phys. reg
RAX	X15
RCX	X17
RBX	X13
RBX	X07
...	...

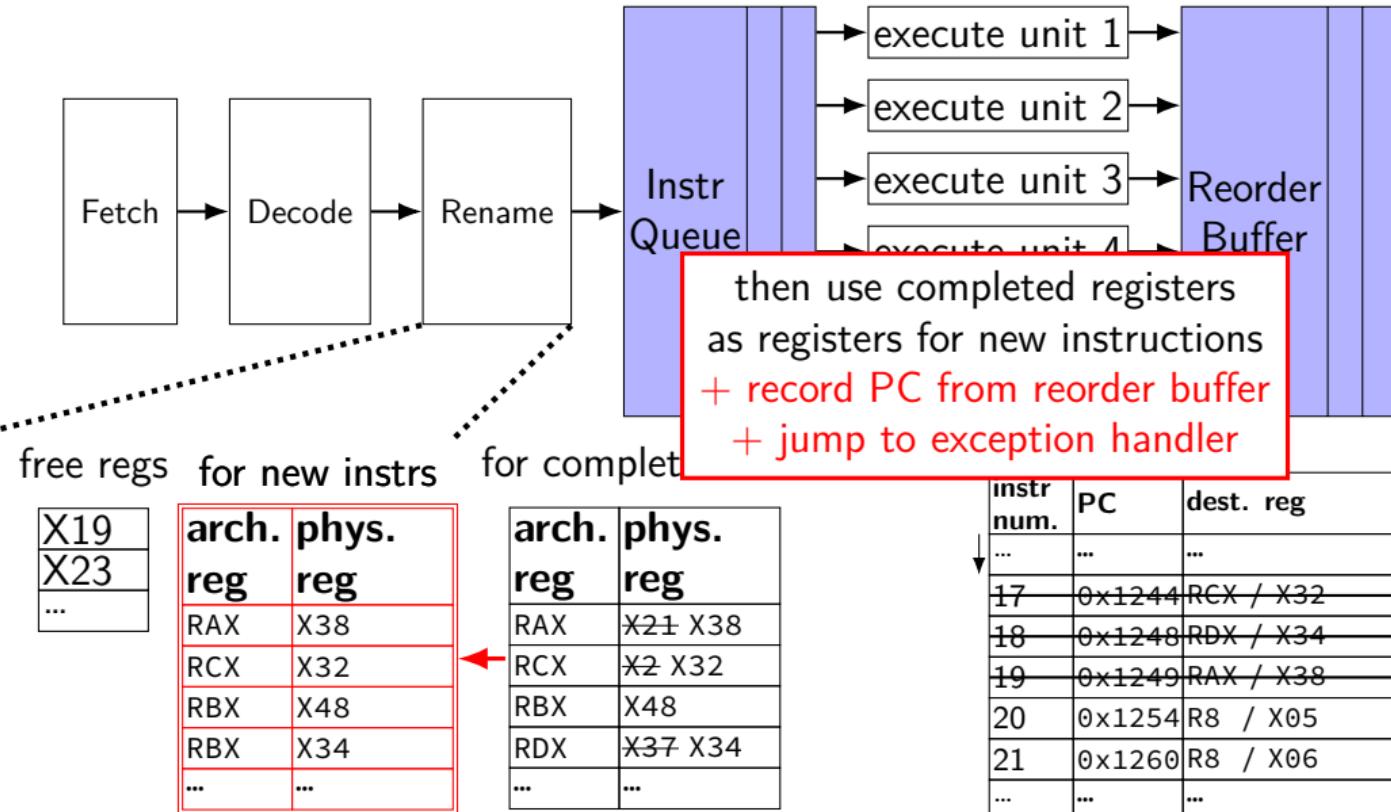
arch. reg	phys. reg
RAX	X21 X38
RCX	X2 X32
RBX	X48
RDX	X37 X34
...	...

instr num.	PC	dest. reg	done?	except?
...
17	0x1244	RCX / X32	✓	
18	0x1248	RDX ./ X34	✓	
19	0x1249	RAX ./ X38	✓	
20	0x1254	R8 / X05	✓	✓
21	0x1260	R8 / X06		
...

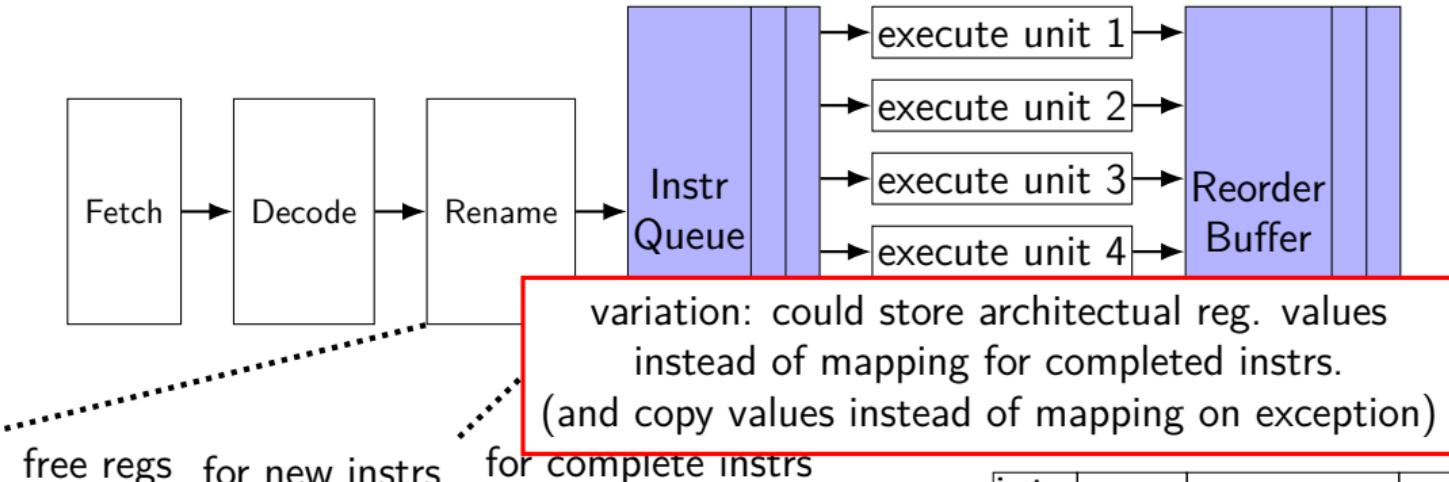
exceptions and OOO (one strategy)



exceptions and OOO (one strategy)



exceptions and OOO (one strategy)



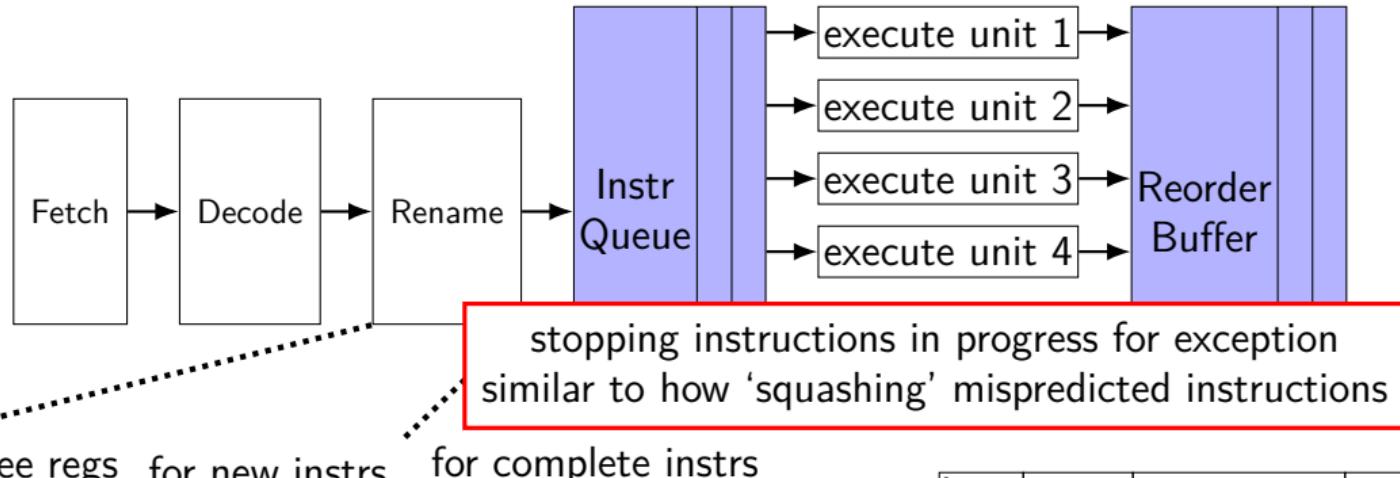
X19
X23
...

arch. reg	phys. reg
RAX	X15
RCX	X17
RBX	X13
RBX	X07
...	...

arch. reg	value
RAX	0x12343
RCX	0x234543
RBX	0x56782
RDX	0xF83A4
...	...

instr num.	PC	dest. reg	done?	except?
...
17	0x1244	RCX / X32	✓	
18	0x1248	RDX / X34	✓	
19	0x1249	RAX / X38	✓	
20	0x1254	R8 / X05	✓	✓
21	0x1260	R8 / X06		
...

exceptions and OOO (one strategy)



free regs for new instrs for complete instrs

X19
X23
...

arch. reg	phys. reg
RAX	X15
RCX	X17
RBX	X13
RBX	X07
...	...

arch. reg	phys. reg
RAX	X21 X38
RCX	X2 X32
RBX	X48
RDX	X37 X34
...	...

instr num.	PC	dest. reg	done?	except?
...
17	0x1244	RCX / X32	✓	
18	0x1248	RDX / X34	✓	
19	0x1249	RAX / X38	✓	
20	0x1254	R8 / X05	✓	✓
21	0x1260	R8 / X06		
...

addressing efficiency

```
for (int kk = 0; kk < N; kk += 2) {  
    for (int i = 0; i < N; ++i) {  
        for (int j = 0; j < N; ++j) {  
            float Cij = C[i * N + j];  
            for (int k = kk; k < kk + 2; ++k) {  
                Cij += A[i * N + k] * B[k * N + j];  
            }  
            C[i * N + j] = Cij;  
        }  
    }  
}
```

tons of multiplies by N??

isn't that slow?

addressing transformation

```
for (int kk = 0; k < N; kk += 2)
    for (int i = 0; i < N; ++i) {
        for (int j = 0; j < N; ++j) {
            float Cij = C[i * N + j];
            float *Bkj_pointer = &B[kk * N + j];
            for (int k = kk; k < kk + 2; ++k) {
                // Bij += A[i * N + k] * A[k * N + j~];
                Bij += A[i * N + k] * Bjk_pointer;
                Bjk_pointer += N;
            }
            C[i * N + j] = Bij;
        }
    }
}
```

transforms loop to **iterate with pointer**

compiler will often do this

increment/decrement by N ($\times \text{sizeof}(\text{float})$)

addressing transformation

```
for (int kk = 0; k < N; kk += 2)
    for (int i = 0; i < N; ++i) {
        for (int j = 0; j < N; ++j) {
            float Cij = C[i * N + j];
            float *Bkj_pointer = &B[kk * N + j];
            for (int k = kk; k < kk + 2; ++k) {
                // Bij += A[i * N + k] * A[k * N + j~];
                Bij += A[i * N + k] * Bjk_pointer;
                Bjk_pointer += N;
            }
            C[i * N + j] = Bij;
        }
    }
}
```

transforms loop to **iterate with pointer**

compiler will often do this

increment/decrement by N ($\times \text{sizeof}(\text{float})$)

addressing efficiency

compiler will **usually** eliminate slow multiplies
doing transformation yourself often slower if so

`i * N; ++i` into `i_times_N; i_times_N += N`

way to check: see if assembly uses lots multiplies in loop

if it doesn't — do it yourself

another addressing transformation

```
for (int i = 0; i < n; i += 4) {  
    C[(i+0) * n + j] += A[(i+0) * n + k] * B[k * n + j];  
    C[(i+1) * n + j] += A[(i+1) * n + k] * B[k * n + j];  
    // ...
```

```
int offset = 0;  
float *Ai0_base = &A[k];  
float *Ai1_base = Ai0_base + n;  
float *Ai2_base = Ai1_base + n;  
// ...  
for (int i = 0; i < n; i += 4) {  
    C[(i+0) * n + j] += Ai0_base[offset] * B[k * n + j];  
    C[(i+1) * n + j] += Ai1_base[offset] * B[k * n + j];  
    // ...  
    offset += n;
```

compiler will sometimes do this, too

another addressing transformation

```
for (int i = 0; i < n; i += 4) {  
    C[(i+0) * n + j] += A[(i+0) * n + k] * B[k * n + j];  
    C[(i+1) * n + j] += A[(i+1) * n + k] * B[k * n + j];  
    // ...
```

```
int offset = 0;  
float *Ai0_base = &A[k];  
float *Ai1_base = Ai0_base + n;  
float *Ai2_base = Ai1_base + n;  
// ...  
for (int i = 0; i < n; i += 4) {  
    C[(i+0) * n + j] += Ai0_base[offset] * B[k * n + j];  
    C[(i+1) * n + j] += Ai1_base[offset] * B[k * n + j];  
    // ...  
    offset += n;
```

compiler will sometimes do this, too

another addressing transformation

```
for (int i = 0; i < n; i += 20) {  
    C[(i+0) * n + j] += A[(i+0) * n + k] * B[k * n + j];  
    C[(i+1) * n + j] += A[(i+1) * n + k] * B[k * n + j];  
    // ...
```

```
int offset = 0;  
float *Ai0_base = &A[0*n+k];  
float *Ai1_base = Ai0_base + n;  
float *Ai2_base = Ai1_base + n;  
// ...  
for (int i = 0; i < n; i += 20) {  
    C[(i+0) * n + j] += Ai0_base[i*n] * B[k * n + j];  
    C[(i+1) * n + j] += Ai1_base[i*n] * B[k * n + j];  
    // ...  
    offset += n;
```

storing 20 A_{iX_base} ? — need the stack

maybe faster (quicker address computation)

maybe slower (can't do enough loads)

another addressing transformation

```
for (int i = 0; i < n; i += 20) {  
    C[(i+0) * n + j] += A[(i+0) * n + k] * B[k * n + j];  
    C[(i+1) * n + j] += A[(i+1) * n + k] * B[k * n + j];  
    // ...
```

```
int offset = 0;  
float *Ai0_base = &A[0*n+k];  
float *Ai1_base = Ai0_base + n;  
float *Ai2_base = Ai1_base + n;  
// ...  
for (int i = 0; i < n; i += 20) {  
    C[(i+0) * n + j] += Ai0_base[i*n] * B[k * n + j];  
    C[(i+1) * n + j] += Ai1_base[i*n] * B[k * n + j];  
    // ...  
    offset += n;
```

storing 20 A_{iX_base} ? — need the stack

maybe faster (quicker address computation)

maybe slower (can't do enough loads)

alternative addressing transformation

instead of:

```
float *Ai0_base = &A[0*n+k];
float *Ai1_base = Ai0_base + n;
// ...
for (int i = 0; i < n; i += 20) {
    C[(i+0) * n + j] += Ai0_base[i*n] * B[k * n + j];
    C[(i+1) * n + j] += Ai1_base[i*n] * B[k * n + j];
    // ...
```

could do:

```
float *Ai0_base = &A[k];
for (int i = 0; i < n; i += 20) {
    float *A_ptr = &Ai0_base[i*n];
    C[(i+0) * n + j] += *A_ptr * A[k * n + j];
    A_ptr += n;
    C[(i+1) * n + j] += *A_ptr * B[k * n + j];
    // ...
```

avoids spilling on the stack, but more dependencies

alternative addressing transformation

instead of:

```
float *Ai0_base = &A[0*n+k];
float *Ai1_base = Ai0_base + n;
// ...
for (int i = 0; i < n; i += 20) {
    C[(i+0) * n + j] += Ai0_base[i*n] * B[k * n + j];
    C[(i+1) * n + j] += Ai1_base[i*n] * B[k * n + j];
    // ...
```

could do:

```
float *Ai0_base = &A[k];
for (int i = 0; i < n; i += 20) {
    float *A_ptr = &Ai0_base[i*n];
    C[(i+0) * n + j] += *A_ptr * A[k * n + j];
    A_ptr += n;
    C[(i+1) * n + j] += *A_ptr * B[k * n + j];
    // ...
```

avoids spilling on the stack, but more dependencies

addressing efficiency generally

mostly: compiler does very good job itself
eliminates multiplications, use pointer arithmetic
often will do better job than if how typically programming would do it manually

sometimes compiler won't take the best option
if spilling to the stack: can cause weird performance anomalies
if indexing gets too complicated — might not remove multiply
if compiler doesn't, you can always make addressing simple yourself
convert to pointer arith. without multiplies