Caching

memory hierarchy assumptions

temporal locality

"if a value is accessed now, it will be accessed again soon"

caches should keep recently accessed values

spatial locality

"if a value is accessed now, adjacent values will be accessed soon"

caches should store adjacent values at the same time

natural properties of programs — think about loops

locality examples

```
double computeMean(int length, double *values) {
    double total = 0.0;
    for (int i = 0; i < length; ++i) {
        total += values[i];
    }
    return total / length;
}
```

temporal locality: machine code of the loop

spatial locality: machine code of most consecutive instructions

temporal locality: `total`, `i`, `length` accessed repeatedly

spatial locality: `values[i+1]` accessed after `values[i]`

building a (direct-mapped) cache

Cache

value
00 00
00 00
00 00
00 00

cache block: 2 bytes

Memory

addresses	bytes
00000–00001	00 11
00010–00011	22 33
00100–00101	55 55
00110–00111	66 77
01000–01001	88 99
01010–01011	AA BB
01100–01101	CC DD
01110–01111	EE FF
10000–10001	F0 F1
…	…

building a (direct-mapped) cache

read byte at 01011?

Cache

value
00 00
00 00
00 00
00 00

cache block: 2 bytes

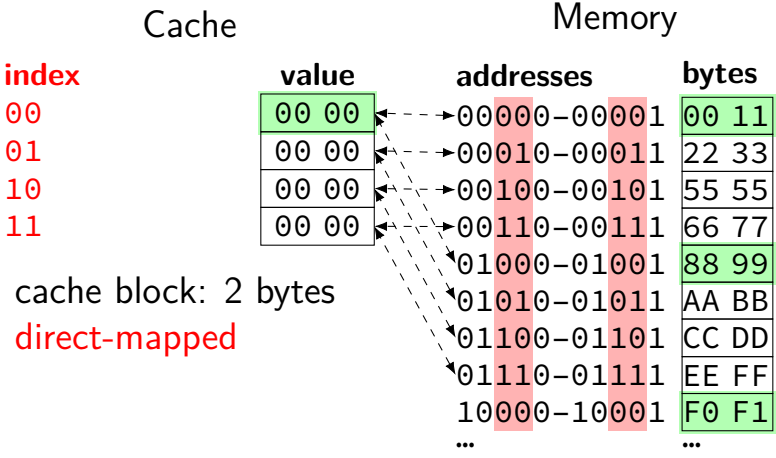
Memory

addresses	bytes
00000–00001	00 11
00010–00011	22 33
00100–00101	55 55
00110–00111	66 77
01000–01001	88 99
01010–01011	AA BB
01100–01101	CC DD
01110–01111	EE FF
10000–10001	F0 F1
…	…

building a (direct-mapped) cache

read byte at 01011?

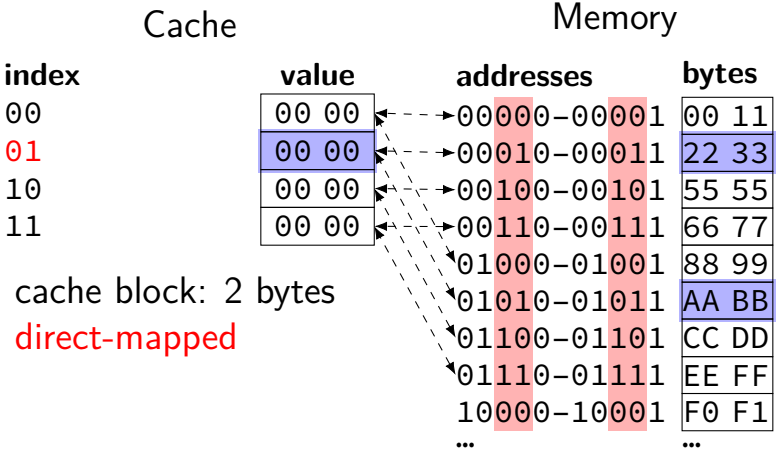
exactly one place for each address
spread out what can go in a block



building a (direct-mapped) cache

read byte at 01011?

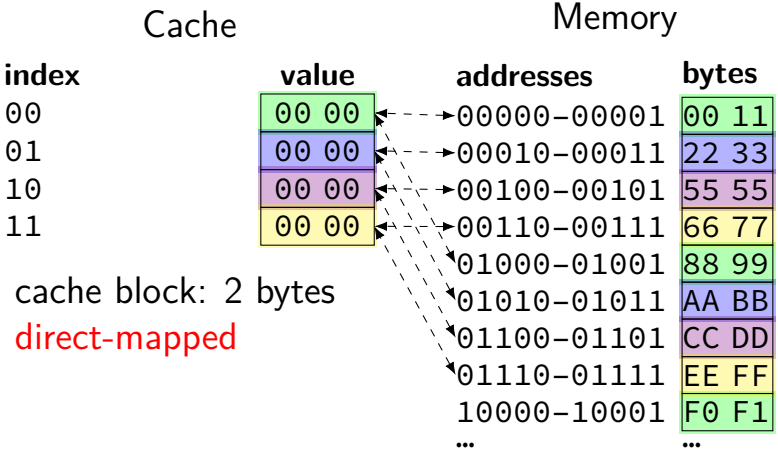
exactly one place for each address
spread out what can go in a block



building a (direct-mapped) cache

read byte at 01011?

exactly one place for each address
spread out what can go in a block

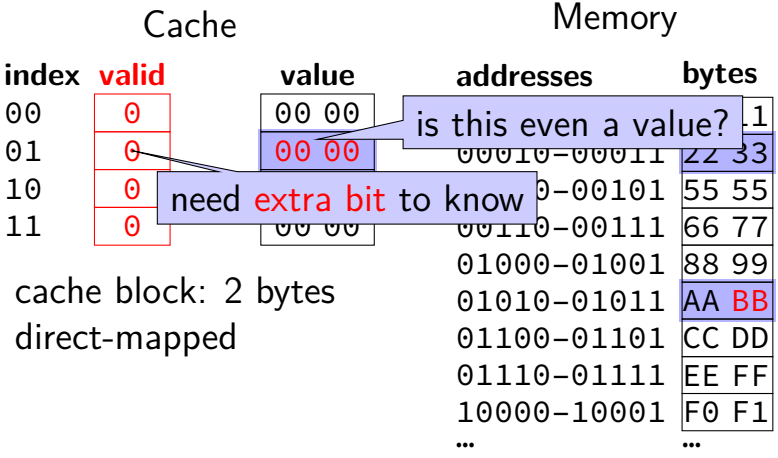


cache block: 2 bytes

direct-mapped

building a (direct-mapped) cache

read byte at 01011?



building a (direct-mapped) cache

read byte at 01011?
invalid, fetch

Cache

index	valid	value
00	0	00 00
01	1	AA BB
10	0	00 00
11	0	00 00

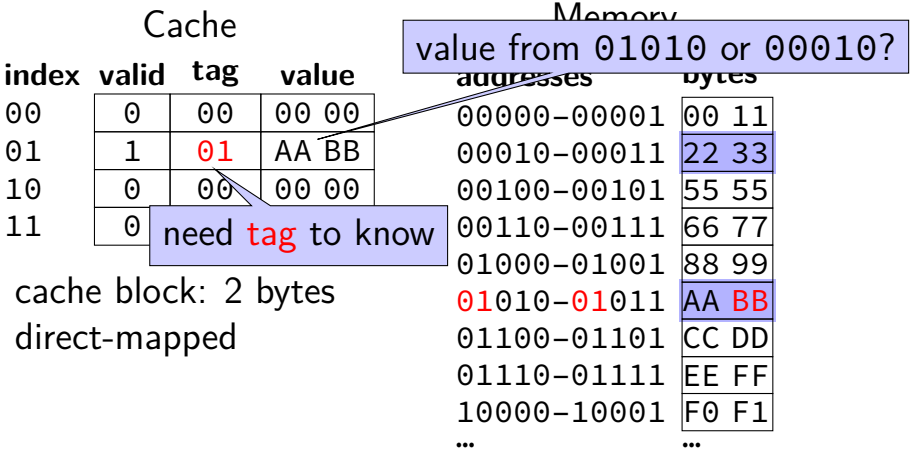
cache block: 2 bytes
direct-mapped

Memory

addresses	bytes
00000–00001	00 11
00010–00011	22 33
00100–00101	55 55
00110–00111	66 77
01000–01001	88 99
01010–01011	AA BB
01100–01101	CC DD
01110–01111	EE FF
10000–10001	F0 F1
…	…

building a (direct-mapped) cache

read byte at 01011?
invalid, fetch



cache block: 2 bytes
direct-mapped

building a (direct-mapped) cache

read byte at 01011?
invalid, fetch

Cache

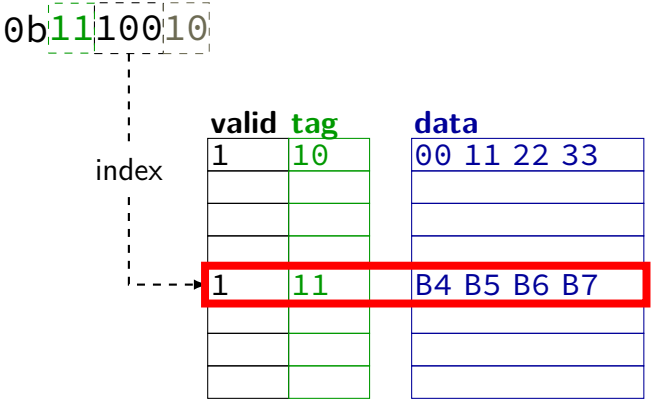
index	valid	tag	value
00	0	00	00 00
01	1	01	AA BB
10	0	00	00 00
11	0	00	00 00

cache block: 2 bytes
direct-mapped

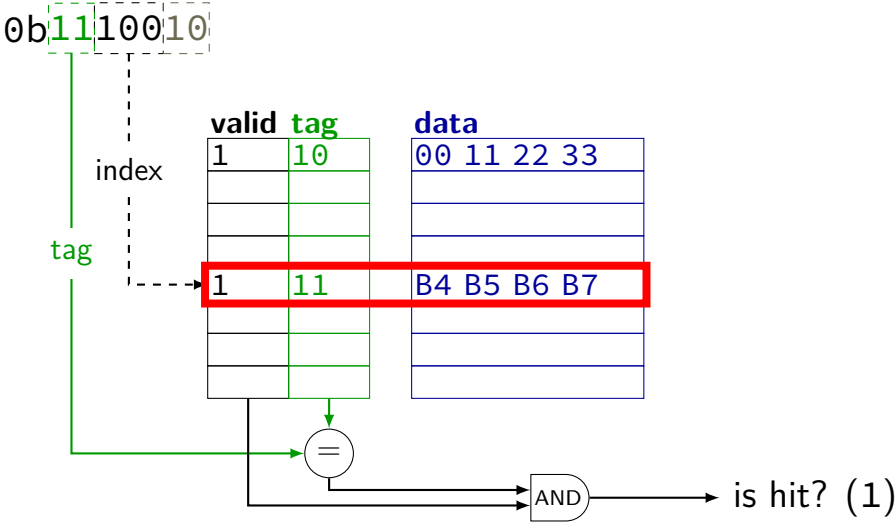
Memory

addresses	bytes
00000–00001	00 11
00010–00011	22 33
00100–00101	55 55
00110–00111	66 77
01000–01001	88 99
01010–01011	AA BB
01100–01101	CC DD
01110–01111	EE FF
10000–10001	F0 F1
…	…

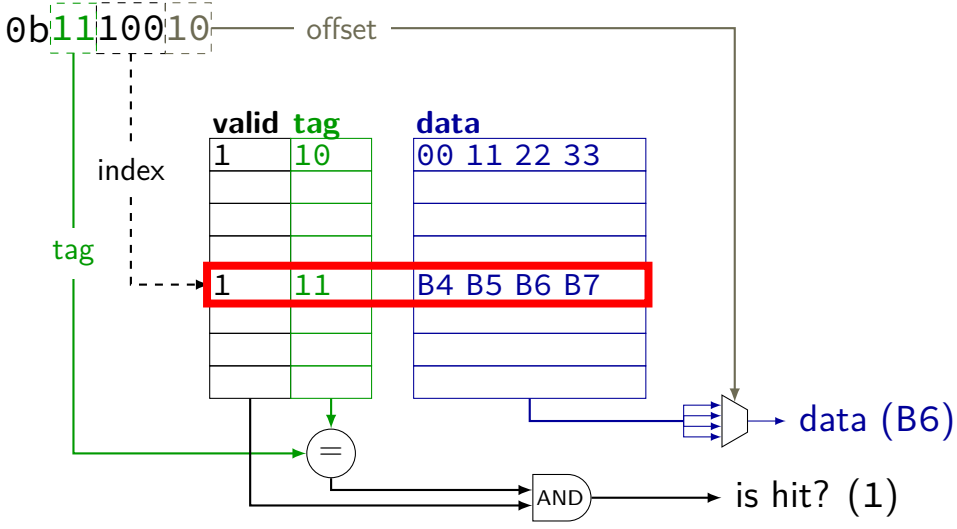
cache operation (read)



cache operation (read)



cache operation (read)



Tag-Index-Offset (TIO)

address `001111` (stores value `0xFF`)

cache	tag	index	offset
2 byte blocks, 4 sets	???	???	???
2 byte blocks, 8 sets	???	???	???
4 byte blocks, 2 sets	???	???	???

2 byte blocks, 4 sets

index	valid	tag	value
00	1	000	00 11
01	1	001	AA BB
10	0	--	-- --
11	1	001	EE FF

4 byte blocks, 2 sets

index	valid	tag	value
0	1	00	00 11 22 33
1	1	01	CC DD EE FF

2 byte blocks, 8 sets

index	valid	tag	value
000	1	00	00 11
001	1	01	F1 F2
010	0	--	-- --
011	0	--	-- --
100	0	--	-- --
101	1	00	AA BB
110	0	--	-- --
111	1	00	EE FF

Tag-Index-Offset (TIO)

address `001111` (stores value `0xFF`)

cache	tag	index	offset
2 byte blocks, 4 sets	???	???	1
2 byte blocks, 8 sets	???	???	1
4 byte blocks, 2 sets	???	???	???

2 byte blocks, 4 sets

index	valid	tag	value
00	1	000	00 11
01	1	001	AA BB
10	0	--	-- --
11	1	001	EE FF

4 byte blocks, 2 sets

index	valid	tag	value
0	1	00	00 11 22 33
1	1	01	CC DD EE FF

2 byte blocks, 8 sets

index	valid	tag	value
000	1	000	00 11
001	1	001	F1 F2
010	0	--	-- --
011	0	--	-- --
100	0	--	-- --
101	1	00	AA BB
110	0	--	-- --
111	1	00	EE FF

$2 = 2^1$ bytes in block
1 bit to say which byte

Tag-Index-Offset (TIO)

address `001111` (stores value `0xFF`)

cache	tag	index	offset
2 byte blocks, 4 sets	???	???	1
2 byte blocks, 8 sets	???	???	1
4 byte blocks, 2 sets	???	???	11

2 byte blocks, 4 sets

index	valid	tag	value
00	1	000	00 11
01	1	001	AA BB
10	0		
11	1		

4 byte blocks, 2 sets

index	valid	tag	value
0	1	00	00 11 22 33
1	1	01	CC DD EE FF

$4 = 2^2$ bytes in block
2 bits to say which byte

2 byte blocks, 8 sets

index	valid	tag	value
000	1	00	00 11
001	1	01	F1 F2
010	0	--	-- --
011	0	--	-- --
100	0	--	-- --
101	1	00	AA BB
110	0	--	-- --
111	1	00	EE FF

Tag-Index-Offset (TIO)

address `001111` (stores value `0xFF`)

cache	tag	index	offset
2 byte blocks, 4 sets	???	11	1
2 byte blocks, 8 sets	???		1
4 byte blocks, 2 sets	???	1	11

2 byte blocks, 4 sets

index	valid	tag	value
00	1	000	00 11
01	1	001	AA BB
10	0	--	-- --
11	1	001	EE FF

4 byte blocks, 2 sets

index	valid	tag	value
0	1	00	00 11 22 33
1	1	01	CC DD EE FF

2 byte blocks, 8 sets

index	valid	tag	value
000	1	00	00 11
001	1	00	F1 F2
010	0	--	-- --
011	0	--	-- --
100	0	--	-- --
101	1	00	AA BB
110	0	--	-- --
111	1	00	EE FF

$2^2 = 4$ sets
2 bits to index set

Tag-Index-Offset (TIO)

address `001111` (stores value `0xFF`)

cache	tag	index	offset
2 byte blocks, 4 sets	???	11	1
2 byte blocks, 8 sets	???	111	1
4 byte blocks, 2 sets	???	1	11

2 byte blocks, 4 sets

index	valid	tag	value
00	1	000	00 11
01	1	001	AA BB
10	0	--	-- --
11	1		

4

index	valid	tag	value
0	1	00	00 11 22 33
1	1	01	CC DD EE FF

$2^3 = 8$ sets

3 bits to index set

2 byte blocks, 8 sets

index	valid	tag	value
000	1	00	00 11
001	1	01	F1 F2
010	0	--	-- --
011	0	--	-- --
100	0	--	-- --
101	1	00	AA BB
110	0	--	-- --
111	1	00	EE FF

Tag-Index-Offset (TIO)

address `001111` (stores value `0xFF`)

cache	tag	index	offset
2 byte blocks, 4 sets	???	11	1
2 byte blocks, 8 sets	???	111	1
4 byte blocks, 2 sets	???	1	11

2 byte blocks, 4 sets

index	valid	tag	value
00	1	000	00 11
01	1	001	AA BB
10	0	--	-- --
11	1	001	EE FF

4 byte blocks, 2 sets

index	valid	tag	value
0	1	00	00 11 22 33
1	1	01	CC DD EE FF

2 byte blocks, 8 sets

index	valid	tag	value
000	1	00	00 11
001	1	01	F1 F2
010	0	--	-- --
01			--
10			--
1			BB
110	0	--	--
111	1	00	EE FF

$2^1 = 2$ sets
1 bit to index set

Tag-Index-Offset (TIO)

address `001111` (stores value `0xFF`)

cache	tag	index	offset
2 byte blocks, 4 sets	001	11	1
2 byte blocks, 8 sets	00	111	1
4 byte blocks, 2 sets	001	1	11

tag — whatever is left over

00	1	000	00 11
01	1	001	AA BB
10	0	--	-- --
11	1	001	EE FF

4 byte blocks, 2 sets

index	valid	tag	value
0	1	00	00 11 22 33
1	1	01	CC DD EE FF

2 byte blocks, 8 sets

index	valid	tag	value
000	1	00	00 11
001	1	01	F1 F2
010	0	--	-- --
011	0	--	-- --
100	0	--	-- --
101	1	00	AA BB
110	0	--	-- --
111	1	00	EE FF

Tag-Index-Offset formulas (direct-mapped only)

m	memory addreses bits (Y86-64: 64)
$S = 2^s$	number of sets
s	(set) index bits
$B = 2^b$	block size
b	(block) offset bits
$t = m - (s + b)$	tag bits
$C = B \times S$	cache size (if direct-mapped)

example access pattern (1)

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

2 byte blocks, 4 sets

index	valid	tag	value
00	0		
01	0		
10	0		
11	0		

example access pattern (1)

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

2 byte blocks, 4 sets

index	valid	tag	value
00	0		
01	0		
10	0		
11	0		

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	0		
01	0		
10	0		
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	0		
10	0		
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	0		
10	0		
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	1	01100	mem[0x62] mem[0x63]
10	0		
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	01100	mem[0x60] mem[0x61]
01	1	01100	mem[0x62] mem[0x63]
10	0		
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	01100	mem[0x60] mem[0x61]
01	1	01100	mem[0x62] mem[0x63]
10	0		
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	1	01100	mem[0x62] mem[0x63]
10	0		
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	miss

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	1	01100	mem[0x62] mem[0x63]
10	1	01100	mem[0x64] mem[0x65]
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	miss

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	1	01100	mem[0x62] mem[0x63]
10	1	01100	mem[0x64] mem[0x65]
11	0		

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	miss

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	1	01100	mem[0x62] mem[0x63]
10	1	01100	mem[0x64] mem[0x65]
11	0		

miss caused by conflict

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

exercise

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

4 byte blocks, 4 sets

index	valid	tag	value
00			
01			
10			
11			

exercise

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

4 byte blocks, 4 sets

index	valid	tag	value
00			
01			
10			
11			

how is the address 61 (01100001) split up into tag/index/offset?

b block offset bits;
$B = 2^b$ byte block size;
s set index bits; $S = 2^s$ sets ;
$t = m - (s + b)$ tag bits (leftover)

exercise

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

4 byte blocks, 4 sets

index	valid	tag	value
00			
01			
10			
11			

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

$B = 4 = 2^b$ byte block size
$b = 2$ (block) offset bits
$t = m - (s + b) = 4$ tag bits

exercise

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

4 byte blocks, 4 sets

index	valid	tag	value
00			
01			
10			
11			

$B = 4 = 2^b$ byte block size
$b = 2$ (block) offset bits
$t = m - (s + b) = 4$ tag bits

exercise

4 byte blocks, 4 sets

address (hex)	result
00000000 (00)	
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

index	valid	tag	value
00			
01			
10			
11			

exercise: how many accesses are hits?

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0000	mem[0x00]- mem[0x03]
01	0		
10			
11			

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0000	mem[0x00]– mem[0x03]
01	0		
10			
11			

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0110	mem[0x60]- mem[0x63]
01	0		
10			
11			

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	hit
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0110	mem[0x60]– mem[0x63]
01	0		
10			
11			

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	hit
01100010 (62)	hit
00000000 (00)	
01100100 (64)	

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0110	mem[0x60]– mem[0x63]
01	0		
10			
11			

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	hit
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0000	mem[0x00]- mem[0x03]
01	0		
10			
11			

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	hit
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	miss

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0000	mem[0x00]- mem[0x03]
01	1	0110	mem[0x64]- mem[0x67]
10			
11			

exercise

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	hit
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	miss

tag index offset

4 byte blocks, 4 sets

index	valid	tag	value
00	1	0000	mem[0x00]-mem[0x03]
01	1	0110	mem[0x64]-mem[0x67]
10			
11			

example access pattern (1)

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	miss
01100100 (64)	miss

tag index offset

$m = 8$ bit addresses
$S = 4 = 2^s$ sets
$s = 2$ (set) index bits

2 byte blocks, 4 sets

index	valid	tag	value
00	1	00000	mem[0x00] mem[0x01]
01	1	01100	mem[0x62] mem[0x63]
10	1	01100	mem[0x64] mem[0x65]
11	0		

miss caused by conflict

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 5$ tag bits

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

multiple places to put values with same index
avoid conflict misses

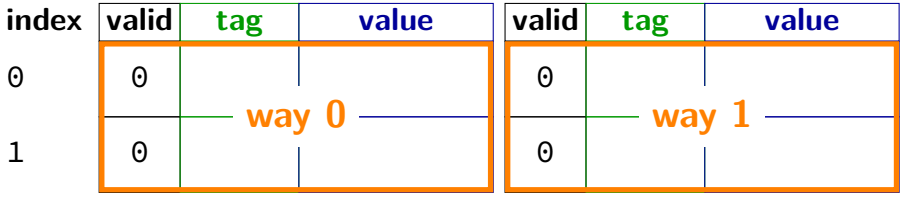
adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0		set 0	0		
1	0		set 1	0		

adding associativity

2-way set associative, 2 byte blocks, 2 sets



adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	0			0		
1	0			0		

$m = 8$ bit addresses
$S = 2 = 2^s$ sets
$s = 1$ (set) index bits

$B = 2 = 2^b$ byte block size
$b = 1$ (block) offset bits
$t = m - (s + b) = 6$ tag bits

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	0			0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	0		
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	
00000000 (00)	
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	

tag index offset

adding associativity

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

needs to replace block in set 0!

tag index offset

adding associativity

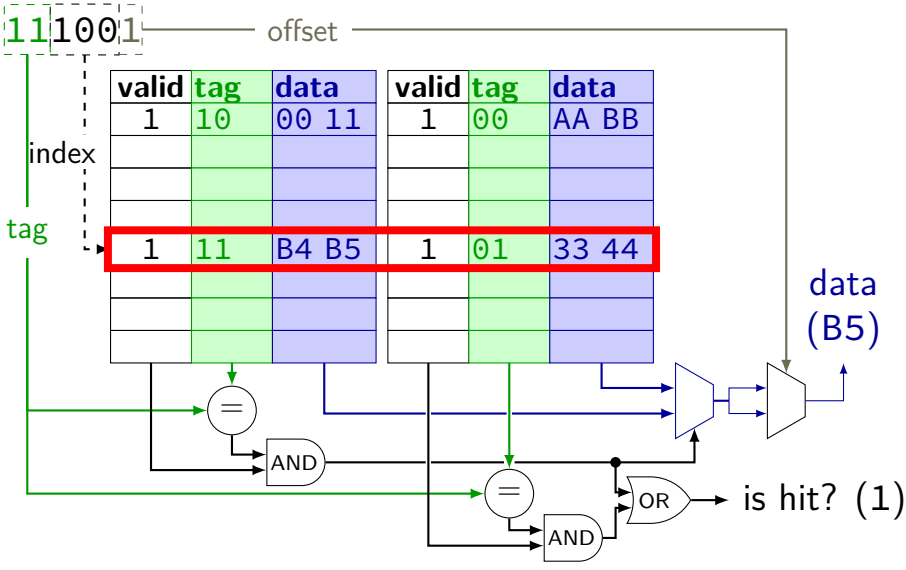
2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

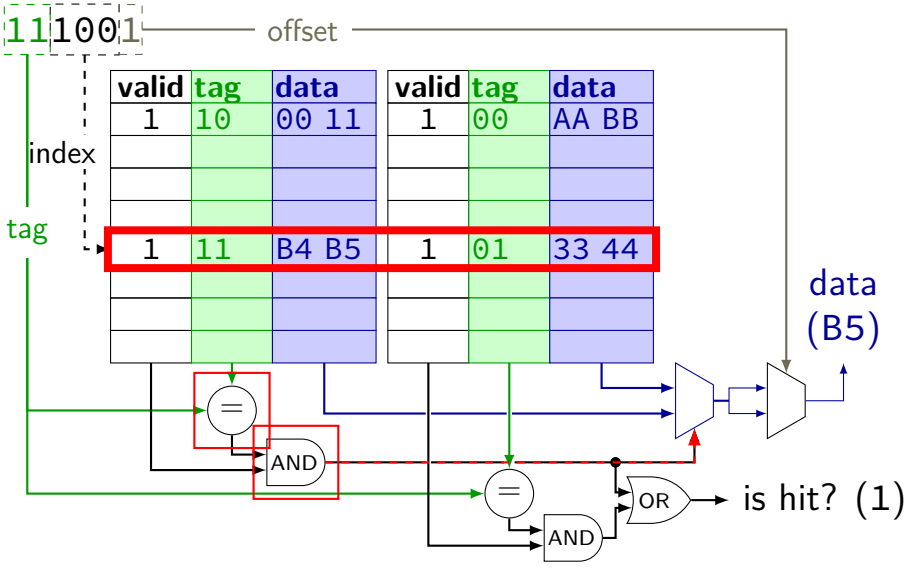
address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

tag index offset

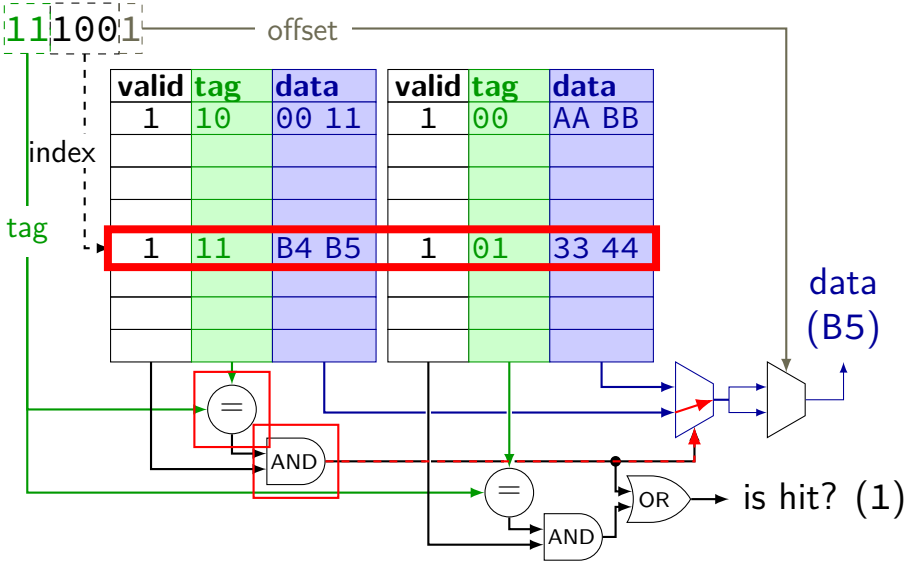
cache operation (associative)



cache operation (associative)



cache operation (associative)



associative lookup possibilities

none of the blocks for the index are valid

none of the valid blocks for the index match the tag

- something else is stored there

one of the blocks for the index is valid and matches the tag

associativity terminology

direct-mapped — one block per set

E-way set associative — E blocks per set

E ways in the cache

fully associative — one set total (everything in one set)

Tag-Index-Offset formulas (complete)

m	memory addreses bits (Y86-64: 64)
E	number of blocks per set ("ways")
$S = 2^s$	number of sets
s	(set) index bits
$B = 2^b$	block size
b	(block) offset bits
$t = m - (s + b)$	tag bits
$C = B \times S \times E$	cache size (excluding metadata)

Tag-Index-Offset exercise

m	memory addreses bits (Y86-64: 64)
E	number of blocks per set ("ways")
$S = 2^s$	number of sets
s	(set) index bits
$B = 2^b$	block size
b	(block) offset bits
$t = m - (s + b)$	tag bits
$C = B \times S \times E$	cache size (excluding metadata)

My desktop:

L1 Data Cache: 32 KB, 8 blocks/set, 64 byte blocks
L2 Cache: 256 KB, 4 blocks/set, 64 byte blocks
L3 Cache: 8 MB, 16 blocks/set, 64 byte blocks

Divide the address `0x34567` into tag, index, offset for each cache.

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64$Byte
	$B = 2^b$ (b: block offset bits)

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64$Byte
	$B = 2^b$ (b: block offset bits)
block offset bits	$b = 6$

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64$Byte
block offset bits	$B = 2^b$ (b: block offset bits) $b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64$Byte
	$B = 2^b$ (b: block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
	$S = \frac{C}{B \times E}$ (S: number of sets)

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
block offset bits	$B = 2^b$ (b: block offset bits) $b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
number of sets	$S = \frac{C}{B \times E}$ (S: number of sets) $S = \frac{32\text{KB}}{64\text{Byte} \times 8} = 64$

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
block offset bits	$B = 2^b$ (b: block offset bits) $b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
number of sets	$S = \frac{C}{B \times E}$ (S: number of sets) $S = \frac{32\text{KB}}{64\text{Byte} \times 8} = 64$
set index bits	$S = 2^s$ (s: set index bits) $s = \log_2(64) = 6$

T-I-O results

	L1	L2	L3
sets	64	1024	8192
block offset bits	6	6	6
set index bits	6	10	13
tag bits		(the rest)	

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

`0x34567`:	3	4	5	6	7
	`0011`	`0100`	`0101`	`0110`	`0111`

bits 0-5 (all offsets): `100111` = `0x27`

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits		(the rest)	

`0x34567`:	3	4	5	6	7
	`0011`	`0100`	`0101`	`0110`	`0111`

bits 0-5 (all offsets): `100111` = `0x27`

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits		(the rest)	

`0x34567`:

3	4	5	6	7
`0011`	`0100`	`0101`	`0110`	`0111`

bits 0-5 (all offsets): `100111` = `0x27`

L1:

bits 6-11 (L1 set): `01 0101` = `0x15`
bits 12- (L1 tag): `0x34`

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

`0x34567`:

3	4	5	6	7
`0011`	`0100`	`0101`	`0110`	`0111`

bits 0-5 (all offsets): `100111` = `0x27`

L1:

bits 6-11 (L1 set): `01 0101` = `0x15`
bits 12- (L1 tag): `0x34`

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

`0x34567`:

3	4	5	6	7
`0011`	`0100`	`0101`	`0110`	`0111`

bits 0-5 (all offsets): `100111` = `0x27`

L2:

bits 6-15 (set for L2): `01 0001 0101` = `0x115`
bits 16-: `0x3`

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

`0x34567`:

3	4	5	6	7
`0011`	`0100`	`0101`	`0110`	`0111`

bits 0-5 (all offsets): `100111` = `0x27`

L2:

bits 6-15 (set for L2): `01 0001 0101` = `0x115`
bits 16-: `0x3`

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		

`0x34567`:

3	4	5	6	7
`0011`	`0100`	`0101`	`0110`	`0111`

bits 0-5 (all offsets): `100111` = `0x27`

L3:

bits 6-18 (set for L3): `0 1101 0001 0101` = `0xD15`
bits 18-: `0x0`

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

how is the address 21 (00100001) split up into tag/index/offset?

b block offset bits;
$B = 2^b$ byte block size;
s set index bits; $S = 2^s$ sets;
$t = m - (s + b)$ tag bits (leftover)

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	0			0			
1	0			0			

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise: how many accesses are hits?
what is the final state of the cache?

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	0			way 1
1	0			0			

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise: how many accesses are hits?
what is the final state of the cache?

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	1	00001	M[0x08] M[0x09] M[0x0A] M[0x0B]	way 0
1	0			0			

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]	1	00001	M[0x08] M[0x09] M[0x0A] M[0x0B]	way 1
1	0			0			

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	way 0
1	1	00000	M[0x0C] M[0x0D] M[0x0E] M[0x0F]	0			way 1

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	miss
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	way 1
1	1	00000	M[0x0C] M[0x0D] M[0x0E] M[0x0F]	0			way 1

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	miss
00100011 (23)	hit

tag index offset

backup slides

cache miss types

compulsory (or *cold*) — first time accessing something

- doesn't matter how big/flexible the cache is

conflict — sets aren't big/flexible enough

- a fully-associtive (1-set) cache of the same size would have done better

capacity — cache was not big enough

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
000	
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

how to decide where to insert 0x64?

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value	LRU
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]	1
1	1	011000	mem[0x62] mem[0x63]	0			1

address (hex)	result
00000000 (00)	mi
00000001 (01)	hit
01100011 (63)	mi
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

track which block was read least recently updated on every access

example replacement policies

least recently used and approximations

- take advantage of temporal locality
- exact: $\lceil \log_2(E!) \rceil$ bits per set for E-way cache
- good approximations: E to $2E$ bits

first-in, first-out

- counter per set — where to replace next

(pseudo-)random

- no extra information!