

Caching

associativity terminology

direct-mapped — one block per set

E -way set associative — E blocks per set

E ways in the cache

fully associative — one set total (everything in one set)

Tag-Index-Offset formulas (complete)

m memory addresses bits (Y86-64: 64)

E number of blocks per set ("ways")

$S = 2^s$ number of sets

s (set) index bits

$B = 2^b$ block size

b (block) offset bits

$t = m - (s + b)$ tag bits

$C = B \times S \times E$ cache size (excluding metadata)

Tag-Index-Offset exercise

m	memory addresses bits (Y86-64: 64)
E	number of blocks per set ("ways")
$S = 2^s$	number of sets
s	(set) index bits
$B = 2^b$	block size
b	(block) offset bits
$t = m - (s + b)$	tag bits
$C = B \times S \times E$	cache size (excluding metadata)

My desktop:

L1 Data Cache: 32 KB, 8 blocks/set, 64 byte blocks

L2 Cache: 256 KB, 4 blocks/set, 64 byte blocks

L3 Cache: 8 MB, 16 blocks/set, 64 byte blocks

Divide the address 0x34567 into **tag**, **index**, **offset** for each cache.

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ (b : block offset bits)

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ (b : block offset bits)
block offset bits	$b = 6$

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ (b : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ (b : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
	$S = \frac{C}{B \times E}$ (S : number of sets)

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ (b : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
	$S = \frac{C}{B \times E}$ (S : number of sets) $\frac{32\text{KB}}{64\text{Byte} \times 8}$
number of sets	$S = \frac{32\text{KB}}{64\text{Byte} \times 8} = 64$

T-I-O exercise: L1

quantity	value for L1
block size (given)	$B = 64\text{Byte}$
	$B = 2^b$ (b : block offset bits)
block offset bits	$b = 6$
blocks/set (given)	$E = 8$
cache size (given)	$C = 32\text{KB} = E \times B \times S$
	$S = \frac{C}{B \times E}$ (S : number of sets) $\frac{32\text{KB}}{64\text{Byte} \times 8} = 64$
number of sets	$S = 64$
	$S = 2^s$ (s : set index bits)
set index bits	$s = \log_2(64) = 6$

T-I-O results

	L1	L2	L3
sets	64	1024	8192
block offset bits	6	6	6
set index bits	6	10	13
tag bits		(the rest)	

T-I-O: splitting

	L1	L2	L3		
block offset bits	6	6	6		
set index bits	6	10	13		
tag bits	(the rest)				
0x34567:	3	4	5	6	7
	0011	0100	0101	0110	0111

bits 0-5 (all offsets): 100111 = 0x27

T-I-O: splitting

	L1	L2	L3		
block offset bits	6	6	6		
set index bits	6	10	13		
tag bits	(the rest)				
0x34567:	3	4	5	6	7
	0011	0100	0101	0110	0111

bits 0-5 (all offsets): **100111** = 0x27

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		
0x34567:	3	4	5
	0011	0100	0101
	0110	0111	0111

bits 0-5 (all offsets): $100111 = 0x27$

L1:

bits 6-11 (L1 set): $01\ 0101 = 0x15$

bits 12- (L1 tag): $0x34$

T-I-O: splitting

	L1	L2	L3		
block offset bits	6	6	6		
set index bits	6	10	13		
tag bits	(the rest)				
0x34567:	3	4	5		
	0011	0100	0101	0110	0111
					7

bits 0-5 (all offsets): $100111 = 0x27$

L1:

bits 6-11 (L1 set): $01\ 0101 = 0x15$

bits 12- (L1 tag): $0x34$

T-I-O: splitting

	L1	L2	L3
block offset bits	6	6	6
set index bits	6	10	13
tag bits	(the rest)		
0x34567:	3	4	5
	0011	0100	0101
	0110	0111	7

bits 0-5 (all offsets): $100111 = 0x27$

L2:

bits 6-15 (set for L2): $01\ 0001\ 0101 = 0x115$
bits 16-: 0x3

T-I-O: splitting

	L1	L2	L3		
block offset bits	6	6	6		
set index bits	6	10	13		
tag bits	(the rest)				
0x34567:	3	4	5	6	7
	0011	0100	0101	0110	0111

bits 0-5 (all offsets): $100111 = 0x27$

L2:

bits 6-15 (set for L2): $01\ 0001\ 0101 = 0x115$
bits 16-: $0x3$

T-I-O: splitting

	L1	L2	L3		
block offset bits	6	6	6		
set index bits	6	10	13		
tag bits	(the rest)				
0x34567:	3	4	5		
	0011	0100	0101	0110	0111

bits 0-5 (all offsets): $100111 = 0x27$

L3:

bits 6-18 (set for L3): $0\ 1101\ 0001\ 0101 = 0xD15$
bits 18-: $0x0$

cache miss types

compulsory (or *cold*) — **first time** accessing something
doesn't matter how big/flexible the cache is

conflict — sets aren't big/flexible enough
a fully-associative (1-set) cache of the same size would have done better

capacity — cache was not big enough

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]
1	1	011000	mem[0x62] mem[0x63]	0		

address (hex)	result
000	how to decide where to insert 0x64?
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

replacement policies

2-way set associative, 2 byte blocks, 2 sets

index	valid	tag	value	valid	tag	value	LRU
0	1	000000	mem[0x00] mem[0x01]	1	011000	mem[0x60] mem[0x61]	1
1	1	011000	mem[0x62] mem[0x63]	0			1

address (hex)	result
00000000 (00)	miss
00000001 (01)	hit
01100011 (63)	miss
01100001 (61)	miss
01100010 (62)	hit
00000000 (00)	hit
01100100 (64)	miss

track which block was read least recently
updated on **every access**

example replacement policies

least recently used and approximations

- take advantage of **temporal locality**

- exact: $\lceil \log_2(E!) \rceil$ bits per set for E -way cache

- good approximations: E to $2E$ bits

first-in, first-out

- counter per set — where to replace next

(pseudo-)random

- no extra information!

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	0		
1	0		

V	tag	value
0		
0		

LRU

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	0		
1	0		

V	tag	value
0		
0		

LRU

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

how is the address 21 (00100001) split up into tag/index/offset?

b block offset bits;

$B = 2^b$ byte block size;

s set index bits; $S = 2^s$ sets;

$t = m - (s + b)$ tag bits (leftover)

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	0		
1	0		

V	tag	value
0		
0		

LRU

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	0		
1	0		

V	tag	value
0		
0		

LRU

address (hex)	hit?
00000000 (00)	
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

exercise: how many accesses are hits?
what is the final state of the cache?

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value	V	tag	value	LRU
0	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	0			way 1
1	0			0			

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	
00001010 (0A)	
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise: how many accesses are hits?
what is the final state of the cache?

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	1	00000	M[0x00] M[0x01] M[0x02] M[0x03]
1	0		

V	tag	value
1	00001	M[0x08] M[0x09] M[0xA] M[0xB]
0		

LRU
way 0

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	
00001100 (0C)	
00000011 (02)	
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]
1	0		

V	tag	value
1	00001	M[0x08] M[0x09] M[0x0A] M[0x0B]
0		

LRU
way 1

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]
1	1	00000	M[0x0C] M[0x0D] M[0x0E] M[0x0F]

V	tag	value	LRU
1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	way 0
0			way 1

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	miss
00100011 (23)	

tag index offset

exercise

4 byte blocks, 2 sets

index	V	tag	value
0	1	00100	M[0x20] M[0x21] M[0x22] M[0x23]
1	1	00000	M[0x0C] M[0x0D] M[0x0E] M[0x0F]

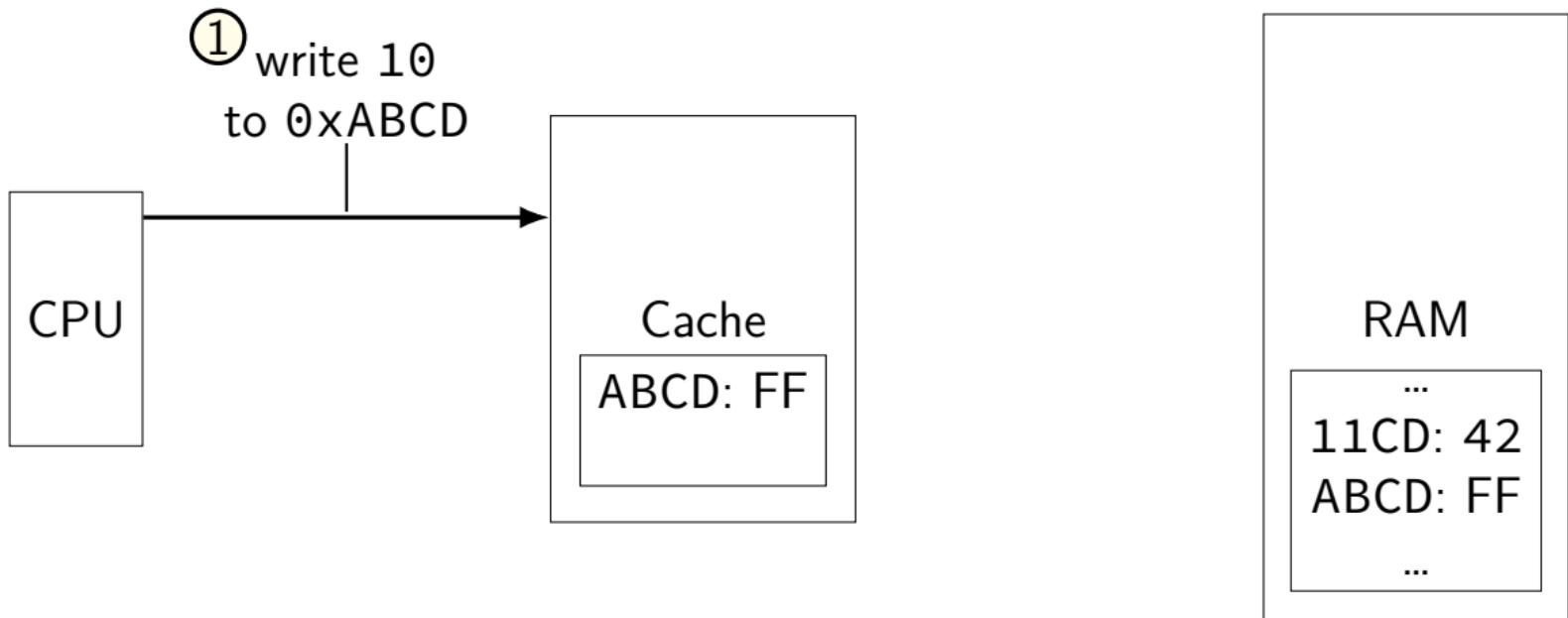
V	tag	value	LRU
1	00000	M[0x00] M[0x01] M[0x02] M[0x03]	way 1
0			way 1

address (hex)	hit?
00000000 (00)	miss
00000001 (01)	hit
00001010 (0A)	miss
00100001 (21)	miss
00001100 (0C)	miss
00000011 (02)	miss
00100011 (23)	hit

tag index offset

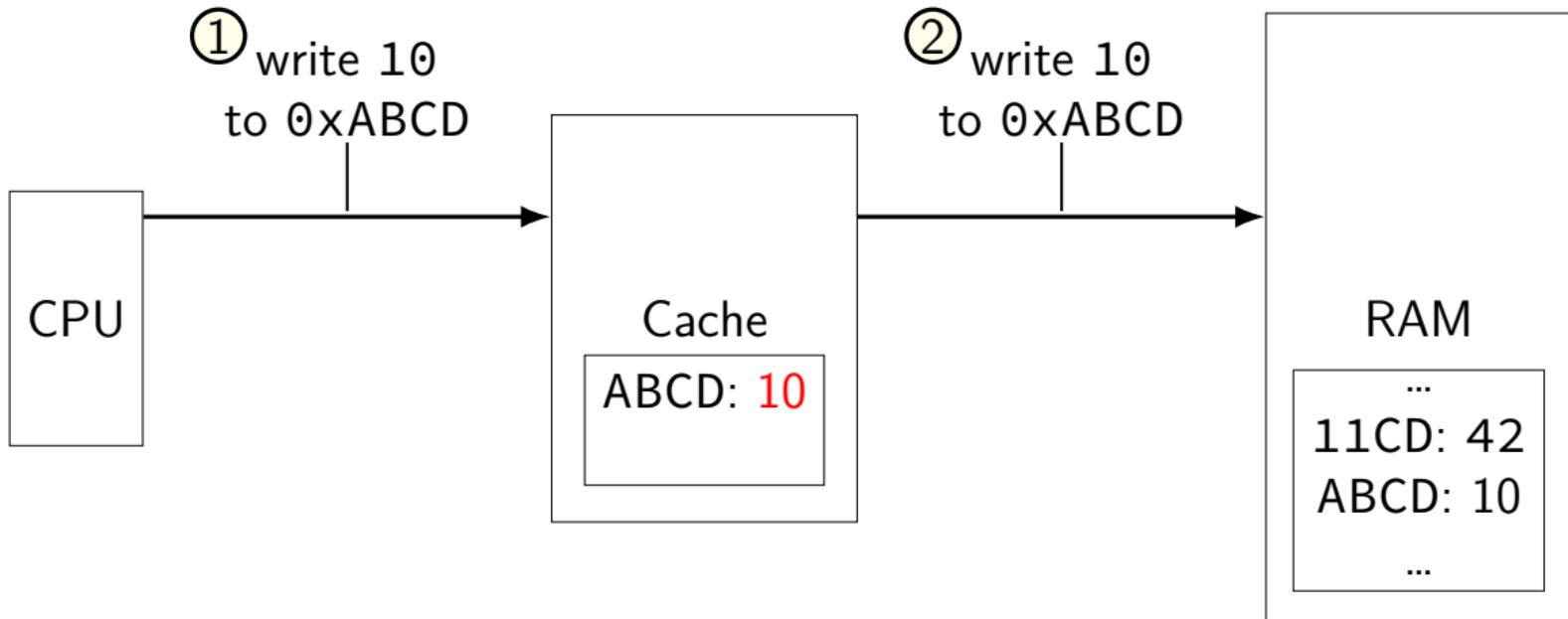
write-through v. write-back

option 1: write-through



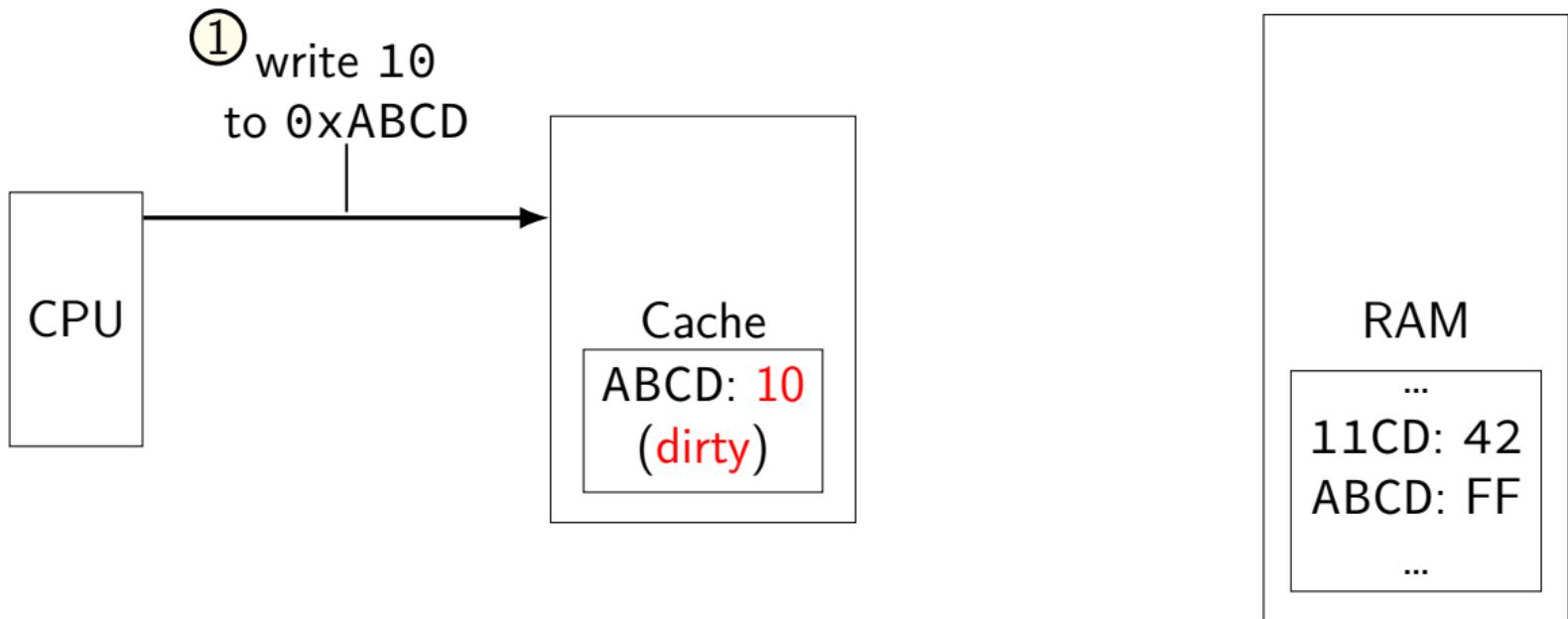
write-through v. write-back

option 1: write-through



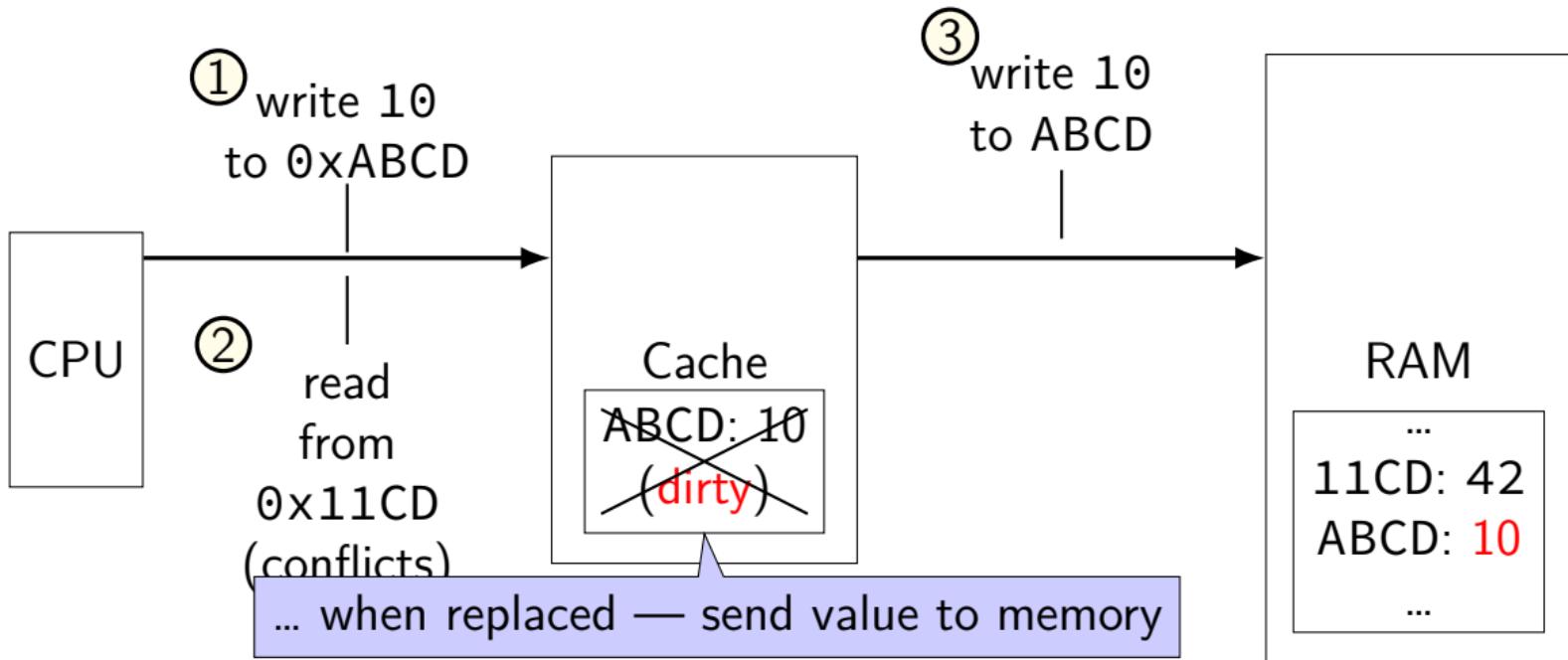
write-through v. write-back

option 2: write-back

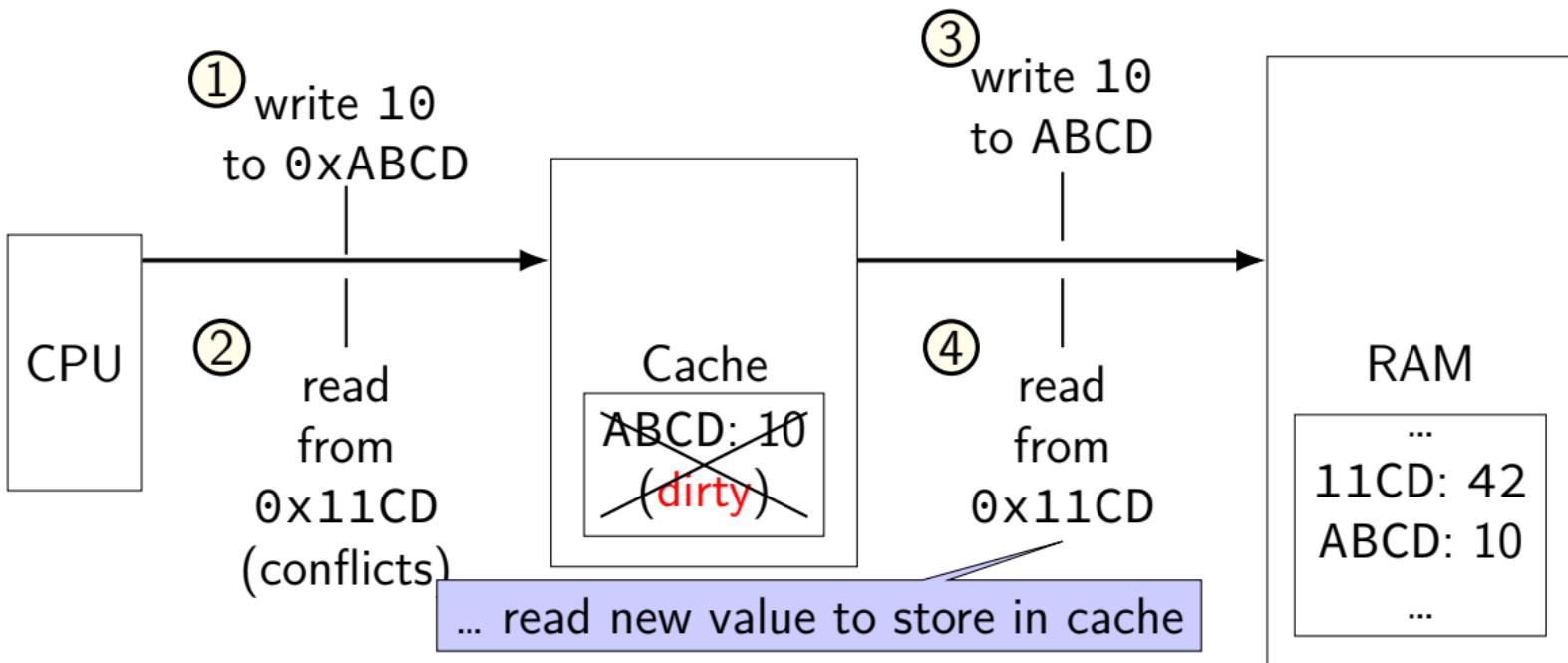


write-through v. write-back

option 2: write-back



write-through v. write-back



writeback policy

changed value!

2-way set associative, 4 byte blocks, 2 sets

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

1 = dirty (different than memory)
needs to be written if evicted

allocate on write?

processor writes **less than whole** cache block

block not yet in cache

two options:

write-allocate

fetch rest of cache block, replace written part

write-no-allocate

send write through to memory

guess: not read soon?

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

write-allocate

2-way set associative, LRU, writeback

index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	0xFF mem[0x05]	1	0
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

index 0, tag 000001

step 1: find least recently used block

step 2: possibly writeback old block

step 3a: read in new block – to get mem[0x05]

step 3b: update LRU information

write-no-allocate

2-way set associative, LRU, writeback

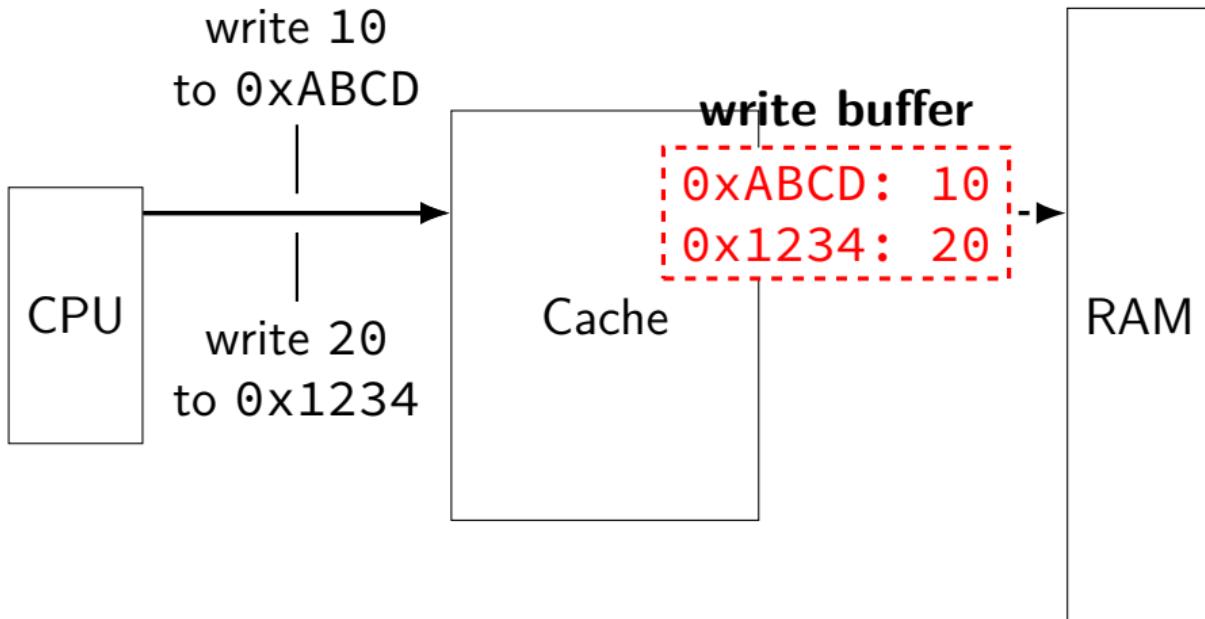
index	valid	tag	value	dirty	valid	tag	value	dirty	LRU
0	1	000000	mem[0x00] mem[0x01]	0	1	011000	mem[0x60]* mem[0x61]*	1	1
1	1	011000	mem[0x62] mem[0x63]	0	0				0

writing 0xFF into address 0x04?

step 1: is it in cache yet?

step 2: no, just send it to memory

fast writes



write appears to complete immediately when placed in buffer
memory can be much slower

cache organization and miss rate

depends on program; one example:

SPEC CPU2000 benchmarks, 64B block size

LRU replacement policies

data cache miss rates:

Cache size	direct-mapped	2-way	8-way	fully assoc.
1KB	8.63%	6.97%	5.63%	5.34%
2KB	5.71%	4.23%	3.30%	3.05%
4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

cache organization and miss rate

depends on program; one example:

SPEC CPU2000 benchmarks, 64B block size

LRU replacement policies

data cache miss rates:

Cache size	direct-mapped	2-way	8-way	fully assoc.
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2KB	5.71%	4.23%	3.30%	3.05%
4KB	3.70%	2.60%	2.03%	1.90%
16KB	1.59%	0.86%	0.56%	0.50%
64KB	0.66%	0.37%	0.10%	0.001%
128KB	0.27%	0.001%	0.0006%	0.0006%

reasoning about cache performance

hit time: time to lookup and find value in cache

L1 cache — typically 1 cycle?

miss rate: portion of hits (value in cache)

miss penalty: extra time to get value if there's a miss
time to access next level cache or memory

miss time: hit time + miss penalty

average memory access time

AMAT = hit time + miss penalty × miss rate

effective speed of memory

making any cache look bad

1. access enough blocks, to fill the cache
2. access an additional block, replacing something
3. access last block replaced
4. access last block replaced
5. access last block replaced

...

but — typical real programs have **locality**

cache optimizations

	miss rate	hit time	miss penalty
increase cache size	better	worse	—
increase associativity	better	worse	worse?
increase block size	depends	worse	worse
add secondary cache	—	—	better
write-allocate	better	—	worse?
writeback	???	—	worse?
LRU replacement	better	?	worse?

$$\text{average time} = \text{hit time} + \text{miss rate} \times \text{miss penalty}$$

cache optimizations by miss type

	capacity	conflict	compulsory
increase cache size	fewer misses	fewer misses	—
increase associativity	—	fewer misses	—
increase block size	—	more misses	fewer misses
(assuming other listed parameters remain constant)			

exercise (1)

initial cache: 64-byte blocks, 64 sets, 8 ways/set

If we leave the other parameters listed above unchanged, which will probably reduce the number of **capacity misses** in a typical program? (Multiple may be correct.)

- A. quadrupling the block size (256-byte blocks, 64 sets, 8 ways/set)
- B. quadrupling the number of sets
- C. quadrupling the number of ways/set

exercise (2)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of **capacity misses** in a typical program? (Multiple may be correct.)

- A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache)
- B. quadrupling the number of ways/set
- C. quadrupling the cache size

exercise (3)

initial cache: 64-byte blocks, 8 ways/set, 64KB cache

If we leave the other parameters listed above unchanged, which will probably reduce the number of **conflict misses** in a typical program?
(Multiple may be correct.)

- A. quadrupling the block size (256-byte block, 8 ways/set, 64KB cache)
- B. quadrupling the number of ways/set
- C. quadrupling the cache size

C and cache misses (1)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2) {
    even_sum += array[i + 0];
    odd_sum += array[i + 1];
}
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 1; i < 1024; i += 2)
    odd_sum += array[i + 1];
```

Assume everything but array is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks? Would a set-associative cache be better?

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, ...

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, ...

...

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, ...

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, ...

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, ...

...

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, ...

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, ...
block at 0: array[0] through array[3]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, ...
block at 16: array[4] through array[7]

...

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, ...
block at 2032: array[508] through array[511]

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, ...

block at 0: array[0] through array[3]

block at 0+2KB: array[512] through array[515]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, ...

block at 16: array[4] through array[7]

block at 16+2KB: array[516] through array[519]

...

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, ...

block at 2032: array[508] through array[511]

block at 2032+2KB: array[1020] through array[1023]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 2KB, 0 + 4KB, ...

set 1: address 16, 16 + 2KB, 16 + 4KB, ...

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 2KB, 0 + 4KB, ...

block at 0: array[0] through array[3]

set 1: address 16, 16 + 2KB, 16 + 4KB, ...

address 16: array[4] through array[7]

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...

address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 2KB, 0 + 4KB, ...

block at 0: array[0] through array[3]

block at 0+1KB: array[256] through array[259]

block at 0+2KB: array[512] through array[515]

...

set 1: address 16, 16 + 2KB, 16 + 4KB, ...

address 16: array[4] through array[7]

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...

address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 2KB, 0 + 4KB, ...

block at 0: array[0] through array[3]

block at 0+1KB: array[256] through array[259]

block at 0+2KB: array[512] through array[515]

...

set 1: address 16, 16 + 2KB, 16 + 4KB, ...

address 16: array[4] through array[7]

...

set 63: address 1008, 2032 + 2KB, 2032 + 4KB ...

address 1008: array[252] through array[255]

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (3, rewritten?)

```
item array[1024]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 1024; i += 128)
    a_sum += array[i];
for (int i = 1; i < 1024; i += 128)
    b_sum += array[i];
```

C and cache misses (4)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 4-way set associative 2KB direct-mapped cache with 16B cache blocks?

backup slides