Cache Performance

C and cache misses (1)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2) {
    even_sum += array[i + 0];
    odd_sum +=  array[i + 1];
}
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (2)

```
int array[1024]; // 4KB array
int even_sum = 0, odd_sum = 0;
for (int i = 0; i < 1024; i += 2)
    even_sum += array[i + 0];
for (int i = 0; i < 1024; i += 2)
    odd_sum +=  array[i + 1];
```

Assume everything but `array` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks? Would a set-associtiave cache be better?

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

- block at 0: array[0] through array[3]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

- block at 16: array[4] through array[7]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

- block at 2032: array[508] through array[511]

thinking about cache storage (1)

2KB direct-mapped cache with 16B blocks —

set 0: address 0 to 15, (0 to 15) + 2KB, (0 to 15) + 4KB, …

- block at 0: array[0] through array[3]
- block at 0+2KB: array[512] through array[515]

set 1: address 16 to 31, (16 to 31) + 2KB, (16 to 31) + 4KB, …

- block at 16: array[4] through array[7]
- block at 16+2KB: array[516] through array[519]

…

set 127: address 2032 to 2047, (2032 to 2047) + 2KB, …

- block at 2032: array[508] through array[511]
- block at 2032+2KB: array[1020] through array[1023]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 1KB, 0 + 2KB, …

set 1: address 16, 16 + 1KB, 16 + 2KB, …

…

set 63: address 1008, 2032 + 1KB, 2032 + 2KB …

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 1KB, 0 + 2KB, …

- block at 0: array[0] through array[3]

set 1: address 16, 16 + 1KB, 16 + 2KB, …

- address 16: array[4] through array[7]

…

set 63: address 1008, 2032 + 1KB, 2032 + 2KB …

- address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 1KB, 0 + 2KB, …

- block at 0: array[0] through array[3]
- block at 0+1KB: array[256] through array[259]
- block at 0+2KB: array[512] through array[515]
- …

set 1: address 16, 16 + 1KB, 16 + 2KB, …

- address 16: array[4] through array[7]

…

set 63: address 1008, 2032 + 1KB, 2032 + 2KB …

- address 1008: array[252] through array[255]

thinking about cache storage (2)

2KB 2-way set associative cache with 16B blocks: block addresses

—

set 0: address 0, 0 + 1KB, 0 + 2KB, …

- block at 0: array[0] through array[3]
- block at 0+1KB: array[256] through array[259]
- block at 0+2KB: array[512] through array[515]
- …

set 1: address 16, 16 + 1KB, 16 + 2KB, …

- address 16: array[4] through array[7]

…

set 63: address 1008, 2032 + 1KB, 2032 + 2KB …

- address 1008: array[252] through array[255]

C and cache misses (3)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 2KB direct-mapped cache with 16B cache blocks?

C and cache misses (3, rewritten?)

```
item array[1024]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 1024; i += 128)
    a_sum += array[i];
for (int i = 1; i < 1024; i += 128)
    b_sum += array[i];
```

C and cache misses (4)

```
typedef struct {
    int a_value, b_value;
    int boring_values[126];
} item;
item items[8]; // 4 KB array
int a_sum = 0, b_sum = 0;
for (int i = 0; i < 8; ++i)
    a_sum += items[i].a_value;
for (int i = 0; i < 8; ++i)
    b_sum += items[i].b_value;
```

Assume everything but `items` is kept in registers (and the compiler does not do anything funny).

How many *data cache misses* on a 4-way set associative 2KB direct-mapped cache with 16B cache blocks?

a note on matrix storage

A — $N \times N$ matrix

represent as array

makes dynamic sizes easier:

```
float A_2d_array[N][N];
float *A_flat = malloc(N * N);
```

```
A_flat[i * N + j] === A_2d_array[i][j]
```

matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

```
/* version 1: inner loop is k, middle is j */
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i * N + j] += A[i * N + k] * A[k * N + j];
```

matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i*N+j] += A[i * N + k] * A[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

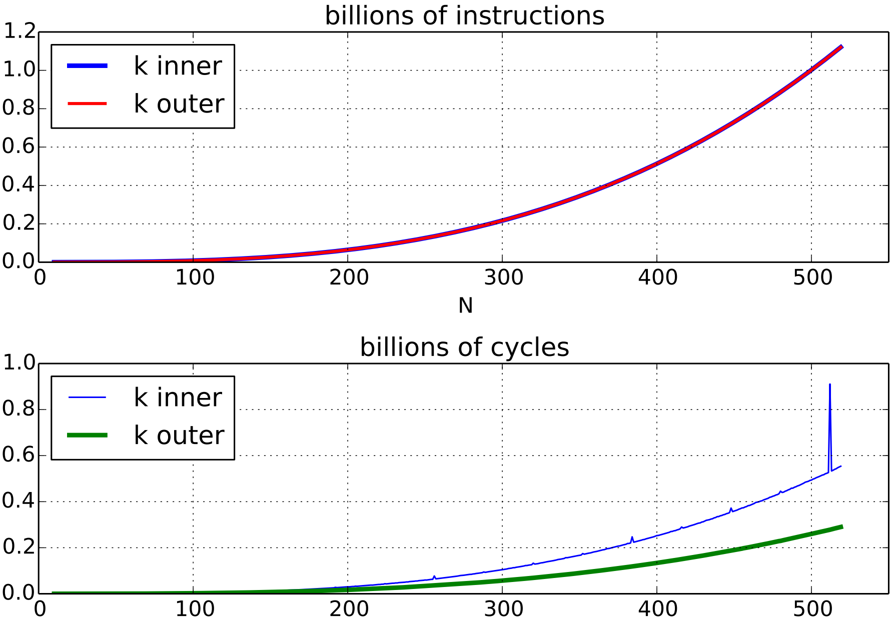
matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

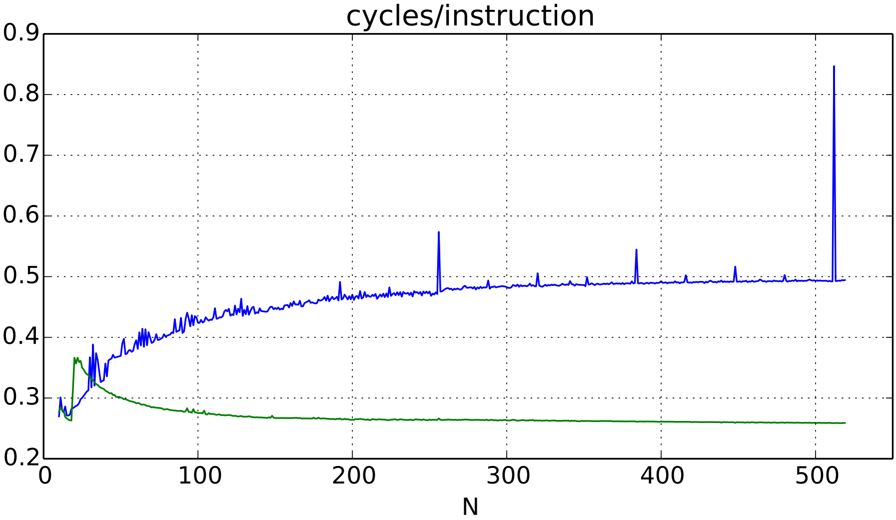
```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i*N+j] += A[i * N + k] * A[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

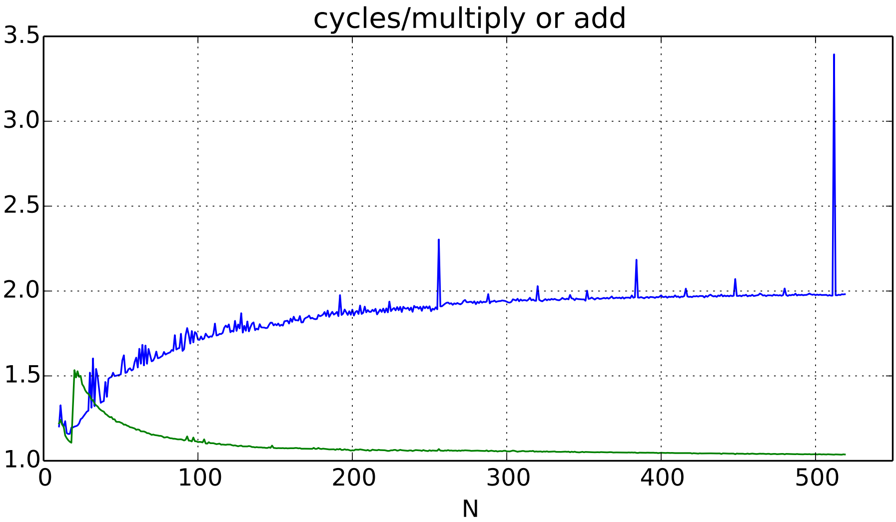
performance



alternate view 1: cycles/instruction



alternate view 2: cycles/operation



loop orders and locality

loop body: $B_{ij} += A_{ik}A_{kj}$

kij order: B_{ij}, A_{kj} have spatial locality

kij order: A_{ik} has temporal locality

… better than …

ijk order: A_{ik} has spatial locality

ijk order: B_{ij} has temporal locality

loop orders and locality

loop body: $B_{ij} {+}= A_{ik} A_{kj}$

kij order: B_{ij}, A_{kj} have spatial locality

kij order: A_{ik} has temporal locality

… better than …

ijk order: A_{ik} has spatial locality

ijk order: B_{ij} has temporal locality

matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i*N+j] += A[i * N + k] * A[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i*N+j] += A[i * N + k] * A[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

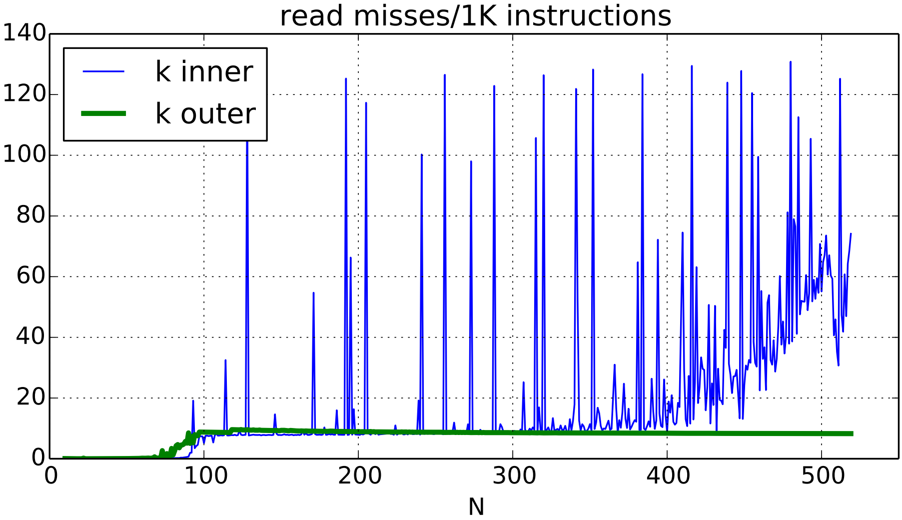
matrix squaring

$$B_{ij} = \sum_{k=1}^{n} A_{ik} \times A_{kj}$$

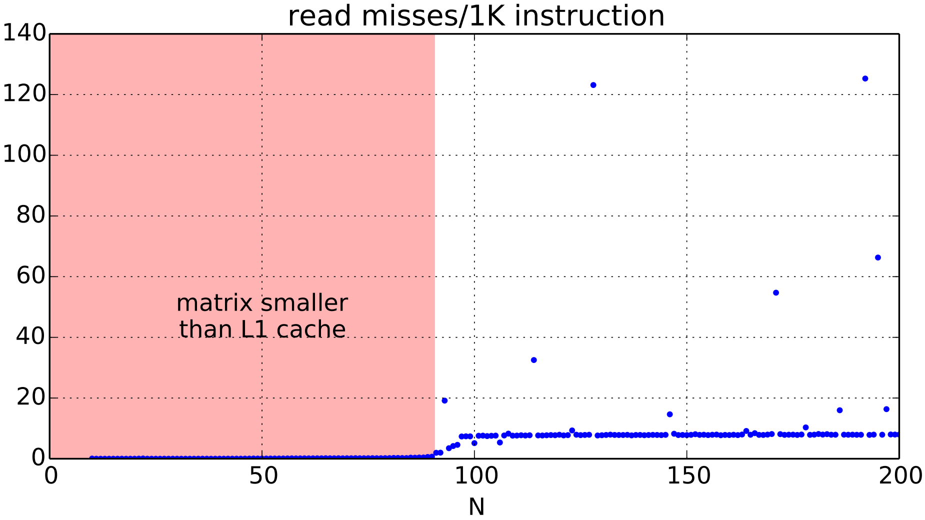
```
/* version 1: inner loop is k, middle is j*/
for (int i = 0; i < N; ++i)
  for (int j = 0; j < N; ++j)
    for (int k = 0; k < N; ++k)
      B[i*N+j] += A[i * N + k] * A[k * N + j];

/* version 2: outer loop is k, middle is i */
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i * N + k] * A[k * N + j];
```

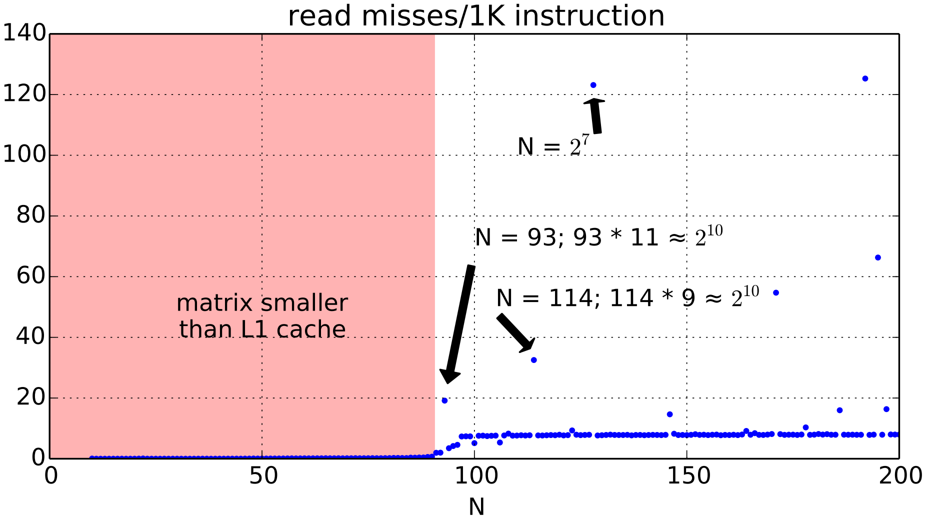
L1 misses


read misses/1K instructions
140
120
100
80
60
40
20
0
0
100
200
300
400
500
N
k inner
k outer

L1 miss detail (1)



L1 miss detail (2)



addresses

```
A[k*114+j]       is at  10 0000 0000 0100
A[k*114+j+1]     is at  10 0000 0000 1000
A[(k+1)*114+j]   is at  10 0011 1001 0100
A[(k+2)*114+j]   is at  10 0101 0101 1100
…
A[(k+9)*114+j]   is at  11 0000 0000 1100
```

addresses

`A[k*114+j]` is at `10` **`0000 00`**`00 0100`
`A[k*114+j+1]` is at `10` **`0000 00`**`00 1000`
`A[(k+1)*114+j]` is at `10` **`0011 10`**`01 0100`
`A[(k+2)*114+j]` is at `10` **`0101 01`**`01 1100`
…
`A[(k+9)*114+j]` is at `11` **`0000 00`**`00 1100`

recall: 6 index bits, 6 block offset bits (L1)

conflict misses

powers of two — lower order bits unchanged

`A[k*93+j]` and `A[(k+11)*93+j]`:

1023 elements apart (4092 bytes; 63.9 cache blocks)

64 sets in L1 cache: usually maps to same set

`A[k*93+(j+1)]` will not be cached (next i loop)

even if in same block as `A[k*93+j]`

reasoning about loop orders

changing loop order changed locality

how do we tell which loop order will be best?

- besides running each one?

systematic approach (1)

```
for (int k = 0; k < N; ++k)
  for (int i = 0; i < N; ++i)
    for (int j = 0; j < N; ++j)
      B[i*N+j] += A[i*N+k] * A[k*N+j];
```

goal: get most out of each cache miss

if N is larger than the cache:

miss for B_{ij} — 1 comptuation

miss for A_{ik} — N computations

miss for A_{kj} — 1 computation

effectively caching just 1 element

keeping values in cache

can't *explicitly* ensure values are kept in cache

...but reusing values *effectively* does this

- cache will try to keep recently used values

cache optimization ideas: choose what's in the cache

- for thinking about it: load values explicitly
- for implementing it: access only values we want loaded

a transformation

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; i += 2)
        for (int j = 0; j < N; ++j)
          B[i*N+j] += A[i*N+k] * A[k*N+j];
```

split the loop over k — should be exactly the same

(assuming even N)

a transformation

```
for (int kk = 0; kk < N; kk += 2)
  for (int k = kk; k < kk + 2; ++k)
      for (int i = 0; i < N; i += 2)
        for (int j = 0; j < N; ++j)
          B[i*N+j] += A[i*N+k] * A[k*N+j];
```

split the loop over k — should be exactly the same
(assuming even N)

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; i += 2)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
```

now reorder split loop — same calculations

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; i += 2)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking

```
for (int kk = 0; kk < N; kk += 2)
  /* was here: for (int k = kk; k < kk + 2; ++k) */
    for (int i = 0; i < N; i += 2)
      for (int j = 0; j < N; ++j)
        /* load Aik, Aik+1 into cache and process: */
        for (int k = kk; k < kk + 2; ++k)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
```

now reorder split loop — same calculations

now handle B_{ij} for $k+1$ right after B_{ij} for k

(previously: $B_{i,j+1}$ for k right after B_{ij} for k)

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

Temporal locality in B_{ij}s

simple blocking – expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

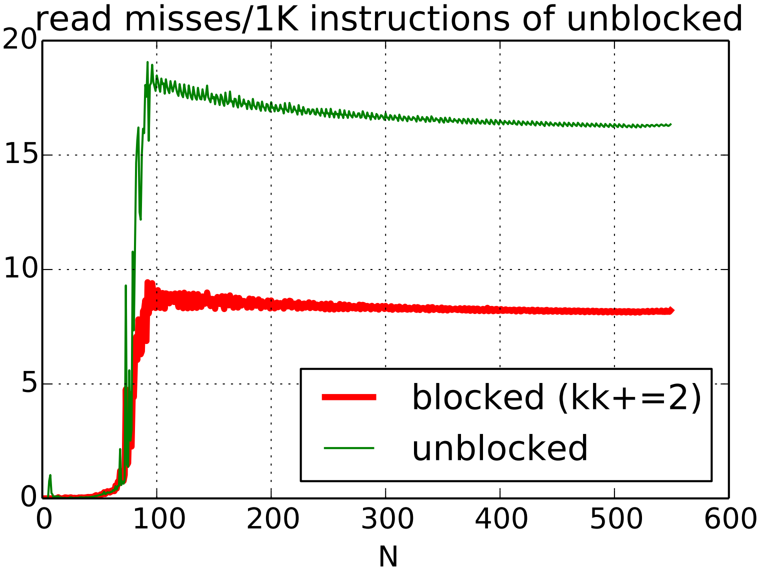
More spatial locality in A_{ik}

simple blocking — expanded

```
for (int kk = 0; kk < N; kk += 2) {
  for (int i = 0; i < N; i += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block" of 2 k values: */
      B[i*N+j] += A[i*N+kk+0] * A[(kk+0)*N+j];
      B[i*N+j] += A[i*N+kk+1] * A[(kk+1)*N+j];
    }
  }
}
```

Still have good spatial locality in A_{kj}, B_{ij}

improvement in read misses



simple blocking (2)

same thing for i in addition to k?

```
for (int kk = 0; kk < N; kk += 2) {
  for (int ii = 0; ii < N; ii += 2) {
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      for (int k = kk; k < kk + 2; ++k)
        for (int i = 0; i < ii + 2; ++i)
            B[i*N+j] += A[i*N+k] * A[k*N+j];
    }
  }
}
```

simple blocking — expanded

```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    /* load a block around Aik */
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      $B_{i+0,j}$ += $A_{i+0,k+0}$ * $A_{k+0,j}$
      $B_{i+0,j}$ += $A_{i+0,k+1}$ * $A_{k+1,j}$
      $B_{i+1,j}$ += $A_{i+1,k+0}$ * $A_{k+0,j}$
      $B_{i+1,j}$ += $A_{i+1,k+1}$ * $A_{k+1,j}$
    }
  }
}
```

simple blocking — expanded

```
for (int k = 0; k < N; k += 2) {
  for (int i = 0; i < N; i += 2) {
    /* load a block around Aik */
    for (int j = 0; j < N; ++j) {
      /* process a "block": */
      $B_{i+0,j}$ += $A_{i+0,k+0}$ * $A_{k+0,j}$
      $B_{i+0,j}$ += $A_{i+0,k+1}$ * $A_{k+1,j}$
      $B_{i+1,j}$ += $A_{i+1,k+0}$ * $A_{k+0,j}$
      $B_{i+1,j}$ += $A_{i+1,k+1}$ * $A_{k+1,j}$
    }
  }
}
```

Now A_{kj} reused in inner loop — more calculations per load!

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                          * A[k * N + j];
```

B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

A_{kj} used I times for one miss — N^2/I misses

catch: $IK + KJ + IJ$ elements must fit in cache

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                          * A[k * N + j];
```

B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

A_{kj} used I times for one miss — N^2/I misses

catch: $IK + KJ + IJ$ elements must fit in cache

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                          * A[k * N + j];
```

B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

A_{kj} used I times for one miss — N^2/I misses

catch: $IK + KJ + IJ$ elements must fit in cache

generalizing cache blocking

```
for (int kk = 0; kk < N; kk += K) {
  for (int ii = 0; ii < N; ii += I) {
    with I by K block of A hopefully cached:
    for (int jj = 0; jj < N; jj += J) {
      with K by J block of A, I by J block of B cached:
      for i in ii to ii+I:
        for j in jj to jj+J:
          for k in kk to kk+K:
            B[i * N + j] += A[i * N + k]
                         * A[k * N + j];
```

B_{ij} used K times for one miss — N^2/K misses

A_{ik} used J times for one miss — N^2/J misses

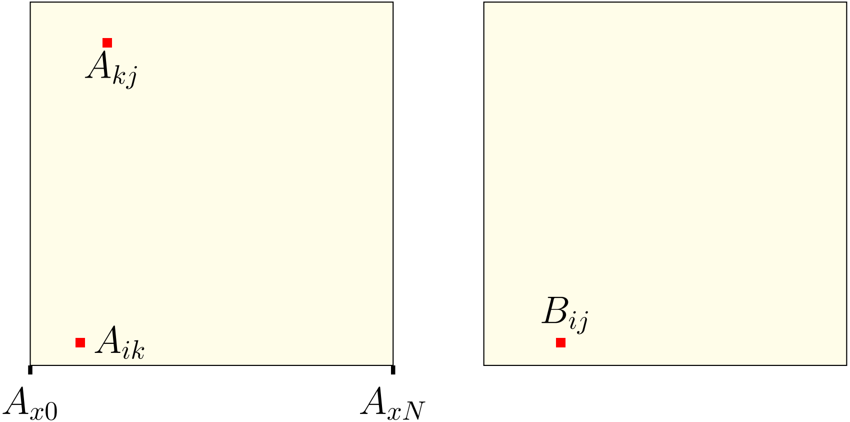
A_{kj} used I times for one miss — N^2/I misses

catch: $IK + KJ + IJ$ elements must fit in cache

view 2: divide and conquer

```
partial_square(float *A, float *B,
               int startI, int endI, ...) {
  for (int i = startI; i < endI; ++i) {
    for (int j = startJ; j < endJ; ++j) {
      ...
}
square(float *A, float *B, int N) {
  for (int ii = 0; ii < N; ii += BLOCK)
    ...
      /* segment of A, B in use fits in cache! */
      partial_square(
            A, B,
            ii, ii + BLOCK,
            jj, jj + BLOCK, ...);
}
```

array usage: kij order

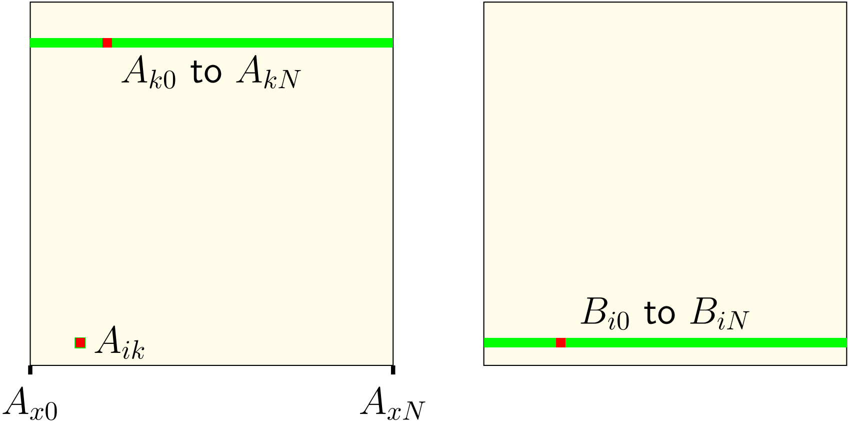


for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

N calculations for A_{ik}

1 for A_{kj}, B_{ij}

array usage: kij order

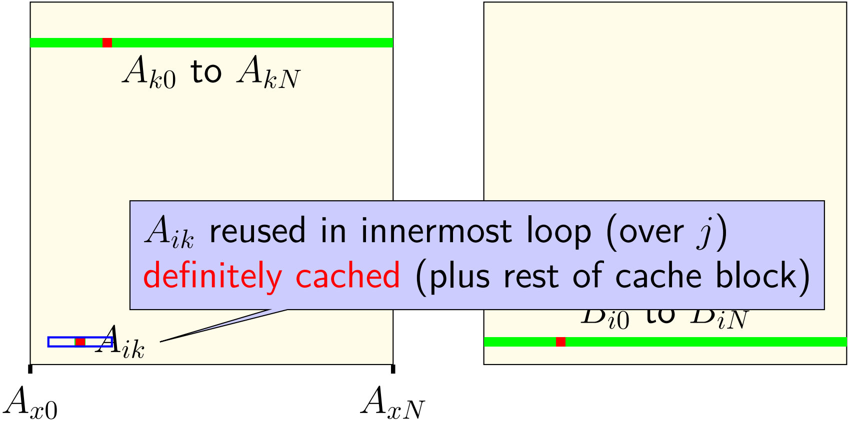


for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

N calculations for A_{ik}

1 for A_{kj}, B_{ij}

array usage: kij order



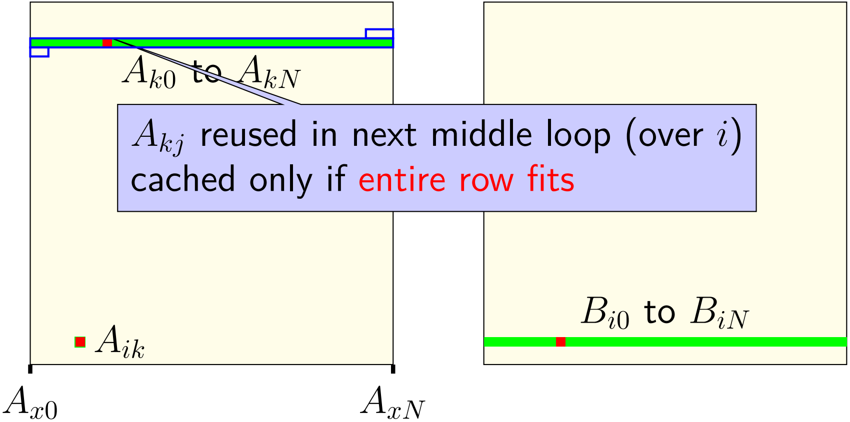
A_{ik} reused in innermost loop (over j)
definitely cached (plus rest of cache block)

for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

N calculations for A_{ik}

1 for A_{kj}, B_{ij}

array usage: kij order



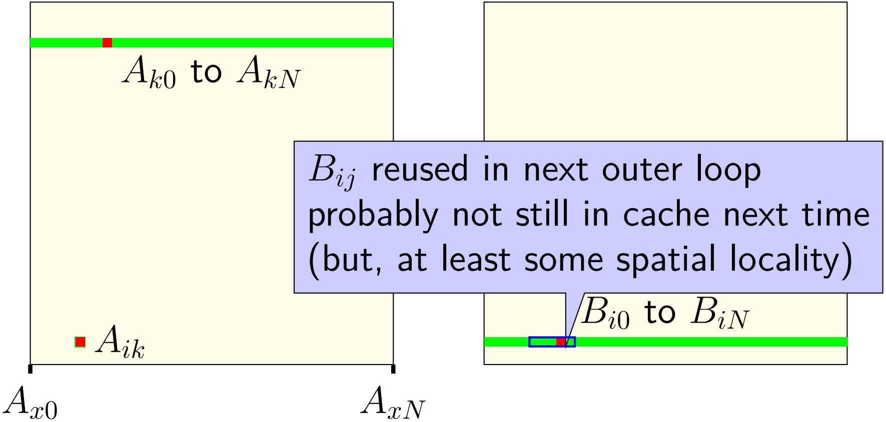
A_{kj} reused in next middle loop (over i)
cached only if entire row fits

for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

N calculations for A_{ik}

1 for A_{kj}, B_{ij}

array usage: kij order



B_{ij} reused in next outer loop
probably not still in cache next time
(but, at least some spatial locality)

for all k: for all i: for all j: $B_{ij} += A_{ik} \times A_{kj}$

N calculations for A_{ik}

1 for A_{kj}, B_{ij}

inefficiencies

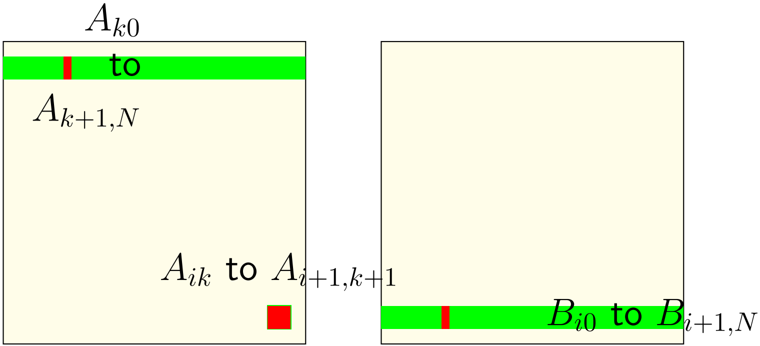
if a row doesn't fit in cache —
cache effectively holds one element

- everything else — too much other stuff between accesses

if a row does fit in cache —
cache effectively holds one row + one element

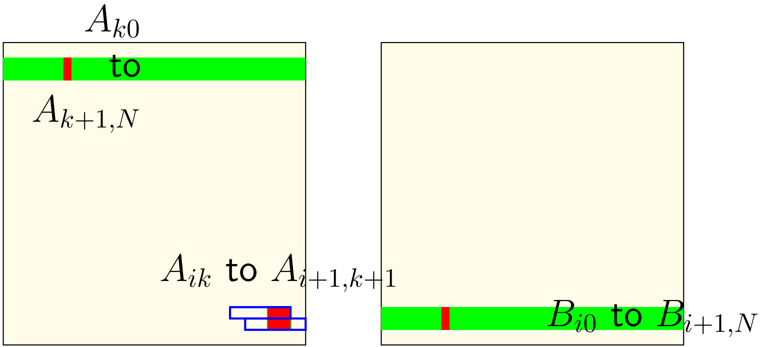
- everything else — too much other stuff between accesses

array usage (better)



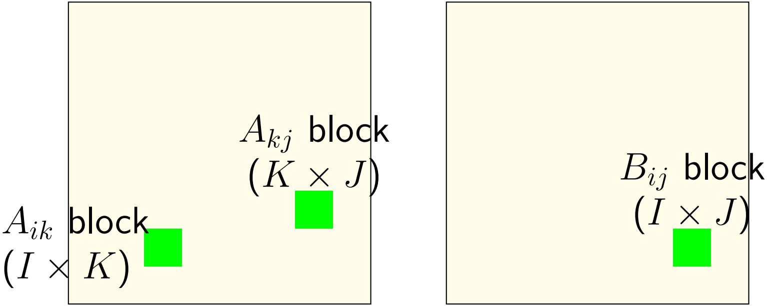
more temporal locality:
N calculations for each A_{ik}
2 calculations for each B_{ij} (for k, $k+1$)
2 calculations for each A_{kj} (for k, $k+1$)

array usage (better)



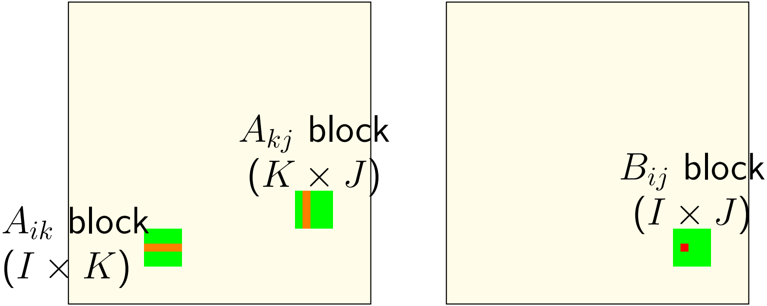
more spatial locality:
calculate on each $A_{i,k}$ and $A_{i,k+1}$ together
both in same cache block — same amount of cache loads

array usage: block



inner loop keeps “blocks” from A, B in cache

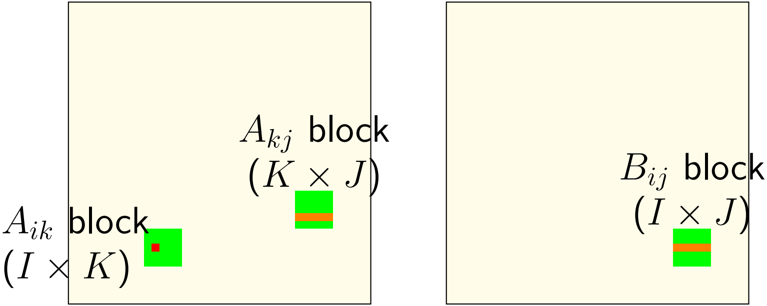
array usage: block



B_{ij} calculation uses strips from A

K calculations for one load (cache miss)

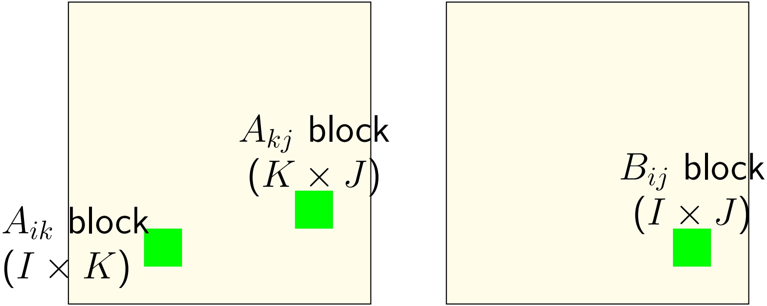
array usage: block



A_{ik} calculation uses strips from A, B

J calculations for one load (cache miss)

array usage: block



(approx.) KIJ fully cached calculations
for $KI + IJ + KJ$ loads
(assuming everything stays in cache)

cache blocking efficiency

load $I \times K$ elements of A_{ik}:

- do $> J$ multiplies with each

load $K \times J$ elements of A_{kj}:

- do I multiplies with each

load $I \times J$ elements of B_{ij}:

- do K adds with each

bigger blocks — more work per load!

catch: $IK + KJ + IJ$ elements must fit in cache

cache blocking rule of thumb

fill the most of the cache with useful data

and do as much work as possible from that

example: my desktop 32KB L1 cache

$I = J = K = 48$ uses $48^2 \times 3$ elements, or 27KB.

assumption: conflict misses aren't important