

| $>$ (fibo 5) $\mid$ (fibo 5) $\mid$ (fibo 4) $\|\mid$ (fibo 3) $\|\mid$ (fibo 2) $\|\mid 1$ $\|\mid$ (fibo 1) $\|\mid 1$ $\|\mid 12$ $\|\|\mid$ (fibo 2) $\|\mid 1$ $\mid 3$ $\mid$ (fibo 3) $\|\mid$ (fibo 2) $\|\mid 1$ $\|\|\mid($ fibo 1$)$ $\|\mid 1$ $\mid 2$ $\mid 5$ 5 |  <br> How many calls to calculate (fibo 60)? |  |
| :---: | :---: | :---: |
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| fast-fibo |  |  |
| :---: | :---: | :---: |
| ```(define (fast-fibo n) (define (fib-helper a b left) (if (<= left 0) b (fib-helper b (+ a b) (- left 1)))) (fib-helper 1 1 (- n 2)))``` |  |  |
|  | 8 | 骨 Computer |

## Fast-Fibo Results

> (fast-fibo 10)
55
> (time (fast-fibo 61))
cpu time: 0 real time: 0 gc time: 0
2504730781961
2.5 Trillion applications
2.5 GHz computer does 2.5 Billion simple operations per second, so 2.5 Trillion applications operations take $\sim 1000$ seconds.
Each application of fibo involves hundreds of simple operations...
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i;; The Earth's mass is $6.0 \times 10^{\wedge} 24 \mathrm{~kg}$
$>$ (define mass-of-earth (* 6 (expt 1024 )))
;i; A typical rabbit's mass is 2.5 kilograms
> (define mass-of-rabbit 2.5)
> (/ (* mass-of-rabbit (fast-fibo 60)) mass-of-earth)
$6.450036483 \mathrm{e}-013$
> (/ (* mass-of-rabbit (fast-fibo 120)) mass-of-earth)
$\mathbf{2 . 2 3 2 6 4 9 6 8 9 5 7 9 5 6 9 3}$
According to Bonacci's model, after less than 10 years, rabbits would out-weigh the Earth!

Beware the Bunnies!! Beware the Sneezewort!!
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## Evaluation Cost

Actual running times
vary according to:

- How fast a processor you have
- How much memory you have
- Where data is located in memory
- How hot it is
- What else is running
- etc...


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## Measuring Cost

- How does the cost scale with the size of the input
- If the input size increases by one, how much longer will it take?
- If the input size doubles, how much longer will it take?

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The Golden Ratio


PS2 Question
(define (find-best-hand hands) (car (sort hands higher-hand?)))
(define (find-best Ist cf)
(if (= 1 (length Ist)) (car Ist)
(pick-better cf (car Ist) (find-best (cdr Ist) cf)))) (define (pick-better cf num1 num2) (if (cf num1 num2) num1 num2))
(define (find-best-hand hands)
(find-best hands higher-hand?))
Which is better and by how much?

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Cost of Fibonacci Procedures
(define (fibo $n$ )
(define (fast-fibo $n$ )
(define (fib-helper a b left)
(if (= left 0 )
(if (or (= n 1) (= n 2))
1 ; ; ; base case
$(+($ fibo $(-n 1))$
(fibo (-n 2)))))
(fib-helper b (+ a b) (- left 1))))
(fib-helper $11(-\mathrm{n} 2)$ ))

| Input | fibo | fast-fibo |
| :---: | :---: | :---: |
| $m$ | $q$ | $z=m k$ |
| $m+1$ | $q^{*} \Phi$ | $(m+1) k$ |
| $m+2$ | at least $q^{2}$ | $(m+2) k$ |

$\Phi=(/(+1$ (sqrt 5)) 2) $=$ "The Golden Ratio" ~ $1.618033988749895 .$. $\sim(/$ (fast-fibo 61) (fast-fibo 60)) $=1.618033988749895$

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| Sorting |  |  |
| :---: | :---: | :---: |
| (define (sort st cf) |  |  |
| (if (null? Ist) Ist <br> (let ((best (find-best lst cf))) <br> (cons best (sort (delete lst best) cf))))) |  |  |
|  |  |  |
|  |  |  |
| (definie (find-best ( (tsf) (f) |  |  |
|  |  |  |
| (define (pick-better cf num1 num2) (if (cf num1 num2) num1 num2) |  |  |
| How much work is sort? |  |  |
|  | 2 | -1. ${ }_{\text {Comm }}$ |


| Sorting Cost |  |  |
| :---: | :---: | :---: |
| (define (sort Ist cf) <br> (if (null? Ist) Ist <br> (let ((best (find-best lst cf))) |  |  |
|  |  |  |
| (define (find-be <br> (if (= 1 (length Ist)) (car Ist) <br> (pick-better cf (car Ist) (find-best (cdr Ist) cf)))) |  |  |
|  |  |  |
| If we double the length of the list, the amount of work approximately quadruples: there are twice as many applications of find-best, and each one takes twice as long |  |  |
| 12.5 ereater | 23 | Campute S Siene |

## Sorting Hands

(define (sort Ist cf) (if (null? Ist) Ist
(let ((best (find-best lst cf)))
(cons
best
(sort (delete Ist best) cf)))))
(define (sort-hands Ist) (sort Ist higher-hand?))
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## Sorting Cost

- What grows?
$-n=$ the number of elements in Ist
- How much work are the pieces?
find-best: work scales as $n$ (increases by one)
delete: work scales as $n$ (increases by one)
- How many times does sort evaluate find-best and delete? $n$
- Total cost: scales as $n^{2}$
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## Charge

- Read Chapter 6: formal notations we will use for this type of analysis
- PS4 out now: you know everything you need for the programming parts; we will cover more on analysis Wednesday and Friday
- Beware the Bunnies and Sneezewort!


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