Lecture 36: Modeling Computing

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How convincing is our Halting Problem proof?

```
(define (contradict-halts x)
  (if (halts? contradict-halts)
      (loop-forever)
      #t))
```

contradicts-halts cannot exist. Everything we used to make it except halts? does exist, therefore halts? cannot exist.

This "proof" assumes Scheme exists and is consistent!

DrScheme

Is DrScheme a proof that Scheme exists?

```
(define (make-huge n)
   (if (= n 0) null
        (cons (make-huge (- n 1))
                  (make-huge (- n 1)))))
(make-huge 10000)
```

Scheme/Charme/Python/etc. all fail to evaluate some program!

Solutions

- Option 1: Prove "Scheme" does exist
  - Show that we could implement all the evaluation rules (if we had "Python", our Charme interpreter would be a good start, but we don't have "Python")
- Option 2: Find a simpler computing model
  - Define it precisely
  - Show that "contradict-halts" can be defined in this model

Modeling Computation

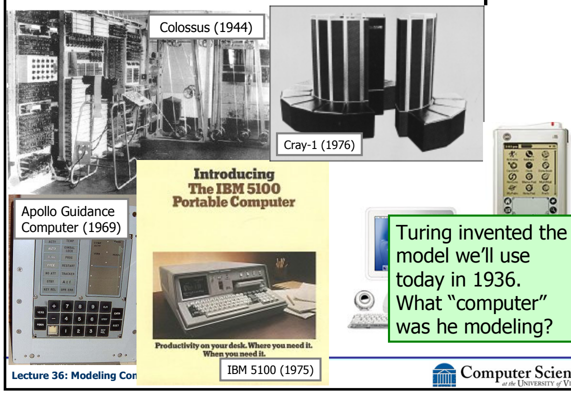
- For a more convincing proof, we need a more precise (but simple) model of what a computer can do
- Another reason we need a model:

  Does complexity really make sense without this? (how do we know what a "step" is? are they the same for all computers?)

What is a model?

$$\nabla \cdot D = \rho$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

How should we model a Computer?



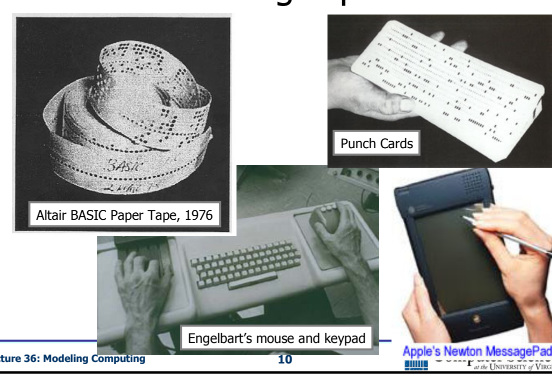
Turing invented the model we'll use today in 1936. What "computer" was he modeling?

"Computers" before WWII

Modeling Computers

- Input
  - Without it, we can't describe a problem
- Output
  - Without it, we can't get an answer
- Processing
  - Need some way of getting from the input to the output
- Memory
  - Need to keep track of what we are doing

Modeling Input



Turing's "Computer"

"Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book."
Alan Turing, *On computable numbers, with an application to the Entscheidungsproblem*, 1936

Simplest Input

- Non-interactive: like punch cards and paper tape
- One-dimensional: just a single tape of values, pointer to one square on tape

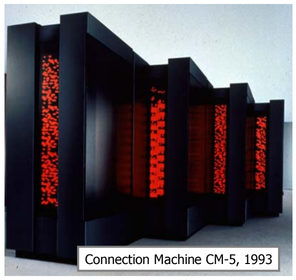
			0	0	1	1	0	0	1	0	0	0						

How long should the tape be?

Infinitely long! We are *modeling* a computer, not building one. Our model should not have silly practical limitations (like a real computer does).

Modeling Output

- Blinking lights are cool, but hard to model
- Output is what is written on the tape at the end of a computation


Connection Machine CM-5, 1993

Modeling Processing (Brains)

"For the present I shall only say that the justification lies in the fact that the human memory is necessarily limited."
Alan Turing

Modeling Processing

- Evaluation Rules
  - Given an input on our tape, how do we evaluate to produce the output
- What do we need:
  - Read what is on the tape at the current square
  - Move the tape one square in either direction
  - Write into the current square

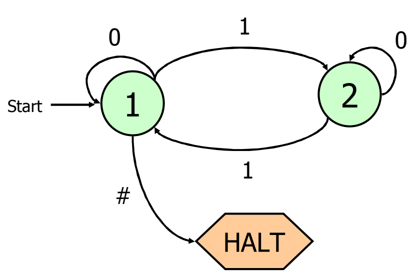
				0	0	1	1	0	0	1	0	0	0						

⇧ Is that enough to model a computer?

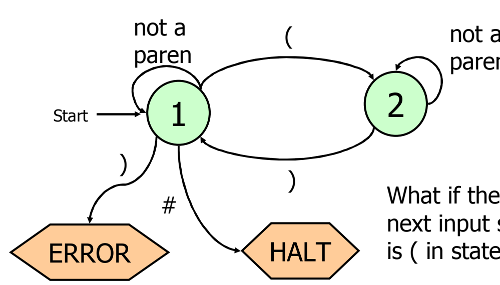
Modeling Processing

- Read, write and move is not enough
- We also need to keep track of what we are doing:
  - How do we know whether to read, write or move at each step?
  - How do we know when we're done?
- What do we need for this?

Finite State Machines

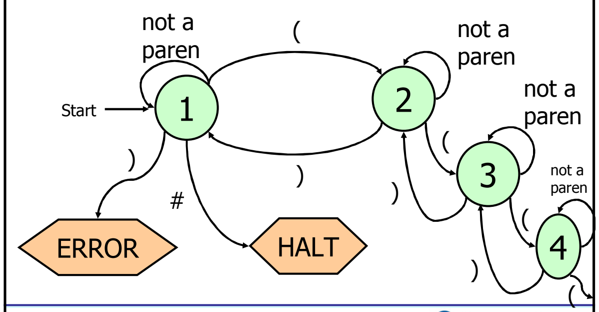


Hmmm…maybe we don't need those infinite tapes after all?



What if the next input symbol is (in state 2?

How many states do we need?



Finite State Machine

- There are lots of things we can't compute with only a finite number of states
- Solutions:
  - Infinite State Machine
    - Hard to describe and draw
  - **Add an infinite tape to the Finite State Machine**

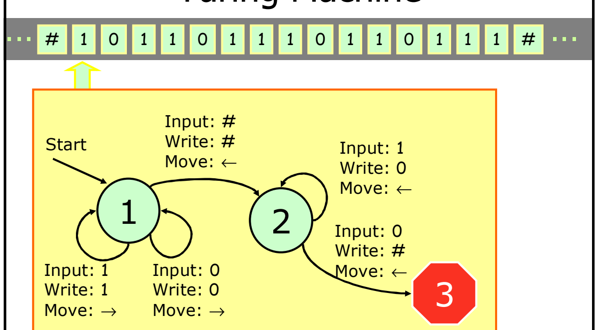
Turing's Explanation

"We have said that the computable numbers are those whose decimals are calculable by **finite** means. ... For the present I shall only say that the justification lies in the fact that the human memory is necessarily limited."

FSM + Infinite Tape

- Start:
  - FSM in Start State
  - Input on Infinite Tape
  - Pointer to start of input
- Step:
  - Read one input symbol from tape
  - Write symbol on tape, and move L or R one square
  - Follow transition rule from current state
- Finish:
  - Transition to halt state

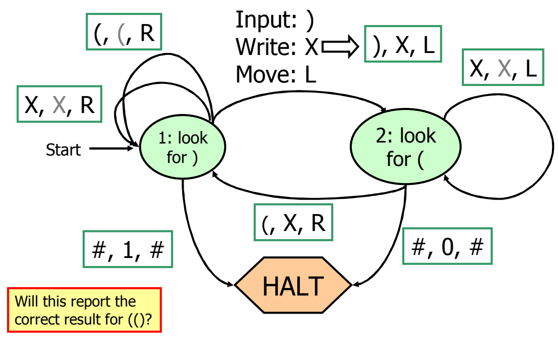
Turing Machine



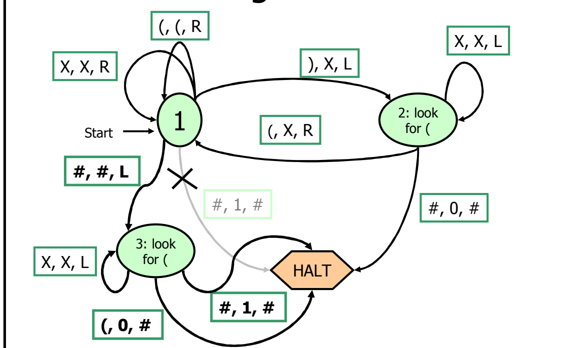
Matching Parentheses

- Find the leftmost)
  - If you don't find one, the parentheses match, write a 1 at the tape head and halt.
- Replace it with an X
- Look left for the first (
  - If you find it, replace it with an X (they matched)
  - If you don't find it, the parentheses didn't match – end write a 0 at the tape head and halt

Matching Parentheses

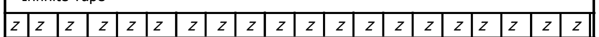


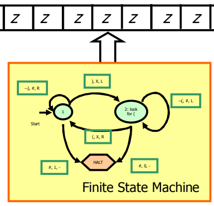
Matching Parentheses



Turing Machine

Infinite Tape





TuringMachine ::= < *Alphabet*, *Tape*, ***FSM*** >
Alphabet ::= { *Symbol** }
Tape ::= < *LeftSide*, *Current*, *RightSide* >
OneSquare ::= *Symbol* | #
Current ::= *OneSquare*
LeftSide ::= [*Square**]
RightSide ::= [*Square**]

Everything to left of *LeftSide* is **#**.
Everything to right of *RightSide* is **#**.

How well does this model your computer?

Exam 2

- Out now, due Friday at 12:02 pm (beginning of class)
- Honor the Honor Code
- It is meant to be short enough that you can also make progress on ps9 this week (but don't wait to start the exam)
  – When you meet with your teammates, don't talk about the exam