

## Class 14: Turing's Model

### Upcoming Schedule

- **Now:** Problem Set 3
- **Monday's class will meet in Rice Hall Bagel Shop Area**
- **Monday, 3 October:** Problem Set 4
- **Wednesday, 12 October:** Exam 1 Due (will be take-home, handed out on Friday, 7 October)

### Notes and Questions

What makes a good *model*?

To model a computer, what are the three things we need to model?

### Turing Machine

A *Turing Machine* is an abstract model of a digital computer. It consists of:

- An **infinitely long tape**, divided into squares. (The tape is usually thought of as infinitely long only in one direction, but it is equivalent in power to a tape that is infinitely long in both directions.)
- A finite **alphabet** of symbols. There is a finite set of symbols that can be written into squares on the tape.
- A **tape head** that can read the alphabet symbol on a single square of the tape. For each step, the tape head reads the symbol at the current tape position, and can move one square either left or right.
- A **finite state machine** (see back) that controls the tape head.

## Finite State Machine

A *Finite State Machine* is a very simple model of a machine that has a finite amount of memory (unlike the Turing Machine model which has an *infinite* amount of memory since the tape is infinitely long). A Finite State Machine consists of:

- A finite **alphabet** of symbols. There is a finite set of symbols that can be written into squares on the tape.
- A finite set of **states**. Some of the states may be distinguished (for the FSM for a Turing Machine, we typically have a distinguished state called “Halt” where the machine stops if that state is reached).
- A set of **decision rules**. Each rule is of the form  $\langle state_0, symbol \rangle \rightarrow state_1$ . If the machine is currently in  $state_0$  and the next input symbol is  $symbol$ , after reading the  $symbol$  the state is now in  $state_1$ .

Unlike a Turing Machine, a Finite State Machine can see each input symbol only once. You can think of it as a machine that starts with the input on a finite tape, and process that input from left to right, reading one square at a time.

Design a finite state machine that checks the parity of a binary number. Your machine should end in state **0** if the input has an even number of **1**'s, and end in state **1** if the input has an odd number of **1**'s.

## Take-Home Problem

Design a Turing Machine that starts with an input tape that starts with a “#”, is followed by a series of “\*” and “♦” symbols, followed by a “#” at the end. The output should be the number of “\*” symbols. A first version should produce the output in unary, leaving the output tape with a sequence of “1” symbols followed by a “#”. For example, if the input tape is #\*♦♦\*♦\*\*♦\*♦♦♦♦# the output tape should be “#11111#”. (A gold-star bonus solution would produce the output in binary notation, instead of unary.)