Class 25: Undecidable Problems

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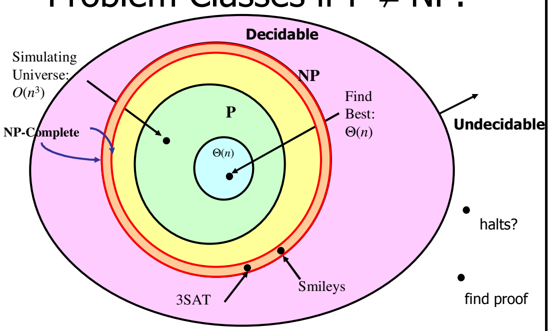
Menu

- Review:
  - Undecidability
  - Halting Problem
- How do we prove a problem is undecidable?
- What do we do when faced with an undecidable problem?

Problem Classes if P ≠ NP:



Halting Problem

Define a procedure halts? that takes a procedure and an input evaluates to #t if the procedure would terminate on that input, and to #f if would not terminate.

(define (halts? procedure input) …)

Informal Proof

```
(define (contradict-halts x)
  (if (halts? contradict-halts null)
      (loop-forever)
      #t))
```

If contradict-halts halts, the if test is true and it evaluates to (loop-forever) - it doesn't halt!

If contradict-halts doesn't halt, the if test if false, and it evaluates to #t. It halts!

Proof by Contradiction

1. Show *X* is nonsensical.
2. Show that if you have *A* and *B* you can make *X*.
3. Show that you can make *A*.
4. Therefore, *B* must not exist.

X = contradict-halts
A = a Scheme interpreter that follows the evaluation rules
B = halts?

"Evaluates to 3" Problem

Input: A procedure *P* and input *I*

Output: **true** if evaluating (*P I*) would result in 3; **false** otherwise.

Is "Evaluates to 3" decidable?

Undecidability Proof

Suppose we could define evaluates-to-3? that decides it. Then we could define halts?:

```
(define (halts? P I)
  (if (evaluates-to-3?
        `(begin (P I) 3))
      #t
      #f))
```

Since it evaluates to 3, we know (P I) must halt.

The only way it could not evaluate to 3, is if (P I) doesn't halt. (Note: assumes (P I) cannot produce an error.)

Hello-World? Problem

Input: A procedure *P* and input *I*

Output: **true** if evaluating (*P I*) would print out "Hello World!"; **false** otherwise.

Is *Hello-World?* decidable?

Undecidability Proof

Suppose we could define hello-world? that decides it. Then we could define halts?:

```
(define (halts? P I)
  (if (hello-world?
        `(begin ((remove-prints P) I)
                (print "Hello World!"))
      #t
      #f))
```


Proof by Contradiction

1. Show *X* is nonsensical.
2. Show that if you have *A* and *B* you can make *X*.
3. Show that you can make *A*.
4. Therefore, *B* must not exist.

X = halts?
A = a Scheme interpreter that follows the evaluation rules
B = hello-world?

From Paul Graham's "Undergraduation":

My friend Robert learned a lot by writing network software when he was an undergrad. One of his projects was to connect Harvard to the Arpanet; it had been one of the original nodes, but by 1984 the connection had died. Not only was this work not for a class, but because he spent all his time on it and neglected his studies, he was kicked out of school for a year. ... When Robert got kicked out of grad school for writing the Internet worm of 1988, I envied him enormously for finding a way out without the stigma of failure. ... It all evened out in the end, and now he's a professor at MIT. But you'll probably be happier if you don't go to that extreme; it caused him a lot of worry at the time.

3 years of probation, 400 hours of community service, $10,000+ fine

Morris Internet Worm (1988)

- *P* = fingerd
  - Program used to query user status
  - Worm also attacked other programs
- *I* = "nop^{400} pushl $68732f pushl $6e69622f movl sp,r10 pushl $0 pushl $0 pushl r10 pushl $3 movl sp,ap chmk $3b"

  (is-worm? P I) should evaluate to #t
- Worm infected several thousand computers (~10% of Internet in 1988)

Worm Detection Problem

Input: A program *P* and input *I*

Output: **true** if evaluating (P I) would cause a remote computer to be "infected".

Virus Detection Problem

Input: A program *P* and input *I*

Output: **true** if evaluating (P I) would cause a file on the host computer to be "infected".

Undecidability Proof

Suppose we could define is-worm? Then:

```
(define (halts? P I)
  (if (is-worm? '(begin ((deworm P) I)
                         worm-code))
      #t
      #f))
```

#t: Since it *is* a worm, we know worm-code was evaluated, and *P* must halt.

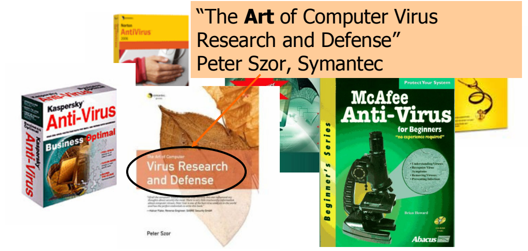
#f)): The worm-code would not evaluate, so *P* must not halt.

Can we make ***deworm***?

Conclusion?

- Anti-Virus programs cannot exist!

"The **Art** of Computer Virus Research and Defense"
Peter Szor, Symantec



"Solving" Undecidable Problems

- No perfect solution exists:
  - Undecidable means there is no procedure that:
    1. Always gives the correct answer
    2. Always terminates
- Must give up one of these to "solve" undecidable problems
  - Giving up #2 is not acceptable in most cases
  - Must give up #1

Actual is-virus? Programs

- Give the wrong answer sometimes
  - "False positive": say P is a virus when it isn't
  - "False negative": say P is safe when it is
- Database of known viruses: if P matches one of these, it is a virus
- Clever virus authors can make viruses that change each time they propagate
  - A/V software ~ finite-proof-finding
  - Emulate program for a limited number of steps; if it doesn't do anything bad, assume it is safe

Proof Recap

- If we had is-virus? we could define halts?
- We know halts? is undecidable
- Hence, we can't have is-virus?
- Thus, we know is-virus? is undecidable

How convincing is our Halting Problem proof?

```
(define (contradict-halts x)
  (if (halts? contradict-halts null)
      (loop-forever)
      #t))
```

If contradict-halts halts, the if test is true and it evaluates to (loop-forever) - it doesn't halt!

If contradict-halts doesn't halt, the if test if false, and it evaluates to #t. It halts!

This "proof" assumes Scheme exists and is consistent!

Charge

- Scheme is very complicated (requires more than 1 page to define):
  - Unlikely we could prove it is consistent
- To have a convincing proof, we need a simpler programming model in which we can write contradict-halts:
  - Next week: Turing's model